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Sum-based dynamic discrete event-triggered mechanism for synchronization of delayed neural networks under deception attacks*

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Abstract: This paper focuses on the design of event-triggered controllers for the synchronization of delayed T-S fuzzy neural networks (NNs) under deception attacks. The traditional event-triggered mechanism (ETM) determines the next trigger based on the current sample, resulting in network congestion. Furthermore, such methods also suffer from the issues of deception attacks and unmeasurable system states. To enhance the system stability, we adaptively detected the occurrence of events over a period of time. In addition, deception attacks are recharacterized to describe general scenarios. Specifically, the following enhancements are implemented. First, we utilized a Bernoulli process to model the occurrence of deception attacks, which can describe a variety of attack scenarios as a type of general Markov process. Second, we introduced a sum-based dynamic discrete event-triggered mechanism (SDDETM), which utilizes a combination of past sampled measurements and internal dynamic variables to determine subsequent triggering events. Finally, we incorporated a dynamic output feedback controller (DOFC) to ensure the system stability. The concurrent design of the DOFC and SDDETM parameters is achieved through the application of the cone complement linearization (CCL) algorithm. We further gave an experiment for validating the effectiveness of the algorithm.

Key words: Sum-based dynamic discrete event-triggered mechanism; T-S fuzzy model; Deception attacks
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1 Introduction

Synchronization of neural networks (NNs), as an important tool, has received extensive research in recent years for NN-based control. In the network environment, the transmission of a large amount of information for controlling NNs leads to network congestion (Wang et al., 2021), making it more difficult for NNs to synchronize. Worse, network attacks (Ma et al., 2024) as well as unmeasurable system state (Liang and Huang, 2021) problems further come up, resulting in the deterioration of the system performance.

For the problem of network congestion, the event trigger mechanism (ETM) is an effective so-

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lution to alleviate the issue. However, designing the event triggering conditions in a continuous space is a significant challenge (Liu et al., 2022). Shen et al. (2023) designed a dynamic ETM with a dynamic threshold parameter (DTP) that can adaptively adjust the triggering condition based on the evolution of system states. This allows the triggering condition to be more responsive to the current system state. Bao et al. (2024) proposed Lyapunov function-based ETM and sampled-data-based ETM to reduce network data transmission. The former introduced a waiting time to avoid unnecessary frequent transmission and incorporates an acknowledgment (ACK) mechanism to promptly transmit data. The latter was to check the triggering condition only at sampling instants. Lei et al. (2024) designed an ETM to ensure the existence of a strictly positive minimum interevent time, thereby preventing Zeno behaviour and reducing unnecessary sampling and control updates. Liu et al. (2023) compared the difference between the current error and the error at the last transmission, and the data were only sent when the difference exceeds the threshold. However, these mechanisms often rely on the current state of the system to determine the next trigger, which can lead to higher trigger frequencies and potential Zeno behavior, where an infinite number of triggers occur in a finite time interval. This can significantly degrade the performance of the system and increase computational and communication overhead. Consequently, this results in ineffective event filtering and fails to reach the trade-off between the network transmission efficiency and the information integrity.

Moreover, in many practical applications, the system states are often unmeasurable. The communication network is vulnerable to external malicious attack due to its openness, sharing, interconnectivity, and versatility. Especially, when the system is subject to uncertainties and nonlinearities, as well as corrupted or incomplete state information. This vulnerability can lead to a serious degradation of system performance and may even cause a system crash. T-S fuzzy NNs (Liu et al., 2019b; Yan et al., 2019; Tan et al., 2020) have emerged and have been widely studied. Nevertheless, the issue of synchronizing T-S fuzzy NNs in the face of cyberattacks has not been extensively explored. Based on this motivation, this paper designs a dynamic output feedback controller (DOFC) for the T-S fuzzy NNs system to counteract deception attacks.

In addition, avoiding Zeno behavior (Wen et al., 2016, 2018) is a critical criterion in event-triggered control systems. Zeno behavior refers to the phenomenon where an infinite number of events occur within a finite time interval in event-triggered control systems. This behavior can lead to several adverse effects, including excessive computational resource consumption and potential system instability. To significantly save communication resources, this study integrates an internal dynamic variable into the sum-based event trigger mechanism, hereby designated as the sum-based dynamic discrete event-triggered mechanism (SDDETM). To handle with the above challenges, fuzzy network is used to model and analyze complex systems relying on the powerful fuzzy inference capabilities of fuzzy networks, which enable fuzzy networks to achieve more precise predictions, identification, and control when faced with nonlinear, time-variant, and uncertain problems. In contrast to traditional ETMs, SDDETM can decrease the frequency of triggers, thereby enhancing the performance of ETM. Considering these advancements, the principal contributions are shown as follows:

- The triggering condition of the proposed SDDETM is composed of two parts: the combination of multiple previous measurement samples and the inclusion of an inherent dynamic variable. With this SDDETM approach, Zeno behavior can be easily excluded.
- Different from the traditional method which uses the Bernoulli distribution to characterize deception attacks, we regard the attack process as a Markov process, and therefore, adopt the Bernoulli process to characterize the occurrence of deceptive attacks.
- A DOFC is proposed to ensure stability of the T-S fuzzy NNs system. Both communication channels from the sensor to DOFC and DOFC to zero-order hold (ZOH) are subject to stochastic deception attacks.

Notation: The symbol "$*$" denotes the symmetric counterparts within a symmetric matrix. The notation $[*]Ux$ is employed to encapsulate the quadratic form, specifically, $[*]Ux = x^TUx$. $He(A) \triangleq A^T + A$. $\mathbb{R}^p$ denotes the p-dimensional

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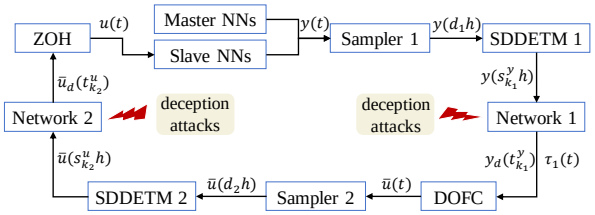

Fig. 1 System schematic representation of the NNs. DOFC: dynamic output feedback controller; NNs: neural networks; SDDETM: sum-based dynamic discrete event-triggered mechanism

Euclidean space, and $\mathbb{R}^{p\times q}$ is the set of real $p\times q$ matrices. $\mathcal{N}_{m+}$ and $\mathcal{N}_m$ denotes the set of positive integers from 1 to m and non-negative integers from 0 to m, respectively. $\mathcal{S}_n^+$ denotes the collection of $n\times n$ symmetric positive definite matrices.

2 System formulation

Fig. 1 depicts the schematic representation of delayed T-S fuzzy NNs subjected to deception attacks. We will introduce each part of the diagram in detail.

2.1 The T-S fuzzy NNs model

The inferred time-delay T-S fuzzy NNs with M plant rule nonlinear system is described as:

Plant Rule l: IF $\theta_1(t)$ denotes υ_1^l, $\cdots$, $\theta_k(t)$ denotes υ_k^l, then

$$\begin{cases}\dot{x}_m(t) = & A_l x_m(t)+B_l\mathscr{F}(x_m(t))+C_l\mathscr{F}(x_m(t-h(t)))\\ & +\mathcal{I}\\ y_m(t) = & D_l x_m(t)\end{cases}$$

where $x_m(t)=\mathrm{col}\{x_{m1}(t),x_{m2}(t),\cdots,x_{ma}(t)\}\in\mathbb{R}^a$ and $y_m(t)\in\mathbb{R}^c$ denote the state vector and the measurable output, respectively. The system matrices A_l, B_l, $C_l\in\mathbb{R}^{a\times a}$, $D_l\in\mathbb{R}^{c\times a}$ are known, and $\mathcal{I}\in\mathbb{R}^a$ represents the input bias, respectively. The function $\mathscr{F}(\cdot)$ represents the neuron activation function. The vector $\theta=[\theta_1(t),\cdots,\theta_k(t)]\in\mathbb{R}^k$ denotes k premise variables. υ_k^l, $l\in\mathcal{N}_L$ is the fuzzy set, where L represents the count of IF-THEN rules. Similarly, the slave system is defined as follows:

Plant Rule l: IF $\theta_1(t)$ denotes υ_1^l, $\cdots$, $\theta_k(t)$ denotes υ_k^l, then

$$\begin{cases}\dot{x}_s(t) = & A_l x_s(t)+B_l\mathscr{F}(x_s(t))+C_l\mathscr{F}(x_s(t-h(t)))\\ & +E_l u(t,r_t,q_t)+\mathcal{I}\\ y_s(t) = & D_l x_s(t)\end{cases}$$

where $x_s(t)$ and $y_s(t)$ present the state vector and measurable output of the response system, respectively. The control input is represented by $u(t,r_t,q_t)$, and E_l is the input matrix. Through fuzzy aggregation, the T-S fuzzy master and slave systems can be formulated in the following manner:

$$\begin{cases}\dot{x}_m(t) = & \sum_{l=1}^{L} h_l(\theta(t))[A_l x_m(t)+B_l\mathscr{F}(x_m(t))\\ & +C_l\mathscr{F}(x_m(t-h(t)))+\mathcal{I}]\\ y_m(t) = & \sum_{l=1}^{L} h_l(\theta(t))D_l x_m(t)\end{cases}\tag{1}$$

and

$$\begin{cases}\dot{x}_s(t) = & \sum_{l=1}^{L} h_l(\theta(t))[A_l x_s(t)+B_l\mathscr{F}(x_s(t))\\ & +C_l\mathscr{F}(x_s(t-h(t)))+E_l u(t,r_t,q_t)+\mathcal{I}]\\ y_s(t) = & \sum_{l=1}^{L} h_l(\theta(t))D_l x_s(t)\end{cases}\tag{2}$$

where $h_l(\theta(t))=\frac{\mu_l(\theta(t))}{\sum_{l=1}^{L}\mu_l(\theta(t))}$, $0\le h_l(\theta(t))\le 1$, $\sum_{l=1}^{M} h_l(\theta(t))=1$, $\mu_l(\theta(t))=\Pi_{d=1}^{k}\varrho_d^l(\theta(t))$, $\varrho_d^l(\theta(t))$ is the grade of membership function of $\theta(t)$ in υ_d^l (Zadeh, 1968). From Eqs (1) and (2), the synchronization error system is represented as follows:

$$\begin{cases}\dot{x}_e(t) = & \sum_{l=1}^{L} h_l(\theta(t))[A_l x_e(t)+B_l\varphi(x_e(t))\\ & +C_l\varphi(x_e(t-h(t)))+E_l u(t,r_t,q_t)]\\ y(t) = & \sum_{l=1}^{L} h_l(\theta(t))D_l x_e(t)\end{cases}$$

where $x_e(t)=x_s(t)-x_m(t)$, $\varphi(x_e(t))=\mathscr{F}(x_s(t))-\mathscr{F}(x_m(t))$, $\varphi(x_e(t-h(t)))=\mathscr{F}(x_s(t-h(t)))-\mathscr{F}(x_m(t-h(t)))$.

2.2 SDDETM

The communication link between the sampler and DOFC is referred to as Network 1, and the channel connecting DOFC to ZOH is named Network 2. The sampler operates with a sampling period of h. The output of the two samplers consists of the data packet series $[d_1,y(d_1h)]$, $[d_2,\bar{u}(d_2h)]$, where $d_1,d_2\in\mathbb{N}^+$, and $\bar{u}(\cdot)$ is discrete-time control signal through a ZOH operation. The delays that vary over time in Network 1 and Network 2 are indicated by $\tau_1(t)\in[0,\tau_{1_M}]$ and $\tau_2(t)\in[0,\tau_{2_M}]$, respectively, with $\tau_{1_M}\in\mathbb{R}^+$ and $\tau_{2_M}\in\mathbb{R}^+$. To design the SDDETM, we make the following assumptions:

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Assumption 1 In the NNs, data transmissions over the network are in single-packet form. There exist time delays, but there are no packet dropouts or disorders in the communication network.

This assumption simplifies the analysis by avoiding the complexities introduced by packet dropouts and disorders, which are common in practical networked control systems. This assumption is widely used in the literature to focus on the impact of time delays.

Assumption 2 The adversary attempts to deteriorate the controller performance by injecting a deception signal into the sampler-DOFC and DOFC-ZOH communication channels. Specifically, the deception signal is modeled as a nonlinear function $G_s(y(t))$ and $G_c(\bar{u}(t))$, respectively, which satisfies the following conditions:

$$\begin{aligned}\|G_s(y(t))\| \leq \|G_s y(t)\|, \quad \text{with} \quad G_s^T G_s > 0\\ \|G_c(\bar{u}(t))\| \leq \|G_c \bar{u}(t)\|, \quad \text{with} \quad G_c^T G_c > 0\end{aligned}$$

where G_s and G_c are known matrices that characterize the deception attacks, and the functions $G_s(y(t))$ and $G_c(\bar{u}(t))$ represent the nonlinearities introduced by the attacks.

