Frontiers of Information Technology & Electronic Engineering
www.jzus.zju.edu.cn; engineering.cae.cn; www.springerlink.com
ISSN 2095-9184 (print); ISSN 2095-9230 (online)
E-mail: jzus@zju.edu.cn

Passivity-based synchronous control of Markov jump systems with actuator saturation

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Received Jan. 16, 2025; Revision accepted May 19, 2025; Crosschecked

Abstract: In this article, the robust control problem of discrete-time Markov jump systems (MJSs) with actuator saturation is investigated via passivity theory. Under the assumption of mode synchronization between the system and the controller, sufficient conditions are established to guarantee the system to be mean square stable and stochastically passive in the domain of attraction via the saturation-dependent Lyapunov function approach and the linear matrix inequality (LMI) technique. The coupling between the system variables is solved, which greatly facilitates the solution of the synchronization controller. Moreover, the estimation of the domain of attraction for the considered MJSs is accomplished through the solution of an optimization problem (OP). By degenerating the mode-dependent controller into its mode-independent counterpart, we derive sufficient conditions to ensure system robustness under the mode-independent control strategy, which are then systematically summarized. Finally, the effectiveness of the proposed integrated design methodology is validated through numerical simulations.

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Key words: Markov jump systems; Synchronous controller; Actuator saturation; Passivity; Linear matrix inequalities

https://doi.org/10.1631/FITEE.2500039 **CLC number:**

1 Introduction

Markov jump systems (MJSs), as a specialized category of stochastic hybrid systems in networked control architectures, have attracted significant academic attention due to their unique capability of concurrently characterizing both external environmental variations and internal parameter transitions. These distinctive features have led to widespread applications across multiple engineering domains (Costa et al., 2005; Cheng et al., 2020; Chen et al., 2022a; Chen et al., 2023; Pang et al., 2024; Wang and Wu, 2024). There exist two fundamental control strategies in MJSs, namely output feedback control and state feedback control. Specifically, output feedback control relies on the output variables of the system to carry out the design of the controller, and it is constrained by the output matrix (Cheng et al., 2021; Tao and Wu, 2024). In contrast, state feedback control relies on the state variables of the system for the design of the controller. Moreover, based on the synchronization between the modes of the state feedback controller and those of the system, the design methods can be classified into three categories.

(i) The mode-independent (MID) control strategy, in which the state feedback gain matrix is fixed and does not change with time (Goncalves et al., 2009; Oliveira et al., 2014; Nie et al., 2024). This strategy can ensure the robustness of the system in the complete absence of mode information, but it has inherent conservatism.

(ii) The asynchronous control strategy, in which the state feedback gain matrix is not fixed, and its

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transition law is inconsistent with the system mode transition (Wu et al., 2014; Wu et al., 2021; Wang et al., 2023). This scheme can improve the system performance in the case of mode mismatch, but at the cost of increased design complexity due to additional variable coupling.

(iii) The mode-dependent (MD) control method, in which the state feedback gain matrix is also not fixed (Zhao et al., 2012; Bolzern et al., 2014; Feng and Shi, 2017; Zhang et al., 2018; Xu et al., 2024). However, different from the asynchronous control strategy, this method makes use of the assumption of complete synchronization. That is, the transition law of the controller mode is completely consistent with the system mode transition. Without additional variable coupling, it is considered a more efficient and convenient approach.

Actuator saturation is an important cause of system nonlinearity and instability (Hu and Lin, 2001). Owing to the inherent physical limitations of actuator components and other factors, the control output of systems exhibiting actuator saturation is constrained. This limitation will cause system performance degradation, and even cause a system crash. A previous paper (Jiang et al., 2021) has investigated an event-triggered H_∞ controller design for discrete-time piecewise-affine systems with actuator saturation. The robust H_∞ MD control for MJSs, that are subject to both actuator saturation and incomplete knowledge of transition probability, has been investigated (Wang et al., 2012). An asynchronous control strategy ensuring mean square stability for MJSs with actuator saturation is introduced in another work (Wang and Wu, 2022). Therefore, studying MJSs with actuator saturation based on passivity is also a novel research direction. The concept of passivity is introduced below.

