Frontiers of Information Technology & Electronic Engineering
www.jzus.zju.edu.cn; engineering.cae.cn; www.springerlink.com
ISSN 2095-9184 (print); ISSN 2095-9230 (online)
E-mail: jzus@zju.edu.cn

Dynamics of a neuron exposed to integer- and fractional-order discontinuous external magnetic flux

Karthikeyan RAJAGOPAL[1,2], Fahimeh NAZARIMEHR[3], Anitha KARTHIKEYAN[1], Ahmed ALSAEDI[4], Tasawar HAYAT[4,5], Viet-Thanh PHAM[†‡6]

[1]*Center for Nonlinear Dynamics, Defence University, Bishoft 6020, Ethiopia*
[2]*Institute of Energy, Mekelle University, Mekelle 231, Ethiopia*
[3]*Biomedical Engineering Department, Amirkabir University of Technology, Tehran 15875-4413, Iran*
[4]*NAAM Research Group, Faculty of Science, King Abdulaziz University, Jeddah 21589, Saudi Arabia*
[5]*Department of Mathematics, Quaid-i-Azam University, Islamabad 45320, Pakistan*
[6]*Modelling Evolutionary Algorithms Simulation and Artificial Intelligence, Faculty of Electrical and Electronics Engineering, Ton Duc Thang University, Ho Chi Minh City 700000, Vietnam*
[†]E-mail: phamvietthanh@tdt.edu.vn

Received June 23, 2018; Revision accepted Sept. 14, 2018; Crosschecked Jan. 25, 2019; Published online Mar. 7, 2019

Abstract: We propose a modified Fitzhugh-Nagumo neuron (MFNN) model. Based on this model, an integer-order MFNN system (case A) and a fractional-order MFNN system (case B) were investigated. In the presence of electromagnetic induction and radiation, memductance and induction can show a variety of distributions. Fractional-order magnetic flux can then be considered. Indeed, a fractional-order setting can be acceptable for non-uniform diffusion. In the case of an MFNN system with integer-order discontinuous magnetic flux, the system has chaotic and non-chaotic attractors. Dynamical analysis of the system shows the birth and death of period doubling, which is a sign of antimonotonicity. Such a behavior has not been studied previously in the dynamics of neurons. In an MFNN system with fractional-order discontinuous magnetic flux, different attractors such as chaotic and periodic attractors can be observed. However, there is no sign of antimonotonicity.

Key words: Fitzhugh-Nagumo; Chaos; Fractional order; Magnetic flux
https://doi.org/10.1631/FITEE.1800389 **CLC number:** O44

1 Introduction

Neurons are excitable cells which transmit electrical and chemical information. For many years, modelling of neurons has been a hot topic. One of the basic neuron models is the Hodgkin and Huxley (HH) model (Hodgkin and Huxley, 1952), which shows the effect of ion channels on membrane potential. A simplified two-dimensional (2D) form of the HH model is the Fitzhugh-Nagumo model (Fitzhugh, 1961; Nagumo et al., 1962). Another well-known neural model was proposed by Izhikevich (2004). These models help scientists study different behaviors of neurons. There are two approaches for modelling the variations in biological systems. The first considers those variations as a statistical part of the model (Perc et al., 2009), while the second considers them as a complex attractor of dynamical systems (McSharry et al., 2003; Haghighi and Markazi, 2017; Panahi et al., 2017). There are some biological facts which prove the value of nonlinear dynamical analysis in the study of neural systems (Freeman, 1988; Schmidt et al., 2013; Gu and Pan, 2015). For example, the study of pattern formation in a

‡ Corresponding author
ORCID: Karthikeyan RAJAGOPAL, http://orcid.org/0000-0003-2993-7182

network of neurons has been a hot topic (Qian et al., 2017; Wang et al., 2017; Lv et al., 2018). The different coupling of neurons has been studied in this area (Wu FQ et al., 2018). A neuron model driven by autapse connection has been investigated with the effect of electromagnetic induction (Ren et al., 2017). A propagating wave initiated in a neuronal network can be controlled by magnetic radiation (Rostami et al., 2018). The collective responses in electrical activities of neurons under field coupling have been studied by Xu et al. (2018). A neuronal circuit can produce the main properties for neuronal acclivities (Ma et al., 2018). Electrical activity in a neuron under a magnetic flow effect has been investigated using a modified Fitzhugh-Nagumo model (Lv et al., 2016). Also, complex electrical activities of cardiac tissue which can set up a time-varying electromagnetic field have been studied using this model (Wu FQ et al., 2016). Implementation of dynamical systems using analog/digital electronics is an interesting topic (Abdolmohammadi et al., 2018; Pham et al., 2018; Yang et al., 2018).

