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An improved error-based observer on repetitive control strategy in electro-optical tracking system

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Abstract: Electro-optical tracking systems have been widely utilized in the cutting-edge domains of free space environment detection and communication, owing to their exceptional performance. However, external disturbances often significantly impact the working accuracy of these systems. As their scope of application continues to broaden, increasingly complex operating conditions are seen introducing more intricate environments and disturbances. This paper introduces an enhanced error-based observer composite control structure, rooted in the repetitive control strategy, tailored for two types of complex disturbances: periodic harmonic disturbance and narrow-band peak periodic disturbance. This structure not only ensures the system's stability but also suppresses periodic disturbances across multiple frequencies, addressing the challenge that current disturbance-suppression methods struggle face in mitigating complex periodic disturbances effectively. Moreover, it provides the necessary proofs and establishes an experimental platform for the electro-optical system to demonstrate the efficacy and reliability of the proposed control methods under various conditions.

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Key words: Disturbance suppression; Error-based observer; Repetitive control; Special frequency point; Electro-optical tracking system

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1 Introduction

The electro-optical tracking system refers to a comprehensive optical instrument that integrates mechanical structures, electronic power, control systems, and other structures. It has been widely used in biomedical applications, aerospace, astronomical observations, quantum computing, microstructure characterization, long-distance information transmission, and other fields (Berkefeld et al., 2010; Beaulieu-Laroche et al., 2021; Madsen et al., 2022; Kalita et al., 2023; Snigirev et al., 2023). For establishing control of the electro-optical tracking systems, the main aim is to improve the stability (Downey, 1989; Zhijun et al., 2022) and to enhance the disturbance-suppression ability of the system. Ensuring the stability of the system throughout is, however, vital (Zhao et al., 2022). As the mobile platform gradually becomes the application carrier of the electro-optical system (Kennedy and Kennedy, 2003; Ricks et al., 2004), the working characteristics of these carriers will, in turn, directly affect the working accuracy of the electro-optical tracking system. The vibration of the mechanical structure, the fluctuation of the mobile platform, the atmospheric disturbance around the work site, and other external factors will act as interference factors in the working process and affect the performance of the electro-optical system (Somaschini et al., 2019). These perturbations have complex properties, which include periodic and aperiodic disturbances. Periodic disturbances, especially,

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tend to seriously affect the working process of the electro-optical tracking system (Deng et al., 2023).

Numerous researchers globally have conducted studies focused on addressing this issue (Caruso, 2001; Lu et al., 2023). For example, the use of disturbance feedforward control has been demonstrated to improve the stability of the stable platform and significantly enhance both the stability and disturbance-suppression performance of the tracking platform (Ren et al., 2018). To address the time delay in the open-loop system, an enhanced disturbance-rejection technique was proposed by Shamsuzzoha and Lee (2009). Furthermore, an adaptive disturbance-rejection method was applied to adjust hard disk drives in the dead zone (Lee et al., 2016). Active disturbance rejection control (ADRC), a method designed to improve both the transient and immunity performances of systems plagued with uncertainties (Dong et al., 2018), incorporates an algorithm proposed to enhance the response speed through a model-assisted ADRC (Sobhy and Lei, 2021). Additionally, the sliding mode control can effectively solve the buffeting problem and ensure the stability of the system in the process of disturbance suppression. An adaptive second-order sliding mode control has also been utilized for enhanced disturbance rejection for grid-connected NPC converters (Shen et al., 2021).

The disturbance observer (DOB) features a simple structure, which enhances the disturbance-suppression capabilities of the system by observing the disturbance signal and providing feedback (Deng et al., 2022). It also maintains the stability, demonstrating excellent comprehensive performance (Ohishi et al., 1987). Building upon this, a DOB control algorithm aimed at high-precision positioning systems and uncertain systems was proposed (Lee and Tomizuka, 1996; Chen et al., 2017). However, despite the initial DOB structure improving the disturbance-suppression performance, it compromised on the stability of the system. To address this problem, Deng et al. (2021) proposed a modified DOB model by adjusting the observer structure and filter parameters, which further enhanced the DOB's performance in both disturbance suppression and target tracking across multiple degrees of freedom.

In the 1980s, Inoue et al. (1981) proposed an early structure akin to repetitive control for managing the computer control of the three main ring magnets of a proton synchrotron with a thyristor power supply. Subsequently, Hara et al (1988) officially introduced this control structure, termed repetitive control, to fulfill the performance requirements of the control system at the repetition frequency. Following this, Lan et al. (2020) and Fedele and Ferrise (2014) introduced an enhanced scheme by combining repetitive control with preview compensation and a fractional order controller, respectively. Furthermore, Chen and Tomizuka (2013) and Tang et al. (2019) developed a new RC structure, incorporating a nominal model. As a control method, repetitive control is capable of achieving high performance with periodic signals through a relatively simple structure. This approach has also been utilized to enhance the disturbance-suppression capability.

Recent years have seen significant research on the modified repetitive controller, which has been proposed to address both theoretical and practical challenges. Longman (2000) suggested that linear iterative learning and repetitive control may have the same design criteria. The study by Zhou et al. (2020) introduced a modified repetitive-control system to enhance the disturbance-rejection performance. Utilizing the principles of repetitive control to refine the extended state observer enables effective disturbance suppression (Nie et al., 2021). This approach is independent of an exact system model, thereby ensuring the system's overall robustness. Furthermore, the concept of repetitive control has been adapted for nonlinear systems. Wang et al. (2019) developed a repetitive controller based on fuzzy observer, employing a T-S fuzzy model to characterize the nonlinear system and address the tracking issue of periodic signals. In the context of the discrete-time system, Tian et al. (2021) applied modified repetitive control to improve ADRC, resulting in enhanced disturbances suppression in PMSM drives. In minimum-phase systems, Astolfi et al. (2021) utilized modified repetitive control to achieve a forward loop in the control system. For permanent magnet synchronous motors, Zheng et al. (2019) introduced a phase advance repetitive control to tackle the challenges posed by various uncertainties. Focusing on system frequency, Ye et al. (2020) proposed an adaptive modified repetitive controller for achieving high-accuracy control in grid-connected inverters.