This assumption is realistic in the context of networked control systems, where communication channels are vulnerable to attacks. Deception attacks are a common type of cyberattack that can significantly degrade system performance.

Assumption 3 Referring (Song et al., 2019), for any $x_1, x_2 \in \mathbb{R}$, $\mathscr{A} = 1, 2, \cdots, a$, the activation function $\mathscr{F}(\cdot)$ satisfies the following conditions:

$$\mathscr{K}_{\mathscr{A}}^{-} \leq \frac{\mathscr{F}}{\mathscr{A}(x_1) - \mathscr{F}_{\mathscr{A}}(x_2)x_1 - x_2} \leq \mathscr{K}_{\mathscr{A}}^{+} \tag{3}$$

where $\mathscr{K}_{\mathscr{A}}^{-}$ and $\mathscr{K}_{\mathscr{A}}^{+}$ are predetermined scalars. For convenience, we define $\mathscr{K}_m = diag\{\mathscr{K}_1^{-}, \ldots, \mathscr{K}_a^{-}\}$, $\mathscr{K}_M = diag\{\mathscr{K}_1^{+}, \ldots, \mathscr{K}_a^{+}\}$.

This assumption ensures that the activation functions are Lipschitz continuous, which is a common requirement in the analysis of NNs. It allows us to derive stability conditions and design controllers that can handle the nonlinearities introduced by the activation functions.

The SDDETM is implemented for both Network 1 and Network 2, respectively, designated as SDDETM 1 and SDDETM 2. Within SDDETM 1, let $y(s_{k_1}^y h)$ denote the most recent sample that has been effectively sent through Network 1, and $y(d_1 h)$ denote the most recent sampled data, where $d_1 \in \mathbb{N}^+$. We establish the following criterion to identify the next triggering moment:

$$s_{k_1+1}^y h = \min_{d_1 \in \mathbb{N}^+} \{d_1 h > s_{k_1}^y h \mid \mathbb{A}_1(t) \geq 0\} \tag{4}$$

with

$$\begin{aligned}\mathbb{A}_1(t) =& \sum_{i=0}^{N_1} [e_{1_i}^T(t) \Phi_{1_i} e_{1_i}(t) - \epsilon_{1_i} y^T(d_1 h - ih) \\ & \times \Phi_{1_i} y(d_1 h - ih)] - \frac{1}{\kappa_1}\eta_1(t) \\ e_{1_i}(t) =& y(s_{k_1}^y h - ih) - y(d_1 h - ih), i \in \mathcal{N}_{N_1}\end{aligned}$$

where $\epsilon_{1_i}, \kappa_1 \in \mathbb{R}^+$, $N_1 \geq 0$, Φ_{1_i} represents the positive matrix that needs to be constructed. In Eq. (4), the $\eta_1(t)$ is generated by

$$\begin{aligned}\dot{\eta}_1(t) =& -\mu_1 \eta_1(t) + \sum_{i=0}^{N_1} [\epsilon_{1_i} y^T(d_1 h - ih) \Phi_{1_i} y(d_1 h - ih) \\ & - e_{1_i}^T(t) \Phi_{1_i} e_{1_i}(t)]\end{aligned}$$

where $\mu_1 > 0$ and $\eta_1(s_0^y h) = 0$ are given parameter and initial condition, respectively. Similarly, for SDDETM 2, we also have

$$s_{k_2+1}^u h = \min_{d_2 \in \mathbb{N}^+} \{d_2 h > s_{k_2}^u h \mid \mathbb{A}_2(t) \geq 0\} \tag{5}$$

where

$$\begin{aligned}\mathbb{A}_2(t) =& \sum_{j=0}^{N_2} [e_{2_j}^T(t) \Phi_{2_j} e_{2_j}(t) - \epsilon_{2_j} \bar{u}^T(d_2 h - jh) \\ & \times \Phi_{2_j} \bar{u}(d_2 h - jh)] - \frac{1}{\kappa_2}\eta_2(t) \\ e_{2_j}(t) =& \bar{u}(s_{k_2}^u h - jh) - \bar{u}(d_2 h - jh), j \in \mathcal{N}_{N_2} \\ \dot{\eta}_2(t) =& -\mu_2 \eta_2(t) + \sum_{j=0}^{N_2} [\epsilon_{2_i} \bar{u}^T(d_2 h - jh) \Phi_{2_j} \bar{u}(d_2 h - jh) \\ & - e_{2_j}^T(t) \Phi_{2_j} e_{2_j}(t)].\end{aligned}$$

From Eqs (4) and (5), we conclude that the SDDETM considers both the latest sample measurement and several previous ones. By modifying the parameters N_1 and N_2, the count of samples considered for determining the subsequent triggering instant can be adjusted.

Remark 1 The proposed SDDETM exhibits some advantages over traditional ETMs:

- It requires no additional hardware for monitoring of the continuous measurements, which thereby reduce computational energy for the sensor units (Donkers and Heemels, 2012; Zhang et al., 2021);

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- In contrast to the periodic ETM (Heemels et al., 2013; Guo et al., 2014; Zhang and Han, 2014), the SDDETM integrates a broader spectrum of system information to decide the sequential triggers by incorporating historical samples;
- The infamous Zeno behavior that needs intricate analysis in (Yan et al., 2019; Zhang et al., 2020) can be readily excluded due to the discrete nature of the SDDETM;
- It is present within a broader framework that encompasses the periodic ETM (Guo et al., 2014; Zhang and Han, 2014) as a specific instance when $N_1 = N_2 = 0$;
- Through the incorporation of internal dynamic variables, the SDDETM increases the minimum operation time for event triggering and further improves the efficiency of network communications.

2.3 The deception attacks

During data transmission, deception attacks are factored into the analysis, which compromises the veracity and reliability of packets through the manipulation of their contents. We postulate that the adversaries have the capability to capture the transmitted data and substitute it with deceptive content. Inspired by (Liu et al., 2018, 2019a), we model the malicious signal within sensor-DOFC and DOFC-ZOH as nonlinear function, denoted as $\mathscr{G}(y(t))$ and $\mathscr{G}(\bar{u}(t))$, respectively.

Let $\{r(t)\}_{t\geq 0}$ and $\{q(t)\}_{t\geq 0}$ denote two distinct right-continuous Markov processes, each taking values within the set $\mathcal{G} = \{1, 2\}$. These processes are dominated by their generators, denoted as $\pi = [\pi_{\mathscr{M}\mathscr{N}}]_{2\times 2}$ and $\rho = [\rho_{\mathscr{P}\mathscr{Q}}]_{2\times 2}$. The transition probabilities are given by (Kazemy et al., 2022)

$$Pr\{r_{t+\delta} = \mathscr{N} \mid r_t = \mathscr{M}\} = \begin{cases} \pi_{\mathscr{M}\mathscr{N}}\delta + o(\delta), & \mathscr{M} \neq \mathscr{N} \\ 1 + \pi_{\mathscr{M}\mathscr{M}}\delta + o(\delta), & \mathscr{M} = \mathscr{N} \end{cases}$$

$$Pr\{q_{t+\delta} = \mathscr{Q} \mid q_t = \mathscr{P}\} = \begin{cases} \rho_{\mathscr{P}\mathscr{Q}}\delta + o(\delta), & \mathscr{P} \neq \mathscr{Q} \\ 1 + \rho_{\mathscr{P}\mathscr{P}}\delta + o(\delta), & \mathscr{P} = \mathscr{Q} \end{cases}$$

where $\delta > 0$, $o(\delta)$ denotes the higher-order infinitesimal of δ, $\pi_{\mathscr{M}\mathscr{N}} \geq 0$, $\mathscr{M} \neq \mathscr{N}$, $\rho_{\mathscr{P}\mathscr{Q}} \geq 0$, $\mathscr{P} \neq \mathscr{Q}$, with $\pi_{\mathscr{M}\mathscr{M}} = -\sum_{\mathscr{M}\neq\mathscr{N}} \pi_{\mathscr{M}\mathscr{N}}$, $\rho_{\mathscr{P}\mathscr{P}} = -\sum_{\mathscr{P}\neq\mathscr{Q}} \rho_{\mathscr{P}\mathscr{Q}}$, for any $\mathscr{M}, \mathscr{P} \in \mathcal{G}$.

For DOFC, due to the attacks on the communication network during data transmission, the input consists of triggered measurements.

$$y_d(s^y_{k_1}h) = b^s(r_t)y(s^y_{k_1}h) + (1 - b^s(r_t))\mathscr{G}_s(y(s^y_{k_1}h)) \quad (6)$$

where $\mathscr{G}_s$ represents the function of characterizing deception attacks within the sampler-DOFC satisfying $\|\mathscr{G}_s(y(t))\| \leq \|G_s y(t)\|$, with $(G_s^T G_s > 0)$, $b^s(1) = 1$, $b^s(2) = 0$.

Similarly, we have the form of the real control input:

$$\bar{u}_d(s^u_{k_2}h) = b^c(q_t)\bar{u}(s^u_{k_2}h) + (1 - b^c(q_t))\mathscr{G}_c(\bar{u}(s^u_{k_c}h)) \quad (7)$$

where $\mathscr{G}_c$ denotes the function that characterizes deception attacks within the DOFC-ZOH and $\|\mathscr{G}_c(\bar{u}(t))\| \leq \|G_c\bar{u}(t)\|$ with $(G_c^T G_c > 0)$, $b^c(1) = 1$, $b^c(2) = 0$.

2.4 The T-S fuzzy DOFC

Let $t^y_{k_1}$ and $t^u_{k_2}$ represent the time series at which the DOFC (ZOH) acquires the triggered sample, with $k_1(k_2) \in \mathbb{N}^+$. Taking into account the influence of the time delay, we obtain

$$y_d(t^y_{k_1}) = y_d(s^y_{k_1}h),\ t \in [t^y_{k_1}, t^y_{k_1+1})$$
$$\bar{u}_d(t^u_{k_2}) = \bar{u}_d(s^u_{k_2}h),\ t \in [t^u_{k_2}, t^u_{k_2+1}).$$

Define

$$\phi_{k_1} \triangleq \min\{o_y \mid t^y_{k_1} + o_y h \geq t^y_{k_1+1},\ o_y \in \mathbb{N}^+\}$$
$$\phi_{k_2} \triangleq \min\{o_u \mid t^u_{k_2} + o_u h \geq t_{k_2+1},\ o_u \in \mathbb{N}^+\}$$
$$\mathcal{B}_{k_y} = [t^y_{k_1} + (k_y - 1)h, t^y_{k_1} + k_y h), k_y = 1, 2, \cdots, \phi_{k_1} - 1$$
$$\mathcal{B}_{\phi_{k_1}} = [t^y_{k_1} + (\phi_{k_1} - 1)h, t^y_{k_1+1})$$
$$\mathcal{C}_{k_u} = [t^u_{k_2} + (k_u - 1)h, t^u_{k_2} + k_u h), k_u = 1, 2, \cdots, \phi_{k_2} - 1$$
$$\mathcal{C}_{\phi_{k_2}} = [t_{k_2} + (\phi_{k_2} - 1)h,\ t^u_{k_2+1}).$$

Subsequently, the interevent interval $[t^y_{k_1}, t^y_{k_1+1})$ and $[t^u_{k_2}, t^u_{k_2+1})$ can be represented as follows:

$$[t^y_{k_1}, t^y_{k_1+1}) = \cup_{k_y=1}^{\phi_{k_1}} \mathcal{B}_{k_y},\ [t^u_{k_2}, t^u_{k_2+1}) = \cup_{k_u=1}^{\phi_{k_2}} \mathcal{B}_{k_u}. \quad (8)$$

We further define some "artificial" functions $\delta_{1_i}(t)$ $(\delta_{2_j}(t))$ and $e_{1_i}(t)$ $(e_{2_j}(t))$ where $i \in \mathcal{N}_{N_1}$ and $j \in \mathcal{N}_{N_2}$, on $[t^y_{k_1}, t^y_{k_1+1})$ $([t^u_{k_2}, t^u_{k_2+1}))$.

$$\delta_{1_i}(t) \triangleq \begin{cases} t - (s^y_{k_1} - i)h, & t \in \mathcal{B}_1 \\ t - (s^y_{k_1} - i)h - h, & t \in \mathcal{B}_2 \\ \quad\vdots & \quad\vdots \\ t - (s^y_{k_1} - i)h - (\phi_{k_1} - 1)h, & t \in \mathcal{B}_{\phi_{k_1}} \end{cases} \quad (9)$$