The calculation of fractional-order systems has been an interesting topic since the 17^{th} century (Stamova and Stamov, 2017). Fractional-order differentiator and integrators have been investigated by Bertsias et al. (2018). Fractional-order systems have many applications in different fields (Jenson and Jeffreys, 1977; Elwakil, 2010). A fractional model of a commercial ear simulator has been studied by Vastarouchas et al. (2018). The existence of chaos in dynamical systems is an obscure phenomenon that has attracted a lot of attention. There is no clear explanation as to how a chaotic attractor can be formed in a system. Fractional-order systems are more complex than integer-order systems. Thus, the existence of chaotic dynamics in fractional-order systems needs more consideration. There have been many studies on chaotic fractional-order systems (Cafagna and Grassi, 2013, 2015; Li and Zhang, 2016). A fractional-order switched system with a chaotic attractor has been discussed by Zambrano-Serrano et al. (2017). Shah et al. (2017) studied field-programmable gate array (FPGA) implementation of fractional-order chaotic systems. A fractional-order chaotic system without equilibria was studied by Pham et al. (2017).

In this study, a modified Fitzhugh-Nagumo neuron (MFNN) model was proposed. An integer-order MFNN system (case A) and a fractional-order MFNN system (case B) were investigated using the proposed model. We discussed the dynamical properties of the MFNN system in cases A and B in this study.

2 Modified Fitzhugh-Nagumo neuron model

It has been shown that there is fluctuation in the ion concentration of potassium, sodium, and calcium. The effects on changes in the membrane potential of myocardial cells can be modelled by introducing an additive magnetic flux variable (Ma et al., 2010) to the well-known Fitzhugh-Nagumo neuron (FNN) model (Ma et al., 2017a). The improved FNN model (Ma et al., 2010) is given by

$$\begin{cases} \dot{x} = -kx(x-a)(x-1) - xy + I_e + k_0\rho(\phi)x, \\ \dot{y} = \left(\varepsilon + \dfrac{\mu_1 y}{x+\mu_2}\right)\left(-y - kx(x-a-1)\right), \\ \dot{\phi} = k_1 x - k_2 z + \phi_e, \end{cases} \tag{1}$$

where state variables x, y, and ϕ describe the membrane potential, slow variable for current, and magnetic flux across the membrane, respectively. The transmembrane current mapped along with the external force is denoted as I_e and modelled as $I_e = I_0\sin(\omega t)$. k_i denots the feedback gains. $k_0\rho(\phi)x$ denotes the current induced by electromagnetic induction in the cell, and the external electromagnetic excitation is given by $\phi_e = E\cos(2\pi f t)$. The memory conductance assumed by Ma et al. (2010) is a memristor magnetic flux relation $\rho(\phi) = \alpha + 3\beta\phi^2$. The magnetic flux effects are discontinuous as the ion concentration thresholds keep shifting. Thus, using discontinuous memductance functions is preferred for complete modelling of the magnetic flux effects on the neurons. Hence, the memory conductance $\rho(\phi)$ is assumed to be a monotonically increasing piecewise linear function (Itoh and Chua, 2008), given by

$$q(\phi) = \beta\phi + 0.5(\alpha-\beta)\left(|\phi+1| - |\phi-1|\right), \tag{2}$$

where $\alpha, \beta > 0$.

The memory conductance can be derived using integer-order differentiation (case A) and fractional-order differentiation (case B).

