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Mixture test strategy optimization for analog systems*#

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Abstract: Since analog systems play an essential role in modern equipment, test strategy optimization for analog systems has attracted extensive attention in both academia and industry. Although many methods exist for the implementation of effective test strategies, diagnosis for analog systems suffers from the impacts of various stresses due to sophisticated mechanism and variable operational conditions. Consequently, the generated solutions are impractical due to the systems' topology and influence of information redundancy. Additionally, independent tests operating sequentially on the generated strategies may increase the time consumption. To overcome the above weaknesses, we propose a novel approach called heuristic programming (HP) to generate a mixture of test strategies. The experimental results prove that HP and Rollout-HP access the strategy with fewer layers and lower cost consumption than state-of-the-art methods. Both HP and Rollout-HP provide more practical strategies than other methods. Additionally, the cost consumption of the strategy based on HP and Rollout-HP is improved compared with those of other methods because of the updating of the test cost and adaptation of mixture OR nodes. Hence, the proposed HP and Rollout-HP methods have high efficiency.

Key words: Fault diagnosis; Heuristic searching; Dynamic programming; Test optimization
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1 Introduction

With the rapid development of electronic technology, analog modules play significant roles in modern devices, such as spacecraft (Li ZW et al., 2013; Liu G et al., 2017; Suryasarman et al., 2018) and airplanes (Tang et al., 2018; Tian et al., 2018; Liu HC et al., 2019). The maintenance and protection of such equipment entail high costs and time limitations. Test strategy optimization (TSO) provides automated diagnosis procedures, optimizes the detection procedures during maintenance, and shortens the time consumed for location of faults (Mei et al., 2022). Therefore, TSO has received considerable attention from both researchers and engineers.

Since TSO can be regarded as a type of multi-objective optimization (MOO) problem, MOO-based methods, such as non-dominated sorting genetic algorithm-II (NSGA-II) (Wang et al., 2019), multi-particle swarm optimization (MPSO) (Li MC et al., 2021), many-objective evolutionary algorithm (MOEA) (Zhang L et al., 2022), and evolutionary algorithm (EA) (Mandaogade and Ingole, 2020), are used to achieve feasible solutions. However, the TSO problem has been proven to be a non-deterministic

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polynomial-time (NP)-hard problem, and the computational complexity of MOO-based methods grows exponentially with the increase of the size of the target systems (Ojstersek et al., 2020). Moreover, the searching space of the TSO problem is nonlinear and nonconvex with Boolean data structure. Such facts increase the difficulty in reaching optimal solutions with MOO strategies. Optimization problem is divided into numbers of deterministic problems, and existing methods focus primarily on solving single-objective problems (Biswas et al., 2014). As a result, the optimization solution may lead to unsatisfactorily obtained single-objective solutions. Finally, MOO-based methods generate only a list of selected test procedures as the optimized solution and require all the selected tests to be operated to obtain the diagnosis results, which can influence the effectiveness of the testability design.

To achieve efficient diagnosis solutions within an acceptable number of search times, the AND/OR (AO*) algorithm was adopted to achieve trade-off between test performance and search complexity by developing a heuristic search under the AND/OR graph (Pattipati and Alexandridis, 1990; Boumen et al., 2009). Since analog systems contain complicated structures, three improvements have been made to AO* algorithms to meet modern maintenance requirements. One approach is optimizing the heuristic mechanism of the search strategy. Considering modern test procedures and system development, revisions to heuristic evaluations, such as introducing execution costs (Zhang SG et al., 2013) and hierarchical structures (Zhang SG et al., 2015), are used to generate practical strategies for real applications. Such improvements enhance the real-world efficiency of diagnosis for aircraft systems and multimode systems. Additionally, heuristic estimation was integrated with information theory, such as entropy (Sun et al., 2019), which reduces the test cost substantially.

In contrast to developing heuristic estimation and search strategies, the other approach is to simplify two procedures to achieve high search speed and produce satisfactory results in highly complicated situations. To avoid an excessive computational burden, a rollout strategy was applied to provide rapid multistep heuristic searching (Tu and Pattipati, 2003). Rollout policies enhance the generation ability of test strategies. Furthermore, Kundakcioglu and Unluyurt (2007) developed an approach to construct a bottom-up decision tree with better performance than that achieved by constructing AND/OR top-down trees. However, this approach fails to generate a practical strategy with an ambiguous fault set.

Although existing methods have achieved high performance in real-world applications, they assume that all diagnosis decisions are made from a single-signal operation and that the test procedure must be sequential. Nevertheless, modern analog systems are equipped with sophisticated mechanisms and are under variable operational conditions, and have various impacts on stress in operational environments (Hoffmann, 1992; Terry et al., 2004; Roy et al., 2019), tolerance of component parameters (Czaja and Zielonko, 2004; Guo and Savir, 2006; Vallette et al., 2007), and complex circuit mechanisms (Butzen et al., 2010; Shima and Kusaga, 2010; Tsukahara et al., 2015). Hence, diagnosis for such systems requires multiple signals based on the structures of the devices. Thus, existing methods have the following weaknesses:

1. Existing methods provide impractical solutions since multi-signal information is required for many detection methods for fault conditions (Yang et al., 2012).

2. Sequential solutions adopt unnecessary information and waste test time due to test dependence (Vasan et al., 2013).

3. The sequential strategy limits the efficiency of parallel detections. Although a mixture strategy optimization method called mixture AO* (MAO*) method has been developed (Mei et al., 2015), the performance of the generated solution is still limited by information redundancy.

To the best of our knowledge, there are no existing methods to overcome these difficulties. Hence, this study aims to apply the relationships between test procedures to test strategy design to provide a mixture test solution considering modern analog system structures. The contributions of this study are as follows:

1. We design a sequence matrix to depict the relationships of tests and evaluate the dynamic cost of the test based on the system topology. By initializing the sequence matrix, we encode the topology of each test procedure and the dependence of the potential cost of test procedures into the structure information

of target systems. The sequential strategy ensures the test completeness of multi-signal information for the diagnosis procedures, and the adjustment process of the conditional cost enhances the flexibility of the test design based on the dependence of the test cost.

