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Design of an eco-gearshift control strategy under a logic system framework*

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Abstract: Good access to traffic information provides enormous potential for automotive powertrain control. We propose a logical control approach for the gearshift strategy, aimed at improving the fuel efficiency of vehicles. The driver power demand in a specific position usually exhibits stochastic features and can be statistically analyzed in accordance with historical driving data and instant traffic conditions; therefore, it offers opportunities for the design of a gearshift control scheme. Due to the discrete characteristics of a gearshift, the control design of the gearshift strategy can be formulated under a logic system framework. To this end, vehicle dynamics are discretized with several logic states, and then modeled as a logic system with the Markov process model. The fuel optimization problem is constructed as a receding-horizon optimal control problem under the logic system framework, and a dynamic programming algorithm with algebraic operations is applied to determine the optimal strategy online. Simulation results demonstrate that the proposed control design has better potential for fuel efficiency improvement than the conventional method.

Key words: Stochastic logic system; Gearshift strategy; Receding-horizon optimization; Traffic information; Eco-driving
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1 Introduction

Development of information technology brings great opportunities and challenges for transportation and automotive control. Control developments for intelligent transportation systems (ITSs) and connected automated vehicles are becoming hot topics. This means that research interest on fuel-saving technologies for road vehicles integrates with traffic information. There is much related literature on traffic-based vehicle control. Kang and Shen (2013) proposed a torque demand strategy for improvement of fuel efficiency of road vehicles in accordance with the traffic flow speed, and the torque demand was determined by traffic status. In Kamal et al. (2009) and Yu et al. (2015), an eco-driving system based on a model predictive controller was designed, where the phase of the traffic signal and the motion of surrounding vehicles were also considered. To improve the fuel efficiency of plug-in hybrid electric vehicles (HEVs), Chen ZY et al. (2017) designed an energy management approach by online prediction of the future driving cycle, and 9.7% of fuel savings could be obtained. Similarly, Zhang YJ et al. (2017) proposed an optimal energy management strategy by analyzing the drivers' behaviors and real-time traffic

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information. Moreover, by analyzing the historical driving speed, an energy distribution algorithm has been designed in Bender et al. (2013) for hybrid hydraulic vehicles to improve fuel efficiency.

However, in the above-mentioned studies, the traffic information was applied mainly to optimize the vehicle speed and energy management of the HEV. Little research has focused on the gearshift control strategy based on traffic information. In fact, gas pedal and gearshift are two basic driver inputs influencing fuel efficiency. In contrast to the stochastic characteristics of the driver's gas pedal, gearshift is an optimizable variable. Yin et al. (2016) investigated the gearshift strategy during the acceleration process by choosing different performance criteria such as power and fuel consumption, and the simulation results indicated that the gearshift could observably influence the fuel consumption and drivability. Ngo V et al. (2012) designed an optimal gearshift control for an HEV using a combination of dynamic programming (DP) and Pontryagin's minimum principle methods. The fuel saving effect was up to 20.3% under the new European drive cycle simulation test. Xu et al. (2018) reviewed the state of the art of automotive transmission technology, and pointed out that the rule-based gearshift strategy was used mainly in road vehicles because of its simplicity and effectiveness. Considering the uncertainty of traffic conditions and driver intentions, real-time optimization for gearshift based on traffic information is still a challenging issue.

Recently, the logic system has become a rapidly developing direction in control and game theory (Guo et al., 2019; Le et al., 2019; Liu Y et al., 2020). Using semi-tensor product (STP) and algebraic state space representation, a systematic theory of the logic system has been established. It consists of (1) topological structure of logical dynamic systems, (2) state space structure of logic control systems (Cheng and Qi, 2010; Yan et al., 2019), (3) controllability and observability (Cheng, 2005; Li FF et al., 2011; Zhang KZ and Zhang, 2014; Lu et al., 2016; Zhu et al., 2019), (4) control design (Liu Y et al., 2016; Li YY et al., 2019; Tong et al., 2019), (5) system identification, (6) stability and stabilization (Li HT et al., 2014; Chen SQ et al., 2019; Li HT and Ding, 2019; Liu JY et al., 2019; Wang et al., 2019; Huang et al., 2020; Zhong et al., 2020), (7) decomposition, including disturbance decoupling, state space decoupling, input-output decoupling, Kalman decomposition (Liu Y et al., 2017; Li YF and Zhu, 2019), etc.

Almost all the major control problems for logical systems have been discussed and solved. Here we summarize some classical results about the topological structure, controllability, and observability of logical dynamical systems. For instance, finite horizon optimization of general logical systems has been discussed by Cheng et al. (2015) and Wu and Shen (2015). Optimization of Boolean networks with the performance criterion was discussed in Fornasini and Valcher (2014), while the same problem for general logical control systems was addressed and solved in Zhao et al. (2011) and Wu et al. (2019). Some new techniques have been developed to solve the optimal control problems for Boolean networks and stochastic logical systems (Zhu et al., 2018; Toyoda and Wu, 2019).

Despite the booming development of the logic system theory, successful applications in engineering practice are still seldom, especially for the automotive control problem. Wu et al. (2016) developed a multi-valued logic-based optimal control strategy, and then successfully applied it to the optimization of combustion engines (Wu and Shen, 2017). Zhang JY and Wu (2018) used a logic control method to design an energy management strategy for hybrid electric vehicles. Kang et al. (2017) designed a gearshift control strategy using the logic control concept, and formulated a finite horizon optimization problem over the whole driving route. The method showed an improvement potential for fuel efficiency, but it is not easy to put into practice because traffic conditions are constantly changing.