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This method effectively mitigates system fluctuations caused by external periodic disturbances.

However, most of the past methods for disturbance suppression have focused on the overall performance of the control system (Zhou et al., 2022), making it challenging to achieve exceptional disturbance-suppression capabilities for periodic harmonic disturbances or narrow-band peak disturbances that occur at a specific frequency point. The integration of a repetitive control strategy with a partial control method has partially addressed the issue of periodic frequency. However, these control structures are limited in that they can only track or suppress the disturbance signal for a single group of harmonic periodic frequencies. As the application environments for electro-optical tracking systems grow increasingly complex, the variety and complexity of disturbances encountered are also escalating. In the context of shipboard and airborne electro-optical systems, encountering multiple sets of harmonic disturbances with different frequencies or the non-periodic relationships of disturbance signals is quite common. In order to achieve more complex disturbance suppression, it is essential to establish a control structure that can not only ensure the basic stability of control performance across the entire frequency spectrum, but also offer flexibility in selecting disturbance-suppression structures at multiple frequencies.

Building upon this foundation, this paper introduces an error-based observer structure grounded in a repetitive control strategy (RCEOB). This structure is designed to meet the stability requirements of the control system while effectively suppressing periodic disturbances. The principal contributions of this work are as follows:

• Considering the complex disturbance scenario in electro-optical tracking systems, an enhanced error-based observer utilizing a repetitive control strategy (RCEOB) is introduced. This approach is capable of suppressing multi-period and harmonic wave disturbances within the control system.

• Addressing the issue of signal amplification at non-notch frequencies inherent in traditional repetitive control, the filter within the repetitive controller is strategically designed to mitigate the waterbed effect to a significant extent.

• To tackle the challenge of multi-harmonic periodic disturbance suppression, the observer structure is refined through a repetitive control strategy, enabling the suppression of harmonic periodic disturbances.

• Guidance on parameter selection and the demonstration of the RCEOB's stability and robustness in practical applications are provided, offering valuable insights for engineering implementations.

The organization of this paper is as follows: Chapter One provides a concise introduction to the operational characteristics of the electro-optical tracking system and outlines various control methodologies. Chapter Two delves deeper into the problem statement, discussing the impact of disturbances on the electro-optical system and introduces the concept of the error-DOB along with the rationale for its enhancement. In Chapter Three, the composite control structure (RCEOB) is unveiled, along with a proof of its stability. This chapter also analyzes the filter structure within the composite framework, addressing the realization of multiple notch filters and the analysis of parameter selection. Chapter Four describes the construction of the experimental platform for the electro-optical tracking system and validates the proposed method through experimental verification. Chapter Five concludes the paper.

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2 Problem formulation

In the electro-optical tracking system depicted in Fig. 1, maintaining the tracking target at the center of the Charge-coupled Device (CCD) target surface requires continuous control based on feedback error. As technology advances, electro-optical tracking systems mounted on mobile platforms are increasingly exposed to complex and variable external disturbances. These disturbances can impact various aspects of the system, ultimately manifesting in the system output, potentially resulting in tracking errors or even deviation from the target.

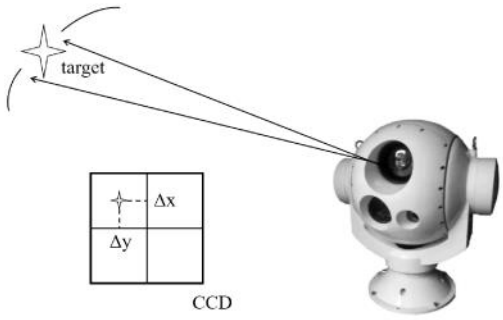

Fig. 1 Schematic diagram of the electro-optical tracking system.

The mathematical expression of system disturbance d in the case of complex disturbance is given below:

$$d(t)=\sum_{i=1}^{n} d_{Pi}(t)+\sum_{i=1}^{m} d_{Aj}(t) \qquad (1)$$

It can be seen that periodic disturbance d_p and nonperiodic disturbance d_A have been included. However, nonperiodic disturbance usually exists in different forms of differentiation in different working environments, which is difficult to be described by the general rule methods and needs to be analyzed in combination with different actual conditions. Periodic disturbance is the form of disturbance that this paper focuses on. Eq. (2) gives the specific form of the periodic disturbance, where A,ω,φ , respectively, represent the amplitude, frequency, and phase of the periodic disturbance.

$$\sum_{i=1}^{n} d_{Pi}(t)=\sum_{i=1}^{n} A_i \sin(\omega_i t+\varphi_i) \qquad (2)$$

In order to address the issue of disturbance suppression in electro-optical systems, the DOB structure is commonly employed within the control systems. DOB serves as a control structure capable of observing disturbance signals. However, DOB has inherent limitations. To overcome these limitations and further enhance the performance of the control structure, an Youla-Kuucera parameterized error-based observer (EOB) method is proposed (Niu et al., 2019), as depicted in Fig. 2. This structure not only meets the disturbance-suppression requirements of the control system but also enhances the system's tracking capability.

The proposed EOB structure achieves the multi-degree-of-freedom control algorithm, which improves the performance of both perturbation and tracking further. However, for the electro-optical tracking system mounted on the mobile platform, the disturbed environment is no longer a simple single-peak or a single-period form. To further improve the performance of EOB structures, it is necessary to improve the disturbance-suppression ability of the system.

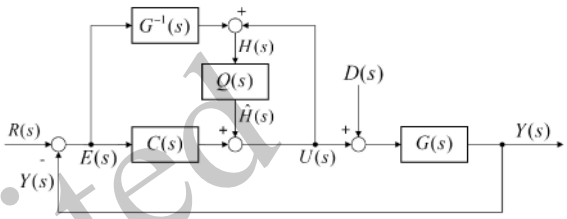

Fig. 2 Error-based observer structure.

Considering the perturbation Eq. (2):

$$\sum_{i=1}^{n} A_i \sin(\omega_i t+\varphi_i).$$

When n = 1, regardless of the value of A, w, the periodic perturbation in this single case can be suppressed by most control methods. When n > 1, w_i appears in the form of frequency doubling. Some improved algorithms using repetitive control strategies can also deal with this situation.