2. A novel heuristic estimator is developed to estimate the cost of potential solutions based on a sequence matrix and dependence information to avoid information redundancy when generating solutions. The proposed method evaluates the heuristic estimation function based on the decision graph and the updated conditional test cost. As a result, the search process not only considers the prior diagnosis results from existing test strategies but also takes the prior test information into account when designing further test strategies. Hence, the generated diagnosis solutions can improve the information usage of the test procedures.

3. Heuristic programming (HP) and Rollout-HP, which combine sequential extension and parallel extension during search tree generation and enhance the diagnostic efficiency for real-world applications, are proposed. HP and Rollout-HP generate both sequential test nodes and parallel test nodes for the test strategy and consider the dependence of different test procedures. Hence, the generated mixture test strategies can save test time consumption and unnecessary test cost compared to the sequential test strategies of classic methods.

Experimental results and real-world applications reveal that the proposed method achieves a lower diagnosis cost and provides final detections within fewer layers than existing solutions. Therefore, this study takes a step toward optimizing diagnosis strategies for modern analog systems under the consideration of multiple information detection and system structures.

2 Problem description

To achieve the most accurate fault location with the minimum cost, there are two essential objects for TSO: the fault state set $S = \{s_1, s_2, \cdots, s_M\}$ and the test point set $T = \{t_1, t_2, \cdots, t_N\}$. Additionally, the fault probability set $P = \{p_1, p_2, \cdots, p_M\}$ and process test cost set $C = \{c_1, c_2, \cdots, c_N\}$ represent the evaluation elements of the solutions. N is the number of test modules and M is the number of fault states. The notations used in this paper are presented in Table A1 in the Appendix.

Most AO*-based approaches (Pattipati and Alexandridis, 1990; Boumen et al., 2009) provide a diagnosis schedule via a generated diagnosis tree, which separates each fault state based on the test performance of the OR nodes. However, the traditional OR node arrangement includes a single test operation during each procedure, causing information redundancy and unnecessary costs for analog system diagnosis. Hence, we introduce multiple OR nodes with parallel test operations in each procedure to generate mixture test strategies. The cost of the root node in the diagnosis tree for the mixture test strategy is calculated as follows:

$$\mathrm{Cost}(S) = \max(c_j | t_j \in \tilde{T}) \sum_{k=1}^{K} \mathrm{Cost}\left(S_k\right), \tag{1}$$

where S_k is the fault state set of the k^{th} AND node and $\tilde{T}$ is the test point set of the corresponding OR nodes.

Similar to the sequential test strategy, the fault isolation rate (FIR) is a significant issue for diagnosis, and is represented as

$$\mathrm{FIR} = \frac{M_{\text{distinguish}}}{M}, \tag{2}$$

where $M_{\text{distinguish}}$ is the number of fault states that can be successfully separated in the solution.

3 The proposed method

3.1 General framework

To solve the TSO problem, the general framework of the proposed method consists of three procedures: abstract model generation, search initialization, and HP.

First, we generate a sequence matrix and a dependence matrix to depict the relationships between tests and fault conditions. Second, the comprehensive cost and corresponding decision graph provide accurate heuristic estimate during the search process. Third, HP is proposed to generate mixture test strategies. Finally, the rollout strategy is adopted for large-scale systems to provide available solutions.

3.2 Abstract model generation

3.2.1 Sequence matrix generation

The sequence matrix, representing the relationships between tests, is generated based on the biograph of the system structures, which is denoted as follows:

$$\begin{aligned} E = \{ & (t_i, s_k) | t_i \text{ is the input of the } k^{\text{th}} \text{ module or} \\ & t_i \text{ is the output of the } k^{\text{th}} \text{ location}\}, \end{aligned} \tag{3}$$

where i is the index of the test procedure.

In the biograph, the weights of the edges represent the directions of the inputs and outputs:

$$w(t_i, s_k) = \begin{cases} -1, & t_i \text{ is the input of } s_k, \\ 1, & t_i \text{ is the output of } s_k. \end{cases} \tag{4}$$

Based on the directions of the tests and fault states, the relationships between tests can be divided into two types: (1) one test is leading another test and one test is behind another test; (2) two tests are independent of each other. Hence, the elements of the sequence matrix $\boldsymbol{L} = [l_{ij}]$ are represented as follows:

$$l_{ij} = \begin{cases} p, & t_i \text{ is } p \text{ steps behind } t_j, \\ q, & t_i \text{ is } q \text{ steps ahead of } t_j, \\ 0, & t_i \text{ and } t_j \text{ have no orders.} \end{cases} \tag{5}$$

According to the biograph of analog systems, $\boldsymbol{L}$ satisfies three properties:

Property 1 For two arbitrary test points t_i and t_j, if there exists a location s_k that satisfies $w(t_i, s_k) = 1$ and $w(t_j, s_k) = -1$, then $l_{ij} = 1$.

Property 2 For three arbitrary test points t_i, t_j, and t_k that satisfy $l_{ik} > 0$ and $l_{kj} > 0$, the distance between t_i and t_j is the sum of the distance between t_i and t_k and the distance between t_k and t_j, which means $l_{ij} = l_{ik} + l_{kj}$.

Property 3 For arbitrary test points t_i and t_j, if $l_{ij} > 0$, then $l_{ji} = -l_{ij}$.

Thus, the sequence matrix can be initialized based on Property 1 and can be extended by Properties 2 and 3.

3.2.2 Dependence matrix generation

Based on the denotation of the dependence matrix $\boldsymbol{D} = [d_{ij}]$ (Pattipati and Alexandridis, 1990), it can be concluded that $d_{ij} = 1$ when arbitrary fault state s_k and test point t_i satisfy $w(t_i, s_k) = 1$. Additionally, for an arbitrary module m_i, if there exist two test points t_j and t_k that satisfy $d_{ij} = 1$ and $l_{kj} > 0$, then $d_{ik} = 1$. Hence, the dependence matrix can also be developed recursively from the previous information.