In this study, an optimal gearshift controller is proposed for improvement of vehicle fuel efficiency on the basis of logic system theory. The major advantages of applying the logic system framework to gearshift control can be defined in three aspects. First, discrete gearshift control can be easily interpreted as a logic variable. Second, the uncertainty due to stochastic traffic may be formulated by a Markov process model and the optimization problem can be designed under a logic system framework. Third, the computational burden of optimization under the logic system framework can be reduced significantly by applying matrix algebraic operations of a DP algorithm. The main contributions can be summarized as follows:

1. By analyzing the stochastic features of the driving behaviors and applying logic quantification technology, vehicle dynamics can be modeled as a Markov process model, which enables us to design the optimization problem under a logic system framework and then reduce the computational burden of the optimization process.

2. Stochastic traffic information is applied to the control design, and with real-time update of the statistical probability, the receding-horizon optimization for the gearshift control law can be solved online.

3. A trade-off optimization criterion between the fuel efficiency and drivability is chosen to avoid traction loss. The simulation validation is implemented on professional vehicle-traffic software to guarantee requisite precision.

2 Control problem description

2.1 Analysis of historical driving data

We focus on the given vehicles such as public buses and logistics trucks, which are daily driving on a fixed commuter route. Historical driving data for a typical commuting vehicle are illustrated in Fig. 1, containing fortnight driving data. Some distinct features for such vehicles are listed below:

1. There are fixed stops along the commuting route, and between adjacent stops the velocity profiles demonstrate similar trends with respect to the driving position.

2. Considering the uncertainty of the traffic environment and driver behaviors, the driving status exhibits stochastic features. However, in different road sections the driver's gas pedal has obvious statistical characteristics.

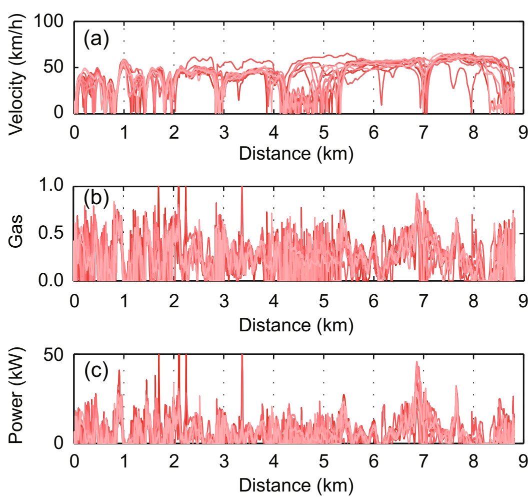

Fig. 1 Sample features of a commuting vehicle: (a) velocity; (b) gas; (c) power

In fact, the driver's gas pedal exactly reflects the power demand in a specific route section, and the engine power will further influence the vehicle dynamics and the fuel consumption performance. Moreover, because the gearshift and gas pedal are the only two external control inputs for most of vehicle powertrain systems and the gearshift strategy is always designed with power demand and vehicle instant velocity, if the gas pedal information (at least the probability distribution) in a future route section is predictable, a possible optimization strategy for gearshift can be developed to improve fuel efficiency.

2.2 Control problem

The objective of this study is to develop an optimal gearshift strategy that minimizes the fuel consumption over a fixed driving route in terms of driver-vehicle-traffic information. Note that, subject to the uncertainty of traffic information, the optimization problem is formulated in the statistical sense.

Given such a purpose, the traffic information and historical driving data are supposed to be stored and obtained via the vehicle-to-everything (V2X) network; namely, the daily driving status of all commuting vehicles at the same route can be collected from cloud storage of the ITS. Meanwhile, the cloud computing platform can analyze the driving features and communicate the updated information to the commuting vehicles. By means of historical data and real-time traffic information, the commuting vehicles can predict the probability distribution of future power demand (i.e., driver gas pedal). To reduce the computational burden, a finite predictive horizon is chosen. At each control step, the optimization for fuel efficiency over such a predictive horizon is implemented to find an optimal gearshift strategy, and then it is advised to the driver taking gearshift control to realize eco-driving. The concept of the receding-horizon optimization problem is illustrated in Fig. 2.

Based on the above concept, we assume that gas pedal at a given route section follows a probability distribution, i.e., $\phi \sim P_{\mathrm{D}}(k)$, and $P_{\mathrm{D}}(k)$ is updated at each control sampling based on the historical data and instant traffic information. The fuel consumption model $\dot{m}_\mathrm{f}(\cdot)$ is associated with the vehicle velocity $v(k)$, gear ratio $i_\mathrm{g}(k)$, and driver gas pedal $\phi(k)$. The mathematical expression for this stochastic optimization problem can be formulated as follows:

$$\min_{\{i_\mathrm{g}^*(i)\}_{i=k}^{k+N}} \mathop{E}_{\phi(i)\sim P_\mathrm{D}(k)} \left\{ \sum_{i=k}^{k+N} \dot{m}_\mathrm{f}\left(v(i),\ i_\mathrm{g}(i),\ \phi(i)\right) \right\} \tag{1}$$

subject to the vehicle motion dynamics, where N denotes the number of prediction steps. To solve the above optimization problem efficiently and in view of the discrete property of the gearshift (there are finite gear ratios in most vehicles that are equipped with a stepped gearbox), a novel design method for gearshift optimization is proposed by means of the logic system framework.