$$\rho(\phi) = \frac{d^\sigma q(\phi)}{d^\sigma \phi}. \tag{3}$$

From Eq. (3), we can easily see that in the order of differentiation $\sigma = 1$, we derive the integer order (case A). The order is fractional in $0 < \sigma < 1$ (case B). By introducing another state to model (1), we propose the modified FNN (MFNN) system as

$$\begin{cases} \dot{x} = -kx(x-a)(x-1) - xy + I_e + k_0 D^\sigma q(\phi_k)x, \\ \dot{y} = \left(\varepsilon + \dfrac{\mu_1 y}{x+\mu_2}\right)(-y - kx(x-a-1)), \\ \dot{\phi} = k_1 x - k_2 z + \phi_e, \\ D^\sigma q(\phi_k) = (\beta + 0.5(\alpha-\beta)(\text{sgn}(\phi_{k-1}+1) \\ \qquad -\text{sgn}(\phi_{k-1}-1)))\, h^\sigma - \displaystyle\sum_{j=1}^{N} B_j^\sigma \phi(t_{k-j}), \end{cases} \tag{4}$$

where σ is the order of differentiation, h is the step size, and B is the predictor coefficient, calculated as described by Baskonus and Bulut (2015). The generalized-order differentiation term $D^\sigma q(\phi_k)$ is derived using the Caputo fractional derivative (Diethelm and Ford, 2002), known as "Caputo derivatives." It is derived using the same initial conditions as those in the integer-order model. The fractional-order discrete form of the incommensurate model of $D^\sigma q(\phi_k)$ is derived using the Adam-Bashforth-Moulton (ABM) approach (Sun et al., 1984; Diethelm, 1997; Diethelm et al., 2004). Using the ABM method (predictor-corrector) (Baskonus and Bulut, 2015), the fractional-order discrete form of $D^\sigma q(\phi_k)$ can be derived as

$$\begin{aligned} q(\phi)_{k+1} = & \sum_{j=0}^{n-1}\left(q(\phi)_0^j \frac{t_{k+1}^j}{j!}\right) + \frac{1}{\Gamma(\sigma_x)}\sum_{j=0}^{k}\Bigg(B_{x,j,k+1} \\ & \cdot\Big(\beta + 0.5(\alpha-\beta)\big(\text{sgn}(\phi_j+1) - \text{sgn}(\phi_j-1)\big)\Big)\Bigg) \\ & \cdot B_{x,k+1,k+1}\Big(\beta + 0.5(\alpha-\beta)\big(\text{sgn}(\phi_{k+1}^p+1) \\ & - \text{sgn}(\phi_{k+1}^p - 1)\big)\Big), \end{aligned} \tag{5}$$

which shows the corrector. The predictor part can be derived as

$$\begin{aligned} q(\phi)_{k+1}^p = & \sum_{j=0}^{n-1}\left(q(\phi)_0^j \frac{t_{k+1}^j}{j!}\right) + \frac{1}{\Gamma(\sigma_x)}\sum_{j=0}^{k}\Bigg(\gamma_{x,j,k+1} \\ & \cdot\Big(\beta + 0.5(\alpha-\beta)\big(\text{sgn}(\phi_j+1) \\ & - \text{sgn}(\phi_j-1)\big)\Big)\Bigg), \end{aligned} \tag{6}$$

$$B_{i,j,n+1} = \begin{cases} k^{q_i+1} - (k-q_i)(k+1)^{q_i}, & j=0, \\ (k-j+2)^{q_i+1} + (k-j)^{q_i+1} \\ \quad -2(k-j+1)^{q_i+1}, & 1 \le j \le k, \\ 1, & j = k+1, \end{cases} \tag{7}$$

$$\gamma_{i,j,n+1} = \frac{h_{q_i}}{q_i}\left((k+1-j)^{q_i} - (k-j)^{q_i}\right). \tag{8}$$

If the generalized order $\sigma = 1$ is used in system (4), the derivative approximations of $D^\sigma q(\phi_k)$ yield the closest integer-order derivative (Diethelm et al., 2004).

The parameters of MFNN system (4) are similar to those described by Ma et al. (2010) with the additional parameters α, β defined as in Eq. (5). The parameters are $a = 0.15$, $\mu_1 = 0.2$, $\mu_2 = 0.3$, $\varepsilon = 0.002$, $\alpha = 0.1$, $\beta = 0.2$, $I_0 = 0.6$, $k_0 = -1$, $k_1 = 0.2$, $k_2 = 1$, $E = 0.1$, $k = 8$, $f = 0.01$, $\omega = 2$, and the initial conditions are taken as [0.2, 0.1, 0.8] for all discussions in this study. The dynamical properties of the MFNN system are derived separately for case A ($\sigma = 1$) and case B ($\sigma < 1$).