However, when n > 1, w_i appears in the form of harmonics at multiple groups of periods or base frequency, and A_i exists, which is far greater than the existing disturbance-suppression capability of the system. It is difficult for existing control methods to deal with these more complex periodic disturbances and disturbances in the form of narrow-band peaks at the same time. In order to maintain the working stability and disturbance-suppression capability of the electro-optical tracking system under such disturbance conditions, it is necessary for the control algorithm to have a better disturbance-suppression ability at a specific frequency point, and must deal with

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multiple groups of harmonic disturbances at different fundamental frequencies.

3 Composite control structure

3.1 Composite structure

It is evident from the analysis presented in Chapter 2 that to continue using the EOB algorithm to meet the operational requirements of the electro-optical tracking system on a mobile platform, we need to address the issue of the multi-periodic disturbance while retaining the structural characteristics of the EOB. In this study, we propose an enhancement to the EOB using a repetitive control strategy, resulting in the RCEOB. This approach aims to maximize the preservation of the EOB's performance while addressing the challenge of multi-periodic disturbances. The overall block diagram (Fig. 3) of the composite control structure, as well as the tracking transfer function (TTF), error attenuation function (EAF), and disturbance-suppression function (DSF) (Eq. 3) are provided herein.

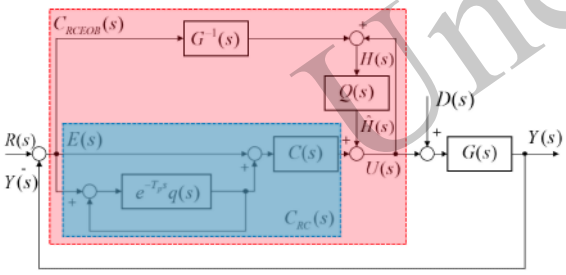

Fig. 3 RCEOB structure.

$$
\begin{aligned}
TTF_{RCEOB} &= \frac{Y(s)}{R(s)} \\
&= \frac{C_{RC}(s)G(s) + G(s)G^{-1}(s)Q(s)}{1 + C_{RC}(s)G(s) + G(s)G^{-1}(s)Q(s) - Q(s)} \\
EAF_{RCEOB} &= \frac{E(s)}{R(s)} \\
&= \frac{1 - Q(s)}{1 + C_{RC}(s)G(s) + G(s)G^{-1}(s)Q(s) - Q(s)} \\
DSF_{RCEOB} &= \frac{Y(s)}{D(s)} \\
&= \frac{G(s) - G(s)Q(s)}{1 + C_{RC}(s)G(s) + G(s)G^{-1}(s)Q(s) - Q(s)}
\end{aligned}
\tag{3}
$$

In the given structure, R is the system input, Y the system output, D the disturbance input of the system, C the controller, G the model of the controlled object, and G^{-1} the nominal inverse model of the controlled object. To distinguish between the two different filters, $q(s)$ is designated for the filter in the repetitive controller, while $Q(s)$ represents the EOB filter. For the sake of simplicity, the $C_{RC}(s)$ structure within the blue block is not fully elaborated upon. The complete form is given in Eq. (5). Detailed analysis can be found in Section 3.2.2.a.

$$
C_{RC} = \frac{C(s)}{1 - e^{-T_p s}q(s)} \tag{4}
$$

Then, the stability and robustness of the RCEOB are analyzed.

Theorem 1 (Stability): In the case that the closed-loop controller $C(s)$ is stable, the following requirements are made for the $Q(s)$ and $q(s)$ filter structures in the control structure. When the following expressions are met, the RCEOB structure is stable.

$$
\left\| \frac{e^{-T_p s}q(s)C(s) - \left(1 - e^{-T_p s}q(s)\right)\left(C(s)Q(s) + G^{-1}(s)Q(s)\right)}{(1 + C(s)G(s))\left[C(s) + G^{-1}(s)Q(s)\left(1 - e^{-T_p s}q(s)\right)\right]} \right\|_{\infty} < 1 \tag{5}
$$

The proof of *Theorem 1* is provided in the Supplementary Materials.

Theorem 2 (Robust Stability): In the RCEOB control system, under the condition that *Theorem 1* has been satisfied, if there is a small disturbance caused by modeling uncertainty in the control ob-

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ject $G(s)$, it is expressed by the following Eq. (6):

$$G_P(s) = G(s)(1 + \Delta(s)) \tag{6}$$

Where $\Delta(s)$ represents the modeling uncertainty, which denotes the system's internal disturbances, and $\Delta(s)$ is stable and bounded. The given system has robust stability if the following expression is true:

$$\|\Delta(s)TTF_{RCEOB}(s)\|_\infty < 1$$

The proof of *Theorem 2* is provided in the Supplementary Materials.

Remark: *Theorem 2* guarantees that the control system can maintain overall stability in the case of small perturbations in the object model.

3.2 Notch structure analysis

After the stability and robustness analysis is completed, the controller and filter in the control structure are described. After analyzing the function and improvement of each structure, the suppression performance of the control structure under the influence of frequency incoherence and multiple frequency multiplier harmonic disturbances is described.

3.2.1 Notch at single and discrete point frequencies

The notch effect at discrete point frequencies in the RCEOB structure is realized by the improved structure of filter $Q(s)$ in the EOB. The specific structure is given below:

$$Q_N(s) = 1 - \prod_{i=1}^{a} \frac{s^2 + \eta_i\omega_i s + \omega_i{}^2}{s^2 + \alpha_i\eta_i\omega_i s + \omega_i{}^2} \cdot (1 - q_{LPF}(s)) \tag{8}$$

$Q_N(s)$ corresponds to multiple filters with different cycle frequencies, and $q_{LPF}(s)$ is a low-pass filter to ensure the stability of the structure. Here, in order to highlight the characteristics of the structure, the default is to use a first-order low-pass filter. w_i is the center frequency of the i-th notch frequency, α_i and η_i are structural parameters, which are analyzed in detail in Section 3.4.