3.3 Initialization and adjustment of cost

3.3.1 Initialization of cost

Since the cost of adding test points is related to the test points, the process cost evaluation of each test is initialized and adjusted based on the individual test costs and the assigned test sets. To initialize the cost, the individual cost for test point t_i is denoted as $\Delta c(t_i)$, and the process cost of the test point is denoted as $c(t_i)$, which represents the price increase when adding t_i to the test strategy. On the basis of the topology of the system, there are two properties for $c(t_i)$:

Property 4 If point t_i is directly linked to the system inputs, the process cost of t_i is equal to its individual cost.

Property 5 For an arbitrary test point t_i, the cost is the combination of its individual cost and the maximum process cost of the directly related test points:

$$c(t_i) = \Delta c(t_i) + \max(c(t_j) | l_{ij} = 1). \tag{6}$$

3.3.2 Adjustment of cost

Because the test assignments change the cost consumption of potential tests, the process cost is adjusted during the TSO process, which satisfies the following properties:

Property 6 If t_k is assigned to the test strategy, the conditional process cost of t_k becomes zero.

Property 7 When t_k is assigned to the test strategy and t_i is directly linked to t_k, if and only if $\max(c(t_j) | l_{ij} = 1) = c(t_k)$, $j = 1, 2, \cdots, N$, the process cost for t_j is adjusted as follows:

$$\hat{c}(t_i | t_k) = c(t_i | t_k). \tag{7}$$

Property 8 When t_k is assigned to the test strategy, for an arbitrary test point t_i that satisfies $l_{ik} = 0$, the conditional process cost equals its process cost.

Property 9 When t_k is assigned to the test strategy, for an arbitrary test point t_i that satisfies

$l_{ik} > 1$, the conditional cost can be calculated as follows:

$$\hat{c}(t_i|t_k) = \Delta c(t_i) + \max(c(t_j|t_k)|l_{ij} = 1, j \neq k). \quad (8)$$

Property 10 When $\tilde{T} = \{t_{k_1}, t_{k_2}, \cdots, t_{k_m}\}$ is added to the test strategy and t_i is directly linked to t_{k_m}, if and only if $\max\left(c\left(t_j|t_{k_1}, t_{k_2}, \cdots, t_{k_{m-1}}\right)| l_{ij} = 1\right) = c\left(t_{k_m}|t_{k_1}, t_{k_2}, \cdots, t_{k_{m-1}}\right)$, then the conditional process cost for t_i is adjusted as follows:

$$c\left(t_i|\tilde{T}\right) = \Delta c(t_i) + c\left(t_{k_m}|t_{k_1}, t_{k_2}, \cdots, t_{k_{m-1}}\right). \quad (9)$$

Property 11 When $\tilde{T} = \{t_{k_1}, t_{k_2}, \cdots, t_{k_m}\}$ is added to the test strategy, for an arbitrary test point t_i that satisfies $l_{ik_m} < 0$, the conditional process cost equals its conditional process cost with $\{t_{k_1}, t_{k_2}, \cdots, t_{k_{m-1}}\}$.

Property 12 When $\tilde{T} = \{t_{k_1}, t_{k_2}, \cdots, t_{k_m}\}$ is added to the test strategy, for an arbitrary test point t_i that satisfies $l_{ik_m} > 1$, the conditional process cost can be calculated as follows:

$$c\left(t_i|\tilde{T}\right) = \max(\Delta c(t_j|t_{k_1}, t_{k_2}, \cdots, t_{k_m})), l_{ij} = 1, j \neq k. \quad (10)$$

Thus, for test point t_{k_m} added to the assigned test point set $\tilde{T}$, the cost of the directly linked test point with t_{k_m} is checked and updated. Then, other test points have their costs updated based on the conditions of their linked points.

The proofs of Properties 1–12 are given in the supplementary materials.

3.4 Initialization and adjustment of decision graph

In addition to the cost evaluation, we develop a decision graph $G = (V, E)$ to represent a further relationship between tests and the diagnosis for each state, represented as follows:

$$V = \{s_i | i = 1, 2, \cdots, N\}, \quad (11)$$

$$E = \{(s_i, s_j) | i \neq j\}, \quad (12)$$

where E is the set of edges. The weight of an edge represents the cost of the optimal test assignment:

$$w(s_i, s_j) = \text{argmin}\{c(t_k)|d_{ik} \oplus d_{jk} = 1\}, \quad (13)$$

where "$\oplus$" represents the XOR operation.

According to Eq. (6), the weight of each edge represents the best decision for testing s_i and s_j considering the cost of the tests and the relationships between faults and tests. Hence, the heuristic estimation of the search process is as follows:

$$h_{S^*} = \sum_{s_j \in S^*} p_j \max_{s_k \in S^*}(w(s_j, s_k)), \quad (14)$$

where S^* is the set of potential fault states.

For $\tilde{T} = \{t_{k_1}, t_{k_2}, \cdots, t_{k_m}\}$ added to the test strategy, if and only if $\min\{c(t_k)|d_{ik} \oplus d_{jk} = 1\} = c(t_{k_m})$, the weight between two faults is adjusted as follows:

$$w(s_i, s_j) = \text{argmin}\left\{c\left(t_k|\tilde{T}\right)|d_{ik} \oplus d_{jk} = 1, k \neq k_m\right\}. \quad (15)$$

3.5 HP and Rollout-HP

3.5.1 HP method

To overcome the obstacles of sequential test strategies, we develop the HP mixture search method, which combines parallel extension and sequential extension in the search process.