3 Dynamical analysis of the modified Fitzhugh-Nagumo neuron system in case A

To investigate the MFNN system with integer-order discontinuous magnetic flux, we take the differentiation order $\sigma = 1$. The parameters of MFNN system (4) are taken as those in Section 2. Fig. 1 shows the 2D phase portraits of the system in the XY and YZ planes.

To investigate the dynamical behavior of the MFNN system (case A), bifurcation plots are derived. The angular frequency ω of the external current is taken as the control parameter. The bifurcation plot (Fig. 2a) confirms that the system takes a period doubling route to chaos. We can also see the birth and death of period doubling, confirming that the MFNN system shows antimonotonicity (Dawson et al., 1992; Blażejczyk-Okolewska and Kapitaniak, 1998; Chudzik et al., 2011; Wu HG et al., 2016; Kengne et al., 2017). The corresponding Lyapunov exponents (LEs) are presented in Fig. 2b.

The process of period doubling and period halving occurring in a bifurcation diagram of a system is termed antimonotonicity (Dawson et al., 1992; Chudzik et al., 2011; Wu HG et al., 2016). To investigate this in detail, we derive the bifurcation of

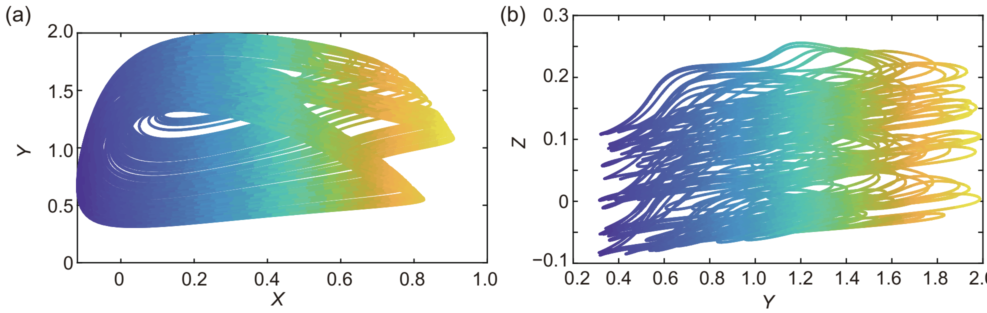

Fig. 1 The 2D phase portraits of the modified Fitzhugh-Nagumo neuron system with integer-order magnetic flux in *XY* (a) and *YZ* (b) planes

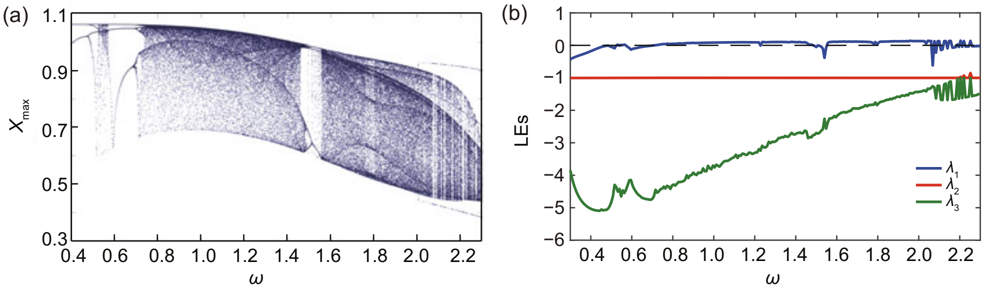

Fig. 2 Bifurcation (a) and Lyapunov exponents (b) of the modified Fitzhugh-Nagumo neuron system in case A with respect to changing ω

the MFNN system with ω taken as the control parameter for various values of α. The novelty of this study is that such a feature occurring in a neuron model has not been discussed in the literature. The evidence for period doubling and its inverse is clearly seen from the bifurcation plots shown in Fig. 3. The dynamical properties of the modified FNN model are highly dependent on the initial value of the magnetic flux variable (Ma et al., 2016, 2017b).

4 Dynamical analysis of the modified Fitzhugh-Nagumo neuron system in case B

To investigate the MFNN system with fractional-order discontinuous magnetic flux, we take the differentiation order $0.9 \leq \sigma < 1$. The parameters of MFNN system (4) are taken as those in Section 2. Fig. 4 shows the 2D phase portraits of the system in the XY and YZ planes with the fractional order $\sigma = 0.99$.

