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Colocated MIMO radar waveform-design based on two-step optimizations in spatial and spectral domains*

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Abstract: We propose an efficient colocated multiple-input multiple-output radar waveform-design method based on two-step optimizations in the spatial and spectral domains. First, a minimum integrated side-lobe level strategy is adopted to obtain the desired beam pattern with spatial nulling. By recovering the hidden convexity of the resulting fractional quadratically constrained quadratic programming non-convex problem, the global optimal solution can be achieved in polynomial time through a semi-definite relaxation followed by spectral factorization. Second, with the transmit waveforms obtained via spatial optimization, a phase changing diagonal matrix is introduced and optimized via power method-like iterations. Without influencing the shape of the optimized beam pattern, the transmit waveforms are further optimized in the spectral domain, and the desired spectral nulling is formed to avoid radar interference on the overlaid licensed radiators. Finally, the superior performance of the proposed method is demonstrated via numerical results and comparisons with other approaches to waveform design.

Key words: Multiple-input multiple-output (MIMO) radar; Waveform design; Spectral factorization; Fractional quadratically constrained quadratic programming (QCQP)
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1 Introduction

The development of multiple-input multiple-output (MIMO) radar has attracted considerable attention recently. Unlike traditional phase array radar, colocated MIMO radar emits different waveforms from different transmit antennas, thus providing extra degrees-of-freedom (DOF). This functionality can be exploited to improve the performance of the radar system with respect to spatial resolution (Bliss and Forsythe, 2003; Bekkerman and Tabrikian, 2004), moving target detection (Forsythe *et al.*, 2004), parameter identification (Fuhrman and San Antonio, 2004), and transmit beam pattern design (Fuhrman and San Antonio, 2008). Therefore, waveform design for colocated MIMO radar has become a research focus over the past few years. In this paper, we propose an efficient waveform design method to form the desired transmit beam pattern with flexible spatial and spectral nullings.

Traditional MIMO radar usually emits orthogonal waveforms from different antennas to form an omnidirectional pattern. However, in many practical scenarios, MIMO radar needs to focus the energy of the transmitted waveforms within one or several predefined angle sections where the targets are likely to exist. At the same time, focusing the transmit energy on the main lobe can also reduce the side-lobe level and weaken the influence of signal-dependent interference. There are usually two steps to achieve a desired transmit beam pattern. First, optimize the covariance matrix of the waveforms according to the desired beam pattern. For this step, various optimization methods can be found in the literature (Stoica *et al.*, 2007; Fuhrman and San Antonio, 2008; Duly *et*

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al., 2013; Hua and Abeysekera, 2013; Pandey and Roy, 2016). The most common design strategy is called the 'beam pattern match' (BM) method (Stoica *et al.*, 2007; Fuhrman and San Antonio, 2008). To solve this problem and obtain the optimal covariance matrix, Fuhrman and San Antonio (2004) adopted a gradient search algorithm, Fuhrman and San Antonio (2008) used a barrier method, and Stoica *et al.* (2007), Hua and Abeysekera (2013), and Pandey and Roy (2016) chose a convex optimization method. Since the side-lobe level is also one of the most important performance indexes, Gong *et al.*, (2014) proposed the minimum peak side-lobe level (PSL) method, Hassanien and Vorobyov (2011) designed the discrete prolate spheroidal sequences-based (DPSS) method, and Xu *et al.* (2015) proposed the minimum integrated side-lobe level (ISL) method, pointing out that the side-lobe levels of the beam pattern obtained by the DPSS method can never be smaller than those obtained by the minimum ISL method. Aubry *et al.* (2016b) studied the robust MIMO radar beam pattern-design method based on ISL/PSL strategies. Once the covariance matrix is obtained, the second step is to synthesize the final waveforms. The most common waveform synthesis method is the cyclic algorithm (CA) proposed by Stoica *et al.* (2008). However, through the CA algorithm, only a sub-optimal solution can be achieved and the synthesized waveforms cannot guarantee forming a nulling in the desired direction to mitigate the signal dependent interferences. To reduce the computational burden of these two steps, Friedlander (2012) and Ehabbazibasmenj *et al.* (2014) proposed a method called transmit beamspace processing (TBP). A weighting matrix for several orthogonal waveforms is introduced in this method, and the original beam pattern-design problem is recast as a quadratic problem of a weighting matrix. Once the covariance matrix is obtained, a randomization method (Luo *et al.*, 2010) can be used to obtain the weighting matrix. Once again, the weighting matrix obtained through randomization is sub-optimal.

Although many solutions and discussions have been put forth for the beam pattern-design problem, very little attention has been paid to beam pattern formation in both spatial and spectral domains. In practical applications, MIMO radar is faced with not only spatial interference from enemies or clutter, but also a spectrally crowded environment due to the increasing volume of spectrum requests from both military sensing applications and civilian wireless services. Thus, optimizing the transmit waveform from previous spatial interference information can achieve superiority in suppressing signal-dependent interference using just an adaptive beamforming technique at the receiver (Chen *et al.*, 2014; Li *et al.*, 2014; Liu *et al.*, 2014). In addition, to improve spectral compatibility with the overlaid licensed radiators, Aubry *et al.* (2015; 2016a) and Remero and Shepherd (2015) proposed a waveform design algorithm for single-input single-output (SISO) radar with improved feasibility with respect to spectral coexistence. Higgins *et al.* (2014) designed an algorithm that mitigates radar interference with the overlaid licensed radiators via spatial and spectral nulling. However, this method just deals with spectral interference mitigation and pays no attention to the beam pattern-design problem. Besides, many current reports in the literature propose waveform design methods for MIMO radar with a suitable frequency allocation (Lindenfeld, 2004; Gerlach *et al.*, 2011; Wang and Lu, 2011; Patton *et al.*, 2012). Yet, none of them take the transmit beam pattern design problem into account.

The goal of this paper is to design colocated MIMO radar waveforms to form a desired beam pattern with flexible spatial and spectral nullings. To the best of our knowledge, there is no report in the literature dealing with this problem. To achieve this goal, the waveform optimizing process can be divided into two steps. First, in the spatial domain, the minimum ISL strategy is adopted to form the optimization model, which can suppress the side-lobe level and flexibly form a spatial nulling. The resulting non-convex fractional programming problem is recast into a general quadratically constrained quadratic programming (QCQP) problem by means of the Charnes–Cooper transformation (Charnes and Cooper, 1962). Then according to the approach proposed by Konar and Sidiropoulos (2015) and Aubry *et al.* (2016c), the hidden convexity of the reformulated QCQP problem with Toeplitz–Hermitian quadratics is recovered, and the global optimal solution is achieved in polynomial time via a semidefinite relaxation (SDR) followed by a spectral factorization theorem (Wu *et al.*, 1996). Second, because changing the initial phase of each sampled transmit sequence will not influence the shape of the beam pattern formed, a phase-changing diagonal matrix is introduced and optimized via iterative optimization by means of power method-like iterations (Soltanalian and Stoica, 2014), and a deep nulling at the overlaid frequency band can be formed to mitigate radar interference with other licensed radiators. Finally, the global optimal waveforms are achieved via sequential optimizations in the spatial and spectral domains, and extensive numerical comparisons are conducted to evaluate the proposed design approach.