The first approach is to extend the current OR nodes and multiply the corresponding substrate AND nodes. The conditional heuristic function for the candidate point is estimated based on the assigned test set for the OR nodes as follows:

$$h_{S^*}\left(t_i|\tilde{T}^*\right) = \text{argmax}\left(c_i|t_i \notin \tilde{T}^*\right) \sum_{s_j \in S^*} p_j + \sum_{r=1}^{m_2} h_{S^*_{m_1,r}}, \quad (16)$$

where $S^*_{m_1,r}$ is the exclusive sub fuzzy set of the test set $\{t_i\} \bigcup \tilde{T}^*$ calculated with Eq. (8). $\tilde{T}^*$ is the set of assigned test procedures.

The second approach is to extend the sub AND nodes and deepen the fault tree, which ends the generation of the current OR node and starts searching for the optimal solution to the extended AND nodes and the corresponding initialized estimation with Eq. (9).

The HP process combining two types of extension works as follows: to avoid search redundancy, the proposed method continues the extension procedure until the parallel extension has an estimated value which is lower than that of the serial extension. For each procedure, the proposed method selects the optimal test point as the target and adds it to the strategy. In addition, for each update of the potential solutions, the cost of the selected tests on the extension nodes is set to 0, and candidate tests are updated based on Property 2. From above, the procedures of HP are as follows:

Step 1: the sequence matrix $\boldsymbol{L} = [l_{ij}]$ is built based on (E, W) with Eq. (5). W is the set of weights with respect to E.

Step 2: the candidate test point set T_{candi} is initialized as T, and the selected set $\tilde{T}$ is set as null. The target fault state set is assigned as S.

Step 3: the estimate of the conditional heuristic function $h_{S^*,\text{seq}}(t_i|\tilde{T}^*)$ is calculated for T_{candi} with Eqs. (15) and (16).

Step 4: the test point $t_{\text{opt}} \in \tilde{T}$ is taken with the optimal conditional heuristic function estimate.

Step 5: t_{opt} is added into $\tilde{T}$, and the adjusted conditional cost is calculated with respect to $\tilde{T}$ with Eqs. (8) and (9).

Step 6: the extended heuristic function $h_{S^*,\text{ext}}(t_i|\tilde{T}^*)$ is estimated with the sub AND nodes based on Eq. (16).

Step 7: if the optimal extended function estimate is lower than the sequential estimate, the test point $t_{\text{opt}}^{'}$ is taken into $\tilde{T}$ and is removed from T_{candi}. S is divided into subsets S_k. Then turn to step 3, and the optimal cost is calculated for providing the diagnosis of $S_k \subset S$. The heuristic function is updated with the real value.

Step 8: if the optimal extended function estimate is higher than or equal to the sequential estimate, t_{opt} is removed from T_{candi} and S is divided into subsets S_k. Then turn to step 3, and the optimal cost is calculated for providing the diagnosis of $S_k \subset S$. The heuristic function is updated with the real value.

Step 9: if the updated solution is still the optimal corresponding to the estimation, take t_{opt} and the diagnosis solution for S_k as the optimal solution. Otherwise, turn to step 3 for searching for other potential solutions.

3.5.2 Rollout-HP

Although HP improves the search strategy compared to heuristic search, the time consumption of heuristic search is likely to be high when applied to large-scale systems. To address this weakness, we introduce a rollout mechanism (Tu and Pattipati, 2003) into heuristic programming to generate available solutions.

The rollout algorithm can be regarded as a finite step of the classic policy iteration method. To constrain the search process, the policy considers only immediate successor nodes for fault tree construction. Therefore, for each AND node with fault state set S^*, the potential test points likely to be added are estimated as follows:

$$T_{\text{potential}} = \{T_{\text{opt},1}, T_{\text{opt},2}, \cdots, T_{\text{opt},K}\}. \quad (17)$$

According to the rollout strategy, the proposed Rollout-HP is as follows:

Step 1: the sequence matrix $\boldsymbol{L} = [l_{ij}]$ is built based on (E, W) with Eq. (5).

Step 2: the candidate test point set T_{candi} is initialized as T, and the selected set $\tilde{T}$ is set as null. The target fault state set is assigned as S.

Step 3: the estimate of the conditional heuristic function $h_{S^*,\text{seq}}(t_i|\tilde{T}^*)$ is calculated for T_{candi} with Eqs. (15) and (16).

Step 4: $T_{\text{potential}}$ is generated according to the K-best conditional heuristic function estimation based on Eq. (17).

Step 5: T_{opt} is taken from $T_{\text{potential}}$ with the minimum $h_{S^*,\text{seq}}$ estimate.

Step 6: the conditional cost of T_{candi} is adjusted with respect to T_{opt}.

Step 7: the extended heuristic function $h_{S^*,\text{ext}}(t_i|\tilde{T}^*)$ is estimated with the sub AND nodes based on Eq. (16).

Step 8: if the optimal extended function estimate is lower than the sequential estimate, the test point $t_{\text{opt}}^{'}$ is taken into T_{opt} and is removed from T_{candi}. S is divided into subsets S_k. Then turn to step 3, and the optimal cost is calculated for providing the diagnosis of $S_k \subset S$. The heuristic function is updated with the real value.

Step 9: if the optimal extended function estimate is higher than or equal to the sequential estimate, T_{opt} is removed from T_{candi}, and S is divided into subsets S_k. Then turn to step 3, and the optimal cost is calculated for providing the diagnosis of $S_k \subset S$. The heuristic function is updated with the real value.

Step 10: if the updated solution is still the optimal corresponding to the estimation with respect to $T_{\text{potential}}$, T_{opt} and the diagnosis solution for S_k

are taken as the optimal solution. Otherwise, turn to step 3 for searching for other potential solutions.