Passivity, regarded as one of the pivotal performance metrics for system robustness, underpins the stable operation of systems amidst intricate operational conditions and interfering environments. It leverages the inherent properties related to energy absorption, storage, and dissipation within the system, effectively enabling the system to withstand uncertainties and perturbations while maintaining its operational integrity and performance stability. In a previous paper (Wu et al., 2017), the mean square stability and stochastic passivity of discrete-time MJSs are achieved through a passivity-based asynchronous controller. A state feedback sampled-data controller independent of time is designed to address the problem of passivity-based robust sampled-data control for continuous-time MJSs (Chen et al., 2020). When deception attacks occur in the system, the security-based passivity problem of discrete-time MJSs is resolved through asynchronous control (Xu et al., 2023). Passivity is one of the properties studied for the system's robustness, and its premise is based on the stability of the system. The system stability criteria and controller design methods proposed in previous articles (Chen et al., 2022b; Chen et al., 2023) have provided research ideas for this paper.

Inspired by previous research, this paper focuses on using the theory of stochastic passivity to explore the robust mean square stability issue for MJSs with actuator saturation. The following are the main contributions of this paper:

(1) By adopting the Lyapunov function analysis method, we have derived sufficient conditions to ensure that the closed-loop system achieves stochastic mean square stability and stochastic passivity within the attraction domain.

(2) By utilizing the linear matrix inequality (LMI) technique, the coupled variables in the sufficient conditions are decoupled.

(3) Through ingenious algebraic manipulations, we have transformed these conditions into a convex optimization problem based on LMI, and accordingly, we have estimated the range of the system's attraction domain.

(4) We degenerate the sufficient conditions to the MID control strategy, enabling the system to be mean square stable and stochastically passive as well.

Notations: Throughout this paper, the superscript 'T' denotes matrix transposition. The symbols $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ represent the n-dimensional Euclidean space and the set of all $n\times m$ real matrices, respectively. For probability space $(\Gamma,\ \Theta,\ \Lambda)$, Γ represents the sample space, Θ is the algebra of events, and Λ stands for the probability measure defined on Θ. The symbol $\mathcal{E}\{\cdot\}$ represents the expectation operator. The notation $P>0$ (respectively, $P\geq 0$) means that P is symmetric and positive definite (respectively, positive semi-definite). The notation $\mathbf{diag}\{A_1, A_2, \ldots, A_N\}$ denotes a block diagonal matrix with diagonal blocks being the matrix A_1, A_2, $\ldots$, A_N. In symmetric block matrices, we use $*$ as

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an ellipsis for terms that are induced by symmetry.

2 Preliminaries

In the context of a specified probability space $(\Gamma,\ \Theta,\ \Lambda)$, the focus is on the consideration of the following MJSs:

$$\begin{cases} x(k+1) = A(r(k))x(k) + B(r(k))\sigma(u(k)) \\ \qquad\qquad +F(r(k))\omega(k), \\ z(k) = C(r(k))x(k) + D(r(k))\sigma(u(k)). \end{cases} \tag{1}$$

The matrices $A(r(k)) \in \mathbb{R}^{n\times n}$, $B(r(k)) \in \mathbb{R}^{n\times m}$, $C(r(k)) \in \mathbb{R}^{p\times n}$, $D(r(k)) \in \mathbb{R}^{p\times m}$, $F(r(k)) \in \mathbb{R}^{n\times f}$ represent the system's parameters. The system state variable is denoted by $x(k) \in \mathbb{R}^n$, while the control input is denoted by $u(k) \in \mathbb{R}^m$. The symbol $\omega(k) \in \mathbb{R}^f$ signifies an external disturbance from the environment, which belongs to the l_2 space over the interval $[0,\ \infty)$, and $z(k) \in \mathbb{R}^p$ denotes the controlled output. We define $\sigma(u_i) = sign(u_i)\min\{1,\ |u_i|\}$. Subsequently, the standard saturation function $\sigma(u(k))$ is presented as follows:

$$\sigma(u(k)) = \begin{bmatrix}\sigma(u_1(k)) & \sigma(u_2(k)) & \dots & \sigma(u_m(k))\end{bmatrix}^T.$$