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$$e_{1_i}(t) \triangleq \begin{cases} y[(s^y_{k_1}-i)h]-y[(s^y_{k_1}-i)h], & t\in\mathcal{B}_1 \\ y[(s^y_{k_1}-i)h]-y[(s^y_{k_1}-i)h+h], & t\in\mathcal{B}_2 \\ \vdots & \vdots \\ y[(s^y_{k_1}-i)h]-y[(s^y_{k_1}-i)h+(\phi_{k_1}-1)h], & t\in\mathcal{B}_{\phi_{k_1}} \end{cases} \tag{10}$$

where $\delta_{2_j}(t)$ is obtained from Eq. (9) by replacing $s^y_{k_1}$ with $s^u_{k_2}$. $e_{2_j}(t)$ is obtained from Eq. (10) by replacing $s^y_{k_1}$ with $s^u_{k_2}$ and y with $\bar{u}$. Based on the definitions of $\delta_{1_i}(t)$ and $\delta_{2_j}(t)$, the following deductions can be made:

$$\underline{\delta}_{1_i} \triangleq ih \le \delta_{1_i}(t) \le (i+1)h+\tau_{1_M} \triangleq \overline{\delta}_{1_i}$$
$$\underline{\delta}_{2_j} \triangleq jh \le \delta_{2_j}(t) \le (j+1)h+\tau_{2_M} \triangleq \overline{\delta}_{2_j}.$$

Then, based on Eqs (9) and (10), the equations hold as follows:

$$y(s^y_{k_1}h-ih) = e_{1_i}(t)+y(t-\delta_{1_i}(t)),\ t\in[t^y_{k_1},t^y_{k_1+1})$$
$$\bar{u}(s^u_{k_2}h-jh) = e_{2_j}(t)+\bar{u}(t-\delta_{2_j}(t)), t\in[t^u_{k_2},t^u_{k_2+1}).$$

Remark 2 With the above analysis, the discrete inputs of DOFC and error system could be represented in continuous form as $\tilde{y}(t) = y_d(s^y_{k_1}h)$, $t\in[t^y_{k_1},t^y_{k_1+1})$, and $u(t,r_t,q_t)=\bar{u}(s^u_{k_2}h)=e_{2_0}(t)+\bar{u}(t-\delta_{2_0}(t))$, $t\in[t^u_{k_2},t^u_{k_2+1})$, respectively. Yue et al. Yue et al. (2013) first introduced the concept of transforming discrete signals into continuous ones. This concept was subsequently refined in Zhang and Han (2014). Motivated by these findings, we formulated a series of functions in Eqs (9) and (10) to manage the discrete signals within the framework of our proposed SDDETM.

Subsequently, we introduce the following T-S fuzzy DOFC.

Control Rule m: IF $\theta_1(t^y_{k_1})$ denotes υ^m_1, $\cdots$, $\theta_k(t^y_{k_1})$ denotes υ^m_k, then

$$\begin{cases} \dot{x}_c(t) = & A_{km}x_c(t)+B_{km}x_c(t-\delta_{1_0}(t)) \\ & +C_{km}x_c(t-\delta_{2_0}(t))+D_{km}\tilde{y}(t) \\ \bar{u}(t) = & F_{km}x_c(t) \end{cases} \tag{11}$$

where $x_c(t)\in\mathbb{R}^a$ is the DOFC's internal state, A_{km}, B_{km}, C_{km}, D_{km} and F_{km} are the controller matrices. Hence, the associated fuzzy DOFC can be expressed as follows:

$$\begin{cases} \dot{x}_c(t) = & \sum_{m=1}^{L} h_m(\theta(t^y_{k1}))[A_{km}x_c(t)+B_{km}x_c(t-\delta_{1_0}(t)) \\ & +C_{km}x_c(t-\delta_{2_0}(t))+D_{km}\tilde{y}(t)] \\ \bar{u}(t) = & \sum_{m=1}^{L} h_m(\theta(t^y_{k1}))F_{km}x_c(t) \end{cases} \tag{12}$$

where $h_m(\theta(t^y_{k_1})) = \frac{\lambda_m(\theta(t^y_{k_1}))}{\sum_{d=1}^{M}\lambda_d(\theta(t^y_{k_1}))}$, $h_m(\theta(t^y_{k_1}))\in[0,1]$, $\sum_{m=1}^{L} h_m(\theta(t^y_{k_1})) = 1$, $\lambda_m(\theta(t^y_{k_1})) = \Pi_{d=1}^{k}\varrho^m_d(\theta(t^y_{k_1}))$, $\varrho^m_d(\theta(t^y_{k_1}))$ is the grade membership of $\theta(t^y_{k_1})$ in υ^m_d.

Assumption 4 It is assumed that there is no direct relationship between system delay and transmission delay during the data transmission process in the closed-loop system.

2.5 The design objective

To simplify the formulations, let $h_l = h_l(\theta(t))$, $h_m = h_m(\theta(t^y_{k1}))$, $l,m\in\mathcal{N}_L$, $b^s(r_t)=b^s$, $b^c(q_t)=b^c$, $1-b^s(r_t)=\bar{b}^s$, $1-b^c(q_t)=\bar{b}^c$. From Eqs (2.1) and (12), for $t\in\mathcal{I}_c=[t^y_{k_1},t^y_{k_1+1})\cup[t^u_{k_2},t^u_{k_2+1})$, the augmented closed-loop system is formulated as below:

$$\begin{aligned}\dot{x}(t) = \sum_{l=1}^{L}\sum_{m=1}^{L} h_l h_m[&A^0_{lm}x(t)+A^1_{lm}x(t-\delta_{1_0}(t))+A^2_{lm} \\ &\times x(t-\delta_{2_0}(t))+A^3_m e_{1_0}(t)+A^4_l e_{2_0}(t)+A^5_l\varphi_1(x(t)) \\ &+A^6_l\varphi_2(x(t-h(t)))+A^7_{lm}g_s(y(t^y_{k_1}))+A^8_l g_c(\bar{u}(t^u_{k_2}))]\end{aligned} \tag{13}$$

where $x(t)=\mathrm{col}\{x_e(t),\ x_c(t)\}$,

$$A^0_{lm}=\begin{bmatrix}A_l & 0\\ 0 & A_{km}\end{bmatrix}, A^1_{lm}=\begin{bmatrix}0 & 0\\ b^sD_{km}D_l & B_{km}\end{bmatrix}$$
$$A^2_{lm}=\begin{bmatrix}0 & b^sE_lF_{km}\\ 0 & C_{km}\end{bmatrix}, A^3_m=\begin{bmatrix}0\\ b^sD_{km}\end{bmatrix}, A^4_l=\begin{bmatrix}b^cE_l\\ 0\end{bmatrix}$$
$$A^5_l=\begin{bmatrix}B_l & 0\\ 0 & 0\end{bmatrix}, A^6_l=\begin{bmatrix}C_l & 0\\ 0 & 0\end{bmatrix}$$
$$A^7_m=\begin{bmatrix}0\\ \bar{b}^sD_{km}\end{bmatrix}, A^8_l=\begin{bmatrix}\bar{b}^cE_l\\ 0\end{bmatrix}.$$

Similar to (Tan et al., 2020), to account for asynchronous premise variables, we assume the presence of $\alpha_m>0$ and $\nu_m>0$, provided that the following inequalities hold:

$$|h_m(\theta(t^y_{k1})-h_m(\theta(t))|\le\nu_m,\ h_m(\theta(t^y_{k1}))=\alpha_m h_m(\theta(t)).$$

It is obvious that $\gamma^m_1 = 1-\frac{\nu_m}{h_l(\theta(t))} \le \alpha_m \le 1+\frac{\nu_m}{h_l(\theta(t))}=\gamma^m_2$. Therefore

$$\begin{aligned}\zeta_1 = \frac{\min\{\gamma^m_1\}}{\max\{\gamma^m_2\}} &\le \frac{\min\{\alpha_l\}}{\max\{\alpha_m\}} \le \frac{\alpha_l}{\alpha_m} \\ &\le \frac{\max\{\alpha_l\}}{\min\{\alpha_m\}} \le \frac{\min\{\gamma^m_1\}}{\max\{\gamma^m_2\}} = \zeta_2.\end{aligned} \tag{14}$$

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The augmented closed-loop system [Eq. (13)] is represented as below:

$$\begin{aligned}\dot{x}(t)=&\sum_{l=1}^{L}\sum_{m=1}^{L}\alpha_m h_l h_m[A_{lm}^0x(t)+A_{lm}^1x(t-\delta_{1_0}(t))+A_{lm}^2\\&\times x(t-\delta_{2_0}(t))+A_{lm}^3e_{1_0}(t)+A_l^4e_{2_0}(t)+A_l^5\varphi_1(x(t))\\&+A_l^6\varphi_2(x(t-h(t)))+A_{lm}^7g_s(y(t_{k_1}^y))+A_l^8g_c(\bar{u}(t_{k_2}^u))].\end{aligned}\tag{15}$$

The primary objective of this study is to concurrently optimize the parameters of the DOFC [Eq. (12)] and the SDDETM Φ_{1_i},Φ_{2_j} in Eqs (4) and (5) to make the closed-loop system [Eq. (15)] asymptotically stable, thereby minimizing unnecessary consumption. This involves ensuring system stability under deception attacks and efficient use of network resources through the SDDETM.

3 Main results

In this section, the stability criteria for the closed-loop system [Eq. (15)] are established in Theorem 1. The sufficient conditions for obtaining stabilizing DOFC and SDDETM parameters are presented in Proposition 1 and Theorem 2.

Theorem 1 For $i\in\mathcal{N}_{N_1}$, $j\in\mathcal{N}_{N_2}$, $\mathscr{B}=\{1,2\}$, $\mathscr{C}=\{3,4\}$, given positive scalars $\underline{\delta}_{1_i}$, $\overline{\delta}_{1_i}$, $\underline{\delta}_{2_j}$, $\overline{\delta}_{2_j}$, ϵ_{1_i}, ϵ_{2_j}, $N_{\mathscr{B}}$, $w_{\mathscr{B}}$, $\mu_{\mathscr{B}}$, $\zeta_{\mathscr{B}}$, and the diagonal matrices $\mathscr{K}_m$, $\mathscr{K}_M$, and real matrix G_s, G_c, the closed-loop system [Eq. (15)] is asymptotically stable with the proposed SDDETM if there exist positive diagonal matrices T_1 and T_2, P, $P_{\mathscr{B}i}$, $P_{\mathscr{C}j}$, $P_5\in\boldsymbol{S}_n^+$, $\Phi_{1_i}\in\boldsymbol{S}_c^+$, $\Phi_{2_j}\in\boldsymbol{S}_b^+$, and matrices X_{1_i}, X_{2_j}, X_3 such that the following inequalities hold:

$$\Theta_l^{11}=\begin{bmatrix}\Pi_{ll}^{11}&*\\ \Pi_{ll}^{12}&\Pi_{13}\end{bmatrix}<0\tag{16}$$

$$\Theta_{lm}^{12}=\begin{bmatrix}\Pi_{lm}^{11}+\zeta_1\Pi_{ml}^{11}&*&*\\ \Pi_{lm}^{12}&\Pi_{13}&*\\ \zeta_2\Pi_{ml}^{12}&0&\zeta_2\Pi_{13}\end{bmatrix}<0\tag{17}$$

$$\Theta_{lm}^{13}=\begin{bmatrix}\Pi_{lm}^{11}+\zeta_2\Pi_{ml}^{11}&*&*\\ \Pi_{lm}^{12}&\Pi_{13}&*\\ \zeta_2\Pi_{ml}^{12}&0&\zeta_2\Pi_{13}\end{bmatrix}<0\tag{18}$$

$$\begin{bmatrix}P_{2_i}&X_{1_i}\\ *&P_{2_i}\end{bmatrix}\geq 0,\quad\begin{bmatrix}P_{4_j}&X_{2_j}\\ *&P_{4_j}\end{bmatrix}\geq 0,\quad\begin{bmatrix}P_5&X_3\\ *&P_5\end{bmatrix}\geq 0$$

where

$$\Pi_{lm}^{11}=\begin{bmatrix}\beta_{lm}^{11}&\beta_{lm}^{12}&\beta_{lm}^{13}&\beta_l^{14}&0&PA_m^7&PA_l^8\\ *&\beta^{22}&0&\beta^{24}&0&0&0\\ *&*&\beta^{33}&0&0&0&0\\ *&*&*&\beta^{44}&0&0&0\\ *&*&*&*&\beta^{55}&0&0\\ *&*&*&*&*&-w_1I&0\\ *&*&*&*&*&*&-w_2I\end{bmatrix}$$