3.5.3 Computational cost analysis

Different from MOO techniques which search for complexity depending on the population and the number of generations, HP and Rollout-HP use heuristic search mechanisms, and the search space is determined by the scale of the target system, the number of potential test procedures, and the relationships between test procedures and fault states. As a result, for the target system with M fault states and N potential test procedures, the maximum computational complexity of HP is $O(N^2\log_2 M)$. The computational complexity of the proposed HP is an order lower comparing with the computational complexity of the classic AO*, which is $O(N\log_2 M)$, since the conditional costs are adjusted during the heuristic search processes. To further reduce the search cost, Rollout-HP limits the search amount by constraining $T_{\text{potential}}$ and improves the maximum computational complexity as $O(NK\log_2 M)$, where $K < N$.

4 Experiments

4.1 Subject

4.1.1 Dataset

Six datasets were adopted for the performance estimation: amplifier (Mei et al., 2022), anti-tank system (Tu and Pattipati, 2003), Apollo aircraft (Pattipati and Alexandridis, 1990), bus system (Wang et al., 2019), filter (Liu HC et al., 2019), and horizon system (Zhang SG et al., 2015). The structure topologies were built based on the block diagram of the experimental systems, and the cost of the system was marked considering the time consumption of each test point and their related tests based on the system topologies.

4.1.2 Real-world applications

We applied the proposed methods to the test strategy design for an intermediate frequency (IF) signal conditioning circuit. Details of the IF signal conditioning circuit are presented in the supplementary materials. Details of the real-world applications are also given in the supplementary materials.

4.2 Existing methods

We compared the proposed method with the classic AO* method (Boumen et al., 2009) and three other advanced methods: AO leaf (AOL) method (Pattipati and Alexandridis, 1990), MAO* method (Mei et al., 2015), and dynamic programming AO* (DP-AO*) (Lu et al., 2018). Additionally, we compared the proposed method with Rollout-AO* (Tu and Pattipati, 2003) to compare the performance under a greedy search operation.

4.3 Implementation details

All algorithms were implemented in MATLAB 2016b on a laptop with a 2.90 GHz central processing unit (CPU) and 4 GB random access memory (RAM). The function of each test of the IF signal conditioning circuit was summarized by the measurement with GPD-330D, PPE-3323 power supply, R&S ESP17 spectrum analyzer, and R&S SMB200A vector signal generator.

5 Results

5.1 Performance on the experimental datasets

Table 1 presents the number of search times and the total cost for the experimental datasets of the seven methods. HP and Rollout-HP reduce the cost consumption by 10.13% to 70.59% for amplifier, bus system, filter, horizon system, and Apollo aircraft, compared with the sequential methods (AO*, AOL, DP-AO*, and Rollout-AO*). HP reduces the cost consumption by 17.95% for filter and by 40.54% for horizon system compared to MAO*. HP reduces the number of search times by 76.60% for amplifier, 16.22% for bus system, and 42.46% for horizon system comparing with Rollout-AO*. Rollout-HP reduces the number of search times by 80.85% for amplifier, 16.22% for bus system, and 43.84% for horizon system compared with Rollout-AO*. In particular, the number of search times of HP is 18.18% higher for amplifier, 83.87% higher for bus system, 43.75% higher for filter compared with AOL. The number of search times of HP is 2.48% higher for anti-tank, 83.87% higher for bus system, 21.25% higher for filter, and 14.28% higher for horizon system compared with DP-AO*. Also, the number of

Table 1 Performance comparison on the experimental datasets

Method	Number of search times					
	Amplifier	Anti-tank system	Bus system	Filter	Horizon system	Apollo aircraft
AO*	20	133	7	74	47	178
AOL	9	187	5	45	15	169
DP-AO*	15	118	5	63	36	167
Rollout-AO*	47	85	37	80	73	39
MAO*	9	84	5	48	29	44
HP	11	121	31	80	42	81
Rollout-HP	9	100	31	66	41	65

Method	Total cost					
	Amplifier	Anti-tank system	Bus system	Filter	Horizon system	Apollo aircraft
AO*	47.4	3.4	0.236	7.8	8.4	3.4
AOL	47.4	3.4	0.236	7.8	8.4	3.4
DP-AO*	47.4	3.4	0.236	7.8	8.4	3.4
Rollout-AO*	47.4	3.4	0.236	7.9	9	3.4
MAO*	42.6	1	0.236	3.9	7.4	1
HP	42.6	1	0.1	3.2	4.4	1
Rollout-HP	42.6	2.6	0.1	4.5	6.4	2.6

search times of HP is 77.42% higher for bus system and 7.50% higher for filter compared with AO*. However, the total cost of HP is lower for those cases above.

The time consumption of the search process is analyzed in Table 2. HP and Rollout-HP obtain comparable time consumption compared with AO*, AOL, and DP-AO*. In addition, the time consumption of Rollout-HP is lower than that of the other methods except for Rollout-AO* for anti-tank system, and lower than that of all the other methods for bus system. However, HP has higher time consumption of 0.0567 s on filter compared with all the seven methods except for MAO*, and Rollout-HP complete the strategy for filter generation within 0.5000 s.

To estimate the performance of the generated strategies, FIR improvement and test assignment of each strategy on the anti-tank system are shown in Fig. 1. HP builds strategy with three levels, while Rollout-HP builds strategy with four levels. In comparison, the sequential and MAO*-based strategies obtain the final diagnosis with five levels. Additionally, all four algorithms take same number of test points to build strategy. The HP-based strategy reaches 25%, 75%, and 100% of FIR in the first three levels respectively, while Rollout-HP achieves 8.3%, 33.3%, and 66.67% of FIR in the first three levels respectively. However, the sequential and MAO*-based strategies achieve <60% of FIR in the first three levels. Hence, HP and Rollout-HP accelerate the improvement of FIR during diagnosis.

5.2 Performance on real-world applications

Table 3 shows the performance of existing and the proposed methods in real-world applications. Similar to the experimental results, all the algorithms reach the highest FIR. The number of search times of HP is 4.14% greater than that of AO*, 37.24% greater than that of AOL, and 34.48% greater than that of Rollout-AO*. HP reduce the number of search times by 48.21% compared with MAO* and Rollout-HP reduce the number of search times by 49.29% compared with MAO*. In particular, HP and Rollout-HP reduce the cost consumption by 41.67% compared with sequential strategies and 47.31% compared with Rollout-AO*.