Notations: scalars are denoted by lowercase italic letters, and lowercase and uppercase letters in bold are used to denote vectors and matrices, respectively. $(\cdot)^{\mathrm{T}}$ and $(\cdot)^{\mathrm{H}}$ represent the transpose and complex conjugate-transpose operators, respectively. $\boldsymbol{I}$ stands for the identity matrix with an appropriate dimension. The absolute value and l_p norm are denoted by $|\cdot|$ and $\|\cdot\|_p$, respectively. $\mathrm{tr}(\cdot)$ denotes the trace operator of a matrix. $\mathrm{diag}(\boldsymbol{A})$ denotes the vector formed by the diagonal elements of matrix $\boldsymbol{A}$, whereas $\mathrm{diag}(\boldsymbol{a})$ indicates the diagonal matrix formed by vector $\boldsymbol{a}$. $\odot$ represents the Hadamard product.

2 Signal model and problem formulation

Consider a colocated MIMO radar with M transmit antennas where the transmit array is a uniform linear array with half-a-wavelength element separation. The nth sampled transmit sequence is denoted as

$$\boldsymbol{s}(n)=[s_1(n),s_2(n),\ldots,s_M(n)]^{\mathrm{T}}\in\mathbb{C}^{M\times 1}. \quad (1)$$

Let the transmit signal matrix be $\boldsymbol{S}\in\mathbb{C}^{M\times N}$; namely,

$$\boldsymbol{S}=[\boldsymbol{s}(0),\boldsymbol{s}(1),\ldots,\boldsymbol{s}(N-1)]=[\boldsymbol{s}_1,\boldsymbol{s}_2,\ldots,\boldsymbol{s}_M]^{\mathrm{T}}, \quad (2)$$

where N is the sample length and $\boldsymbol{s}_m=[s_m(0), s_m(1), \ldots, s_m(N-1)]^{\mathrm{T}}\in\mathbb{C}^{N\times 1}$ denotes the transmit sequence of the mth antenna. Assume the transmit waveforms are narrowband, and the steering vector is expressed as

$$\boldsymbol{a}(\theta)=[1,\mathrm{e}^{-\mathrm{j}\pi\sin\theta},\ldots,\mathrm{e}^{-\mathrm{j}(M-1)\pi\sin\theta}]^{\mathrm{T}}. \quad (3)$$

The transmit beam pattern for the nth instant is defined as the power distribution along different steering directions, i.e.,

$$\begin{aligned}P(n,\theta)&=\boldsymbol{a}^{\mathrm{H}}(\theta)\boldsymbol{s}(n)\boldsymbol{s}^{\mathrm{H}}(n)\boldsymbol{a}(\theta)\\&=\boldsymbol{s}^{\mathrm{H}}(n)\boldsymbol{a}(\theta)\boldsymbol{a}^{\mathrm{H}}(\theta)\boldsymbol{s}(n)\\&=\boldsymbol{s}^{\mathrm{H}}(n)\boldsymbol{A}\boldsymbol{s}(n),\end{aligned} \quad (4)$$

where $\boldsymbol{A}=\boldsymbol{a}(\theta)\boldsymbol{a}^{\mathrm{H}}(\theta)$ is the steering matrix. Let the whole scanning region be $\varTheta=[-\pi/2, \pi/2]$. The main-lobe region where targets likely exist is $\varTheta_{\mathrm{ml}}=[\theta_{\mathrm{low}}, \theta_{\mathrm{up}}]$ and $\theta_0=(\theta_{\mathrm{low}}+\theta_{\mathrm{up}})/2$, and the rest of $\varTheta$ is denoted as the side-lobe region, i.e., $\varTheta_{\mathrm{sl}}=\varTheta-\varTheta_{\mathrm{ml}}$. Suppose that there is a signal-dependent interference at the θ_{int} direction. To mitigate this interference definitely, a deep and wide spatial nulling needs to be formed and the width of the nulling can be set as $\varTheta_{\mathrm{int}}=[\theta_{\mathrm{int}}-\varDelta, \theta_{\mathrm{int}}+\varDelta]$, where $\varDelta$ decides the width of the nulling. To concentrate the transmit power into $\varTheta_{\mathrm{ml}}$ as much as possible and form the desired 3 dB beamwidth main lobe, we can formulate the following optimization problem by the minimum ISL strategy:

$$\begin{aligned}&\min_{\boldsymbol{x}\in\mathbb{C}^{M\times 1}} \frac{\boldsymbol{x}^{\mathrm{H}}\boldsymbol{A}_{\mathrm{sl}}(\theta)\boldsymbol{x}}{\boldsymbol{x}^{\mathrm{H}}(\boldsymbol{A}_{\mathrm{ml}}(\theta)+\boldsymbol{I}_M)\boldsymbol{x}}\\ &\text{s.t.}\ \ \|\boldsymbol{x}\|^2=E,\ \boldsymbol{x}^{\mathrm{H}}\boldsymbol{A}(\tilde{\theta})\boldsymbol{x}\geq\frac{1}{2}\boldsymbol{x}^{\mathrm{H}}\boldsymbol{A}(\theta_0)\boldsymbol{x},\\ &\qquad \boldsymbol{x}^{\mathrm{H}}\boldsymbol{A}(\tilde{\theta})\boldsymbol{x}\leq\boldsymbol{x}^{\mathrm{H}}\boldsymbol{A}(\theta_0)\boldsymbol{x},\ \forall\tilde{\theta}\in\varTheta_{\mathrm{ml}},\\ &\qquad \boldsymbol{x}^{\mathrm{H}}\boldsymbol{A}_{\mathrm{int}}(\theta)\boldsymbol{x}\leq\varepsilon,\end{aligned} \quad (5)$$

where $\boldsymbol{x}=\boldsymbol{s}(n)$, $\boldsymbol{A}_{\mathrm{sl}}=\int_{\varTheta_{\mathrm{sl}}}\boldsymbol{A}(\theta)\mathrm{d}\theta$, $\boldsymbol{A}_{\mathrm{ml}}=\int_{\varTheta_{\mathrm{ml}}}\boldsymbol{A}(\theta)\mathrm{d}\theta$, and $\boldsymbol{A}_{\mathrm{int}}(\theta)=\int_{\varTheta_{\mathrm{int}}-\varDelta}^{\varTheta_{\mathrm{int}}+\varDelta}\boldsymbol{A}(\theta)\mathrm{d}\theta$. ε is the predefined nulling depth. Note that compared with the beam pattern-design method proposed by Xu *et al*. (2015), the objective function and the steering matrix are both different. In particular, the solution described in the following sections is totally different from that proposed by Xu *et al*. (2015). The first constraint of problem (5) denotes the maximum transmit power at the nth instant. The second and third constraints are used to guarantee that the width of the formed main lobe is 3 dB. The last constraint can form a wide and deep spatial nulling toward the interference direction. It is obvious that problem (5) is non-convex and in general NP-hard. Fortunately, the theorem proposed by Aubry *et al.* (2016c) can be used to solve this problem in polynomial time and obtain the global optimal solution. Furthermore, with the global optimal solution, spectral optimization can be performed further to optimize the power spectral density (PSD) of the transmit waveforms.