Theorem 3 For $Q_N(s)$, when $a = 1$, the complementary sensitivity function $ESF_{QN}(s)$ is at $w_i = w_1$, and $\exists\delta > 0, \delta \to 0$ exists to make $ESF_{Q_N}(\omega_1) \ll ESF_{Q_N}(\omega_1+\delta)ESF_{Q_N}(\omega_1) \ll ESF_{Q_N}(\omega_1-\delta)$ true.

The proof of *Theorem 3* is provided in the Supplementary Materials.

Remark: *Theorem 3* states that for Q_N, a high-performance notch effect at w_1 can be achieved.

Combined with the image of the complement sensitivity function, the filter notch point is set to 0.5 Hz for easy observation. Fig. 4 can confirm the theoretical analysis.

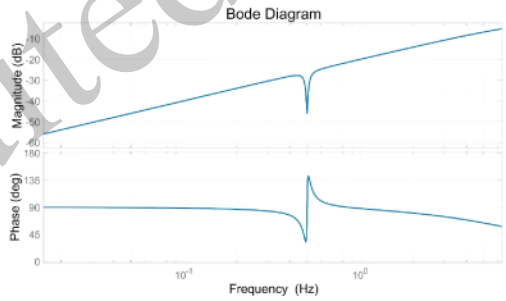

Fig. 4 The complementary sensitivity function of Q_N when $a = 1$, $\alpha_1 = 10$, $\eta_1 = 0.01$, and $w_1 = \pi$.

Theorem 4 For $Q_N(s)$, when $a \neq 1$, the complementary sensitivity function $ESF_{QN}(s)$ is at $w_i = w_n$, and $\exists\delta > 0, \delta \to 0$ exists to make $ESF_{Q_N}(\omega_n) \ll ESF_{Q_N}(\omega_n+\delta)ESF_{Q_N}(\omega_n) \ll ESF_{Q_N}(\omega_n-\delta)$ true.

The proof of *Theorem 4* is provided in the Supplementary Materials.

Remark: *Theorem 4* states that for the above $Q_N(s)$ filter structure, it can be implemented simultaneously at multiple frequency points.

At $\omega = \omega_1, \omega_2 ... \omega_i$, the frequency response is minimized. At the same time, the frequency response keeps monotonically increasing between different notch points until the next notch point takes the

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minimum again. Therefore, the filter structure can be extended from a single frequency to multiple notch frequency points, and even realize multiple specific frequency notches with discrete periodic frequencies. In this case, the curve of the complementary sensitivity function of the trap structure is given.

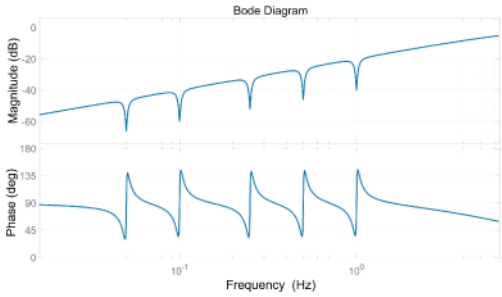

Fig. 5 The complementary sensitivity function of Q_N when a = 5, $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = 10$, $\eta_1 = \eta_2 = \eta_3 = \eta_4 = \eta_5 = 0.01$, and $w_1 = 0.1\pi$, $w_2 = 0.2\pi$, $w_3 = 0.5\pi$, $w_4 = \pi$, $w_5 = 2\pi$.

The reliability of Theorem 4 can be verified by the curve shown in Fig. 5 for the complementary sensitivity function.

Theorems 3 and 4 show how the RCEOB structure realizes the notch of the control structure at a single point frequency and at discrete frequency points. In addition, the complex disturbance environment must contain harmonic disturbances in the form of periodic frequencies.

3.2.2 Notch at the harmonic periodic frequency

In the RCEOB structure, there are two structures that can realize the notch at the harmonic periodic frequency, namely the repetition controller and another EOB filter structure.

a. Repetitive controller

The specific control structure of the repetitive controller is shown in Fig. 6. By using this structure, the closed-loop controller can achieve high performance at the repetition frequency and solve the problem of suppression of the harmonic signal disturbance at the repetition frequency.

In Eq. (4), $1 - e^{-T_P s}$ provides the theoretical limit gain at the harmonic frequency based on the fundamental frequency $1/T_p 1$ and the integer multiple of the fundamental frequency, and $q(s)$ in the classical repetitive control structure is a lowpass filter, so that the controller as a whole can have better disturbance immunity in the main operating frequency band.

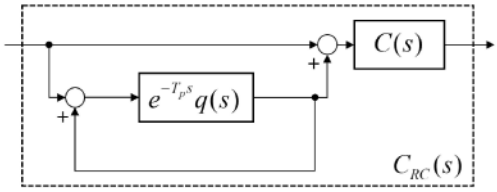

Fig. 6 Repetitive control structure.

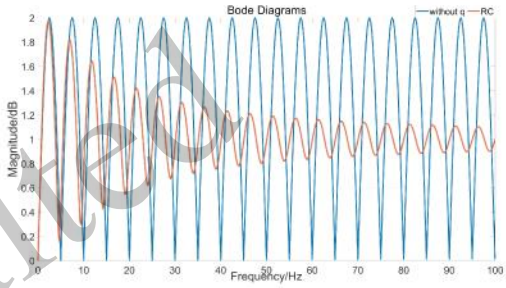

Fig. 7 Comparison between the sensitivity function $1 - e^{-T_P s}$ and the effect of $1 - e^{-T_P s}q(s)$, wherein it is the reciprocal of the base frequency f = 5 Hz and the system sampling frequency f_s = 200 Hz.

As can be seen from Fig. 7 of the comparison results, repetitive control, as a classical control method, has excellent disturbance-suppression performance under the disturbance input scenarios of repetition frequency. However, according to Bird's integral theorem (Zhang and Sun, 2003), the continuous theory method completely avoids the amplification effect at other non-trap frequencies caused by the waterbed effect, and such a comb effect will lead to disturbance amplification at $\omega_i \neq n(1/T_P)$, n = 1, 2, ..., n. It may also cause the notch frequency point to be inaccurate.