The sequential strategy with AO* and the developed strategy with HP are depicted in Figs. 2 and 3, respectively. According to the sequential strategy, V_{6pp} is measured before V_{4dc}, which causes incorrect detection during diagnosis. The test points for the HP-based strategy are assigned after the analysis of all related tests. Meanwhile, 53.85% of the fault states require fewer levels of test procedures than the sequential strategy. Therefore, the HP-based strategy is practical and efficient for real-world applications.

Table 2 Time consumption for the experimental datasets

Method	Time consumption (s)					
	Amplifier	Anti-tank system	Apollo aircraft	Bus system	Filter	Horizon system
AO*	0.0460	0.0632	0.0432	0.0234	0.0322	0.0183
AOL	0.0200	0.0543	0.0422	0.0234	0.0543	0.0342
DP-AO*	0.0320	0.0465	0.0415	0.0253	0.0473	0.0132
MAO*	0.1030	0.1540	0.0231	0.0184	0.0824	0.0678
Rollout-AO*	0.0300	0.0345	0.0182	0.0232	0.0314	0.0120
HP	0.0350	0.0453	0.0345	0.0230	0.0567	0.0342
Rollout-HP	0.0210	0.0421	0.0232	0.0120	0.0345	0.0213

Table 3 Performance comparison on real-world applications

Method	Number of search times	Number of tests	Fault isolation rate (%)	Total cost
AO*	139	10	69.23	8.4
AOL	91	10	69.23	8.4
DP-AO*	120	10	69.23	8.4
MAO*	280	9	69.23	8.4
Rollout-AO*	95	13	69.23	9.3
HP	145	9	69.23	4.9
Rollout-HP	142	9	69.23	4.9

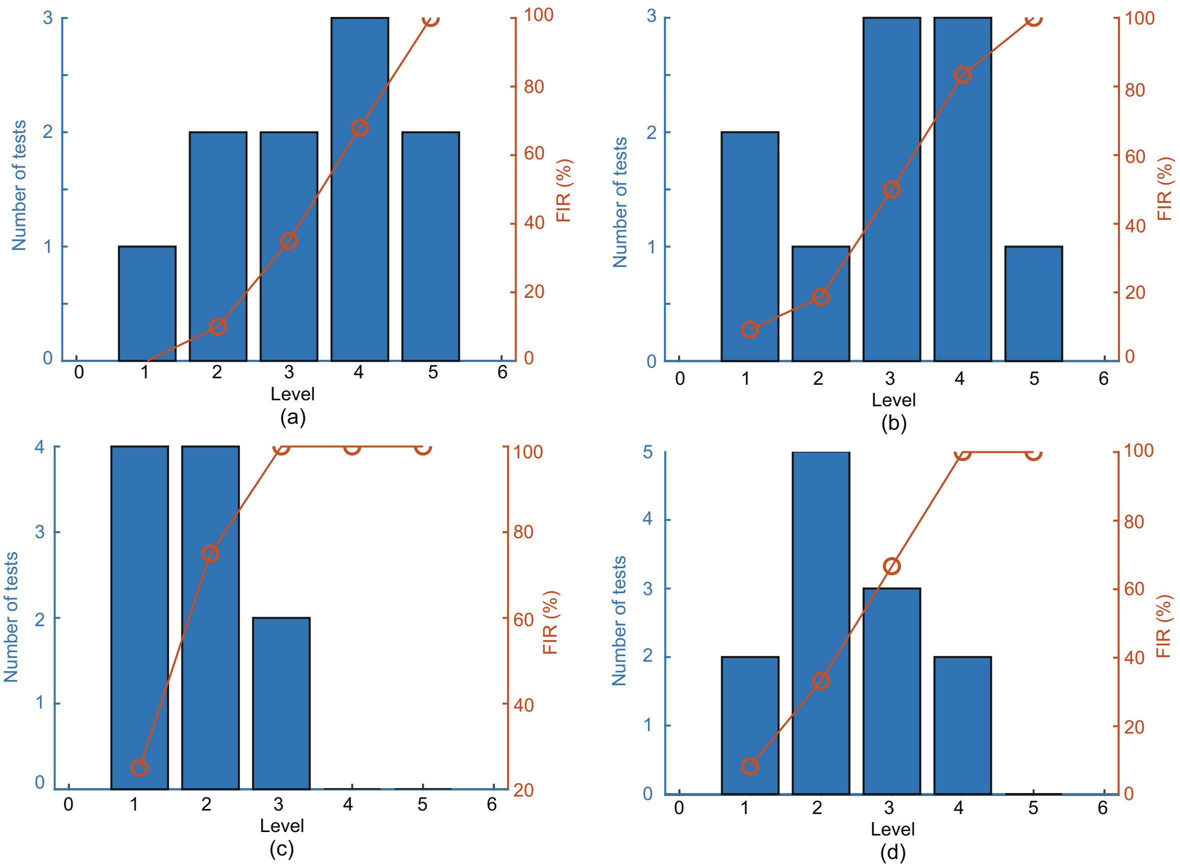

Fig. 1 Fault isolation rate (FIR) improvement and test assignment on the anti-tank system dataset: (a) sequential strategy; (b) MAO*-based strategy; (c) HP-based strategy; (d) Rollout-HP-based strategy