$$\begin{aligned}\beta_{lm}^{11}=&He(PA_{lm}^0)-\sum_{i=0}^{N_1}P_{1i}-\sum_{j=0}^{N_2}P_{3j}-P_5\\&-2E_1^T\mathscr{K}_m^TT_1\mathscr{K}_ME_1,\ E_1=[I_n\ 0_n],\ E_2=[0_n\ I_n]\\ \beta_{lm}^{12}=&[P\mathcal{A}_{lm}^1+[\vartheta_{lm}^{10}\ \cdots\ \vartheta_{lm}^{1N_1}],\ P\mathcal{A}_{lm}^2+[\vartheta_{lm}^{20}\ \cdots\ \vartheta_{lm}^{2N_2}]\\&P_5-X_3^T,\ X_3^T],\ \vartheta_{lm}^{1i}=[P_{1i}\ 0_{n,2n}],\ \vartheta_{lm}^{2j}=[P_{3j}\ 0_{n,2n}]\\ \mathcal{A}_{lm}^1=&[0_n,\ A_{lm}^1,\ 0_{n,(3N_1+1)n}]\\ \mathcal{A}_{lm}^2=&[0_n,\ A_{lm}^2,\ 0_{n,(3N_2+1)n}]\\ \beta_{lm}^{13}=&P[\mathcal{A}_m^3\ \mathcal{A}_l^4],\ \beta_l^{14}=[PA_l^5+E_1^TT_1(k_m+k_M)\ PA_l^6]\\ \mathcal{A}_m^3=&[A_m^3\ 0_{n,N_1c}],\ \mathcal{A}_l^4=[A_l^4\ 0_{n,N_2b}]\\ \beta^{22}=&\mathrm{diag}\{M_{10},M_{11},\cdots,M_{1N_1},M_{20},M_{21},\cdots,M_{2N_2},M_3\}\\ M_{1i}=&\begin{bmatrix}-P_{1i}-P_{2i}&P_{2i}-X_{1i}^T&X_{1i}^T\\ *&-2P_{2i}+He(X_{1i})&P_{2i}-X_{1i}^T\\ *&*&-P_{2i}\end{bmatrix}\\ M_{2j}=&\begin{bmatrix}-P_{3j}-P_{4j}&P_{4j}-X_{2j}^T&X_{2j}^T\\ *&-2P_{4j}+He(X_{2j})&P_{4j}-X_{2j}^T\\ *&*&-P_{4j}\end{bmatrix}\\ M_3=&\begin{bmatrix}-2P_5+He(X_3)-2E_1^T\mathscr{K}_m^TT_2\mathscr{K}_ME_1&P_5-X_3^T\\ *&-P_5\end{bmatrix}\\ \beta^{33}=&-\mathrm{diag}\{\Phi_{10},\Phi_{11},\cdots,\Phi_{1N_1},\Phi_{20},\Phi_{21},\cdots,\Phi_{2N_2}\}\\ \beta_l^{24}=&\begin{bmatrix}0_{3(N_1+N_2+2)n,a}&0_{3(N_1+N_2+2)n,a}\\ 0_{2n,a}&[(\mathscr{K}_m+\mathscr{K}_M)^TT_2^TE_1^T\ 0_{a,n}]^T\end{bmatrix}\\ \beta^{44}=&\mathrm{diag}\{-2T_1,-2T_2\},\ \beta^{55}=\mathrm{diag}\{-\mu_1I,-\mu_2I\}\\ \Pi_{lm}^{12}=&[(F_{lm}^1)^T\mathcal{D}\ \ (F_{lm}^2)^T\ \ (F_l^3)^T\ \ (F_m^4)^T]^T\\ F_{lm}^1=&[A_{lm}^0\ \mathcal{A}_{lm}^1\ \mathcal{A}_{lm}^2\ 0_{n,2n}\ \mathcal{A}_m^3\ \mathcal{A}_l^4\ A_l^5\ A_l^6\ 0_{n,2}\ A_m^7\ A_l^8]\\ \mathcal{D}=&[\underline{\delta}_{1_0},\cdots,\underline{\delta}_{1N_1},(\overline{\delta}_{1_0},-\underline{\delta}_{1_0}),\cdots,(\overline{\delta}_{1N_1},-\underline{\delta}_{1N_1}),\underline{\delta}_{2_0},\\&\cdots,\underline{\delta}_{2N_2},(\overline{\delta}_{2_0},-\underline{\delta}_{2_0}),\cdots,(\overline{\delta}_{2N_2},-\underline{\delta}_{2N_2}),h_M]\\ F_{lm}^2=&\mathrm{col}\{H_{10},\ H_{11},\ \cdots,\ H_{1N_1},\ H_{20},\ H_{21},\ \cdots,\ H_{2N_2}\}\\ H_{1i}=&[0_{c,(3i-1)n}\ D_lE_1\ 0_{c,[3(N_1+N_2-i)+10]n+N}],\ i\in\mathcal{N}_{N_1}\\ H_{2j}=&[0_{b,[3(N_1+j)+2]n}\ F_{km}E_2\ 0_{b,[3(N_2-j)+7]n+N}],\ j\in\mathcal{N}_{N_2}\\ F_l^3=&[0_{c,2n}\ G_sD_lE_1\ 0_{c,[3(N_1+N_2)+6]n}\ G_s\ 0_{c,N+n-c}]\\ F_m^4=&[0_{b,(3N_1+5)n}\ G_CF_{km}E_2\ 0_{b,(3N_2+3)n+(N_1+1)c}\\&G_c\ 0_{b,(N_2+1)b+n+c+2}]\\ N=&(N_1+2)C+(N_2+2)b+2\\ \Pi_{13}=&\mathrm{diag}\{\mathcal{E},(\Delta\beta_l^{33})^{-1},-I,-I\}\\ \mathcal{E}=&-\mathrm{diag}\{P_{10}^{-1},\cdots,P_{1N_1}^{-1},P_{20}^{-1},\cdots,P_{2N_1}^{-1},P_{30}^{-1},\cdots,\\&P_{3N_2}^{-1},P_{40}^{-1},\cdots,P_{4N_2}^{-1},P_5^{-1}\}\\ \Delta=&\mathrm{diag}\{\epsilon_{1_0},\epsilon_{1_1},\cdots,\epsilon_{1N_1},\epsilon_{2_0},\epsilon_{2_1},\cdots,\epsilon_{2N_2}\}.\end{aligned}$$

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Proof. Choose a Lyapunov functional candidate as follows:

$$V(t) = V_1(t) + \sum_{i=0}^{N_1} V_2^i(t), + \sum_{j=0}^{N_2} V_3^j(t) + V_4(t) \tag{19}$$

where

$$\begin{aligned}
V_1(t) =& x^T(t)Px(t) + \eta_1(t) + \eta_2(t)\\
V_2^i(t) =& \underline{\delta}_{1_i} \int_{-\underline{\delta}_{1_i}}^{0} \int_{t+\theta}^{t} \dot{x}^T(s) P_{1_i} \dot{x}(s) ds d\theta\\
&+ (\overline{\delta}_{1_i} - \underline{\delta}_{1_i}) \int_{-\overline{\delta}_{1_i}}^{-\underline{\delta}_{1_i}} \int_{t+\theta}^{t} \dot{x}^T(s) P_{2_i} \dot{x}(s) ds d\theta\\
V_3^j(t) =& \underline{\delta}_{2_j} \int_{-\underline{\delta}_{2_j}}^{0} \int_{t+\theta}^{t} \dot{x}^T(s) P_{3_j} \dot{x}(s) ds d\theta\\
&+ (\overline{\delta}_{2_j} - \underline{\delta}_{2_j}) \int_{-\overline{\delta}_{2_j}}^{-\underline{\delta}_{2_j}} \int_{t+\theta}^{t} \dot{x}^T(s) P_{4_j} \dot{x}(s) ds d\theta\\
V_4(t) =& h_M \int_{-h_M}^{0} \int_{t+\theta}^{t} \dot{x}^T(s) P_5 \dot{x}(s) ds d\theta.
\end{aligned} \tag{20}$$

From Eqs (6) and (7), for $t \in \mathcal{I}_c$, we have

$$\begin{aligned}
\mathcal{A}_3 =& y(t)^T w_1 G_s^T G_s y(t) - w_1 \mathscr{G}_s^T(y(t)) \mathscr{G}_s(y(t)) \geq 0\\
\mathcal{A}_4 =& \bar{u}(t)^T w_2 G_c^T G_c \bar{u}(t) - w_2 \mathscr{G}_c(\bar{u}(t)) \mathscr{G}_c^T(\bar{u}(t)) \geq 0.
\end{aligned}$$

Thus, the time derivative of $V_k(t)$, $(k \in \mathcal{N}_{4+})$ satisfies

$$\begin{aligned}
\mathbb{E}\{\dot{V}(t)\} \leq & \sum_{l=1}^{M} \sum_{m=1}^{L} 2\alpha_m h_l h_m \mathbb{E}\{x^T(t) P \dot{x}(t)\} + \mathcal{A}_3 + \mathcal{A}_4\\
&+ \mathbb{E}\{\dot{x}^T(t) \mathcal{F} \dot{x}(t)\} + \dot{\eta}_1 + \dot{\eta}_2\\
&- \sum_{i=0}^{N_1} [\underline{\delta}_{1_i} \int_{t-\underline{\delta}_{1_i}}^{t} \dot{x}^T(s) P_{1_i} \dot{x}(s) ds\\
&+ (\overline{\delta}_{1_i} - \underline{\delta}_{1_i}) \int_{t-\overline{\delta}_{1_i}}^{t-\underline{\delta}_{1_i}} \dot{x}^T(s) P_{2_i} \dot{x}(s) ds]\\
&- \sum_{j=0}^{N_2} [\underline{\delta}_{2_j} \int_{t-\underline{\delta}_{2_j}}^{t} \dot{x}^T(s) P_{3_j} \dot{x}(s) ds\\
&- (\overline{\delta}_{2_j} - \underline{\delta}_{2_j}) \int_{t-\overline{\delta}_{2_j}}^{t-\underline{\delta}_{2_j}} \dot{x}^T(s) P_{4_j} \dot{x}(s) ds]\\
&- h_M \int_{t-h_M}^{t} \dot{x}^T(s) P_5 \dot{x}(s) ds
\end{aligned} \tag{21}$$

where $\mathcal{F} = \sum_{i=0}^{N_1} [\underline{\delta}_{1_i}^2 P_{1_i} + (\overline{\delta}_{1_i} - \underline{\delta}_{1_i})^2 P_{2_i}] + h_M^2 G_5 + \sum_{j=0}^{N_2} [\underline{\delta}_{2_j}^2 P_{3_j} + (\overline{\delta}_{2_j} - \underline{\delta}_{2_j})^2 P_{4_j}]$. From Jensen's inequality (Boyd et al., 1994), it follows that

$$\begin{aligned}
&-\underline{\delta}_{1_i} \int_{t-\underline{\delta}_{1_i}}^{t} \dot{x}^T(s) P_{1_i} \dot{x}(s) ds \leq\\
&\qquad -[*]^T P_{1_i} [x(t) - x(t - \underline{\delta}_{1_i})]\\
&-\underline{\delta}_{2_j} \int_{t-\underline{\delta}_{2_j}}^{t} \dot{x}^T(s) P_{3_j} \dot{x}(s) ds \leq\\
&\qquad -[*]^T P_{3_j} [x(t) - x(t - \underline{\delta}_{2_j})].
\end{aligned}$$

Subsequently, utilizing Lemma 3 in (Zhang et al., 2020), the integral terms in Eq. (19) can be relaxed as:

$$\begin{aligned}
&(\underline{\delta}_{1_i} - \overline{\delta}_{1_i}) \int_{t-\overline{\delta}_{1_i}}^{t-\underline{\delta}_{1_i}} \dot{x}^T(s) P_{2_i} \dot{x}(s) ds\\
&\qquad \leq [*] \mathcal{M}_1^i \begin{bmatrix} x(t - \underline{\delta}_{1_i}) \\ x(t - \delta_{1_i}(t)) \\ x(t - \overline{\delta}_{1_i}) \end{bmatrix}\\
&(\underline{\delta}_{2_j} - \overline{\delta}_{2_j}) \int_{t-\overline{\delta}_{2_j}}^{t-\underline{\delta}_{2_j}} \dot{x}^T(s) P_{4_j} \dot{x}(s) ds\\
&\qquad \leq [*] \mathcal{M}_2^j \begin{bmatrix} x(t - \underline{\delta}_{2_j}) \\ x(t - \delta_{2_j}(t)) \\ x(t - \overline{\delta}_{2_j}) \end{bmatrix}
\end{aligned}$$

where $\mathcal{M}_1^i = M_1^i + \mathrm{diag}\{P_{1i}, 0, 0\}$, $\mathcal{M}_2^j = M_2^j + \mathrm{diag}\{P_{3j}, 0, 0\}$.