3 Waveform design based on joint optimization in spatial and spectral domains

3.1 Waveform design via spatial optimization

The optimization problem (5) is a fractional QCQP problem about $\boldsymbol{x}$, which is non-convex and in general NP-hard. With the Charnes–Cooper transformation theorem, problem (5) can be recast as

$$\begin{aligned} &\min_{\boldsymbol{x}\in\mathbb{C}^{M\times 1},\gamma\in\mathbb{R}} \boldsymbol{x}^{\mathrm{H}}\boldsymbol{A}_{\mathrm{sl}}(\theta)\boldsymbol{x} \\ &\text{s.t.}\ \ \boldsymbol{x}^{\mathrm{H}}(\boldsymbol{A}_{\mathrm{ml}}(\theta)+\boldsymbol{I}_M)\boldsymbol{x}=1, \\ &\quad\ \ \boldsymbol{x}^{\mathrm{H}}\boldsymbol{I}_M\boldsymbol{x}=\gamma E,\ \gamma\geq 0, \\ &\quad\ \ \boldsymbol{x}^{\mathrm{H}}(1/2\,\boldsymbol{A}(\theta_0)-\boldsymbol{A}(\tilde{\theta}))\boldsymbol{x}\leq 0, \\ &\quad\ \ \boldsymbol{x}^{\mathrm{H}}(\boldsymbol{A}(\tilde{\theta})-\boldsymbol{A}(\theta_0))\boldsymbol{x}\leq 0,\ \forall\tilde{\theta}\in\Theta_{\mathrm{ml}}, \\ &\quad\ \ \boldsymbol{x}^{\mathrm{H}}\boldsymbol{A}_{\mathrm{int}}(\theta)\boldsymbol{x}\leq\gamma\varepsilon. \end{aligned} \tag{6}$$

The following proposition provides the equivalence between problems (5) and (6):

Proposition 1 Problems (5) and (6) are equivalent. Additionally, any optimal solution to problem (5) is also an optimal solution to problem (6).

Proof Let $\tilde{\boldsymbol{x}}$ be the optimal solution to problem (5). Because matrix $\boldsymbol{A}_{\mathrm{ml}}(\theta)=\int_{\Theta_{\mathrm{ml}}}\boldsymbol{A}(\theta)\mathrm{d}\theta\geq 0$, the Hermitian matrix $\boldsymbol{A}_{\mathrm{ml}}(\theta)+\boldsymbol{I}_M$ is a positive matrix, namely, $\boldsymbol{A}_{\mathrm{ml}}(\theta)+\boldsymbol{I}_M>0$. For any complex vector $\boldsymbol{x}\in\mathbb{C}^{M\times 1}$, there is $\boldsymbol{x}^{\mathrm{H}}(\boldsymbol{A}_{\mathrm{ml}}(\theta)+\boldsymbol{I}_M)\boldsymbol{x}>0$. With the Charnes–Cooper transformation theorem, we can have $\tilde{\boldsymbol{x}}^{\mathrm{H}}(\boldsymbol{A}_{ml}(\theta)+\boldsymbol{I}_M)\tilde{\boldsymbol{x}}=1/\gamma$, $\gamma\geq 0$, and considering $\boldsymbol{x}=\sqrt{\gamma}\tilde{\boldsymbol{x}}$, problem (5) boils down to problem (6). Thus, $(\tilde{\boldsymbol{x}},(\tilde{\boldsymbol{x}}^{\mathrm{H}}\boldsymbol{A}_{\mathrm{ml}}(\theta)\tilde{\boldsymbol{x}})^{-1})$ is an optimal solution to problem (6). On the other hand, if $(\bar{\boldsymbol{x}},\bar{\gamma})$ is an optimal solution to problem (6), it is easily shown that $\bar{\boldsymbol{x}}/\sqrt{\bar{\gamma}}$ is an optimal solution to problem (5). Therefore, problems (5) and (6) are equivalent and Proposition 1 is proved.

Set $\boldsymbol{X}=\boldsymbol{x}\boldsymbol{x}^{\mathrm{H}}$. According to the property of the matrix trace operator, problem (6), the QCQP problem about $\boldsymbol{x}$, can be reformulated as a semidefinite programming problem (SDP) with a rank-1 constraint as follows:

$$\begin{aligned} &\min_{\boldsymbol{X}\in\mathbb{C}^{M\times M},\ \gamma\in\mathbb{R}} \operatorname{tr}(\boldsymbol{A}_{\mathrm{sl}}(\theta)\boldsymbol{X}) \\ &\text{s.t.}\ \ \operatorname{tr}\left[(\boldsymbol{A}_{\mathrm{ml}}(\theta)+\boldsymbol{I}_M)\boldsymbol{X}\right]=1, \\ &\quad\ \ \operatorname{tr}(\boldsymbol{X})=\gamma E,\ \gamma\geq 0, \\ &\quad\ \ \operatorname{tr}\left[(\boldsymbol{A}(\tilde{\theta})-\boldsymbol{A}(\theta_0))\boldsymbol{X}\right]\leq 0, \\ &\quad\ \ \operatorname{tr}\left[(1/2\,\boldsymbol{A}(\theta_0)-\boldsymbol{A}(\tilde{\theta}))\boldsymbol{X}\right]\leq 0,\ \forall\tilde{\theta}\in\Theta_{\mathrm{ml}}, \\ &\quad\ \ \operatorname{tr}(\boldsymbol{A}_{\mathrm{int}}(\theta)\boldsymbol{X})\leq\gamma\varepsilon, \\ &\quad\ \ \boldsymbol{X}\geq 0,\ \operatorname{rank}(\boldsymbol{X})=1. \end{aligned} \tag{7}$$

Because in problem (6), the Hermitian matrices of constraints, $\boldsymbol{A}(\tilde{\theta})-\boldsymbol{A}(\theta_0)$ and $1/2\,\boldsymbol{A}(\theta_0)-\boldsymbol{A}(\tilde{\theta})$, cannot be guaranteed as semidefinite, problem (6) is still non-convex and in general an NP-hard problem. The difficulty in solving problem (6) is presented by the rank-1 constraint in problem (7). Thus, to solve problem (7) in polynomial time, the SDR method can be adopted and the relaxed version of problem (7) is

$$\begin{aligned} &\min_{\boldsymbol{X}\in\mathbb{C}^{M\times M},\ \gamma\in\mathbb{R}} \operatorname{tr}(\boldsymbol{A}_{\mathrm{sl}}(\theta)\boldsymbol{X}) \\ &\text{s.t.}\ \ \operatorname{tr}\left[(\boldsymbol{A}_{\mathrm{ml}}(\theta)+\boldsymbol{I}_M)\boldsymbol{X}\right]=1, \\ &\quad\ \ \operatorname{tr}(\boldsymbol{X})=\gamma E,\ \gamma\geq 0, \\ &\quad\ \ \operatorname{tr}\left[(\boldsymbol{A}(\tilde{\theta})-\boldsymbol{A}(\theta_0))\boldsymbol{X}\right]\leq 0, \\ &\quad\ \ \operatorname{tr}\left[(1/2\,\boldsymbol{A}(\theta_0)-\boldsymbol{A}(\tilde{\theta}))\boldsymbol{X}\right]\leq 0,\ \forall\tilde{\theta}\in\Theta_{\mathrm{ml}}, \\ &\quad\ \ \operatorname{tr}(\boldsymbol{A}_{\mathrm{int}}(\theta)\boldsymbol{X})\leq\gamma\varepsilon,\ \boldsymbol{X}\geq 0. \end{aligned} \tag{8}$$

Fortunately, the relaxed problem (8) is tight and its solution is the global optimal result.

Proposition 2 According to the definition of matrices $\boldsymbol{A}_{\mathrm{sl}}(\theta)$, $\boldsymbol{A}_{\mathrm{ml}}(\theta)$, and $\boldsymbol{A}_{\mathrm{int}}(\theta)$, problems (7) and (8) are equivalent.