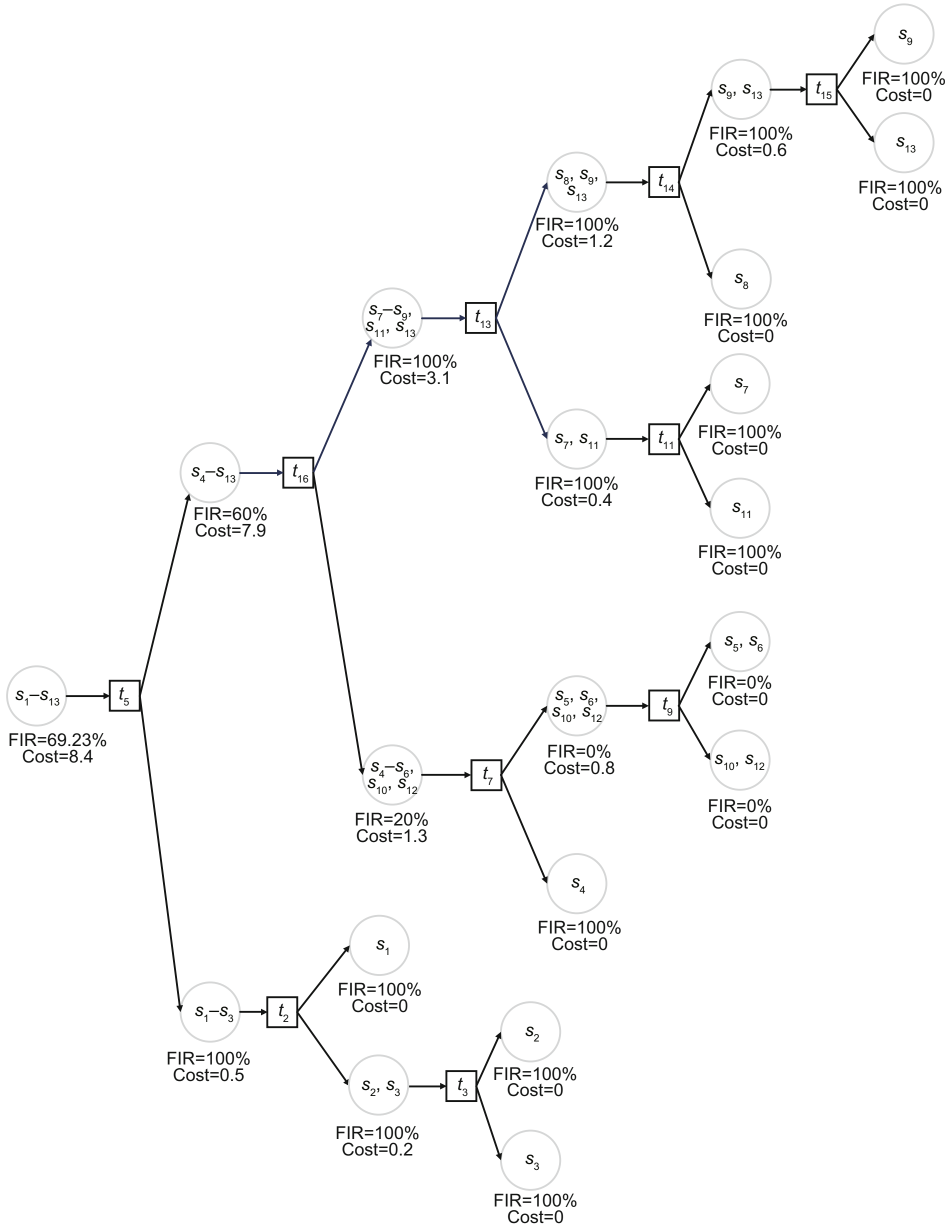

Fig. 2 Sequential strategy for real-world applications by AO* (FIR: fault isolation rate)

Finally, the cost comparison for each fault state is shown in Fig. 4. The HP-based strategy achieves lower cost and requires a smaller number of layers than sequential strategy for s_2 and s_4–s_9.

6 Discussion

This research aims to improve the efficiency of diagnosis strategies for modern analog systems considering multiple information requirements. Therefore, HP and Rollout-HP are proposed to use a sequential matrix that considers the system's topology during the search process. Additionally, the proposed methods combine sequential and parallel strategies for solution generation to fully use the test procedures. The results reveal that HP and Rollout-HP are practical to test strategies and can enhance the detection efficiency of diagnosis strategies.

First, the real-world applications demonstrate that HP and Rollout-HP based solutions avoid incorrect detection during analysis, in contrast to sequential solutions. Different from sequential strategy

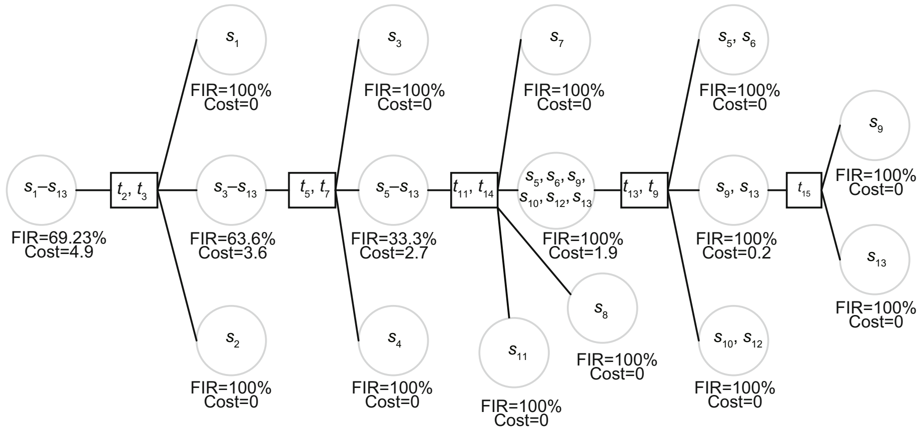

Fig. 3 The developed strategy for real-world applications by the HP algorithm (FIR: fault isolation rate)