From Assumption 3, there exist positive diagonal matrices T_1 and T_2 making the following inequalities hold:

$$[*] \begin{bmatrix} -2\mathscr{K}_m T_1 \mathscr{K}_M & T_1(\mathscr{K}_m + \mathscr{K}_M) \\ * & -2T_1 \end{bmatrix} \begin{bmatrix} x_e(t) \\ f(x_e(t)) \end{bmatrix} \geq 0$$

$$[*] \begin{bmatrix} -2\mathscr{K}_m T_2 \mathscr{K}_M & T_2(\mathscr{K}_m + \mathscr{K}_M) \\ * & -2T_2 \end{bmatrix} \begin{bmatrix} x_e(t - h(t)) \\ f(x_e(t - h(t))) \end{bmatrix} \geq 0.$$

Define an augmented state vector as

$$\begin{aligned}
\xi(t) = \mathrm{col}\{&x(t), \mathcal{X}_1(t), \mathcal{X}_2(t), \mathcal{X}_3(t), \mathscr{E}_1(t), \mathscr{E}_2(t), \varphi(x(t)),\\
&\varphi(x(t - h(t))), \sqrt{\eta_1(t)}, \sqrt{\eta_2(t)}, \mathscr{G}_s(y(t)), \mathscr{G}_c(\bar{u}(t))\}
\end{aligned} \tag{22}$$

where

$$\begin{aligned}
\mathcal{X}_1(t) =& [x(t - \underline{\delta}_{10})\ x(t - \delta_{10}(t))\ x(t - \overline{\delta}_{10})\ \cdots\\
&\quad x(t - \underline{\delta}_{1N_1})\ x(t - \delta_{1N_1}(t))\ x(t - \overline{\delta}_{1N_1})]\\
\mathcal{X}_2(t) =& [x(t - \underline{\delta}_{20})\ x(t - \delta_{20}(t))\ x(t - \overline{\delta}_{20})\ \cdots\\
&\quad x(t - \underline{\delta}_{2N_2})\ x(t - \delta_{2N_2}(t))\ x(t - \overline{\delta}_{2N_2})]\\
\mathcal{X}_3(t) =& [x(t - h(t))\ x(t - h_M)]\\
\mathscr{E}_1(t) =& [e_{10}(t)\ e_{11}(t)\ \cdots\ e_{1N_1}(t)]\\
\mathscr{E}_2(t) =& [e_{20}(t)\ e_{21}(t)\ \cdots\ e_{2N_2}(t)].
\end{aligned}$$

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Then, from Eqs (21) and (22), we have $\dot{V}(t) \leq 0$ if and only if the subsequent inequalities are satisfied:

$$\sum_{l=1}^{M-1}\sum_{m\neq l}^{M}\alpha_m h_l h_m\{\Pi_{lm}^{11}-(\Pi_{lm}^{12})^T\Pi_{13}^{-1}\Pi_{lm}^{12}+\frac{\alpha_l}{\alpha_m}[\Pi_{ml}^{11}-(\Pi_{ml}^{12})^T\Pi_{13}^{-1}\Pi_{ml}^{12}]\}+\sum_{l=1}^{M}\alpha_l h_l^2[\Pi_{ll}^{11}-(\Pi_{ll}^{12})^T\Pi_{13}^{-1}\Pi_{ll}^{12}]<0.$$

Utilizing the Schur complement [Eqs (16)–(18)], SDDETM [Eqs (4)–(5)], and DOFC [Eq. (12)] implies that

$$\begin{aligned}&\Pi_{ll}^{11}-(\Pi_{ll}^{12})^T\Pi_{13}^{-1}\Pi_{ll}^{12}<0\\&\Pi_{lm}^{11}-(\Pi_{lm}^{12})^T\Pi_{13}^{-1}\Pi_{lm}^{12}+\zeta_1\Pi_{ml}^{11}-\zeta_2(\Pi_{ml}^{12})^T\Pi_{13}^{-1}\Pi_{ml}^{12}<0\\&\Pi_{lm}^{11}-(\Pi_{lm}^{12})^T\Pi_{13}^{-1}\Pi_{lm}^{12}+\zeta_2\Pi_{ml}^{11}-\zeta_2(\Pi_{ml}^{12})^T\Pi_{13}^{-1}\Pi_{ml}^{12}<0.\end{aligned}$$

Considering $(\zeta_2-\frac{\alpha_l}{\alpha_m})(\Pi_{ml}^{12})^T\Pi_{13}^{-1}\Pi_{ml}^{12}<0$, we can obtain

$$\begin{aligned}&\Pi_{lm}^{11}-(\Pi_{lm}^{12})^T\Pi_{13}^{-1}\Pi_{lm}^{12}+\zeta_1\Pi_{ml}^{11}\\&\qquad-\frac{\alpha_l}{\alpha_m}(\Pi_{ml}^{12})^T\Pi_{13}^{-1}\Pi_{ml}^{12}<0\\&\Pi_{lm}^{11}-(\Pi_{lm}^{12})^T\Pi_{13}^{-1}\Pi_{lm}^{12}+\zeta_2\Pi_{ml}^{11}\\&\qquad-\frac{\alpha_l}{\alpha_m}(\Pi_{ml}^{12})^T\Pi_{13}^{-1}\Pi_{ml}^{12}<0.\end{aligned}\tag{23}$$

From Eqs (14) and (23), we have $\Pi_{lm}^{11}-(\Pi_{lm}^{12})^T\Pi_{13}^{-1}\Pi_{lm}^{12}+\frac{\alpha_l}{\alpha_m}[\Pi_{ml}^{11}-(\Pi_{ml}^{12})^T\Pi_{13}^{-1}\Pi_{ml}^{12}]<0$. The asymptotic stability of the closed-loop system [Eq. (15)] can be ensured by Eqs (16)–(18). Consequently, the proof is thereby established. □

Remark 3 By referring to the research progress on delay systems in recent papers (Zhang et al., 2018; Chen et al., 2022), the Lyapunov functional candidate [Eq. (19)] in this study can be further improved from the following two aspects. First, the incorporation of a quadratic matrix that is affinely dependent on the delay offers a more generalized framework than the conventional constant matrix. Secondly, the utilization of the two double-integral terms in $V_4(t)$ to represent the interval $[t-h_M,t]$ as $[t-h_M,t-h(t)]$ and $[t-h(t),t]$, which introduces two different matrices G_{51} and G_{52}. This augmented Lyapunov functional scheme has the potential to further reduce the conservatism of the design results, which will be explored in future research.

In Theorem 1, it is observed that the DOFC parameters A_{km}, B_{km}, C_{km}, D_{km}, and F_{km} are intertwined with the Lyapunov matrix P, rendering the problem nonconvex and the DOFC unresolvable. Specifically, the resolution is divided into two parts. First, we decouple of the DOFC parameters and matrix P in Proposition 1. Furthermore, the nonconvex LMIs are transformed into convex formulations, which referring to cone complement linearization (CCL) approach in Theorem 2.

Proposition 1 For $i\in\mathcal{N}_{N_1}$, $j\in\mathcal{N}_{N_2}$, $\mathscr{B}=\{1,2\}$, $\mathscr{C}=\{3,4\}$, given positive scalars $\underline{\delta}_{1_i}$, $\overline{\delta}_{1_i}$, $\underline{\delta}_{2_j}$, $\overline{\delta}_{2_j}$, ϵ_{1_i}, ϵ_{2_j}, $N_{\mathscr{B}}$, $w_{\mathscr{B}}$, $\mu_{\mathscr{B}}$, $\zeta_{\mathscr{B}}$, the diagonal matrices $\mathscr{K}_m$, $\mathscr{K}_M$, and real matrix G_s, G_c, the close-loop system [Eq. (15)] is asymptotically stable with the proposed SDDETM, if there exist positive diagonal matrices T_1 and T_2, and X, $Y\in\boldsymbol{S}_a^+$, $\tilde{P}_{\mathscr{B}_i}$, $\tilde{P}_{\mathscr{C}_j}$, $\tilde{P}_5\in\boldsymbol{S}_n^+$, $\Phi_{1_i}\in\boldsymbol{S}_c^+$, $\tilde{\Phi}_{2_j}\in\boldsymbol{S}_b^+$, matrices $\tilde{X}_{1_i}$, $\tilde{X}_{2_j}$ and $\tilde{X}_3$ of appropriate dimensions such that

$$\Theta_l^{21}=\begin{bmatrix}\Pi_{ll}^{21} & *\\ \Pi_{ll}^{22} & \Pi^{23}\end{bmatrix}\leq 0\tag{24}$$

$$\Theta_{lm}^{22}=\begin{bmatrix}\Pi_{lm}^{21}+\zeta_1\Pi_{ml}^{21} & * & *\\ \Pi_{lm}^{22} & \Pi^{23} & *\\ \zeta_2\Pi_{ml}^{22} & 0 & \zeta_2\Pi^{23}\end{bmatrix}\leq 0\tag{25}$$

$$\Theta_{lm}^{23}=\begin{bmatrix}\Pi_{lm}^{21}+\zeta_2\Pi_{ml}^{21} & * & *\\ \Pi_{lm}^{22} & \Pi^{23} & *\\ \zeta_2\Pi_{ml}^{22} & 0 & \zeta_2\Pi^{23}\end{bmatrix}\leq 0\tag{26}$$

$$\begin{aligned}&Z=\begin{bmatrix}X & I\\ * & Y\end{bmatrix}\geq 0,\ \begin{bmatrix}\tilde{P}_{2_i} & \tilde{X}_{1_i}\\ * & \tilde{P}_{2_i}\end{bmatrix}\geq 0\\&\begin{bmatrix}\tilde{P}_{4_j} & \tilde{X}_{2_j}\\ * & \tilde{P}_{4_j}\end{bmatrix}\geq 0,\ \begin{bmatrix}\tilde{P}_5 & \tilde{X}_3\\ * & \tilde{P}_5\end{bmatrix}\geq 0\end{aligned}\tag{27}$$

where

$$\Pi_{lm}^{21}=\begin{bmatrix}\gamma_{lm}^{11} & \gamma_{lm}^{12} & \gamma_{lm}^{13} & \gamma_l^{14} & 0 & \psi_l^7 & \psi_l^8\\ * & \gamma_l^{22} & 0 & \gamma_l^{24} & 0 & 0 & 0\\ * & * & \beta_l^{33} & 0 & 0 & 0 & 0\\ * & * & * & \beta_l^{44} & 0 & 0 & 0\\ * & * & * & * & \beta_l^{55} & 0 & 0\\ * & * & * & * & * & -w_1 I & 0\\ * & * & * & * & * & * & -w_2 I\end{bmatrix}$$

$$\begin{aligned}&\gamma_{lm}^{11}=He(\psi_l^0)-\sum_{i=0}^{N_1}\tilde{P}_{1i}-\sum_{j=0}^{N_2}\tilde{P}_{3j}-\tilde{P}_5\\&\qquad-2J_1^TE_1^T\mathscr{K}_m^TT_1\mathscr{K}_mJ_1E_1\\&\gamma_{lm}^{12}=[P\tilde{\mathcal{A}}_{lm}^1+[\tilde{\vartheta}_{lm}^{10}\ \cdots\ \tilde{\vartheta}_{lm}^{1N_1}],\ P\tilde{\mathcal{A}}_{lm}^2+[\tilde{\vartheta}_{lm}^{20}\ \cdots\ \tilde{\vartheta}_{lm}^{2N_2}],\\&\qquad\tilde{P}_5-\tilde{X}_3^T,\ \tilde{X}_3^T]\\&\vartheta_{lm}^{1i}=[\tilde{P}_{1i}\ 0_{n,2n}],\ \vartheta_{lm}^{2j}=[\tilde{P}_{3j}\ 0_{n,2n}]\end{aligned}$$

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$\tilde{\mathcal{A}}^1_{lm} = [0_n\ \psi^1_{lm}\ 0_{n,(3N_1+1)n}],\ \tilde{\mathcal{A}}^2_{lm} = [0_n\ \psi^2_{lm}\ 0_{n,(3N_2+1)n}]$