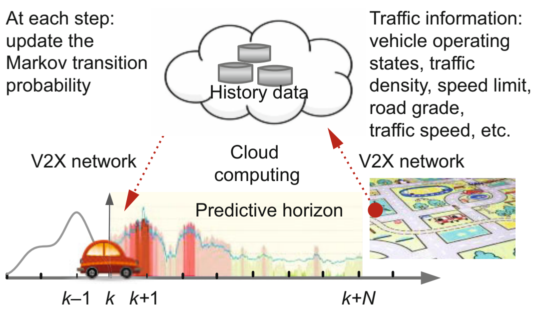

Fig. 2 Receding-horizon optimization for automotive powertrain control based on traffic data

3 Modeling

A typical longitudinal dynamics of the vehicle is given by

$$Jm\frac{\mathrm{d}v}{\mathrm{d}t} = F_\mathrm{d}(\tau_\mathrm{e}, i_\mathrm{g}) - F_\mathrm{a}(v) - F_\mathrm{i}(\theta) - F_\mathrm{r}(\theta), \tag{2}$$

where v is the vehicle speed, J is the inertia of all rotating parts, and m is the vehicle mass. The driving force on the tires $F_\mathrm{d}(\cdot)$ can be expressed by

$$F_\mathrm{d}(\tau_\mathrm{e}, i_\mathrm{g}) = \frac{\tau_\mathrm{e} i_\mathrm{g} i_0 \eta}{R}, \tag{3}$$

where τ_e denotes the engine torque output, i_g is the gear ratio of the selected gear, i_0 is the final ratio, η is the transmission efficiency, and R is the radius of the driving tires. The resistance working on the vehicle includes air resistance $F_\mathrm{a}(\cdot)$, gradient resistance $F_\mathrm{i}(\cdot)$, and rolling resistance $F_\mathrm{r}(\cdot)$, and their detailed expressions are given by

$$\begin{cases} F_\mathrm{a}(v) = \dfrac{1}{2}\rho C_\mathrm{d} A v^2, \\ F_\mathrm{i}(\theta) = mg\sin\theta, \\ F_\mathrm{r}(\theta) = mgf\cos\theta, \end{cases}$$

where ρ is the air density, C_d is the air resistance coefficient, A is the frontal area of the vehicle, g is the gravity coefficient, f is the rolling resistance coefficient, and θ denotes the road grade.

Furthermore, engine torque output is usually regarded as a polynomial function associated with engine speed n_e and gas pedal position ϕ. The engine speed can be derived from the vehicle speed and gear ratio ($n_\mathrm{e} = K i_0 i_\mathrm{g} v/R$); therefore, the engine torque can be written as

$$\tau_\mathrm{e} = \mathrm{map}_1(v, i_\mathrm{g}, \phi). \tag{4}$$

The fuel consumption rate $\dot{m}_\mathrm{f}$ relies mainly on the engine operation condition, and the fuel consumption model can be represented by

$$\dot{m}_\mathrm{f} = \mathrm{map}_2(v, i_\mathrm{g}, \phi). \tag{5}$$

Eqs. (4) and (5) are usually calibrated by the steady-state experimental data of the engine.

In this study, we focus mainly on the commuting vehicle on a fixed driving route, and the vehicle dynamics with respect to the driving distance is more helpful for controller design. To this end, the above time-based model (2) has to be converted into a spatial-domain model by applying the following transformations (Ivarsson, 2009):

$$\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}v}{\mathrm{d}l}\frac{\mathrm{d}l}{\mathrm{d}t} = v\frac{\mathrm{d}v}{\mathrm{d}l}.$$

Then the vehicle dynamic model becomes

$$Jmv\frac{\mathrm{d}v}{\mathrm{d}l} = F_\mathrm{d}(\tau_\mathrm{e}, i_\mathrm{g}) - F_\mathrm{a}(v) - F_\mathrm{i}(\theta) - F_\mathrm{r}(\theta). \tag{6}$$

For simplicity, we define $F_\Sigma(v, \phi, i_\mathrm{g})$ to replace the right side of Eq. (6), and discretize Eq. (6) with a small distance interval using the forward-difference method. Then we have

$$v(k+1) = v(k) + \frac{l}{Jmv(k)} F_\Sigma\left(v(k), \phi(k), i_\mathrm{g}(k)\right), \tag{7}$$

where l denotes the discrete distance interval.

It should be pointed out that dynamic model (7) is a stochastic model containing one random variable (gas pedal position ϕ), one control input (gear ratio i_{g}), and one state (vehicle speed v). As aforementioned, the probability distribution of the gas pedal status can be statistically calculated for the commuting vehicle with its driving distance, denoted as

$$\phi \sim P_{\mathrm{D}}(k). \tag{8}$$

Up to this point, the longitudinal dynamics of the commuting vehicle has been derived. To design the gearshift controller under the logic system framework, the vehicle dynamics (Eq. (7)) should be converted to the logic system with finite logic states and logic control. Denote V as the reachable range of vehicle speed and divide the reachable speed range into finite disjoint intervals, e.g., $\{S^1, S^2, \cdots, S^m\}$. In addition, the following conditions are satisfied:

$$\begin{cases} S^i \cap S^j = \varnothing, \quad i \neq j,\ i,j = 1,2,\ldots,m, \\ \bigcup_{i=1}^{m} S^i \equiv V. \end{cases} \tag{9}$$

Then the vehicle speed can be quantified as the discrete state, and further transformed to the logic state, i.e.,

$$v(k) \in S^i \ \rightarrow\ \boldsymbol{x}_k = \boldsymbol{\delta}_v^i,\ i = 1,2,\ldots,m, \tag{10}$$

where $\boldsymbol{\delta}_v^i$ denotes the i^{th} column of the identity matrix $\boldsymbol{I}_{m\times m}$. The above relationship implies that a sole corresponding logic state $\boldsymbol{x}_k$ can be found for any given value of vehicle speed $v(k)$.