Proof As the transmit array of MIMO radar is a uniform linear array, the steering matrix $\boldsymbol{A}(\theta)=\boldsymbol{a}(\theta)\boldsymbol{a}^{\mathrm{H}}(\theta)$ is a Toeplitz–Hermitian matrix. For convenient expression, denote $\boldsymbol{A}_0=\boldsymbol{A}_{\mathrm{sl}}(\theta)$, $\boldsymbol{A}_1=\boldsymbol{A}_{\mathrm{ml}}(\theta)+\boldsymbol{I}_M$, $\boldsymbol{A}_2=\boldsymbol{I}_M$, $\boldsymbol{A}_3=\boldsymbol{A}(\tilde{\theta})-\boldsymbol{A}(\theta_0)$, $\boldsymbol{A}_4=1/2\boldsymbol{A}(\theta_0)-\boldsymbol{A}(\tilde{\theta})$, and $\boldsymbol{A}_5=\boldsymbol{A}_{\mathrm{int}}(\theta)$. It is obvious that each matrix $\boldsymbol{A}_m$, $m\in\{0, 1, \ldots, 5\}$, is a Toeplitz–Hermitian matrix. Based on the property of a Toeplitz matrix, each $\boldsymbol{A}_m$ can be written as

$$\boldsymbol{A}_m = \sum_{k=-M+1}^{M-1} a_{m,k}\boldsymbol{J}_k, \tag{9}$$

where $a_{m,k}$ is the element of the kth diagonal of $\boldsymbol{A}_m$, and setting k=0 corresponds to the main diagonal of $\boldsymbol{A}_m$. For any $k\in S=\{-M+1, -M+2, \ldots, M-1\}$, define the shift matrix as

$$\boldsymbol{J}_k(l,r) = \begin{cases} 1, & l-r=k, \\ 0, & l-r\neq k, \end{cases} \quad (l,r)\in\{1,2,\ldots,M\}^2. \tag{10}$$

With the help of Eqs. (9) and (10), the constraints of problem (8) can be rewritten as

$$\mathrm{tr}(\boldsymbol{A}_m(\theta)\boldsymbol{X}) = \boldsymbol{x}^{\mathrm{H}}\boldsymbol{A}_m(\theta)\boldsymbol{x} = \mathrm{Re}(\boldsymbol{a}_m^{\mathrm{T}}\boldsymbol{r}),\ m=0,1,\ldots,5, \tag{11}$$

where $\boldsymbol{a}_m=[a_{m,-M+1}, a_{m,-M+2}, \ldots, a_{m,M-1}]^{\mathrm{T}}\in\mathbb{C}^{2M-1}$, $\boldsymbol{r}=[r_{-M+1}, r_{-M+2}, \ldots, r_{M-1}]^{\mathrm{T}}\in\mathbb{C}^{2M-1}$, and $r_i=\mathrm{tr}(\boldsymbol{J}_i\boldsymbol{X})$, $i\in S$. Thus, problem (7) can be equivalently expressed as

$$\begin{aligned} &\min_{\boldsymbol{X}\in\mathbb{C}^{M\times M},\ \gamma\in\mathbb{R}} \mathrm{Re}(\boldsymbol{a}_0^{\mathrm{T}}\boldsymbol{r}) \\ &\text{s.t.}\ \mathrm{Re}(\boldsymbol{a}_1^{\mathrm{T}}\boldsymbol{r})=1,\ \mathrm{Re}(\boldsymbol{a}_2^{\mathrm{T}}\boldsymbol{r})=\gamma E,\ \gamma\geq 0, \\ &\quad \mathrm{Re}(\boldsymbol{a}_3^{\mathrm{T}}\boldsymbol{r})\leq 0,\ \mathrm{Re}(\boldsymbol{a}_4^{\mathrm{T}}\boldsymbol{r})\leq 0, \\ &\quad \mathrm{Re}(\boldsymbol{a}_5^{\mathrm{T}}\boldsymbol{r})\leq\lambda\varepsilon,\ r_i=\mathrm{tr}(\boldsymbol{J}_i\boldsymbol{X}),\ i\in S, \\ &\quad \boldsymbol{X}\geq 0,\ \mathrm{rank}(\boldsymbol{X})=1. \end{aligned} \tag{12}$$

Additionally, problem (8) shares the following form:

$$\begin{aligned} &\min_{\boldsymbol{X}\in\mathbb{C}^{M\times M},\ \gamma\in\mathbb{R}} \mathrm{Re}(\boldsymbol{a}_0^{\mathrm{T}}\boldsymbol{r}) \\ &\text{s.t.}\ \mathrm{Re}(\boldsymbol{a}_1^{\mathrm{T}}\boldsymbol{r})=1,\ \mathrm{Re}(\boldsymbol{a}_2^{\mathrm{T}}\boldsymbol{r})=\gamma E,\ \gamma\geq 0, \\ &\quad \mathrm{Re}(\boldsymbol{a}_3^{\mathrm{T}}\boldsymbol{r})\leq 0,\ \mathrm{Re}(\boldsymbol{a}_4^{\mathrm{T}}\boldsymbol{r})\leq 0, \\ &\quad \mathrm{Re}(\boldsymbol{a}_5^{\mathrm{T}}\boldsymbol{r})\leq\lambda\varepsilon,\ r_i=\mathrm{tr}(\boldsymbol{J}_i\boldsymbol{X}),\ i\in S, \\ &\quad \boldsymbol{X}\geq 0. \end{aligned} \tag{13}$$

As we can see, the constraints $r_i=\mathrm{tr}(\boldsymbol{J}_i\boldsymbol{X})$, $\boldsymbol{X}\geq 0$, and $\mathrm{rank}(\boldsymbol{X})=1$ constitute the linear matrix inequalities (LMI) and parameterization of the autocorrelation sequence r_i of an MA process of order $2M-1$. Alkire and Vandenberghe (2002) have proved that the rank-1 constraint is redundant, which means that problems (12) and (13) are equivalent and therefore, problems (7) and (8) are equivalent. Proposition 2 is proved.

With Propositions 1 and 2, the hidden convexity of problem (5), with Toeplitz–Hermitian quadratic forms, can be recovered and problem (5) can be exactly solved in polynomial time using the convex method. Once the global optimal solution $\tilde{\boldsymbol{X}}$, $\mathrm{rank}(\tilde{\boldsymbol{X}})\geq 2$, is obtained by solving problem (8), there exists a rank-1 matrix $\bar{\boldsymbol{X}}$ which defines the same sequence $\boldsymbol{r}$ as $\tilde{\boldsymbol{X}}$ does, i.e.,

$$r_i = \mathrm{tr}(\boldsymbol{J}_i\bar{\boldsymbol{X}}) = \mathrm{tr}(\boldsymbol{J}_i\tilde{\boldsymbol{X}}),\ \forall i\in S. \tag{14}$$

With sequence $\boldsymbol{r}$, we can achieve the rank-1 matrix $\bar{\boldsymbol{X}}$ via the minimum phase spectral factor algorithm. Through discrete Fourier transforms, we can obtain the radar's power spectral density at the nth instant:

$$R(\omega) = \sum_{i=-M+1}^{M-1} r_i \mathrm{e}^{-\mathrm{j}\omega i} = \left\|X(\omega)\right\|^2. \tag{15}$$

To obtain the rank-1 matrix $\bar{\boldsymbol{X}}$ we need to introduce the Fejér–Riesz theorem (Wu *et al.*, 1996):

If a complex function $W(z)$: $\mathbb{C}\rightarrow\mathbb{C}$ satisfies

$$W(z) = \sum_{n=-m}^{m} w(n)z^{-n},\ W(z)\geq 0,\ \forall\left\|z\right\|=1, \tag{16}$$

then there exists $Y(z)$: $\mathbb{C}\rightarrow\mathbb{C}$ and the complex sequence $\boldsymbol{y}=\{y_0, y_1, \ldots, y_m\}\in\mathbb{C}^{m+1}$ such that

$$Y(z) = \sum_{n=0}^{m} y(n)z^{-n},\ W(z) = \left\|Y(z)\right\|^2,\ \forall\left\|z\right\|=1. \tag{17}$$