The transition probability matrix (TPM) of the system is expressed as matrix $\Pi = \{\pi_{ij}\}$. The system's mode $r(k)$ depends on a homogeneous Markov process $\{r(k), k \geq 0\}$. It is defined as follows:

$$\Pr\{r(k+1) = j | r(k) = i\} = \pi_{ij}, \tag{2}$$

where $i, j \in \mathcal{N}$, $0 \leq \pi_{ij} \leq 1$, $\sum_{j=1}^{N} \pi_{ij} = 1$, positive integer set $\mathcal{N} = \{1, 2, \dots, N\}$ and N is pre-known. The definition of the synchronous controller is given as follows:

$$u(k) = K(r(k))x(k). \tag{3}$$

where $K(r(k)) \in \mathbb{R}^{m\times n}$.

In order to further develop the research in this paper, we introduce some crucial concepts and definitions. They are listed below:

Considering a symmetric polyhedron set $\mathcal{L}(H_i)$, its definition is considered as follows:

$$\mathcal{L}(H_i) = \{x(k) \in \mathbb{R}^n : |h_{iq}x(k)| \leq 1\}. \tag{4}$$

Further, the definition of an ellipsoid set $\Omega(P_{il})$ is given as follows:

$$\Omega(P_{il}) = \left\{x(k) \in \mathbb{R}^n : x^T(k)P_{il}x(k) \leq 1\right\}. \tag{5}$$

where matrix $P_{il} \geq 0$, h_{iq} expresses the elements in row q of $\mathcal{L}(H_i)$.

Definition 1 (Liu et al., 2006) Set $\mathcal{D} \subset \mathbb{R}^n$ is considered the domain of attraction of mean square scene (DoA-MSS) of MJSs in Eq. (1), if the following inequality is true

$$\mathcal{E}\left\{\sum_{k=0}^{\infty} \|x(k, x_0, r_0)\|^2 | x_0, r_0\right\} < \infty \tag{6}$$

and $x(0)$ belongs to set $\mathcal{D}$.

Definition 2 The MJSs in Eq. (1) is said to be stochastically passive, if there exists a scalar $\gamma > 0$ such that the following inequality

$$2\sum_{s=0}^{k_0} \mathcal{E}\{z(s)^T\omega(s)\} \geqslant -\gamma \sum_{s=0}^{k_0} ||\omega(s)||^2, k_0 \geq 0, \tag{7}$$

is true under the zero initial condition.

Lemma 1 (Hu and Lin, 2001) If $x(k) \in \mathcal{L}(H_i)$, the standard actuator saturation function $\sigma(u(k)) = \sigma(K_i x(k))$ is further defined as follows:

$$\sigma(K_i x(k)) = \sum_{l=1}^{2^m} \eta_l(k)(M_l K_i + M_l^- H_i)x(k), \tag{8}$$

The variables $\eta_l(k)$ satisfy the conditions $0 \leq \eta_l(k) \leq 1$ and $\sum_{l=1}^{2^m} \eta_l(k) = 1$. The matrices K_i and H_i are control gain matrices. The diagonal matrices M_l belong to $\mathbb{R}^{m\times m}$ and have diagonal elements that are either '1' or '0'. And, matrices M_l^- are defined as $M_l^- = I - M_l$, where I is the identity matrix.

Lemma 2 (Cao et al., 2002) For matrices $P_i \in \mathbb{R}^{m\times m} > 0$, $\phi_i \in \mathbb{R}^{m\times m}$, and $i \in \mathcal{M}$, positive integer set $\mathcal{M} = \{1, 2, \dots, M\}$, if $\sum_{i=1}^{M} \alpha_i = 1, 0 \leq \alpha_i \leq 1$, the following inequalities are true:

$$(\sum_{i=1}^{M} \alpha_i\phi_i)^{\mathrm{T}} P_i (\sum_{i=1}^{M} \alpha_i\phi_i) \leq \sum_{i=1}^{M} \alpha_i\phi_i^{\mathrm{T}} P_i\phi_i. \tag{9}$$

3 Main results

In this section, some sufficient conditions for ensuring the mean square stability and stochastic passivity of MJSs are given, utilizing saturation-dependent Lyapunov functions and LMI techniques.