Moreover, for a vehicle equipped with a stepped gearbox, only finite gear ratios $\{i_{\mathrm{g}}^1, i_{\mathrm{g}}^2, \ldots, i_{\mathrm{g}}^n\}$ can be chosen in the driving process; therefore, the gear ratio can be naturally represented as the logic variable:

$$i_{\mathrm{g}}(k) = i_{\mathrm{g}}^r \ \leftrightarrow\ \boldsymbol{u}_k = \boldsymbol{\delta}_u^r,\ r = 1,2,\ldots,n, \tag{11}$$

where $\boldsymbol{\delta}_u^r$ denotes the r^{th} column of the identity matrix $\boldsymbol{I}_{n\times n}$.

Based on the definitions of Eqs. (10) and (11), a logical dynamic system can be derived, i.e., $\boldsymbol{x}_{k+1} = f(\boldsymbol{x}_k, \boldsymbol{u}_k, \phi_k)$. However, it is not easy to directly obtain the exact evolutionary relationship of the logic state from Eq. (7). Alternately, a Markov process approach with transition probability can be adopted to describe the logic system dynamics. Denote $p_{ij}(\boldsymbol{\delta}_u^r)$ as the transition probability from the i^{th} logic state to the j^{th} logic state under a given gear position. It can be calculated by

$$p_{ij}(\boldsymbol{\delta}_u^r) = \sum P_{\mathrm{D}}(\phi) P\left(\boldsymbol{x}_{k+1} = \boldsymbol{\delta}_v^j | \boldsymbol{x}_k = \boldsymbol{\delta}_v^i, \boldsymbol{u}_k = \boldsymbol{\delta}_u^r, \phi\right), \tag{12}$$

where $P_{\mathrm{D}}(\phi)$ is the occurrence probability of the driver gas pedal ϕ statistically calculated based on historical data at a given distance section. Here, subscript "D" denotes a distance section for statistical calculation. Then the conditional probability in Eq. (12) is given by

$$\begin{aligned} &P\left(\boldsymbol{x}_{k+1} = \boldsymbol{\delta}_v^j | \boldsymbol{x}_k = \boldsymbol{\delta}_v^i, \boldsymbol{u}_k = \boldsymbol{\delta}_u^r, \phi\right) \\ &\quad = P\left(v(k+1) \in S^j | v(k) \in S^i, i_{\mathrm{g}}(k) = i_{\mathrm{g}}^r, \phi\right) \\ &\quad = \frac{P\left(v(k+1) \in S^j, v(k) \in S^i | i_{\mathrm{g}}(k) = i_{\mathrm{g}}^r, \phi\right)}{P\left(v(k) \in S^i | \boldsymbol{u} = \boldsymbol{\delta}_u^r, \phi\right)}. \end{aligned} \tag{13}$$

In fact, Eq. (13) can be easily calculated by Eq. (7). Hence, the logical dynamic system can be finally formulated by the Markov process model as Eq. (12). In addition, Eqs. (4) and (5) can be interpreted with the logic state and logic control with the following relationships:

$$\bar{\tau}_{\mathrm{e}}(k) = \mathrm{map}_1\left(\bar{v}(\boldsymbol{x}_k), i_{\mathrm{g}}(\boldsymbol{u}_k), \phi(k)\right), \tag{14}$$

$$\dot{\bar{m}}_{\mathrm{f}}(k) = \mathrm{map}_2\left(\bar{v}(\boldsymbol{x}_k), i_{\mathrm{g}}(\boldsymbol{u}_k), \phi(k)\right), \tag{15}$$

where $\bar{v}(\boldsymbol{x}_k)$ denotes the mean value of the vehicle speed interval corresponding to logic state $\boldsymbol{x}_k$ and $i_{\mathrm{g}}(\boldsymbol{u}_k)$ represents the gear ratio corresponding to logic control $\boldsymbol{u}_k$.

4 Controller design

4.1 Optimization criterion

As mentioned in Section 2, the objective of this study is to optimize the fuel efficiency of the commuting vehicle by choosing an appropriate gear position. Based on the logic system framework, the original optimization problem (1) can be reformulated as follows. For any given initial state $\boldsymbol{x}_k$, find an optimal policy π^* by minimizing the following cost:

$$J_\pi(x_0) = \underset{\phi_k \sim P_{\mathrm{D}}(k)}{E} \left\{ \sum_{k}^{k+N} g(\boldsymbol{x}_k, \boldsymbol{u}_k) \right\} \tag{16}$$

subject to Eq. (12), where $g(\boldsymbol{x}_k, \boldsymbol{u}_k)$ is the per-step cost of indicating the fuel consumption.

Since gearshift control not only influences the fuel consumption but also changes the driving force, a suitable trade-off between fuel efficiency and drivability should be considered. In general, the upshift gear leads the engine to the highly efficient working area, but it also reduces the driving force on the tires owing to the lower gear ratio. Then the per-step cost will be designed as follows:

$$g(\boldsymbol{x}_k,\boldsymbol{u}_k)=\alpha\dot{\bar{m}}_{\rm f}(k)\frac{l}{\bar{v}(\boldsymbol{x}_k)}+\frac{\beta}{(\tau_{\rm e,max}-\bar{\tau}_{\rm e}(k))\,\bar{v}(\boldsymbol{x}_k)i_{\rm g}(\boldsymbol{u}_k)}. \quad (17)$$

In the right side of Eq. (17), the first item denotes the fuel consumption amount and the second item implies the drivability that is represented by the reciprocal of the driving force on the tires. It should be pointed out that the denominator of the second item in Eq. (17) reflects the power reserve (Ngo DV, 2012). The higher power reserve means better acceleration capability. Parameters α and β are the weighting factors for fuel consumption and drivability indices, respectively. $l/\bar{v}(\boldsymbol{x}_k)$ denotes the average passing time through each distance interval, and $\tau_{\rm e,max}$ is the maximum engine torque which is treated as a constant in this study.