Let $z=\mathrm{e}^{\mathrm{j}\omega}$. According to the minimum phase spectral factorization, we can denote the minimum phase factor of $R(z)$ by

$$\log X_{\mathrm{mp}}(z) = \alpha(z) + \mathrm{j}\beta(z). \tag{18}$$

Since $X_{\mathrm{mp}}(z)$ is a minimum phase, $\log X_{\mathrm{mp}}(z)$ is analytic in $\Omega=\left\{z\,\middle|\,\left\|z\right\|\geq 1\right\}$ and its power series expansion can be written as

$$\log X_{\mathrm{mp}}(z)=\sum_{n=0}^{\infty}a_n z^{-n},\ \ \|z\|\geq 1. \quad (19)$$

With sequence $\boldsymbol{r}$, $R(z)$ and $\alpha(z)$=1/2log $R(z)$ can be obtained directly. With $z=\mathrm{e}^{\mathrm{j}\omega}$, $\alpha(\omega)=\sum_{n=0}^{\infty}a_n\cos(\omega n)$ and $\beta(\omega)=-\sum_{n=0}^{\infty}a_n\sin(\omega n)$ are Hilbert transform pairs. We can obtain $\beta(\omega)$ from $\alpha(\omega)$ through the Hilbert transform. Then the minimum phase factor $X_{\mathrm{mp}}(z)$ can be constructed from $\beta(\omega)$ and $\alpha(\omega)$. The final transmit waveform of the nth instant, $\bar{\boldsymbol{x}}(n)$, can be achieved through the inverse discrete Fourier transform. As the power spectral density $R(z)$ contains only the amplitude information of $X(z)$, spectral factorization is non-unique. Furthermore, if $\|\theta(z)\|$=1, there is

$$R(z)=\|X(z)\|^2=\|\theta(z)X(z)\|^2. \quad (20)$$

Therefore, based on the minimum phase spectral factor $X_{\mathrm{mp}}(z)$, the transmit sequence at the mth instant can be obtained via the inverse discrete Fourier transform of $\theta_m(z)X_{\mathrm{mp}}(z)$, where $\theta_m(z)$ is random and $\|\theta_m(z)\|$=1. Finally, after N times spectral factorization, the optimal transmit waveforms via spectral optimization are achieved, which can be given as

$$\boldsymbol{S}=[\bar{\boldsymbol{x}}(0),\bar{\boldsymbol{x}}(1),\dots,\bar{\boldsymbol{x}}(N-1)]\in\mathbb{C}^{M\times N}. \quad (21)$$

It is obvious that for each instant, the transmit beam pattern of MIMO radar has the desired shape and in this way, the stay time of the radar signal on targets is guaranteed compared with the time division method proposed by Duly *et al*. (2013). The computational complexities of solving problem (8) and the spectral factorization algorithm are $O(M^{3.5})$ and $O(MN\log M)$, respectively.

3.2 Waveform design via spectral optimization

Through the above waveform design method based on the minimum ISL strategy, we can obtain the global optimal waveform matrix $\boldsymbol{S}$, which can form the desired beam pattern with a wide and deep nulling to mitigate spatial interferences. However, to mitigate MIMO radar interference to other licensed radiators which share an overlaid frequency band with the MIMO radar, it is necessary to optimize the transmit waveforms in the spectral domain. The transmit beam pattern of MIMO radar is concerned only with the correlation of the two antennas' transmit sequences; thus, with the optimized signal matrix $\boldsymbol{S}\in\mathbb{C}^{M\times N}$, a phase-changing diagonal matrix $\boldsymbol{\Lambda}=\mathrm{diag}(\mathrm{e}^{\mathrm{j}\varphi_0},\mathrm{e}^{\mathrm{j}\varphi_1},\dots,\mathrm{e}^{\mathrm{j}\varphi_{N-1}})$ is introduced and the new signal matrix obtained is

$$\tilde{\boldsymbol{S}}=\boldsymbol{S}\boldsymbol{\Lambda}=[\mathrm{e}^{\mathrm{j}\varphi_0}\boldsymbol{x}(0),\mathrm{e}^{\mathrm{j}\varphi_1}\boldsymbol{x}(1),\dots,\mathrm{e}^{\mathrm{j}\varphi_{N-1}}\boldsymbol{x}(N-1)]. \quad (22)$$

Thus, we have

$$\tilde{\boldsymbol{S}}\tilde{\boldsymbol{S}}^{\mathrm{H}}=\boldsymbol{S}\boldsymbol{\Lambda}\boldsymbol{\Lambda}^{\mathrm{H}}\boldsymbol{S}^{\mathrm{H}}=\boldsymbol{S}\boldsymbol{S}^{\mathrm{H}}, \quad (23)$$

which means that the transmit beam pattern will never be influenced by diagonal matrix $\boldsymbol{\Lambda}$. Therefore, the transmit sequence of the mth antenna can be written as

$$\tilde{\boldsymbol{s}}_m=\boldsymbol{s}_m\boldsymbol{\Lambda}=[x_m(0),x_m(1),\dots,x_m(N-1)]\boldsymbol{\Lambda}. \quad (24)$$

We can obtain the power spectral density (PSD) of the mth antenna's transmit sequence, i.e.,

$$\begin{aligned}\tilde{S}_m(f)&=\left\|\sum_{n=0}^{N-1}x_m(n)\mathrm{e}^{\mathrm{j}\varphi_n}\mathrm{e}^{-\mathrm{j}2\pi fn}\right\|^2\\&=\left\|\boldsymbol{v}^{\mathrm{H}}\boldsymbol{s}_m\odot\boldsymbol{F}(f)\right\|^2\\&=\boldsymbol{v}^{\mathrm{H}}(\boldsymbol{s}_m\odot\boldsymbol{F}(f))(\boldsymbol{s}_m\odot\boldsymbol{F}(f))^{\mathrm{H}}\boldsymbol{v},\end{aligned} \quad (25)$$

where $\boldsymbol{F}(f)=[1,\mathrm{e}^{-\mathrm{j}2\pi f},\dots,\mathrm{e}^{-\mathrm{j}2\pi f(N-1)}]^{\mathrm{T}}$ stands for the Fourier transform vector and $\boldsymbol{v}=[\mathrm{e}^{-\mathrm{j}\varphi_0},\mathrm{e}^{-\mathrm{j}\varphi_1},\dots,\mathrm{e}^{-\mathrm{j}\varphi_{N-1}}]^{\mathrm{T}}$. Suppose the overlaid bandwidth is $\Omega=[f_1,f_2]$. The total transmit power on this band can be given as

$$\begin{aligned}\int_{f_1}^{f_2}\tilde{S}_m(f)\mathrm{d}f&=\boldsymbol{v}^{\mathrm{H}}\int_{f_1}^{f_2}(\boldsymbol{s}_m\odot\boldsymbol{F}(f))(\boldsymbol{s}_m\odot\boldsymbol{F}(f))^{\mathrm{H}}\mathrm{d}f\boldsymbol{v}\\&=\boldsymbol{v}^{\mathrm{H}}\boldsymbol{R}_J^m\boldsymbol{v},\end{aligned} \quad (26)$$

where $\boldsymbol{R}_J^m=\int_{f_1}^{f_2}(\boldsymbol{s}_m\odot\boldsymbol{F}(f))(\boldsymbol{s}_m\odot\boldsymbol{F}(f))^{\mathrm{H}}\mathrm{d}f$. To minimize radar interference to other radiators, we need to minimize the transmit power of M antennas on this frequency band, i.e.,

$$\begin{aligned}&\min_{\boldsymbol{v}} \boldsymbol{v}^{\mathrm{H}}\left(\sum_{m=1}^{M}\boldsymbol{R}_J^m\right)\boldsymbol{v}\\ &\text{s.t.}\quad \|\boldsymbol{v}(n)\|=1,\ n=1,2,\dots,N.\end{aligned} \tag{27}$$