To solve the waterbed effect, the $q(s)$ in the repeating controller is improved as follows:

$$q_{new}(s) = \frac{kq(s)}{1-(1-k)e^{-T_P s}} k \in (0,1) \qquad (9)$$

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The k are structural parameters, and T_p influence the notch frequency.

Theorem 5 When $q(s) = q_{new}(s)$ is satisfied in the repetition controller, the sensitivity function value can be made to approach 1 at the non-notch frequency, that is

$$\omega_i \neq n(1/T_P), n = 1, 2, ..., n.$$

Remark: *Theorem 5* states that the improved filter can effectively suppress the waterbed effect and reduce the frequency amplification at non-notch points.

The proof of *Theorem 5* is provided in the Supplementary Materials.

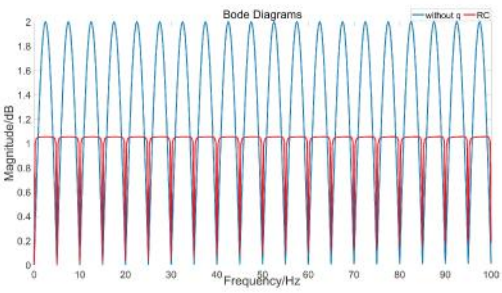

Fig. 8 When $k = 0.3$, comparing the effect of $1-e^{-T_P s}$ and the improved $1-e^{-T_P s}q(s)$ sensitivity function, where T_p is the reciprocal of the fundamental frequency $f = 5$ Hz, and the system sampling frequency $fs = 200$ Hz.

Note: The improved repetition controller suppresses the waterbed effect and realizes the suppression effect of recurring harmonic disturbances.

It is not difficult to see from the comparison in Fig. 8 that, taking the fundamental frequency near $f = 5$ Hz as an example, after improvement, the disturbance-amplification effect at the non-notch frequency is significantly reduced, and the amplification effect on the disturbance is only 1.05 times, with a decrease of 47.5%.

b. Repetitive control strategy improves the EOB filter

By improving the $q(s)$ filter structure in the controller, a group of harmonic disturbances with fundamental frequency and its frequency doubling can be solved. In the control process of the electro-optical tracking system, there may be a group of harmonics and a single fundamental wave, or even multiple groups of harmonic disturbances. Next, we propose another improvement and theoretical analysis of the $Q(s)$ structure. By introducing the repetitive control strategy into the EOB filter structure, we try to further expand the disturbance-suppression ability of the control structure in the above working environment.

The specific structure of the improved filter is given below:

$$Q_{RC}(s) = [1 - \beta(1 - e^{-T_P{}'s})] \cdot q_{LPF}(s) \qquad (10)$$

Where, $Q_{RC}(S)$ introduces the idea of repetition controller into the filter structure and also achieves the suppression of harmonic disturbances in the filter. β is the structural parameter, which is analyzed in detail in Section 3.4.

At the same time, it is independent of the repetition control structure in the controller. In order to distinguish the parameters in the repetition controller, $T_P{}'$ is used; $q_{LPF}(s)$ is a low-pass filter to ensure the stability of the structure, and a first-order low-pass filter is used here.

Lemma 1 For $Q_{RC}(s) = [1 - \beta(1 - e^{-T_P{}'s})]$ $Q_{LPF}(s)$, it can realize the notch at the repetition frequency in the EOB filter structure.

As for $Q_{RC}(s) = [1 - \beta(1 - e^{-T_P{}'s})] \cdot q_{LPF}(s)$, it has a simpler structure and fewer parameters than $Q_N(S)$, and then its principle is analyzed.

By observing the structure of the filter, it can be seen that the improved structure is based on the original low-pass filter structure by adding a structure similar to the traditional repetitive control.

Similar to the repetitive controller idea, the RC-modified filter also uses the structure of $1 - e^{-T_P{}'s}$ to achieve high performance at the fundamental frequency and its integer multiples.

Remark: The changes proposed in *Lemma 1* give the EOB structure the ability to handle a set of harmonic disturbances independently.

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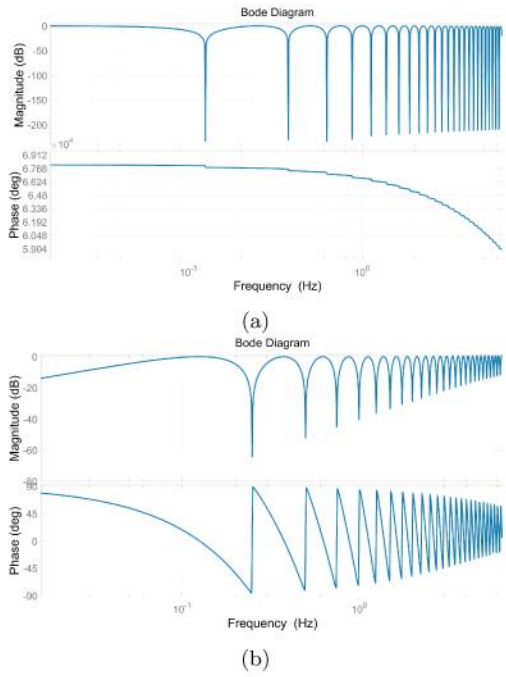

Fig. 9 (a) $Q_{RC}(s)$ sensitivity function curve; (b) $1-Q_{RC}(S)$ complement sensitivity function curve.

According to the curves of the sensitivity function and the complement sensitivity function in Fig. 9, it can be seen that the filter $Q_{RC}(s)$ improved by RC can easily suppress the disturbance when w_i is a group of harmonics in the periodic disturbance, and at the same time ensure the stability of the overall control structure.

3.3 Implementation of multiple notch structures

After the analysis of the control structure, this section will focus on the analysis of how to use the composite control structure to realize multiple notches, so as to achieve the purpose of disturbance suppression of periodic harmonics and nonharmonic discrete periodic disturbances at multiple frequency points.

In Section 3.1, the complete control structure of the RCEOB is given. In order to analyze how this structure implements multiple notches, the part before the control object G is now regarded as an overall controller, which is shown in the red box line in Fig. 5.