At each control sampling, the probability distribution of the gas pedal $P_{\rm D}(k)$ is updated based on historical data and instant traffic information. Then the transition probability $p_{ij}(\boldsymbol{\delta}_u^r)$ is renewed accordingly. By solving the optimization problem (16), the optimal policy π^* can be obtained in real time. Finally, the optimal gear ratio at step k will be deduced by

$$\boldsymbol{u}_k^*=\mu_k^*(\boldsymbol{x}_k),\ \mu_k^*\in\pi^*. \quad (18)$$

4.2 Dynamic programming algorithm

To solve the above receding-horizon optimization problem, the DP algorithm is adopted. Owing to the logic system framework, the DP algorithm can be expressed using the algebraic operation method (Wu and Shen, 2015). The STP and increment dimensional method are adopted in the DP algorithm to reduce the computational burden. In particular, the calculation procedure can be summarized as follows:

Step 1: Set the number of predictive steps N, and set the terminal cost function $\boldsymbol{\Phi}$ as a zero vector $\boldsymbol{\Phi}=\mathbf{0}_{m\times 1}$. At step k, update the probability of gas pedal $P_{\rm D}(k)$.

Step 2: Calculate the transition probability $p_{ij}(\boldsymbol{\delta}_u^r)$ based on Eq. (12) and construct the transition probability matrix:

$$\mathbb{P}=\left((\boldsymbol{P}(\boldsymbol{\delta}_u^1))^{\rm T},(\boldsymbol{P}(\boldsymbol{\delta}_u^2))^{\rm T},\ldots,(\boldsymbol{P}(\boldsymbol{\delta}_u^n))^{\rm T}\right)^{\rm T}, \quad (19)$$

where $\boldsymbol{P}(\boldsymbol{\delta}_u^r)$ is the transition probability matrix $(m\times m)$ under fixed logic control $(r=1,2,...,n)$:

$$\boldsymbol{P}(\boldsymbol{\delta}_u^r)=\begin{bmatrix} p_{11}(\boldsymbol{\delta}_u^r) & p_{12}(\boldsymbol{\delta}_u^r) & \ldots & p_{1m}(\boldsymbol{\delta}_u^r)\\ p_{21}(\boldsymbol{\delta}_u^r) & p_{22}(\boldsymbol{\delta}_u^r) & \ldots & p_{2m}(\boldsymbol{\delta}_u^r)\\ \vdots & \vdots & & \vdots\\ p_{m1}(\boldsymbol{\delta}_u^r) & p_{m2}(\boldsymbol{\delta}_u^r) & \ldots & p_{mm}(\boldsymbol{\delta}_u^r)\end{bmatrix}. \quad (20)$$

Step 3: Calculate the per-step cost matrix $\boldsymbol{G}$ $(m\times n)$. Actually, the per-step cost function $g(\cdot)$ can be expressed in the form

$$g(\boldsymbol{x},\boldsymbol{u})=\boldsymbol{xGu},$$

with

$$\boldsymbol{G}=\begin{bmatrix} g(\boldsymbol{\delta}_v^1,\boldsymbol{\delta}_u^1) & g(\boldsymbol{\delta}_v^1,\boldsymbol{\delta}_u^2) & \ldots & g(\boldsymbol{\delta}_v^1,\boldsymbol{\delta}_u^n)\\ g(\boldsymbol{\delta}_v^2,\boldsymbol{\delta}_u^1) & g(\boldsymbol{\delta}_v^2,\boldsymbol{\delta}_u^2) & \ldots & g(\boldsymbol{\delta}_v^2,\boldsymbol{\delta}_u^n)\\ \vdots & \vdots & & \vdots\\ g(\boldsymbol{\delta}_v^m,\boldsymbol{\delta}_u^1) & g(\boldsymbol{\delta}_v^m,\boldsymbol{\delta}_u^2) & \ldots & g(\boldsymbol{\delta}_v^m,\boldsymbol{\delta}_u^n)\end{bmatrix}.$$

Step 4: Solve the optimal cost function for the one-horizon problem denoted by $\boldsymbol{T}_\mu$:

$$\boldsymbol{T}_\mu=\begin{bmatrix} \min\limits_{r=1,2,\ldots,n}\left\{\boldsymbol{G}(1,r)+(\boldsymbol{\delta}_v^1)^{\rm T}\ltimes(\boldsymbol{\delta}_u^r)^{\rm T}\mathbb{P}\boldsymbol{\Phi}\right\}\\ \min\limits_{r=1,2,\ldots,n}\left\{\boldsymbol{G}(2,r)+(\boldsymbol{\delta}_v^2)^{\rm T}\ltimes(\boldsymbol{\delta}_u^r)^{\rm T}\mathbb{P}\boldsymbol{\Phi}\right\}\\ \vdots\\ \min\limits_{r=1,2,\ldots,n}\left\{\boldsymbol{G}(m,r)+(\boldsymbol{\delta}_v^m)^{\rm T}\ltimes(\boldsymbol{\delta}_u^r)^{\rm T}\mathbb{P}\boldsymbol{\Phi}\right\}\end{bmatrix} \quad (21)$$

and obtain the corresponding optimal control μ_{N-1-k}^*:

$$\mu_{N-1-k}^*(\boldsymbol{x})=[\boldsymbol{\delta}_u^{q_1},\boldsymbol{\delta}_u^{q_2},\ldots,\boldsymbol{\delta}_u^{q_m}]\boldsymbol{x}, \quad (22)$$

where superscript q_i $(i=1,2,\ldots,m)$ is the optimal index obtained by solving each optimization problem in Eq. (21), and notation "$\ltimes$" denotes the STP operator.