We can obtain the largest eigenvalue $\lambda_{\max}$ via an eigenvalue decomposition of matrix $\sum_{m=1}^{M}\boldsymbol{R}_J^m$. Let $\bar{\boldsymbol{R}}=\lambda_{\max}\boldsymbol{I}_N-\sum_{m=1}^{M}\boldsymbol{R}_J^m$. With the help of auxiliary matrix $\bar{\boldsymbol{R}}$, problem (27) can be equivalently recast as

$$\begin{aligned}&\max_{\boldsymbol{v}} \boldsymbol{v}^{\mathrm{H}}\bar{\boldsymbol{R}}\boldsymbol{v}\\ &\text{s.t.}\quad \|\boldsymbol{v}(n)\|=1,\ n=1,2,\dots,N.\end{aligned} \tag{28}$$

With the semidefinite Hermitian matrix $\bar{\boldsymbol{R}}$ and the feasible region of vector $\boldsymbol{v}$, $\varDelta=\{\boldsymbol{v}\in\mathbb{C}^{N\times 1}\,|\,\|\boldsymbol{v}(n)\|=1,\ n=0,1,\dots,N-1\}$, problem (28) is a unimodular quadratic programming (UQP) problem. Power method-like iteration optimization (Soltanalian and Stoica, 2014) can be used to solve problem (28). Assume that after k iterations, we obtain the optimized vector $\boldsymbol{v}^{(k)}$. Then the (k+1)th iteration optimization problem can be given as

$$\min_{\boldsymbol{v}^{(k+1)}\in\varDelta}\left\|\boldsymbol{v}^{(k+1)}-\bar{\boldsymbol{R}}\boldsymbol{v}^{(k)}\right\|_2^2. \tag{29}$$

Through a power method-like algorithm, we can obtain the solution to problem (29):

$$\boldsymbol{v}^{(k+1)}=\exp(\mathrm{j}\arg(\bar{\boldsymbol{R}}\boldsymbol{v}^{(k)})). \tag{30}$$

Because $\boldsymbol{v}\in\varDelta$, we have

$$\left\|\boldsymbol{v}^{(k+1)}-\bar{\boldsymbol{R}}\boldsymbol{v}^{(k)}\right\|_2^2=\text{const.}-2\,\mathrm{Re}(\boldsymbol{v}^{(k+1)\mathrm{H}}\bar{\boldsymbol{R}}\boldsymbol{v}^{(k)}), \tag{31}$$

and $\boldsymbol{v}^{(k+1)}$ should maximize $\mathrm{Re}(\boldsymbol{v}^{(k+1)\mathrm{H}}\bar{\boldsymbol{R}}\boldsymbol{v}^{(k)})$. If $\boldsymbol{v}^{(k+1)}\neq\boldsymbol{v}^{(k)}$, because $\bar{\boldsymbol{R}}$ is a Hermitian semidefinite matrix, there is

$$(\boldsymbol{v}^{(k+1)}-\boldsymbol{v}^{(k)})^{\mathrm{H}}\bar{\boldsymbol{R}}(\boldsymbol{v}^{(k+1)}-\boldsymbol{v}^{(k)})>0. \tag{32}$$

Because $\mathrm{Re}(\boldsymbol{v}^{(k+1)\mathrm{H}}\bar{\boldsymbol{R}}\boldsymbol{v}^{(k)})>\boldsymbol{v}^{(k)\mathrm{H}}\bar{\boldsymbol{R}}\boldsymbol{v}^{(k)}$, we have the following result:

$$\begin{aligned}\boldsymbol{v}^{(k+1)\mathrm{H}}\bar{\boldsymbol{R}}\boldsymbol{v}^{(k+1)}&>2\,\mathrm{Re}(\boldsymbol{v}^{(k+1)\mathrm{H}}\bar{\boldsymbol{R}}\boldsymbol{v}^{(k)})-\boldsymbol{v}^{(k)\mathrm{H}}\bar{\boldsymbol{R}}\boldsymbol{v}^{(k)}\\&>\boldsymbol{v}^{(k)\mathrm{H}}\bar{\boldsymbol{R}}\boldsymbol{v}^{(k)}.\end{aligned} \tag{33}$$

Because sequence $\{\boldsymbol{v}^{(k)\mathrm{H}}\bar{\boldsymbol{R}}\boldsymbol{v}^{(k)}\}$ is convergent, problem (28) is also convergent through iterations. After k iterations, if $\|\boldsymbol{v}^{(k+1)}-\boldsymbol{v}^{(k)}\|\le\eta$, the iteration process will stop and the optimized vector $\boldsymbol{v}^{\star}$ is obtained. Finally, after spectral optimization, the waveform matrix obtained is given as

$$\tilde{\boldsymbol{S}}^{\star}=\boldsymbol{S}\mathrm{diag}(\boldsymbol{v}^{\star}). \tag{34}$$

The iteration number L is decided by η and the computational complexity of the power method-like iterations is $O(LN^2)$.

4 Numerical results

We provide several numerical results to demonstrate the superior performance of the proposed waveform design method. The parameters for simulation are set as follows: the whole search region is $\varTheta$=[−90°, 90°]; the angle section for energy focus is $\varTheta_{\mathrm{ml}}$=[−15°, 15°]; suppose a strong reflection point at the direction of arrival (DOA) θ_{int}=40°; to mitigate the interference conservatively, we set the width and depth of the formed nulling as $2\varDelta$=6° and $\varepsilon=10^{-5}$, respectively. The carrier frequency and bandwidth of the transmitted waveforms are f_0=1 GHz and B=1 MHz, respectively. The total transmit power of the radar system is E=1.

4.1 Analysis of optimized waveforms obtained by the proposed method

First, the impact of the variables N (length of the transmit sequence) and M (number of transmit antennas) is analyzed. Figs. 1a and 1b show the normalized transmit beam patterns formed with different values of M and N, respectively. Fig. 1a shows that the proposed waveform design method can form the desired steady main lobe with the desired width of 30° and a deep and wide nulling toward the DOA θ_{int}=40°. Furthermore, the MIMO radar's transmit beam pattern has no relationship with the length of the transmit sequence. This is because the theoretical definition of a beam pattern in a coherent processing interval is given by

$$\begin{aligned}P(\theta)&=\sum_{n=0}^{N-1}P(n,\theta)=\boldsymbol{a}^{\mathrm{H}}(\theta)\sum_{n=0}^{N-1}\boldsymbol{s}(n)\boldsymbol{s}^{\mathrm{H}}(n)\boldsymbol{a}(\theta)\\&=N\boldsymbol{a}^{\mathrm{H}}(\theta)\boldsymbol{R}\boldsymbol{a}(\theta),\end{aligned} \tag{35}$$

where $\boldsymbol{R}=E\{\boldsymbol{s}(n)\boldsymbol{s}^{\mathrm{H}}(n)\}$ is the covariance matrix of the transmit sequence at the nth instant. The beam pattern is entirely decided by matrix $\boldsymbol{R}$, which has no relationship with the length of the transmit sequence. As a result, the beam patterns formed with different values of N are identical. However, the dimension of covariance matrix $\boldsymbol{R}$ is decided by the number of transmit antennas. The larger the number of transmit antennas, the higher the dimension of matrix $\boldsymbol{R}$ and the larger the DOFs of the waveforms. Thus, when we increase the number of antennas, the beam pattern focuses more energy in the main lobe, the side-lobe level decreases, and the nulling becomes deeper and wider (Fig. 1b).