According to the Eq. (9), the transfer function of the composite controller is given by Eq. (11):

$$\begin{aligned} C_{RCEOB} &= \frac{C_{RC}(s)+G^{-1}(s)Q(s)}{1-Q(s)} \\ &= \frac{C(s)+G^{-1}(s)Q(s)(1-e^{-T_P s}q(s))}{(1-e^{-T_P s}q(s))(1-Q(s))} \end{aligned} \tag{11}$$

The denominator of the controller is analyzed using the same analysis method as the repetitive controller.

$$den_{C_{RCEOB}} = (1-e^{-T_P s}q(s))(1-Q(s)) \tag{12}$$

Lemma 2 When $q(s)$ and $Q(s)$ given in Eqs (11) and (12) are $q_{new}(s)$ and $Q_N(s)$ given in Section 2 of this chapter, simultaneous trapping at discrete frequency points and single harmonic periodic frequency points can be achieved.

The two factors in Eq. (12) correspond exactly to the $1-e^{-T_P s}q(s)$ of the repetition controller that generates the harmonic theoretical limiting gain and to the complement sensitivity function $ESF_Q = 1-Q(s)$ of the filter structure in the EOB. This structure can realize the notch effect of the two structures analyzed before at the same time. Firstly, on the basis of repeating the notch at one cycle frequency of the controller, $Q_N(s)$ is added to realize the effect of increasing discrete notch points.

Fig. 10 shows the controller effect of the repeated controller and $Q_N(s)$ acting simultaneously in the main operating frequency band.

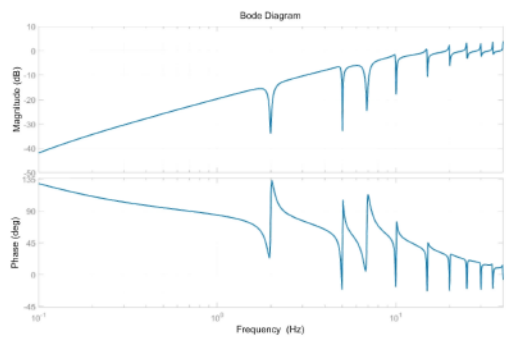

Fig. 10 In the composite control structure, the $(1-e^{-T_P s}q(s))(1-Q_N(s))$ sensitivity function curve when the notch of the repeating controller is set to 5 Hz and its frequency doubling, and the notch point of $Q_N(s)$ is set to 1 Hz and 7 Hz.

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It can be seen that by adjusting the filter structure, the composite control structure can achieve the notch effect for multiple cycles.

Remark: *Lemma 2* shows that the composite control structure can realize the disturbance suppression of the harmonic periodic frequency and the discrete frequency point periodic disturbance simultaneously.

However, at the same time, it must be noted that $Q_N(s)$ at this point is under the condition $k = 2$. In combination with the analysis of stability conditions given in Eq. (5), it can be seen that the increase of the value of k will gradually increase the complexity of the multiplicative part of the filter structure, which will definitely lead to the failure of the control system to meet the stability conditions. In other words, the composite structure with $Q_N(s)$ can indeed improve the disturbance-suppression performance when dealing with a single periodic frequency doubling harmonic and a few discrete frequency disturbances. However, when the second group of periodic frequency doubling harmonic disturbances occur, in order to ensure the stability of the system, this structure can only cover the disturbance signal at the low-frequency narrow band peak. It is not possible to suppress the periodic frequency doubling harmonic disturbances in all operating frequency domains.

Lemma 3 When $q(s)$ and $Q(s)$ given in Eqs (11) and (12) are $q_{new}(s)$ and $Q_{RC}(s)$ given in Section 2 of this chapter, simultaneous trapping at multiple harmonic periodic frequency points can be achieved.

The improved $Q_{RC}(s)$ combined with the repetitive control strategy can solve the situation where there are two groups of periodic frequency doubling harmonic disturbances at the same time, in which case Eq. (12) becomes:

$$den_{C_{RCEOB}} = (1 - e^{-T_P s} q(s))(1 - Q_{RC}(s)) \quad (13)$$

At this point, another factor in Eq. (13) becomes the complement sensitivity function of $Q_{RC}(s)$. The two factors in the structure have repeated control structures and independent parameters, which means that the compound control structure can realize the notch at two groups of periodic frequency doubling harmonic points at the same time. Comparing $Q_N(s)$ with a larger k value, the overall structure of $Q_{RC}(S)$ controller is simple, and it is easy to meet the stability condition given by Theorem 1, which ensures the stability of the overall control structure.

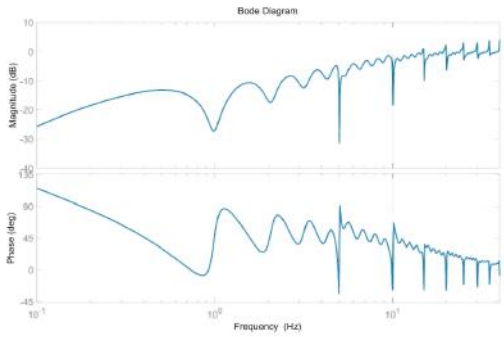

Fig. 11 In the composite control structure, $(1 - e^{-T_P s} q(s))(1 - Q_{RC}(s))$ sensitivity function curve when the trap is set to 5 Hz and its frequency doubling by repeating controller and when set to 2 Hz and its frequency doubling by $Q_{RC}(s)$.

As can be seen from Fig. 11, the composite controller at this time completes the notch at the two groups of harmonic frequencies w_a = 1, 2, 3…Hz and w_b = 5, 10, 15…Hz.

Lemma 3 shows that the complex control structure can realize the suppression of multiple independent harmonic periodic frequency disturbances simultaneously.

To sum up, the composite control structure given by Eq. (11) can realize multiple notch waves. At the same time, with different $Q(s)$, the structure can realize notch waves for single periodic harmonics, narrow band peak disturbance at discrete frequencies and two independent periodic harmonics respectively, so as to meet the disturbance-suppression requirements of different situations in the actual electro-optical system.