Step 5: Set $k=k+1$, $\boldsymbol{\Phi}_k^*=\boldsymbol{T}_\mu$, and $\boldsymbol{\Phi}=\boldsymbol{T}_\mu$. If $k<N$, go back to step 4; if $k=N$, then $J^*=\boldsymbol{\Phi}_N^*$ and the final optimal control policy at the current control step is

$$\pi^*(k)=\{\mu_k^*,\mu_{k+1}^*,\ldots,\mu_{k+N-1}^*\}. \quad (23)$$

Step 6: Apply the control $\boldsymbol{u}_k^*=\mu_k^*(\boldsymbol{x}_k)$ to the system during step k, and move to the next control step. Return to step 1.

5 Simulation validation

5.1 Simulation configuration

To simulate the vehicle dynamics in a stochastic traffic scenario, the validation was implemented on the professional software CarMaker®. The software provides an accurate driver model that can drive a car in any given stochastic traffic condition. The logic control scheme was programmed in Simulink and then the co-simulation between CarMaker and Simulink was realized via the S-function interface. Fig. 3 shows the structure diagram of the simulation environment.

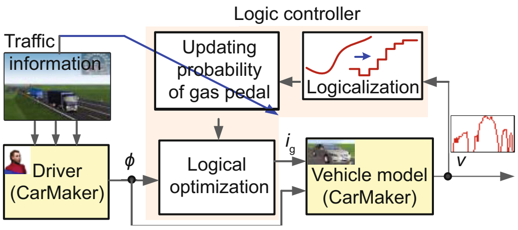

Fig. 3 Structure of the simulation environment

We set up a commuting route including several stops. The traffic density can be assigned to generate the stochastic traffic flow. The vehicle model adopts a passenger car equipped a five-gear automatic transmission, and then the logic control input has five elements, i.e., $\{\boldsymbol{\delta}_u^1, \boldsymbol{\delta}_u^2, \boldsymbol{\delta}_u^3, \boldsymbol{\delta}_u^4, \boldsymbol{\delta}_u^5\}$. According to the definition in Eq. (10), we divided the vehicle speed range into 20 sections with 5 km/h an interval, and the highest speed section was defined as $S^{20} = \{v \geq 95\}$ to ensure that all speeds were covered. Therefore, the vehicle speed was transformed to the logic state with 20 elements, i.e., $\{\boldsymbol{\delta}_v^1, \boldsymbol{\delta}_v^2, \ldots, \boldsymbol{\delta}_v^{20}\}$. In addition, the discrete distance interval (i.e., control sampling) was set as $l = 5$ m.

5.2 Online updating for the Markov transition probability

A crucial parameter for receding-horizon optimization is the statistical probability of the gas pedal $P_{\mathrm{D}}(\phi)$. This probability is calculated by the cloud computing platform in real time once the new information is updated. In this study, we renew the occurrence probability of the gas pedal with the following strategy: collect the gas pedal information of all vehicles passing through the specific predictive horizon $D = [k, k+N]l$, and statistically analyze the occurrence frequency of the certain gas pedal value in this horizon with

$$P_{\mathrm{D}} = \frac{N_i\left(\phi \in [\Delta-\epsilon, \Delta+\epsilon]\right)}{N_i}, \tag{24}$$

where N_i is the total number of statistical samples and $N_i\left(\phi \in [\Delta-\epsilon, \Delta+\epsilon]\right)$ is the count of occurrences that the gas pedal position belongs to $[\Delta-\epsilon, \Delta+\epsilon]$.

In this case, the prediction horizon was set as 100 m (i.e., $N = 20$), and the range of the gas pedal position was discretized in 100 grids at 1% interval percentage. Based on the above updating strategy, a sample representing the variation of the occurrence probability of the gas pedal position is illustrated in Fig. 4.

The Markov transition probability (Eq. (12)) can then be obtained based on the probability of the gas pedal. To better understand the Markov process, an example of transition probability (Eq. (20)) in one predictive horizon is shown in Fig. 5. The changes in color in the figure indicate the transition probability from logic state $\boldsymbol{x}_k$ to $\boldsymbol{x}_{k+1}$.

At each control sampling, an optimal control policy $\pi^*(k) = \{\mu_k^*, \mu_{k+1}^*, \ldots, \mu_{k+N-1}^*\}$ can be obtained. Fig. 6 shows an optimal gearshift control mapping with respect to the vehicle speed within one optimization horizon. The weighting factors for fuel efficiency and drivability indexes are chosen as 1 and 0.1, respectively. Fig. 6 indicates that in a certain road section, the optimal gear number can be found with respect to vehicle speed.

5.3 Discussion of validation results

Essentially, the proposed logic control enables the traffic information to be used in the driving control process. To validate the effectiveness of the proposed controller, we compared the simulation results between the logic and baseline control. The baseline control strategy adopts the conventional gearshift method, which means that the gearshift is a mapping function with respect to the vehicle speed and driver power demand. The baseline control is usually calibrated by numerical driving tests in practice.

In addition, the proposed logic controller just provides an optimal gearshift reference. In a real driving scenario, a supervisory gearshift strategy is pre-set to avoid gearshift shock and ensure driving safety and comfort. The supervisory algorithm controls the final gearshift with the following rules:

First, the gear number command provided by the logic controller must ensure that the engine speed is within the possible range; otherwise, the baseline gearshift strategy is activated. Second, the logic controller releases the control during the braking condition because the vehicle dynamics (Eq. (2)) will be no longer satisfied for deriving the logic control law, and in this case the final gear command will maintain its last value until the baseline control is activated (when the baseline gear command changes). Third, the automatic transmission will always upshift stepwise to the target gear.