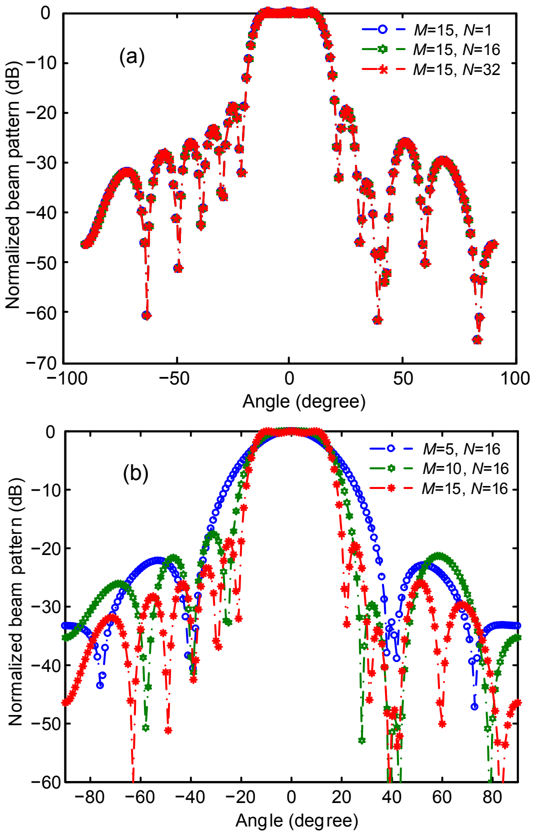

Fig. 1 Normalized beam patterns with fixed M and different N (a) and fixed N and different M (b)

Next, we check the performance of the waveform design method in the spectral domain. Let M=15 and N=256, and the other simulation conditions are the same as the previous ones. To clearly demonstrate the effectiveness of the proposed waveform design scheme, we simulate the two-dimensional transmit beam pattern in the spatial and spectral domains for different cases (Fig. 2). Waveforms obtained only through spatial optimization can form a spatial nulling at θ=40° but cannot form a spectral nulling at the overlaid frequency band (Fig. 2a), which means that the general spatial waveform design method cannot

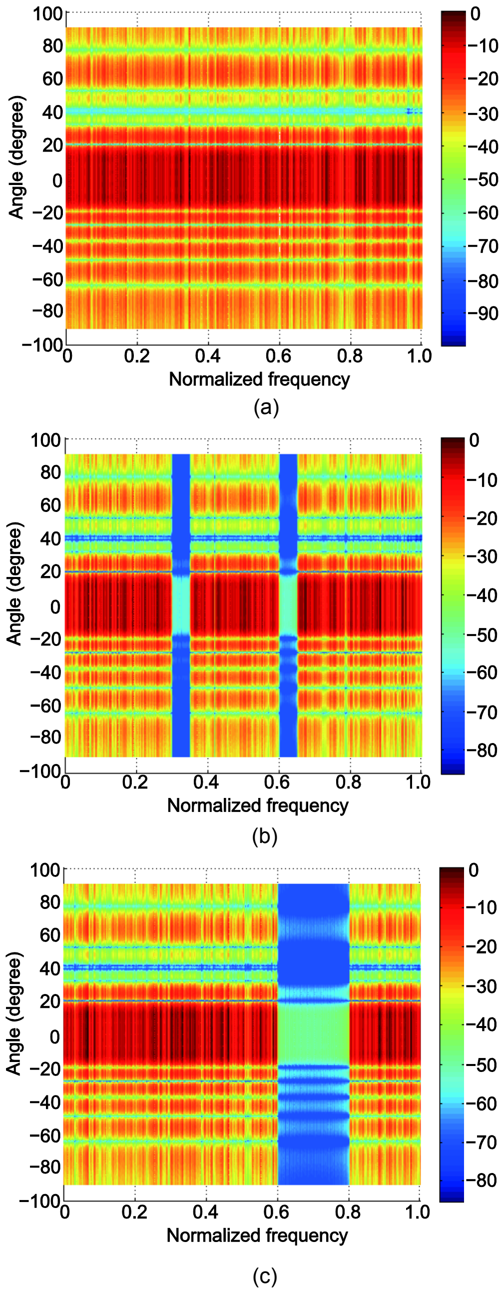

Fig. 2 Two-dimensional transmit beam pattern formed by optimized waveforms via only spatial optimization (a), two-step optimization with the overlaid bands [0.3, 0.35] and [0.6, 0.65] (b), and two-step optimization with the overlaid band [0.6, 0.8] (c)

mitigate spectral interferences. However, after the second spectral optimization process, the waveforms obtained can form a spectral nulling with the desired width at the desired frequency band (Figs. 2b and 2c). Furthermore, when the spectral nulling formed becomes wider, the spectral nulling shallows. The depths of the spectral nullings in Figs. 2b and 2c are about −55 dB and −45 dB, respectively.

To check the convergence performance of the power method-like iterations, denote the log-norm of the difference between two contiguous optimized vectors $\boldsymbol{v}^{(k)}$ and $\boldsymbol{v}^{(k+1)}$ as

$$\delta = \log\left\|\boldsymbol{v}^{(k+1)} - \boldsymbol{v}^{(k)}\right\|. \tag{36}$$

The relationship between the value of δ and the iteration number is shown in Fig. 3. Note that, as expected, the difference decreases as the iteration number increases. Since the auto-correlation and cross-correlation of the transmit waveforms are important performance indexes for a radar system, we thus compare the optimized waveforms with those synthesized ones via the CA method in terms of the auto- and cross-correlation performances (Fig. 4).

It is obvious that the optimized waveforms via two-step optimization in the spatial and spectral domains have similar auto- and cross-correlation performances to the synthesized waveforms via the CA method. Since the price of transmit beam pattern design is the orthogonality of the waveforms, the cross-correlation of optimized waveforms is degraded just like the synthesized waveforms via the CA method. However, with the same cost in auto- and cross-correlation performances, the proposed method achieves more benefits in the transmit beam pattern design, as shown in the following section.

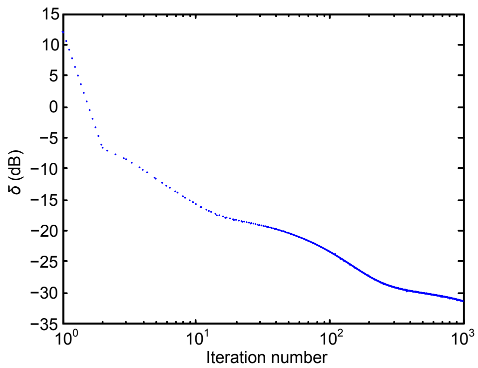

Fig. 3 The δ of two contiguous optimized vectors $v^{(k)}$ and $v^{(k+1)}$ versus the iteration number