$\gamma^{13}_l = [\psi^3_{lm}\ 0_{n,N_1C}\ \psi^4_l\ 0_{n,N_2b}]$

$\gamma^{14}_l = [\psi^5_l + J_1^T E_1^T T_1(\mathscr{K}_m + \mathscr{K}_M)\ \psi^6_l]$

$\gamma^{22}_l = \mathrm{diag}\{\tilde{M}_{10}, \tilde{M}_{11}, \cdots, \tilde{M}_{1N_1}, \tilde{M}_{20}, \tilde{M}_{21}, \cdots, \tilde{M}_{2N_2}, \tilde{M}_3\}$

$$\tilde{M}_{1i} = \begin{bmatrix} -\tilde{P}_{1i} - \tilde{P}_{2i} & \tilde{P}_{2i} - \tilde{X}^T_{1i} & \tilde{X}^T_{1i} \\ * & -2\tilde{P}_{2i} + He(\tilde{X}_{1i}) & \tilde{P}_{2i} - \tilde{X}^T_{1i} \\ * & * & -\tilde{P}_{2i} \end{bmatrix}$$

$$\tilde{M}_{2j} = \begin{bmatrix} -\tilde{P}_{3j} - \tilde{P}_{4j} & \tilde{P}_{4j} - \tilde{X}^T_{2j} & \tilde{X}^T_{2j} \\ * & -2\tilde{P}_{4j} + He(\tilde{X}_{2j}) & \tilde{P}_{4j} - \tilde{X}^T_{2j} \\ * & * & -\tilde{P}_{4j} \end{bmatrix}$$

$$\tilde{M}_3 = \left[\begin{array}{cc|c} -2\tilde{P}_5 + He(\tilde{X}_3) & & \\ -2J_1^T E_1^T \mathscr{K}_m^T T_2 \mathscr{K}_M E_1 J_1 & & \tilde{P}_5 - \tilde{X}^T_3 \\ \hline * & & -\tilde{P}_5 \end{array}\right]$$

$$\gamma^{24}_l = \begin{bmatrix} 0_{3(N_1+N_2+2)n,a} & 0_{3(N_1+N_2+2)n,a} \\ 0_{2n,a} & [(\mathscr{K}_m + \mathscr{K}_M)^T T_2^T E_1^T J_1^T\ 0_{a,n}]^T \end{bmatrix}$$

$\Pi^{22}_{lm} = [(\tilde{F}^1_{lm})^T \mathcal{D}\ \ (\tilde{F}^2_{lm})^T\ \ (\tilde{F}^3_l)^T\ \ (\tilde{F}^4_m)^T]^T$

$\tilde{F}_1 = [\psi^0_l\ \tilde{\mathcal{A}}^1_{lm}\ \tilde{\mathcal{A}}^2_{lm}\ 0_{n,2n}\ \gamma^{13}_l\ \psi^5_l\ \psi^6_l\ 0_{n,2}\ \psi^7_l\ \psi^8_l]$

$\tilde{F}^2_{lm} = \mathrm{col}\{\tilde{H}_{10},\ \tilde{H}_{11},\ \cdots,\ \tilde{H}_{1N_1},\ \tilde{H}_{20},\ \tilde{H}_{21},\ \cdots,\ \tilde{H}_{2N_2}\}$

$\tilde{H}_{1i} = [0_{c,(3i-1)n}\ \psi^9_l\ 0_{c,[3(N_1+N_2-i)+10]n+N}],\ i \in \mathcal{N}_{N_1}$

$\tilde{H}_{2j} = [0_{b,[3(N_1+j)+2]n}\ \psi^{10}_m\ 0_{b,[3(N_2-j)+7]n+N}],\ j \in \mathcal{N}_{N_2}$

$\tilde{F}^T_3 = [0_{c,2n}\ G_s\psi^9_l\ 0_{c,[3(N_1+N_2)+6]n}\ G_s\ 0_{c,N+n-c}]$

$\tilde{F}^T_4 = [0_{b,(3N_1+5)n}\ G_c\psi^{10}_m\ 0_{b,(3N_2+3)n+(N_1+1)c}$
$G_c\ 0_{b,(N_2+1)b+n+c+2}]$

$\Pi^{23} = \mathrm{diag}\{\tilde{\mathcal{E}}, (\Delta\beta^{33})^{-1}, -I, -I\}$

$\tilde{\mathcal{E}} = -\mathrm{diag}\{Z\tilde{P}_{10}^{-1}Z, \cdots, Z\tilde{P}_{1N_1}^{-1}Z, Z\tilde{P}_{20}^{-1}Z, \cdots, Z\tilde{P}_{2N_1}^{-1}Z,$
$Z\tilde{P}_{30}^{-1}Z, \cdots, Z\tilde{P}_{3N_2}^{-1}Z, Z\tilde{P}_{40}^{-1}Z, \cdots, Z\tilde{P}_{4N_2}^{-1}Z, Z\tilde{P}_5^{-1}Z\}$

with

$$J_1 = \begin{bmatrix} X & I \\ X & 0 \end{bmatrix},\ J_2 = \begin{bmatrix} I & Y \\ 0 & N^T \end{bmatrix},\ \psi^0_{lm} = \begin{bmatrix} A_lX & A_l \\ \varpi^1_{lm} & YA_l \end{bmatrix}$$

$$\psi^1_{lm} = \begin{bmatrix} 0 & 0 \\ \varpi^3_{lm} & b^s\varpi^2_m D_l \end{bmatrix},\ \psi^2_{lm} = \begin{bmatrix} E_l\varpi^4_m & 0 \\ \varpi^5_{lm} & 0 \end{bmatrix}$$

$$\psi^3_m = \begin{bmatrix} 0 \\ b^s\varpi^2_m \end{bmatrix},\ \psi^4_l = \begin{bmatrix} b^cE_l \\ b^cYE_l \end{bmatrix}, \psi^5_l = \begin{bmatrix} B_l & 0 \\ YB_l & 0 \end{bmatrix}$$

$$\psi^6_l = \begin{bmatrix} C_l & 0 \\ YC_l & 0 \end{bmatrix},\ \psi^7_m = \begin{bmatrix} 0 \\ (1-b^s)\varpi^2_m \end{bmatrix}$$

$$\psi^8_l = \begin{bmatrix} (1-b^s)E_l \\ (1-b^s)YE_l \end{bmatrix}, \psi^9_l = \begin{bmatrix} D_lX & D_l \end{bmatrix}, \psi^{10}_m = \begin{bmatrix} \varpi^4_m & 0 \end{bmatrix}.$$

Proof. Partition the Lyapunov matrix $P = \begin{bmatrix} Y & N \\ N^T & Y_1 \end{bmatrix}$, where $N = X^{-1} - Y$ and $Y_1 = N^T(Y - X^{-1})^{-1}N$. Without loss of generality, we assume N is nonsingular (Zhang and Han, 2014). Obviously, since $Y_1 > 0$, $Y - X^{-1} > 0$, thus $Z > 0$ by Schur complement. It is noted that, considering the definitions of J_1, J_2 and N, the equation $J_2 = PJ_1$ holds. Moreover, we define

$$\mathcal{J}_1 \triangleq \mathrm{diag}\{\overbrace{J_1, \cdots, J_1}^{3(N_1+N_2)+9}, \overbrace{I_c, \cdots, I_c}^{N_1+1}, \overbrace{I_b, \cdots, I_b}^{N_2+1}, I_n, I_n, I, I,$$
$$I_c, I_b, \overbrace{J_2, \cdots, J_2}^{2(N_1+N_2)+5}, \overbrace{I_c, \cdots, I_c}^{N_1+1}, \overbrace{I_b, \cdots, I_b}^{N_2+1}, I_c, I_b\}$$

$$\mathcal{J}_2 \triangleq \mathrm{diag}\{\mathcal{J}_1, \overbrace{J_2, \cdots, J_2}^{2(N_1+N_2)+5}, \overbrace{I_c, \cdots, I_c}^{N_1+1}, \overbrace{I_b, \cdots, I_b}^{N_2+1}, I_c, I_b\}$$

and $\varpi^1_{lm} \triangleq (YA_l - NA_{km})X$, $\varpi^2_m \triangleq ND_{km}$,$\varpi^3_{lm} \triangleq (b^s\varpi^2_m D_l + NB_{km})X$, $\varpi^4_m \triangleq F_{km}X$, $\varpi^5_{lm} \triangleq YE_l\varpi^4_m + NC_{km}X$.

Perform congruence transformation on Θ^2_l utilizing the matrix $\mathcal{J}$, such that $\tilde{P}_{\mathscr{B}_1} = J_1^T P_{\mathscr{B}_i} J_1$, $\tilde{P}_{\mathscr{C}_j} = J_1^T P_{\mathscr{C}_j} J_1$, $\tilde{P}_5 = J_1^T P_5 J_1$, $\tilde{X}_{1_i} = J_1^T X_{1_i} J_1$, $\tilde{X}_{2_j} = J_1^T X_{2_j} J_1$, $\tilde{X}_3 = J_1^T X_{2_j} J_1$. Subsequently, after some simple algebraic manipulations, we have

$$\mathcal{J}_1^T \Theta^{11}_l \mathcal{J}_1 = \Theta^{21}_l,\ \mathcal{J}_2^T \Theta^{12}_{lm} \mathcal{J}_2 = \Theta^{22}_{lm},\ \mathcal{J}_2^T \Theta^{13}_{lm} \mathcal{J}_2 = \Theta^{23}_{lm}.$$

Denote $P = J_2J_1^{-1}$, $P_{\mathscr{B}_i} = J_1^{-T}\tilde{P}_{\mathscr{B}_i}J_1^{-1}$, $P_{\mathscr{C}_j} = J_1^{-T}\tilde{P}_{\mathscr{C}_j}J_1^{-1}$, $P_5 = J_1^{-T}\tilde{P}_5J_1^{-1}$,$X_{1_i} = J_1^{-T}\tilde{X}_{1_i}J_1^{-1}$, $X_{2_j} = J_1^{-T}\tilde{X}_{2_j}J_1^{-1}$,$X_3 = J_1^{-T}\tilde{X}_3J_1^{-1}$, and

$$A_{km} = N^{-1}(\varpi^1_{lm} - YA_lX)X^{-1},\ D_{km} = N^{-1}\varpi^2_m,$$
$$B_{km} = N^{-1}(\varpi^3_{lm} - b^s\varpi^2_m D_lX)X^{-1},\ F_{km} = \varpi^4_m X^{-1},$$
$$C_{km} = N^{-1}(\varpi^5_{lm} - YE_l\varpi^4_m)X^{-1}.$$

Then, it evident that $P > 0$, $\mathcal{J}_1^{-T}\Theta^{21}_l\mathcal{J}_1^{-1} = \Theta^{11}_l$, $\mathcal{J}_2^{-T}\Theta^{22}_{lm}\mathcal{J}_2^{-1} = \Theta^{12}_{lm}$, $\mathcal{J}_2^{-T}\Theta^{23}_{lm}\mathcal{J}_2^{-1} = \Theta^{13}_{lm}$. That is, Schur complement [Eqs (16)–(18)], SDDETM [Eqs (4)–(5)], and DOFC [Eq. (12)] can be ensured by Eqs (24)–(26), which completes the proof. □

Theorem 2 For $i \in \mathcal{N}_{N_1}$, $j \in \mathcal{N}_{N_2}$, $\mathscr{B} = \{1, 2\}$, $\mathscr{C} = \{3, 4\}$, given positive scalars $\underline{\delta}_{1_i}$, $\overline{\delta}_{1_i}$, $\underline{\delta}_{2_j}$, $\overline{\delta}_{2_j}$, ϵ_{1_i}, ϵ_{2_j}, $N_{\mathscr{B}}$, $w_{\mathscr{B}}$, $\mu_{\mathscr{B}}$, $\zeta_{\mathscr{B}}$, and the diagonal matrices $\mathscr{K}_m$, $\mathscr{K}_M$, and real matrix G_s, G_c, if there exist positive diagonal matrices T_1 and T_2, and $P_{\mathscr{B}_i}$, $\bar{P}_{\mathscr{B}_i}$, $Q_{\mathscr{B}_i}$, $\bar{Q}_{\mathscr{B}_i}$, $P_{\mathscr{C}_j}$, $\bar{P}_{\mathscr{C}_j}$, $Q_{\mathscr{C}_j}$, $\bar{Q}_{\mathscr{C}_j}$, Q_5 $\bar{Q}_5$, Q_6, $\bar{Q}_6$, Q_7, $\bar{Q}_7$, $\bar{Z} \in \boldsymbol{S}^+_n$, $\bar{\Phi}_{1_i} \in \boldsymbol{S}^+_c$, $\bar{\Phi}_{2_j} \in \boldsymbol{S}^+_b$, then the stabilization problem of close-loop system [Eq. (15)] integrating DOFC and SDDETM is solvable if Eq.