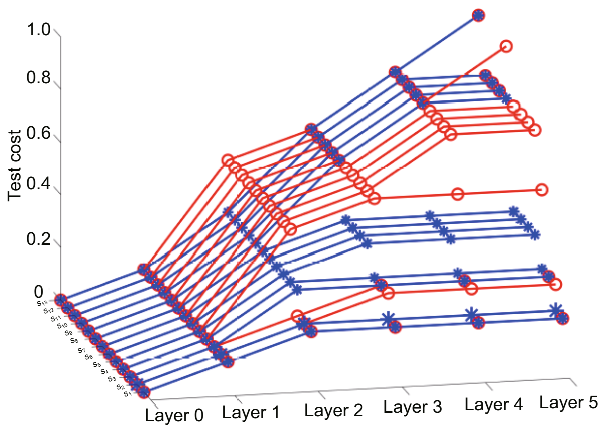

Fig. 4 Test cost on real-world applications (The red lines refer to the test cost of AO*-based methods and the blue lines refer to the test cost of HP-based methods. References to color refer to the online version of this figure)

optimization, which assumes that the diagnosis is concluded based on single test analysis, the proposed HP and Rollout-HP methods adapt a sequential matrix to depict the relationships among the test points. The heuristic estimation is calculated and updated based on the sequential matrix and guides the search process. Therefore, the generated solutions obey the system structure, and are practical for modern analog systems.

In addition, the proposed HP and Rollout-HP methods significantly decrease the cost of the generated strategies compared to existing methods. For most fault states, the diagnosis cost of HP and Rollout-HP based methods is lower than that of the sequential strategy and MAO*. By generating mixture strategies, the efficiency of most fault states is improved compared with that of other methods.

Meanwhile, HP-based strategies are proven to isolate most faults with higher speeds than existing solutions and to achieve the final diagnosis with lower levels of tests. On one hand, the improvement is likely to be a function of the combination of the sequential strategy and parallel strategy during generation, which increases the flexibility of the test solution and the detection speed of diagnosis. On the other hand, updating the dependent cost for each test eliminates the assigned cost and prevents information redundancy during detection, which also improves test efficiency.

Moreover, the time consumption of HP is higher than that of existing methods on several datasets because the search space of the mixture test strategies is extended compared to the search space of sequential test strategies. However, taking advantage of HP, the overall time consumption of the proposed HP and Rollout-HP methods is comparable to that of state-of-the-art methods, and is acceptable for most real-world applications.

In summary, this study has taken a step toward optimizing diagnosis strategies for modern analog systems under the consideration of multiple information detection and system structures. Additionally, the study provides an efficient approach to diagnosis by generating mixture test procedures.

However, it considers only the stable situation of analog systems and ignores dynamic characteristics of reliability maintenance, such as changes in failure probabilities and the adjustment of test consumption. The approaches proposed here should be replicated with dynamic adjustment of the test strategies during the entire lifetime of analog systems.

7 Conclusions

This study focuses on the TSO problem with multiple analog system diagnosis information requirements. Although heuristic search methods, such as AO*, AOL, and DP-AO*, balance between test performance and search complexity, the sequential test strategies adopt unnecessary information and waste test time since they consider the test procedure as an independent event. Furthermore, the generated methods may provide impractical solutions when multi-signal information is required for many detection methods of fault conditions.

To overcome the information redundancy and correlation, we develop mixed search strategies called HP and Rollout-HP to generate the optimal test strategy considering the relationship between test procedures for diagnosis. In addition, the sequence matrix is introduced to heuristic estimation based on a decision graph to guarantee the information usage efficiency during solution construction. The sequential strategy ensures the test completeness of multi-signal information for the diagnosis procedures, and the adjustment process of the conditional cost enhances the flexibility of the test design based on the dependence of the test cost. We also design a novel heuristic estimator based on the sequence matrix and dependence information. As a result, the search process not only considers the prior diagnosis results from existing test strategies but also takes the prior test information into account when designing further test strategies. Combining sequential and parallel extensions in the search process, HP and Rollout-HP consider the dependence of different test procedures that save the number of tests and unnecessary test cost compared to the sequential test strategies of the classic methods.

Both experimental results and real-world applications prove that the generated solutions of the proposed methods decrease the global cost by enhancing the efficiency of each test procedure of the corresponding OR nodes in the fault tree. Taking advantage of the parallel extension during the search process, HP achieves the comparable number of search times with other methods and generates optimal solutions within an acceptable time duration. Additionally, the search efficiency of HP is competitive with that of the sequential methods when applying rollout strategies. Therefore, the proposed HP and Rollout-HP methods provide more efficient diagnosis solutions for analog systems than state-of-the-art approaches.

Contributors

Wenjuan MEI and Zhen LIU designed the research. Ouhang LI, Yusong MEI, and Yongji LONG processed the data. Wenjuan MEI drafted the paper. Zhen LIU and Yuanzhang SU helped organize the paper. Wenjuan MEI, Zhen LIU, and Ouhang LI revised and finalized the paper.

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Compliance with ethics guidelines

Wenjuan MEI, Zhen LIU, Ouhang LI, Yuanzhang SU, Yusong MEI, and Yongji LONG declare that they have no conflict of interest.

Data availability

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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Appendix: Notations used in this paper

Table A4 Notations used in this paper

Notation	Description
C	Process test cost set
$\mathrm{Cost}(S)$	Cost of the root node in the diagnosis tree
$c(t_i)$	Process cost of the test point t_i
$\Delta c(t_i)$	Individual cost for the test point t_i
$c(t_i\|\tilde{T})$	Conditional process cost for t_i
$\boldsymbol{D}$	Dependence matrix
E	Edge set of the biograph of the system structures
FIR	Fault isolation rate
G	Decision graph
h_{S^*}	Heuristic estimation of the search process
$h_{S^*}(t_i\|\tilde{T}^*)$	Conditional heuristic function for the candidate point
$\boldsymbol{L}$	Sequence matrix based on the biograph of the system structures
$N_{\mathrm{distinguish}}$	Number of fault states that can be successfully separated in the solution
P	Fault probability set
S	Fault state set
S_k	Fault state set of the k^{th} AND node
T	Test point set
$\tilde{T}$	Set of the corresponding OR nodes
$T_{\mathrm{potential}}$	Potential test point set
$w(t_i, s_k)$	Weights of the edges of (t_i, s_k)

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