The dynamical behavior of the MFNN (case B) system is investigated with two control parameters. The first bifurcation is with respect to changing parameter α with the fractional order $\sigma = 0.99$ and the other parameters unchanged. Fig. 5a shows the bifurcation of the MFNN system with fractional-order discontinuous flux linkage. It can be seen that case B of the MFNN system has a period halving after chaotic dynamic and shows no evidence of antimonotonicity. The corresponding LEs are shown in Fig. 5b. The second bifurcation is with respect to changing the fractional-order σ with the other parameters unchanged. The bifurcation diagram is shown in Fig. 5c. The system has a period doubling route to chaos at lower σ values and shows chaotic oscillations for $\sigma > 0.95$.

5 Conclusions

The dynamical behaviors of neurons are very important. In this paper, a modified Fitzhugh-Nagumo neuron model was proposed. The model was studied with two approaches. In the first approach (case A), the magnetic flux was considered of integer order. In the second approach (case B), it was considered of fractional order. Electromagnetic induction and radiation can cause variation in the distribution of memductance and induction. Thus, studies of fractional-order magnetic flux are

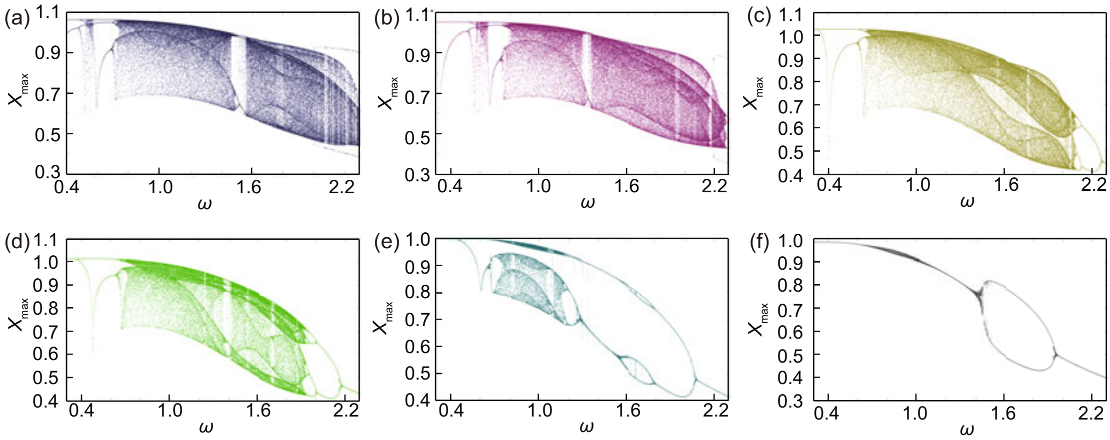

Fig. 3 Bifurcation of the modified Fitzhugh-Nagumo neuron system in case A with respect to changing ω for $\alpha = 0.1$ (a), $\alpha = 0.2$ (b), $\alpha = 0.4$ (c), $\alpha = 0.5$ (d), $\alpha = 0.6$ (e), and $\alpha = 0.7$ (f)

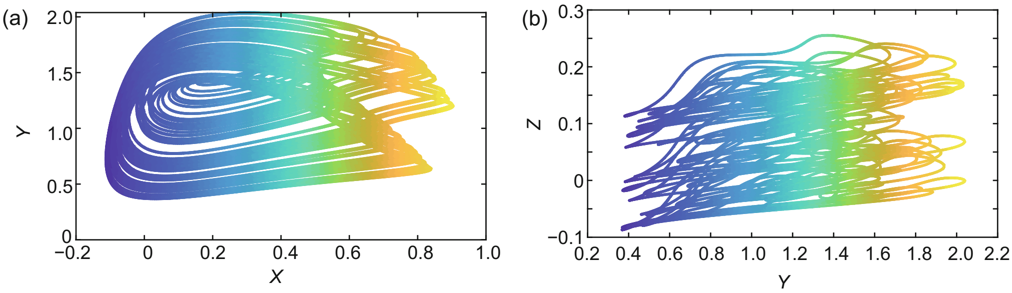

Fig. 4 The 2D phase portraits of the modified Fitzhugh-Nagumo neuron system with fractional-order magnetic flux in XY (a) and YZ (b) planes