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(27) and the following inequalities are satisfied:

$$\Theta_l^{31} = \begin{bmatrix} \Pi_{ll}^{31} & * \\ \Pi_{ll}^{22} & \Pi^{33} \end{bmatrix} \leq 0 \tag{28}$$

$$\Theta_{lm}^{32} = \begin{bmatrix} \Pi_{lm}^{31} + \zeta_1 \Pi_{ml}^{21} & * & * \\ \Pi_{lm}^{22} & \Pi^{33} & * \\ \zeta_2 \Pi_{ml}^{22} & 0 & \zeta_2 \Pi^{33} \end{bmatrix} \leq 0 \tag{29}$$

$$\Theta_{lm}^{33} = \begin{bmatrix} \Pi_{lm}^{31} + \zeta_2 \Pi_{ml}^{21} & * & * \\ \Pi_{lm}^{22} & \Pi^{33} & * \\ \zeta_2 \Pi_{ml}^{22} & 0 & \zeta_2 \Pi^{33} \end{bmatrix} \leq 0 \tag{30}$$

where

$$\Pi_{lm}^{31} = \begin{bmatrix} \varsigma_{lm}^{11} & \gamma_{lm}^{12} & \gamma_{lm}^{13} & \gamma_l^{14} & 0 & \psi_l^7 & \psi_l^8 \\ * & \varsigma_l^{22} & 0 & \gamma_l^{24} & 0 & 0 & 0 \\ * & * & \beta_l^{33} & 0 & 0 & 0 & 0 \\ * & * & * & \beta_l^{44} & 0 & 0 & 0 \\ * & * & * & * & \beta_l^{55} & 0 & 0 \\ * & * & * & * & * & -w_1 I & 0 \\ * & * & * & * & * & * & -w_2 I \end{bmatrix}$$

$$\varsigma_{lm}^{11} = He(\psi_l^0) - \sum_{i=0}^{N_1} \tilde{P}_{1i} - \sum_{j=0}^{N_2} \tilde{P}_{3j} - \tilde{P}_5 - 2Q_6$$

$$\varsigma_l^{22} = \mathrm{diag}\{\tilde{M}_{10}, \tilde{M}_{11}, \cdots, \tilde{M}_{1N_1}, \tilde{M}_{20}, \tilde{M}_{21}, \cdots, \tilde{M}_{2N_2}, \bar{M}_3\}$$

$$\bar{M}_3 = \begin{bmatrix} -2\tilde{P}_5 + He(\tilde{X}_3) - 2Q_7 & \tilde{P}_5 - \tilde{X}_3^T \\ * & -\tilde{P}_5 \end{bmatrix}$$

$$\Pi^{33} = \mathrm{diag}\{\bar{\mathcal{E}}, (\Delta\bar{\beta}^{33})^{-1}, -I, -I\}$$

$$\bar{\mathcal{E}} = -\mathrm{diag}\{Q_{10}, Q_{11}, \cdots, Q_{1N_1}, Q_{20}, Q_{21}, \cdots, Q_{2N_1}, Q_{30}, Q_{31}, \cdots, Q_{3N_2}, Q_{40}, Q_{41}, \cdots, Q_{4N_2}, Q_5\}$$

$$\bar{\beta}^{33} = \mathrm{diag}\{-\bar{\phi}_{10}, -\bar{\phi}_{11}, \cdots, -\bar{\phi}_{1N_1}, -\bar{\phi}_{20}, -\bar{\phi}_{21}, \cdots, -\bar{\phi}_{2N_2}\}$$

and the following equations hold:

$$\begin{aligned} &J_1\bar{J}_1 = I, Z\bar{Z} = I, Q_{\mathscr{B}_i}\bar{Q}_{\mathscr{B}_i} = I, Q_{\mathscr{C}_j}\bar{Q}_{\mathscr{C}_j} = I \\ &\tilde{P}_{\mathscr{B}_i}\bar{P}_{\mathscr{B}_i} = I, \tilde{P}_{\mathscr{C}_j}\bar{P}_{\mathscr{C}_j} = I, \tilde{P}_5\bar{P}_5 = I \\ &\Phi_{1_i}\bar{\Phi}_{1_i} = I, \Phi_{2_j}\bar{\Phi}_{2_j} = I. \end{aligned} \tag{31}$$

Proof. Eqs (24)–(26) contain the nonlinear terms $-Z\tilde{P}_{\mathscr{B}_i}^{-1}Z$, $-Z\tilde{P}_{\mathscr{C}_j}^{-1}Z$, $-Z\tilde{P}_5^{-1}Z$, $\Phi_{1_i}^{-1}$, and $\Phi_{2_j}^{-1}$, where $\mathscr{B} = \{1, 2\}$, $\mathscr{C} = \{3, 4\}$, $i \in \mathcal{N}_{N_1}$, $j \in \mathcal{N}_{N_2}$. We utilized the CCL algorithm (El Ghaoui et al., 1997) to transform nonconvex linear matrix inequalities into convex optimization problems. Accordingly, the stabilization of the system [Eq. (15)] can be solved, leading to obtain the DOFC. The proof is thereby established. □

Remark 4 The use of the CCL algorithm in this paper significantly enhances the computational efficiency and effectiveness of the proposed method. By transforming nonconvex linear matrix inequalities into convex optimization problems, the CCL algorithm allows for the simultaneous optimization of the DOFC and SDDETM parameters. This approach not only simplifies the computational process but also yields less conservative results, ensuring robust stability and efficient resource utilization in the presence of deception attacks and time delays.

4 Simulation examples

In this section, we demonstrated two cases to evidence the effectiveness of the SDDETM. To illustrate the difference between SDDETM ($N_1 > 0$, $N_2 > 0$) and traditional ETMs ($N_1 = N_2 = 0$), consider the following examples. In traditional ETMs, the trigger frequency is high due to reliance on the current sample, leading to potential network congestion and Zeno behavior. In contrast, SDDETM uses past samples and internal dynamic variables, reducing trigger frequency and avoiding Zeno behavior.

4.1 Case 1

Consider NN system [Eq. (15)] with system matrices (Liu et al., 2019b):

$$A_1 = -\begin{bmatrix} 1.06 & 0 \\ 0 & 1.06 \end{bmatrix}, A_2 = -\begin{bmatrix} 1.6 & 0 \\ 0 & 2.3 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0.3 & -0.42 \\ -0.42 & 0.3 \end{bmatrix}, B_2 = \begin{bmatrix} 0.2 & -0.32 \\ -0.32 & 0.2 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} 0.3 & 0.3 \\ 0.3 & 0.3 \end{bmatrix}, C_2 = \begin{bmatrix} 0.4 & 0.4 \\ 0.4 & 0.4 \end{bmatrix}$$

$$E_1 = \begin{bmatrix} -3 & -0.15 \\ -0.75 & -2 \end{bmatrix}, E_2 = \begin{bmatrix} -0.95 & -0.85 \\ -1.75 & -0.55 \end{bmatrix}.$$

Suppose that the transition probability are

$$\begin{aligned} \pi &= [\pi_{\mathscr{M}\mathscr{N}}]_{2\times 2} = \begin{bmatrix} -0.3 & 0.3 \\ 0.6 & -0.6 \end{bmatrix} \\ \rho &= [\rho_{\mathscr{P}\mathscr{Q}}]_{2\times 2} = \begin{bmatrix} -0.4 & -0.4 \\ 0.65 & -0.65 \end{bmatrix} \end{aligned} \tag{32}$$

and $f(x_e(t)) = \mathrm{col}\{f_1(x_e(t)), f_2(x_e(t))\}$ is activation function, where $f_1(x_e(t)) = 0.5x_1(t) - \tanh(0.2x_1(t) + 0.2x_1(t))$, $f_2(x_e(t)) = 0.95x_2(t) - \tanh(0.75x_1(t))$, and $\mathcal{I} = 0$ is the input bias. $x_e(t) = [1.7, -2.6]^T$ and $x_c(t) = [0.9, -1.9]^T$ are the corresponding initial parameters of the system [Eq. (15)]. The sample period h is set at 0.1 s, with the upper bound of time delays denoted as $h_M = 0.1$ s, and

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Table 1 Total triggers generated by SDDETM with different N_1, N_2 (CASE 1)

	$N_1,\ N_2$	0 (Liu et al., 2019b)	1	2	3	4
DATA 1	SDDETM 1	56	54	52	50	49
	SDDETM 2	69	65	64	63	60
	Total number of triggers	125	119	116	113	109
	ϵ_{1_i} $(i \in \mathcal{N}_{N_1})$	0.05	0.05, 0.10	0.05, 0.10, 0.10	0.05, 0.10, 0.10, 0.20	0.05, 0.10, 0.10, 0.20, 0.15
	ϵ_{2_j} $(j \in \mathcal{N}_{N_2})$	0.05	0.05, 0.30	0.05, 0.30, 0.30	0.05, 0.30, 0.30, 0.30, 0.15	0.05, 0.30, 0.30, 0.15, 0.50
DATA 2	SDDETM 1	56	55	50	48	39
	SDDETM 2	69	66	65	61	60
	Total number of triggers	125	121	115	109	99
	ϵ_{1_i} $(i \in \mathcal{N}_{N_1})$	0.05	0.05, 0.50	0.05, 0.50, 0.50	0.05, 0.50, 0.50, 0.50	0.05, 0.50, 0.50, 0.50, 0.45
	ϵ_{2_j} $(j \in \mathcal{N}_{N_2})$	0.05	0.05, 0.30	0.05, 0.30, 0.30	0.05, 0.30, 0.30, 0.30, 0.15	0.05, 0.30, 0.30, 0.15, 0.50

SDDETM: sum-based dynamic discrete event-triggered mechanism

$\tau_{AM} = \tau_{BM} = 0.01\,\mathrm{s}$. Utilizing the LMI Toolbox within MATLAB, we obtain

$$
\begin{aligned}
A_{1k} &= \begin{bmatrix} -2.911 & 0.150 \\ 0.204 & -2.980 \end{bmatrix},\ A_{2k} = \begin{bmatrix} -5.069 & -0.096 \\ -0.080 & -5.043 \end{bmatrix} \\
B_{1k} &= \begin{bmatrix} -0.0715 & -0.049 \\ -0.068 & -0.043 \end{bmatrix},\ B_{2k} = \begin{bmatrix} 0.010 & 0.006 \\ 0.008 & 0.007 \end{bmatrix} \\
C_{k1} &= \begin{bmatrix} -0.002 & 0.003 \\ 0.002 & -0.003 \end{bmatrix},\ C_{k2} = \begin{bmatrix} 0.003 & -0.005 \\ -0.003 & 0.004 \end{bmatrix} \\
D_{k1} &= \begin{bmatrix} -0.021 & -0.015 \\ -0.013 & -0.017 \end{bmatrix},\ D_{k2} = \begin{bmatrix} 0.008 & 0.015 \\ 0.018 & 0.006 \end{bmatrix} \\
F_{k1} &= \begin{bmatrix} -0.002 & 0.003 \\ 0.002 & -0.002 \end{bmatrix},\ F_{k2} = \begin{bmatrix} 0.002 & -0.003 \\ -0.002 & 0.003 \end{bmatrix}
\end{aligned} \tag{33}
$$

and the SDDETM parameters are

$$
\Phi_{1_0} = \begin{bmatrix} 25.024 & 11.991 \\ 11.991 & 24.495 \end{bmatrix}, \Phi_{2_0} = \begin{bmatrix} 65.921 & 40.767 \\ 40.767 & 65.966 \end{bmatrix} \tag{34}
$$

With Eqs (33) and (34), the state trajectories of the close system [Eq. (15)] with $N_1 = N_2 = \{0, 2, 4\}$ is depicted in Fig. 2, respectively. Clearly, the state trajectories converge to zero within 10 s, demonstrating the efficacy of the designed DOFC in stabilizing the system.