The comparison results are plotted in Fig. 7. In the simulation, the vehicle drove from 825 m to 1330 m in the stochastic traffic scenario. Note that the logic controller works only in the

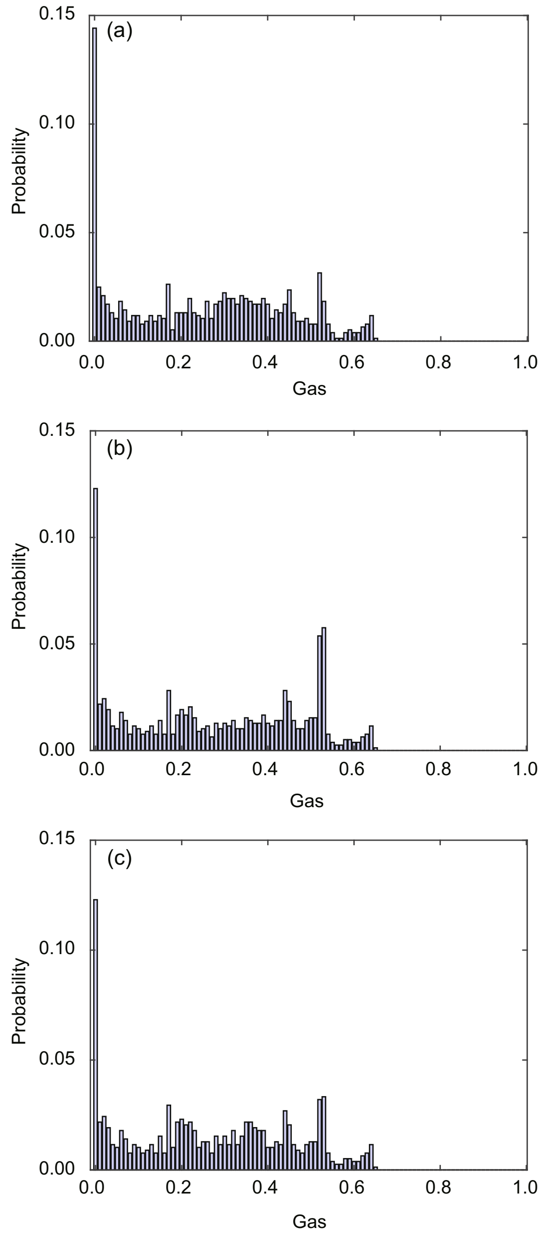

Fig. 4 A variation example of the probability of the gas pedal position in one prediction horizon at the first (a), second (b), and third (c) updates

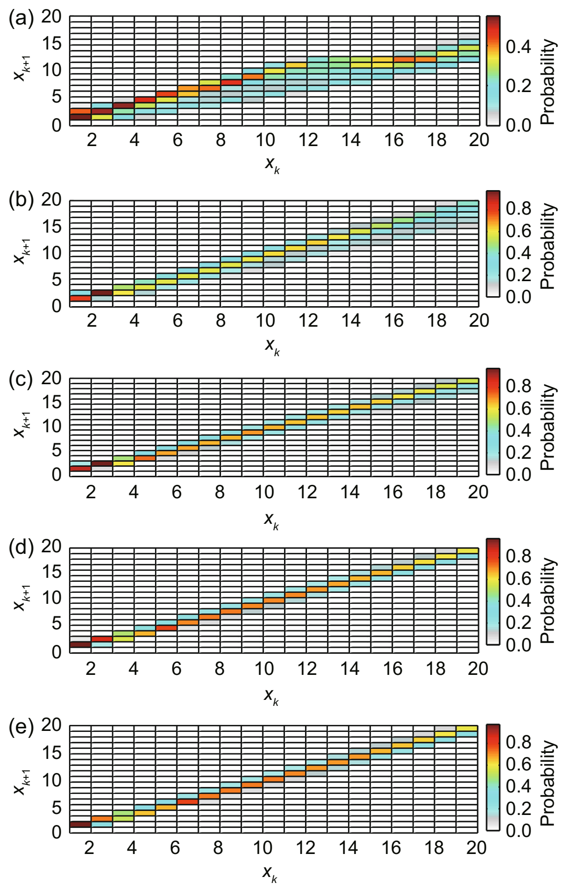

Fig. 5 An example of the state transition probability in one optimization horizon: (a) $u_k = \delta_u^1$ (gear 1); (b) $u_k = \delta_u^2$ (gear 2); (c) $u_k = \delta_u^3$ (gear 3); (d) $u_k = \delta_u^4$ (gear 4); (e) $u_k = \delta_u^5$ (gear 5). References to color refer to the online version of this figure