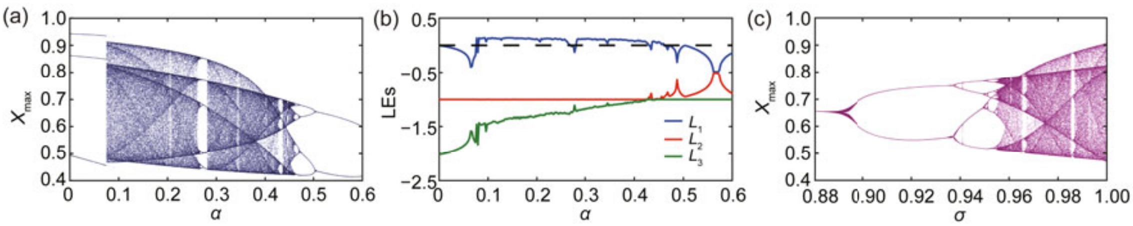

Fig. 5 (a) and (b) are the bifurcation and Lyapunov exponents of the modified Fitzhugh-Nagumo neuron system in case B with respect to changing α, respectively, and (c) is the bifurcation of the modified Fitzhugh-Nagumo neuron system in case B with respect to changing σ

important. Indeed, a fractional-order setting can be used to investigate non-uniform diffusion. The dynamical properties of these two cases (A and B) were investigated in this study. The results showed that only the integer-order system has an antimonotonicity feature. The initial condition of the magnetic flux variable is very important in the dynamical behavior of the MFNN model. Multistability and coexisting attractors in the two cases of MFNN can be investigated in future studies.

References

Abdolmohammadi HR, Khalaf AJM, Panahi S, et al., 2018. A new 4D chaotic system with hidden attractor and its engineering applications: analog circuit design and field programmable gate array implementation. *Pramana*, 90(6):70. https://doi.org/10.1007/s12043-018-1569-2

Baskonus HM, Bulut H, 2015. On the numerical solutions of some fractional ordinary differential equations by fractional Adams-Bashforth-Moulton method. *Open Math*, 13(1):547-556. https://doi.org/10.1515/math-2015-0052

Bertsias P, Safari L, Minaei S, et al., 2018. Fractional-order differentiators and integrators with reduced circuit complexity. Int Symp on Circuits and Systems, p.1-4. https://doi.org/10.1109/ISCAS.2018.8351452

Blażejczyk-Okolewska B, Kapitaniak T, 1998. Co-existing attractors of impact oscillator. *Chaos Sol Fract*, 9(8):1439-1443. https://doi.org/10.1016/S0960-0779(98)00164-7

Cafagna D, Grassi G, 2013. Elegant chaos in fractional-order system without equilibria. *Math Probl Eng*, Article 380436. https://doi.org/10.1155/2013/380436

Cafagna D, Grassi G, 2015. Fractional-order systems without equilibria: the first example of hyperchaos and its application to synchronization. *Chin Phys B*, 24(8):080502. https://doi.org/10.1088/1674-1056/24/8/080502

Chudzik A, Perlikowski P, Stefanski A, et al., 2011. Multistability and rare attractors in van der Pol-Duffing oscillator. *Int J Bifurc Chaos*, 21(7):1907-1912. https://doi.org/10.1142/S0218127411029513

Dawson SP, Grebogi C, Yorke JA, et al., 1992. Antimonotonicity: inevitable reversals of period-doubling cascades. *Phys Lett A*, 162(3):249-254. https://doi.org/10.1016/0375-9601(92)90442-O

Diethelm K, 1997. An algorithm for the numerical solution of differential equations of fractional order. *Electron Trans Numer Anal*, 5:1-6.

Diethelm K, Ford NJ, 2002. Analysis of fractional differential equations. *J Math Anal Appl*, 265(2):229-248. https://doi.org/10.1006/jmaa.2000.7194

Diethelm K, Ford NJ, Freed AD, 2004. Detailed error analysis for a fractional Adams method. *Numer Algor*, 36(1):31-52. https://doi.org/10.1023/B:NUMA.0000027736.85078.be

Elwakil AS, 2010. Fractional-order circuits and systems: an emerging interdisciplinary research area. *IEEE Circ Syst Mag*, 10(4):40-50. https://doi.org/10.1109/MCAS.2010.938637

Fitzhugh R, 1961. Impulses and physiological states in theoretical models of nerve membrane. *Biophys J*, 1(6):445-466. https://doi.org/10.1016/S0006-3495(61)86902-6