3.4 Parameter selection of the multiple notch structure

After introducing how the composite control structure achieves the process of multiple notches, this section will focus on the problem of the selection of parameters in the composite controller, and finally give a group of reference for the selection of parameters of the filter in the multiple notch structure.

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First list all the adjustable parameters in the $q(s)$ and two $Q(s)$ structures in Table 1:

Table 1 All adjustable parameters in the composite controller

Source	Name
$C_{RC}(s)$	k
	T_P
	α_i
$Q_N(s)$	η_i
	w_i
$Q_{RC}(s)$	β
	T_P'

For the repetitive controller, the main adjustable parameter is k, and the influence of different k values on the control effect under the condition of fixed base frequency is compared in Fig. 12.

By comparison, it can be seen that in the range of $k \in (0, 1)$, the value will significantly affect the suppression effect of the repetitive control filter on the waterbed effect. The smaller the value of k, the more obvious the suppression effect on the error amplification at the non-notch frequencies will be, while the notch effect on the fundamental and harmonic frequencies will not be affected.

In $Q_N(s)$, there are three adjustable parameters, among which is the frequency at the i single notch. In the case of fixed w_i, the influence of the value ranges of the other two structural parameters on the overall effect of the filter is compared, where $\alpha_i > 1$ must be ensured.

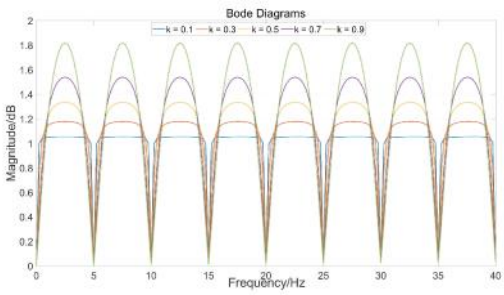

Fig. 12 Comparison of $1 - e^{-T_P s} q(s)$ sensitivity functions when k takes different values. The base frequency is f = 5 Hz = $1/T_p$ and the sampling frequency is 100 Hz.

Firstly, analyze the influence of α_i on the filter effect in Fig. 13:

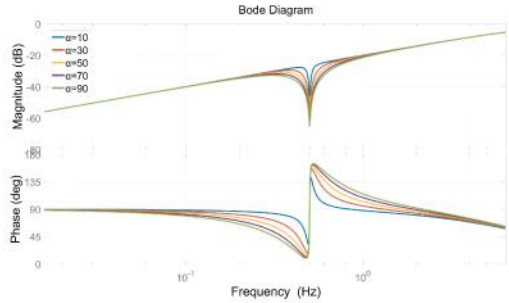

Fig. 13 Changes of $Q_N(s)$ complement sensitivity function with different α values. The sampling frequency is 100 Hz, the notch frequency w = 5, and the other structural parameter η is fixed as 0.01.

Through theoretical analysis, it can be seen that α_i mainly affects the peak depth of the notch point in the notch structure of the filter. A larger value of α_i can provide better notch effect at the set frequency, but it will also change the system characteristics at the surrounding frequency.

And for another structural parameter η_i in Fig. 14:

It can also be seen from the comparison that η_i does not affect the notch depth at the trap point, but only acts on the waveform around the trap point. The smaller the value η_i is, the more concentrated the trapping effect will be and the less the effect will be at the nontrapping frequencies.

To sum up, for $Q_N(s)$, on the premise of meeting the response stability and robustness, each notch point $w = w_i$, $i = 1, 2,\ldots, k$, α_i should be as large as possible, and η_i should be as small as possible, so as to realize the notch effect on the set frequency points relatively concentrated.

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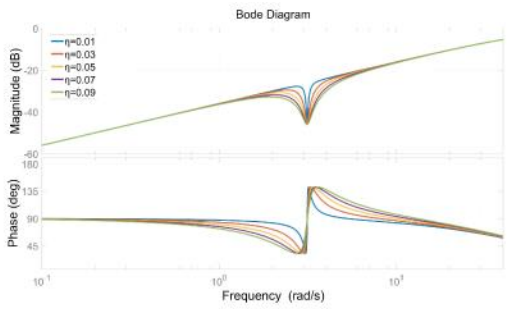

Fig. 14 Changes of the $Q_N(s)$ complement sensitivity function with different values of η. The sampling frequency is 100 Hz, the notch frequency $w = 5$, and the other structural parameter α is fixed as 10.

For $Q_{RC}(S)$ with improved repetitive control, except for setting the notch frequency, there is only one adjustable parameter β, which is analyzed:

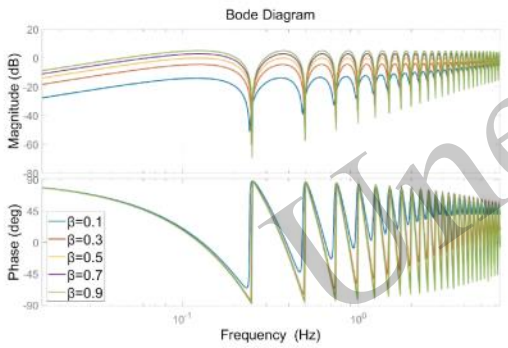

Fig. 15 Comparison of $Q_{RC}(S)$ complement sensitivity function under different value ranges of β. The sampling frequency is 100 Hz and the notch base frequency is 0.25 Hz.

β has a complex influence on the performance of $Q_{RC}(S)$. As shown in Fig. 15, the increase of β will provide significantly better notch performance, but it may cause the waterbed effect at other non-notch frequencies to amplify the disturbance. According to Bird's integral theorem, this property is consistent with the characteristics of repetitive control. At the same time, too small a β value may lead to an inaccurate frequency at the trap point, which will also influence the effect of disturbance suppression.

Finally, the reference values of each parameter are analyzed in this section, and the actual effects of the multiple notch under the modified reference values are given in Table 2 and Fig. 16.