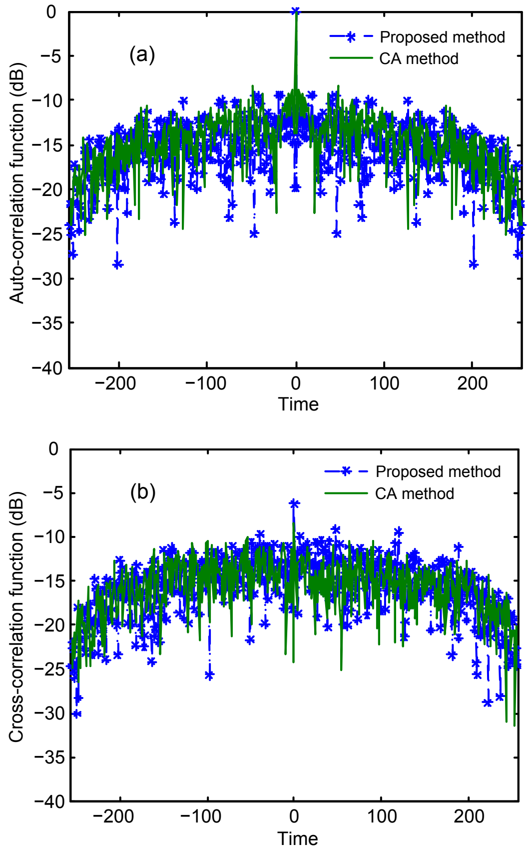

Fig. 4 Auto-correlation function of the transmit waveforms (a) and cross-correlation function of the transmit waveforms (b)

4.2 Comparisons among different design methods

Simulations have been conducted to demonstrate the superior performance of our design method. First, the classic BM method and CA algorithm are compared with our proposed method. Let M=15 and N=16. The iteration number of the CA algorithm is L=1000, and other simulation parameters do not change. Note that p-order derivative constraints are imposed in the BM method to form the desired wide nulling. Based on the optimized covariance matrix $\boldsymbol{R}$, the CA algorithm is used to synthesize the final constant-modulus waveforms. Besides, to demonstrate the effectiveness of the spectral factorization method, we compare the beam pattern formed by the final synthesized waveform obtained by minimum phase spectral factorization and the beam pattern formed by the optimal covariance matrix $\boldsymbol{X}$ achieved through problem (8), which we will call the SDR method. Fig. 5 shows the beam patterns formed by these different methods. It is obvious that the CA method cannot guarantee the desired nulling toward the direction of interference. This is because the synthesized waveforms via the CA method only approximately realize $\boldsymbol{R}^{1/2}\boldsymbol{U}$ and cannot satisfy the nulling constraint. Our proposed method can not only achieve the global optimal waveforms but also have a lower side-lobe level. Furthermore, compared with the computational complexities of the BM method and the CA algorithm, which are $O(M^{3.5})$ and $O(LMN^2)$, respectively, our proposed method is more efficient.

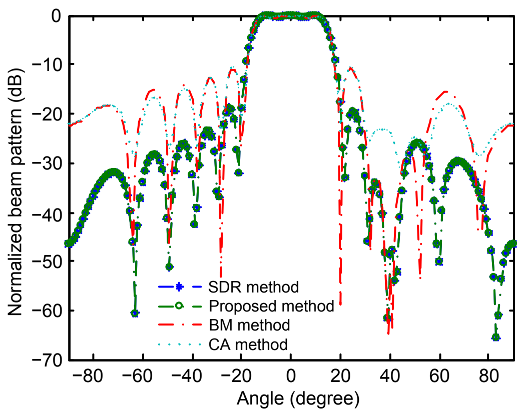

Fig. 5 Beam pattern comparisons between the proposed method and the BM and CA methods

Next, we compare the proposed method with the DPSS method. Let the numbers of waveforms be four and six for the DPSS method, respectively. To form a spatial nulling, we project the transmit beam space obtained on the orthogonal projection matrix of the interference's steering vector, i.e.,

$$\bar{\boldsymbol{C}} = \boldsymbol{P}_{\perp}\boldsymbol{C}, \tag{37}$$

where $\boldsymbol{C}$ is the transmit weighting matrix obtained and $\boldsymbol{P}_{\perp}=\boldsymbol{I}-\boldsymbol{a}(\theta_{\text{int}})(\boldsymbol{a}^{\text{H}}(\theta_{\text{int}})\boldsymbol{a}(\theta_{\text{int}}))^{-1}\boldsymbol{a}^{\text{H}}(\theta_{\text{int}})$ is the projection matrix. Fig. 6 shows the numerical results of the two methods. The DPSS method can form only a narrow nulling, and when the number of waveforms increases, the beam pattern formed is out of the desired shape. Even when the number of waveforms is k=4, our proposed method still has an obvious superior performance. This simulation results prove the conclusion that the side-lobe levels of the beam pattern obtained by the DPSS method can never be smaller than those obtained by the minimum ISL method.

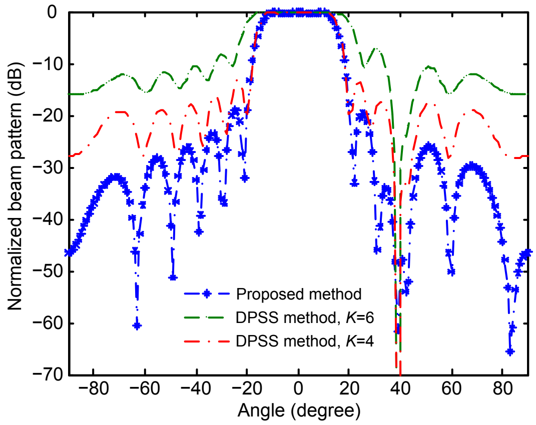

Fig. 6 Beam pattern comparisons between the proposed method and DPSS method

Finally, since Xu's method and our proposed method are both based on the minimum-ISL strategy, we compare the performance of these two methods. Let the number of waveforms be K=4 for Xu's method. Fig. 7 shows their formed beam patterns. In Xu's method, the final transmit waveform synthesized by the randomization algorithm is sub-optimal and the beam pattern formed with the synthesized waveforms is distorted. The computational complexity of Xu's method is $O((MK)^{3.5})+O(Q(MK)^2)$, where Q is the randomization number. The computational burden is huge, especially when the number of waveforms is large. Like the DPSS method, Xu's method just optimizes the transmit weighting matrix, so it cannot obtain the constant-modulus waveform. In contrast, our proposed method overcomes these problems perfectly.

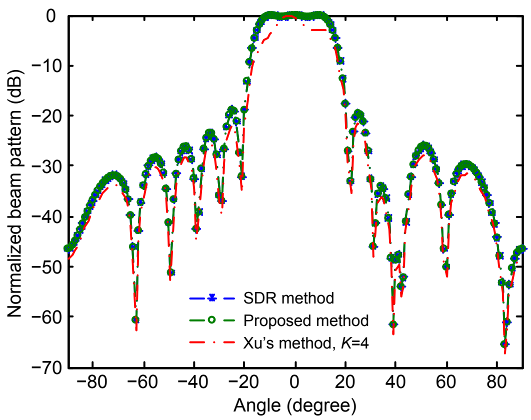

Fig. 7 Beam pattern comparisons between the proposed method and Xu's method

5 Conclusions

An efficient waveform design method has been considered for colocated MIMO radar. In contrast to existing waveform design approaches, the transmit waveforms are optimized in the spatial domain to form a beam pattern with the desired shape and nulling to mitigate signal-dependent interferences. With the resulting non-convex fractional QCQP problem with Toeplitz–Hermitian quadratics, the hidden convexity of the resulting problem is recovered and the global optimal solution is achieved in polynomial time via SDR followed by spectral factorization. With the optimal waveforms obtained via spatial optimization, a further spectral optimization is performed by means of power method-like iterations to mitigate the spectral interferences and improve the spectral compatibility with other overlaid licensed radiators. The performances of the proposed method in beam pattern formation and spectrum optimization are considered. Furthermore, numerical comparisons have been conducted to demonstrate the superior performance of our method over existing methods.

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