When $N_1 > 0$ and $N_2 > 0$, the SDDETM incorporates previous samples, and there exist multiple triggering thresholds ϵ_{1_i} and ϵ_{2_j}, where $i \in \mathcal{N}_{N_1}$ and $j \in \mathcal{N}_{N_2}$. As these thresholds are predefined, we initiate a loop for each threshold. Specifically, we search for thresholds within the range of $[0.05, 0.5]$ with the step size of 0.05. For each pair of ϵ_{1_i} and ϵ_{2_j}, the triggers generated by SDDETM 1 and SDDETM 2 are documented. Table 1 presents two cases from the comprehensive results. For DATA 2 in Table 1, we have the following observations: when $N1 = N2 = \{0, 2, 4\}$, the SDDETM reduces the signal transmission in Network 1 by 44%, 50%, 61%, and in Network 2 by 31%, 35%, 40%, respectively. It is evident that as N_1 and N_2 rise, the communication resources can be saved. The release intervals that illustrate the results of SDDETM 1 and SDDETM 2 for $N_1 = N_2 = \{0, 2, 4\}$ are presented in Fig. 2. Analysis of Table 1 and Fig. 2 reveals that the number of triggers released by both SDDETMs decreases as N_1 and N_2 increase. This means that with appropriately selected triggering thresholds, network resources can be preserved as N_1 and N_2 rise. The efficiency of the SDDETM is thus demonstrated.

4.2 Case 2

Consider two IF-THEN rules to simulate the T-S fuzzy neural networked system, that is $l = 2$. Let $h_1(\theta_1(t)) = \sin^2(t)$, $h_2(\theta_1(t)) = 1 - h_1(\theta_1(t))$ and the parameters for T-S neural networked system

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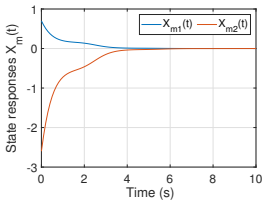
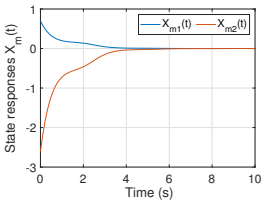
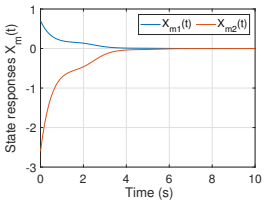


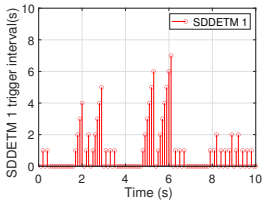
State trajectories ($N_1 = N_2 = 0$).



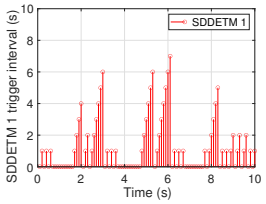
State trajectories ($N_1 = N_2 = 2$).



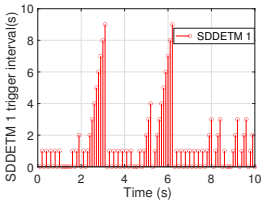
State trajectories ($N_1 = N_2 = 4$).



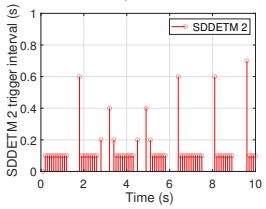
SDDETM 1 ($N_1 = N_2 = 0$).



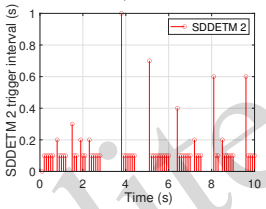
SDDETM 1 ($N_1 = N_2 = 2$).



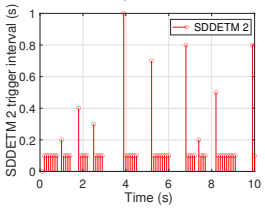
SDDETM 1 ($N_1 = N_2 = 4$).



SDDETM 2 ($N_1 = N_2 = 0$).



SDDETM 2 ($N_1 = N_2 = 2$).



SDDETM 2 ($N_1 = N_2 = 4$).

Fig. 2 State trajectories and release time intervals of SDDETM 1 and SDDETM 2. SDDETM: sum-based dynamic discrete event-triggered mechanism

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[Eq. (15)] are as follows (Tan et al., 2020):

$$A_1 = -\begin{bmatrix} 3.9 & 0 \\ 0 & 3.1 \end{bmatrix}, A_2 = -\begin{bmatrix} 4.1 & 0 \\ 0 & 5.9 \end{bmatrix}$$
$$B_1 = \begin{bmatrix} 0.9 & -0.3 \\ -0.2 & 0.5 \end{bmatrix}, B_2 = \begin{bmatrix} 0.8 & -0.1 \\ -0.3 & 0.6 \end{bmatrix}$$
$$C_1 = \begin{bmatrix} 0.8 & 0.2 \\ 0.2 & 0.3 \end{bmatrix}, C_2 = \begin{bmatrix} 0.7 & 0.1 \\ 0.1 & 0.4 \end{bmatrix}$$
$$E_1 = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.8 \end{bmatrix}, E_2 = \begin{bmatrix} 0.6 & 0 \\ 0 & 0.5 \end{bmatrix}.$$

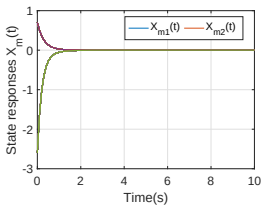
Suppose that the neuron activation function is given as $f(x_e(t)) = \mathrm{col}\{f_1(x_e(t)), f_2(x_e(t))\}$, where $f_1(x_e(t)) = f_2(x_e(t)) = 10 \times [\sin(0.03x) - \tanh(0.02x)]$, and the input bias is given as $J = 0$. The transition probability is given as Eq. (32). Here we set $h_M = 0.1\,\mathrm{s}$, $h = 0.1\,\mathrm{s}$, $\mathscr{K}_m = 0$, $\mathscr{K}_M = I$. The nonlinear functions of the deception attack in sampler-DOFC and DOFC-ZOH are $g_s(y_e(t)) = -\tanh(0.2y_e(t))$ and $g_c(\bar{u}(t)) = -\tanh(0.2\bar{u}(t))$, respectively. According to Theorem 2 and the LMI Toolbox for Eq. (27) and Eqs (28)–(30), the DOFC parameters are obtained as follows:

$$A_{1k} = \begin{bmatrix} -4.973 & 0.064 \\ -0.087 & -4.449 \end{bmatrix}, A_{2k} = \begin{bmatrix} -9.287 & -0.013 \\ 0.009 & -7.444 \end{bmatrix}$$
$$B_{1k} = \begin{bmatrix} -0.002 & 2.150 \\ 0.000 & -0.007 \end{bmatrix}, B_{2k} = \begin{bmatrix} 0.005 & -0.003 \\ -7.001 & 0.001 \end{bmatrix}$$
$$C_{k1} = \begin{bmatrix} 0.017 & 0.000 \\ 0.000 & -0.016 \end{bmatrix}, C_{k2} = \begin{bmatrix} -0.036 & 0.001 \\ 2.344 & 0.033 \end{bmatrix}$$
$$D_{k1} = \begin{bmatrix} 0.010 & 0.000 \\ 0.000 & 0.007 \end{bmatrix}, D_{k2} = \begin{bmatrix} 0.011 & 0.000 \\ 0.000 & -0.001 \end{bmatrix}$$
$$F_{k1} = \begin{bmatrix} 0.0124 & 0.000 \\ 0.000 & -0.013 \end{bmatrix}, F_{k2} = \begin{bmatrix} -0.016 & 0.000 \\ 0.000 & 0.016 \end{bmatrix}$$

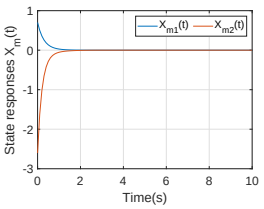
and the SDDETM parameters are

$$\Phi_{1_0} = \begin{bmatrix} 62.696 & 25.499 \\ 25.499 & 21.687 \end{bmatrix}, \Phi_{2_0} = \begin{bmatrix} 11154.919 & 7235.158 \\ 7235.158 & 11075.499 \end{bmatrix}.$$

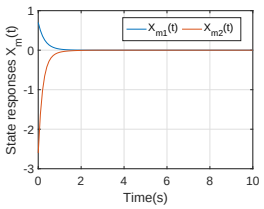
The state responses of the NN and the released time intervals of SDDETM 1 and SDDETM 2 when $N_1 = N_2 = \{0, 2, 4\}$ are depicted in Fig. 3. Total triggers released by SDDETM 1 and SDDETM 2 with different N_1 and N_2 are presented in Table 2. The effectiveness of the designed DOFC in stabilizing the system can be obtained from the above results.



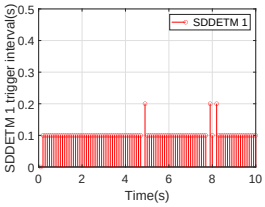
State trajectories ($N_1 = N_2 = 0$).



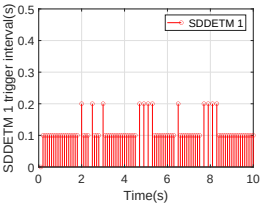
State trajectories ($N_1 = N_2 = 2$).



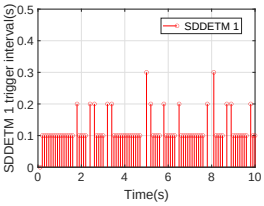
State trajectories ($N_1 = N_2 = 4$).



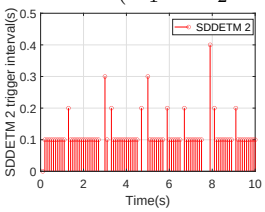
SDDETM 1 ($N_1 = N_2 = 0$).



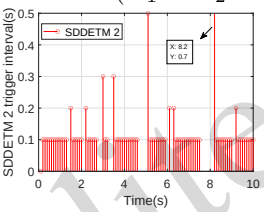
SDDETM 1 ($N_1 = N_2 = 2$).



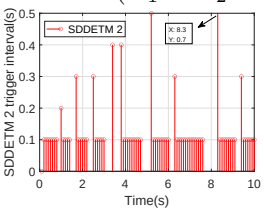
SDDETM 1 ($N_1 = N_2 = 4$).



SDDETM 2 ($N_1 = N_2 = 0$).



SDDETM 2 ($N_1 = N_2 = 2$).



SDDETM 2 ($N_1 = N_2 = 4$).

Fig. 3 The state trajectories and release time intervals of SDDETM 1 and SDDETM 2. SDDETM: sum-based dynamic discrete event-triggered mechanism

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5 Conclusions

The objective of this paper is to design a DOFC and a SDDETM to ensure the synchronization of master and slave NNs under deception attacks. The proposed SDDETM offers significant advantages over traditional ETMs by reducing trigger frequency, avoiding Zeno behavior, and providing more efficient use of network resources. Through the application of the CCL algorithm, the concurrent design of the DOFC and SDDETM parameters is achieved, further enhancing the stability and performance of the system. The simulation results validate the effectiveness of the proposed SDDETM in achieving faster and more stable convergence compared to traditional ETMs.

Future research should consider the design under conditions where multi-packet transmission and disorder are common, as the current work assumes that packets are transmitted individually and orderly.

Contributors

Zhongjing YU and Duo ZHANG designed the research. Zhongjing YU processed the data. Zhongjing YU and Duo ZHANG drafted the paper. Shihan KONG, Deqiang OUYANG, and Hongfei LI helped organize the paper. Junzhi YU revised and finalized the paper.

Conflict of interest

All the authors declare that they have no conflict of interest.

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Table 2 Total triggers generated by SDDETM with different N_1, N_2 (CASE 2)

	$N_1,\ N_2$	0 (Tan et al., 2020)	1	2	3	4
DATA 1	SDDETM 1	86	82	80	74	72
	SDDETM 2	82	77	76	74	72
	Total number of triggers	168	159	156	148	144
	$\epsilon_{1_i}\ (i \in \mathcal{N}_{N_1})$	0.05	0.05, 0.15	0.05, 0.15, 0.30	0.05, 0.15, 0.30, 0.30	0.05, 0.15, 0.30, 0.30, 0.30
	$\epsilon_{2_j}\ (j \in \mathcal{N}_{N_2})$	0.05	0.05, 0.25	0.05, 0.25, 0.40	0.05, 0.25, 0.40, 0.40	0.05, 0.25, 0.40, 0.40, 0.40
DATA 2	SDDETM 1	86	83	82	80	78
	SDDETM 2	82	79	78	76	74
	Total number of triggers	168	162	160	156	152
	$\epsilon_{1_i}\ (i \in \mathcal{N}_{N_1})$	0.05	0.05, 0.10	0.05, 0.10, 0.15	0.05, 0.10, 0.15, 0.20	0.05, 0.10, 0.15, 0.20, 0.30
	$\epsilon_{2_j}\ (j \in \mathcal{N}_{N_2})$	0.05	0.05, 0.25	0.05, 0.25, 0.25	0.05, 0.25, 0.25, 0.25, 0.40	0.05, 0.25, 0.25, 0.40, 0.40

SDDETM: sum-based dynamic discrete event-triggered mechanism

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