Table 2 All adjustable parameters in the composite controller (with value)

Source	Name	Value
$C_{RC}(s)$	k	0.1
	T_P	1/5
$Q_N(s)$	α_i	10
	η_i	0.01
	w_i	2 Hz
	β	0.5
$Q_{RC}(s)$	T_P'	1

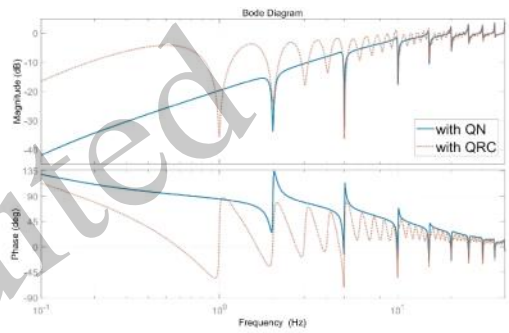

Fig. 16 Performance of the $(1-e^{-T_P s}q(s))(1-Q(s))$ sensitivity function in the composite controller structure under the parameters in Table 2.

4 Experimental verification

4.1 Experimental platform construction

In order to verify the effectiveness of the method, combined with the experimental conditions, the following system was built, as is shown in Fig. 17.

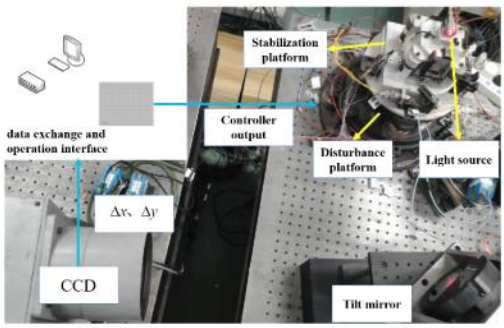

Fig. 17 Composition of the experimental platform.

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On the basis of the platform, the controlled object is fitted first, and the controller in the control structure is set according to the fitting results to ensure the stability of the single closed-loop process.

Through the open-loop measurement, it can be seen that in the main operating frequency band of 0.50 Hz, the control object can be regarded as a linear object, and a second-order oscillation link is used to fit it:

$$G(s) = \frac{12320}{37.7s^2 + 619.7s + 53580} \tag{14}$$

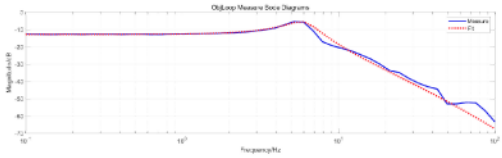

Fig. 18 Fitting of the open-loop characteristics of experimental subjects.

Based on the fitting results in Fig. 18, in order to ensure the stability of the system, the multi-notch effect of the composite controller is mainly tested, and a simple controller $C(s)$ is designed:

$$C(s) = \frac{9425s^2 + 154900s + 13390000}{142.1s^2 + 53580s} \tag{15}$$

4.2 Closed-loop experimental results

In order to verify the effectiveness of the method and show the performance improvement of the RCEOB method at the trap point, the actual transfer functions, error-tracking functions, and DSFs of different methods are measured, and the experimental results are given in the form of a Bode diagram. Refer to Table 2 for the RCEOB structure parameters.

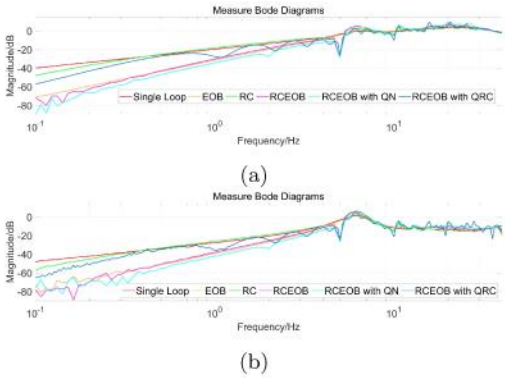

Fig. 19 (a) Comparison of error-tracking performance of different control methods on the experimental platform; (b) comparison of disturbance-suppression performance of the different control methods on the experimental platform.

The analysis of the experimental results is provided in the Supplementary Materials.

4.3 Single-frequency disturbance

The experimental results are provided in the Supplementary Materials.

4.4 Multi-frequency disturbance

The experimental results are provided in the Supplementary Materials.

5 Conclusion

In this paper, a multi-notch controller structure (RCEOB) with multiple selectable parameters is proposed for dealing with the periodic disturbances in the process of the electro-optical tracking system; especially, the multi-periodic harmonic and narrow-band peak disturbances are dealt with, which are difficult to be suppressed by the existing disturbance-suppression methods. The RCEOB structure is proposed to suppress the disturbance on the premise of ensuring the stability of the system also, and the filter in the structure is improved by repeated control strategy, which achieves the disturbance suppression in the case of complex disturbances. In addition, the parameter selection in the composite structure is analyzed, and the multiple notch disturbance suppression is realized. Finally, an experimental platform is set up to verify the effectiveness of the control method and the disturbance-suppression performance, which proves that the proposed method has excellent performance with respect to the theoretical analysis.

However, according to Bird's integral theorem, the waterbed effect must exist in any repetition controller or filter improved by repetition control strategy, which is completely eliminated by the theory. How to further suppress the waterbed effect on the existing basis is the direction of future thinking. At the same

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time, the existence of multiple notch structures may also affect the accuracy of each notch structure, and how to continue to improve the structure to make each notch point more accurate is another problem to be solved.

The follow-up work plan is based on the multi-degree-of-freedom control structure, combined with the adaptive recognition process, and strives to realize the disturbance-suppression algorithm with self-adjusting ability.

Contributors

Mai TANG designed the research. Mai TANG and Wen-qiang XIA processed the data. Mai TANG drafted the manuscript. Jiu-qiang DENG helped organize the manuscript. Mai TANG and Yao MAO revised and finalized the paper.

Compliance with ethics guidelines

Mai TANG, Wen-qiang XIA, Jiu-qiang DENG and Yao MAO declare that they have no conflict of interest.

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List of Supplementary Materials

1 Proof of Theorem 1

2 Proof of Theorem 2

3 Proof of Theorem 3

4 Proof of Theorem 4

5 Proof of Theorem 5

6 Closed-loop experimental results analysis

7 Experimental results of single-frequency disturbance

8 Experimental results of multi-frequency disturbance

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