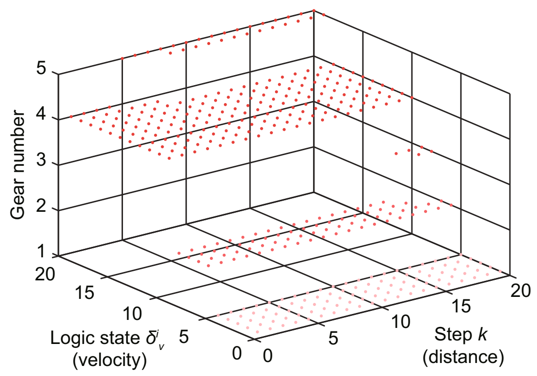

Fig. 6 An example of the optimal control policy in one optimization horizon

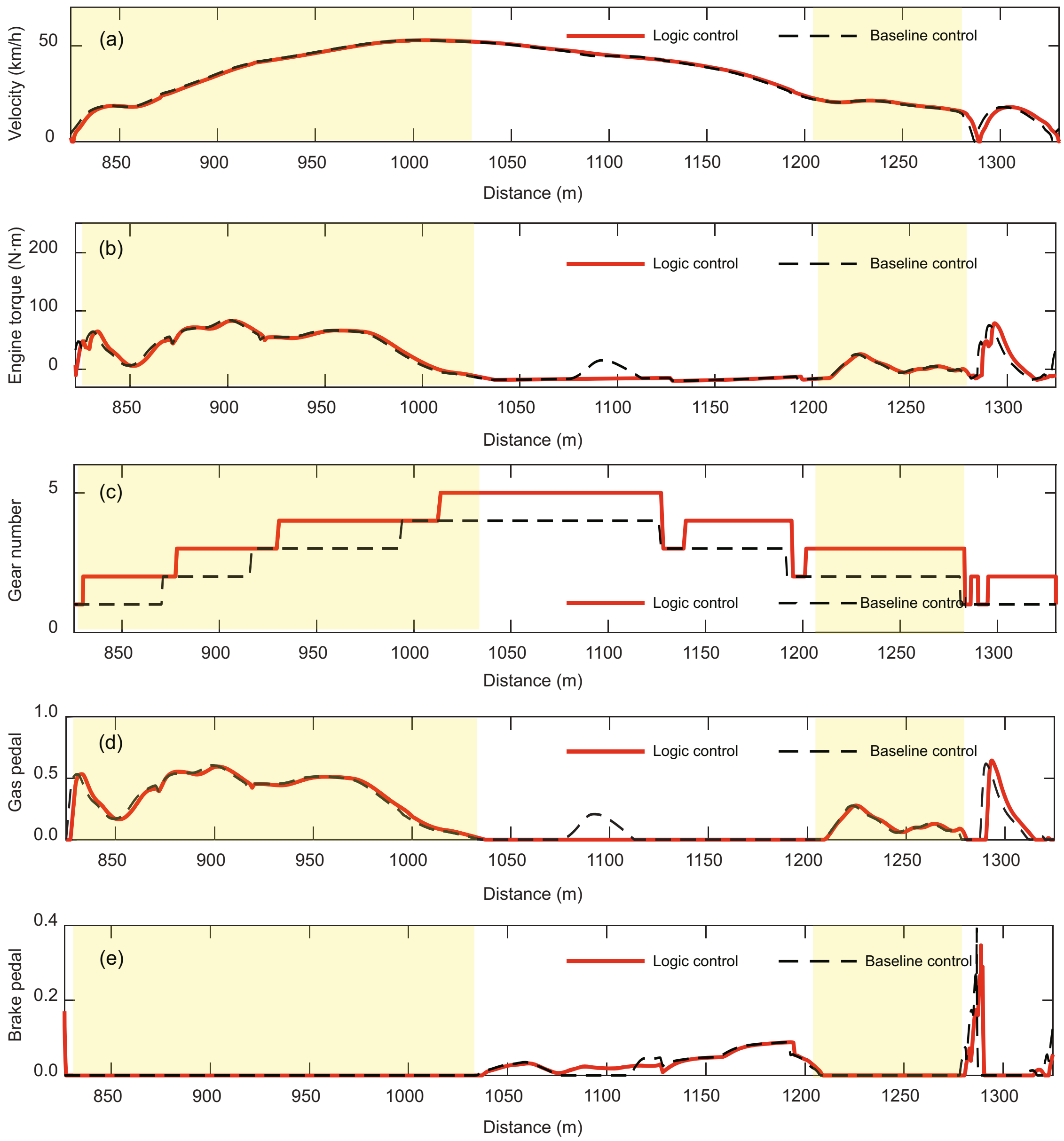

Fig. 7 Simulation performance comparison between logic control and baseline control (colorful areas indicate the acceleration processes where the logic controller is activated): (a) velocity; (b) engine torque; (c) gear number; (d) gas pedal; (e) brake pedal. References to color refer to the online version of this figure

acceleration or cruising process according to the supervisory gearshift strategy. Clearly, given the prediction of future driving conditions, the logic control always exhibited early upshift behavior to lead the engine to the highly efficient work area. As a consequence, logic control must achieve better fuel efficiency than the baseline strategy. The results reflected the effectiveness of the proposed control design.

6 Conclusions

A novel gearshift control approach under a stochastic logic system framework has been proposed in this paper. The control objective is to improve the fuel efficiency of commuting vehicles using historical data and traffic information. The discrete nature of the gearshift causes the logic control design to have more merits. Considering that the probability of the driver power demand at the specific position can be statistically analyzed based on traffic information, the spatial-domain based stochastic dynamic model for vehicle speed can be derived and further transformed under the logic system framework. The receding-horizon optimization algorithm for the gearshift strategy has been developed. The optimization problem can be solved online by means of the algebraic operations of the DP algorithm with lower complexity. The simulation results demonstrated the effectiveness of the proposed control design.

In general, there are two important design means determining the control performance. One is the transformation of the vehicle dynamics from the continuous domain to a discrete logic system framework. To achieve this transformation, the vehicle speed has been discretized into several sections to achieve logicalization, and the Markov transition probability method has been adopted to characterize the dynamic evolutionary process of the logic state. Indeed, different discretization methods will change the Markov model and therefore influence the control performance. We just provided a simple discretization method with equal intervals for vehicle speed to demonstrate the effectiveness of the logicalization method. The other important design is the update of the Markov process model in real time. We just gave a simple updating algorithm using historical driving data. Actually, the probability of the driver gas pedal is associated not only with historical traffic information, but also to the instant traffic conditions. A more sophisticated updating algorithm and discretization method for vehicle speed will be studied in the future work.

Contributors

Ming-xin KANG designed the research. Ming-xin KANG and Jin-wu GAO processed the data. Ming-xin KANG drafted the manuscript. Jin-wu GAO helped organize the manuscript. Ming-xin KANG and Jin-wu GAO revised and finalized the manuscript.

Compliance with ethics guidelines

Ming-xin KANG and Jin-wu GAO declare that they have no conflict of interest.

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