Freeman WJ, 1988. Strange attractors that govern mammalian brain dynamics shown by trajectories of electroencephalographic (EEG) potential. *IEEE Trans Circ Syst*, 35(7):781-783. https://doi.org/10.1109/31.1822

Gu HG, Pan BB, 2015. A four-dimensional neuronal model to describe the complex nonlinear dynamics observed in the firing patterns of a sciatic nerve chronic constriction injury model. *Nonl Dynam*, 81(4):2107-2126. https://doi.org/10.1007/s11071-015-2129-7

Haghighi HS, Markazi AHD, 2017. A new description of epileptic seizures based on dynamic analysis of a thalamocortical model. *Sci Rep*, 7:13615. https://doi.org/10.1038/s41598-017-13126-4

Hodgkin AL, Huxley AF, 1952. A quantitative description of membrane current and its application to conduction and excitation in nerve. *J Physiol*, 117(4):500-544. https://doi.org/10.1113/jphysiol.1952.sp004764

Itoh M, Chua LO, 2008. Memristor oscillators. *Int J Bifurc Chaos*, 18(11):3183-3206. https://doi.org/10.1142/S0218127408022354

Izhikevich EM, 2004. Which model to use for cortical spiking neurons? *IEEE Trans Neur Netw*, 15(5):1063-1070. https://doi.org/10.1109/TNN.2004.832719

Jenson VG, Jeffreys GV, 1977. Mathematical Methods in Chemical Engineering. Elsevier, Amsterdam, the Netherlands.

Kengne J, Negou AN, Tchiotsop D, 2017. Antimonotonicity, chaos and multiple attractors in a novel autonomous memristor-based jerk circuit. *Nonl Dynam*, 88(4):2589-2608. https://doi.org/10.1007/s11071-017-3397-1

Li CL, Zhang J, 2016. Synchronisation of a fractional-order chaotic system using finite-time input-to-state stability. *Int J Syst Sci*, 47(10):2440-2448. https://doi.org/10.1080/00207721.2014.998741

Lv M, Wang CN, Ren GD, et al., 2016. Model of electrical activity in a neuron under magnetic flow effect. *Nonl Dynam*, 85(3):1479-1490. https://doi.org/10.1007/s11071-016-2773-6

Lv M, Ma J, Yao YG, et al., 2018. Synchronization and wave propagation in neuronal network under field coupling. *Sci China Technol Sci*, in press. https://doi.org/10.1007/s11431-018-9268-2

Ma J, Wang CN, Tang J, et al., 2010. Eliminate spiral wave in excitable media by using a new feasible scheme. *Commun Nonl Sci Numer Simul*, 15(7):1768-1776. https://doi.org/10.1016/j.cnsns.2009.07.013

Ma J, Wu FQ, Wang CN, 2016. Synchronization behaviors of coupled neurons under electromagnetic radiation. *Int J Mod Phys B*, 31(2):1650251. https://doi.org/10.1142/S0217979216502519

Ma J, Wang Y, Wang CN, et al., 2017a. Mode selection in electrical activities of myocardial cell exposed to electromagnetic radiation. *Chaos Sol Fract*, 99:219-225. https://doi.org/10.1016/j.chaos.2017.04.016

Ma J, Wu FQ, Ren GD, et al., 2017b. A class of initials-dependent dynamical systems. *Appl Math Comput*, 298:65-76. https://doi.org/10.1016/j.amc.2016.11.004

Ma J, Wu FQ, Alsaedi A, et al., 2018. Crack synchronization of chaotic circuits under field coupling. *Nonl Dynam*, 93(4):2057-2069. https://doi.org/10.1007/s11071-018-4307-x

McSharry PE, Smith LA, Tarassenko L, 2003. Prediction of epileptic seizures: are nonlinear methods relevant? *Nat Med*, 9(3):241-242. https://doi.org/10.1038/nm0303-241

Nagumo J, Arimoto S, Yoshizawa S, 1962. An active pulse transmission line simulating nerve axon. *Proc IRE*, 50(10):2061-2070. https://doi.org/10.1109/JRPROC.1962.288235

Panahi S, Aram Z, Jafari S, et al., 2017. Modeling of epilepsy based on chaotic artificial neural network. *Chaos Sol Fract*, 105:150-156. https://doi.org/10.1016/j.chaos.2017.10.028

Perc M, Rupnik M, Gosak M, et al., 2009. Prevalence of stochasticity in experimentally observed responses of pancreatic acinar cells to acetylcholine. *Chaos*, 19(3):037113. https://doi.org/10.1063/1.3160017

Pham VT, Kingni ST, Volos C, et al., 2017. A simple three-dimensional fractional-order chaotic system without equilibrium: dynamics, circuitry implementation, chaos control and synchronization. *AEU Int J Electron Commun*, 78:220-227. https://doi.org/10.1016/j.aeue.2017.04.012

Pham VT, Volos C, Jafari S, et al., 2018. A novel cubic-equilibrium chaotic system with coexisting hidden attractors: analysis, and circuit implementation. *J Circ Syst Comput*, 27(4):1850066. https://doi.org/10.1142/S0218126618500664

Qian Y, Liu F, Yang KL, et al., 2017. Spatiotemporal dynamics in excitable homogeneous random networks composed of periodically self-sustained oscillation. *Sci Rep*, 7(1):11885.
https://doi.org/10.1038/s41598-017-12333-3

Ren GD, Zhou P, Ma J, et al., 2017. Dynamical response of electrical activities in digital neuron circuit driven by autapse. *Int J Bifurc Chaos*, 27:1750187.
https://doi.org/10.1142/S0218127417501875

Rostami Z, Pham VT, Jafari S, et al., 2018. Taking control of initiated propagating wave in a neuronal network using magnetic radiation. *Appl Math Comput*, 338:141-151.
https://doi.org/10.1016/j.amc.2018.06.004

Schmidt C, Grant P, Lowery M, et al., 2013. Influence of uncertainties in the material properties of brain tissue on the probabilistic volume of tissue activated. *IEEE Trans Biomed Eng*, 60(5):1378-1387.
https://doi.org/10.1109/TBME.2012.2235835

Shah DK, Chaurasiya RB, Vyawahare VA, et al., 2017. FPGA implementation of fractional-order chaotic systems. *AEU Int J Electron Commun*, 78:245-257.
https://doi.org/10.1016/j.aeue.2017.05.005

Stamova I, Stamov G, 2017. Functional and Impulsive Differential Equations of Fractional Order: Qualitative Analysis and Applications. CRC Press, Boca Raton, USA.

Sun H, Abdelwahab A, Onaral B, 1984. Linear approximation of transfer function with a pole of fractional power. *IEEE Trans Autom Contr*, 29(5):441-444.
https://doi.org/10.1109/TAC.1984.1103551

Vastarouchas C, Psychalinos C, Elwakil AS, 2018. Fractional-order model of a commercial ear simulator. IEEE Int Symp on in Circuits and Systems, p.1-4.
https://doi.org/10.1109/ISCAS.2018.8351400

Wang CN, Lv M, Alsaedi A, et al., 2017. Synchronization stability and pattern selection in a memristive neuronal network. *Chaos*, 27(11):113108.
https://doi.org/10.1063/1.5004234

Wu FQ, Wang CN, Xu Y, et al., 2016. Model of electrical activity in cardiac tissue under electromagnetic induction. *Sci Rep*, 6:28.
https://doi.org/10.1038/s41598-016-0031-2

Wu FQ, Wang Y, Ma J, et al., 2018. Multi-channels coupling-induced pattern transition in a tri-layer neuronal network. *Phys A*, 493:54-68.
https://doi.org/10.1016/j.physa.2017.10.041

Wu HG, Bao BC, Liu Z, et al., 2016. Chaotic and periodic bursting phenomena in a memristive Wien-bridge oscillator. *Nonl Dynam*, 83(1-2):893-903.
https://doi.org/10.1007/s11071-015-2375-8

Xu Y, Jia Y, Ma J, et al., 2018. Collective responses in electrical activities of neurons under field coupling. *Sci Rep*, 8(1):1349.
https://doi.org/10.1038/s41598-018-19858-1

Yang SM, Wei XL, Deng B, et al., 2018. Efficient digital implementation of a conductance-based globus pallidus neuron and the dynamics analysis. *Phys A*, 494:484-502.
https://doi.org/10.1016/j.physa.2017.11.155

Zambrano-Serrano E, Muñoz-Pacheco J, Campos-Cantón E, 2017. Chaos generation in fractional-order switched systems and its digital implementation. *AEU Int J Electron Commun*, 79:43-52.
https://doi.org/10.1016/j.aeue.2017.05.032