Theorem 1 If there exist $P_{il} > 0$, scalar $\gamma > 0$ satisfying expressions (10)-(12),

$$\tilde{A}_{il}^T - \tilde{B}_{il}^T \tilde{F}_i^{-1} \tilde{B}_{il} - P_{il} < 0, \tag{10}$$

$$\tilde{F}_i \leq 0, \tag{11}$$

$$\Omega(P_{il}) \subset \mathcal{L}(H_i), \tag{12}$$

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the MJSs in Eq. (1) are mean square stable and stochastically passive. Additionally, the set $\bigcap_{i=1}^{N}\bigcap_{l=1}^{2^m}\Omega(P_{il})$ is referred to as an estimation of the DoA-MSS, where

$$\begin{aligned}
\tilde{A}_{il} &= A_{il}^T\sum_{j=1}^{N}\pi_{ij}P_{jt}A_{il},\\
\tilde{B}_{il}^T &= A_{il}^T\sum_{j=1}^{N}\pi_{ij}P_{jt}F_i - C_{il}^T,\\
\tilde{F}_i &= F_i^T\sum_{j=1}^{N}\pi_{ij}P_{jt}F_i - \gamma I,\\
A_{il} &= A_i + B_i(M_lK_i + M_l^-H_i),\\
C_{il} &= C_i + D_i(M_lK_i + M_l^-H_i).
\end{aligned}$$

Proof. It is assumed that $x(0)$ belongs to the set $\bigcap_{i=1}^{N}\bigcap_{l=1}^{2^m}\Omega(P_{il})$. According to expression (12), it can be deduced that $x(0)$ is within the $\mathcal{L}(H_i)$. To fully leverage the available information, a stochastic Lyapunov function that depends on both saturation and system mode is introduced as follows:

$$\mathcal{V}(k) = x^T(k)\left(\sum_{l=1}^{2^m}\eta_l(k)P_{il}\right)x(k). \tag{13}$$

Before further derivation, some important parameters are calculated below:

$$\begin{aligned}
x(k+1) &= A(r(k))x(k) + F(r(k))\omega(k)\\
&\quad + B(r(k))\sum_{l=1}^{2^m}\eta_l(k)(M_lK_i + M_l^-H_i)x(k)\\
&= \sum_{l=1}^{2^m}\eta_l(k)A_{il}x(k) + F_i\omega(k).
\end{aligned} \tag{14}$$

$$\begin{aligned}
z(k) &= (C(r(k)) + D(r(k))\sum_{l=1}^{2^m}\eta_l(k)\\
&\quad (M_lK_i + M_l^-H_i))x(k)\\
&= \sum_{l=1}^{2^m}\eta_l(k)C_{il}x(k)
\end{aligned} \tag{15}$$

For simplicity, the symbol $A(r(k))$ is abbreviated to A_i, and other system parameters are simplified in the same manner. Combining Eq. (13), Eq. (14), and *Lemma* 2, the following definitions can be obtained:

$$\begin{aligned}
&\mathcal{E}\{\mathcal{V}(x(k+1),\theta(k+1)=j \mid x(k),\theta(k)=i)\}\\
&= x^T(k+1)\sum_{t=1}^{2^m}\eta_t(k+1)\sum_{j=1}^{N}\pi_{ij}P_{jt}x(k+1)\\
&= \sum_{l=1}^{2^m}\eta_l(k)\begin{bmatrix}x(k)\\ \omega(k)\end{bmatrix}^T\begin{bmatrix}A_{il}^T\\ F_i^T\end{bmatrix}\sum_{t=1}^{2^m}\eta_t(k+1)\\
&\quad \sum_{j=1}^{N}\pi_{ij}P_{jt}\sum_{l=1}^{2^m}\eta_l(k)\begin{bmatrix}A_{il}^T\\ F_i^T\end{bmatrix}^T\begin{bmatrix}x(k)\\ \omega(k)\end{bmatrix}\\
&\le \sum_{t=1}^{2^m}\eta_t(k+1)\sum_{l=1}^{2^m}\eta_l(k)\begin{bmatrix}x(k)\\ \omega(k)\end{bmatrix}^T\\
&\quad \begin{bmatrix}A_{il}^T\\ F_i^T\end{bmatrix}\sum_{j=1}^{N}\pi_{ij}P_{jt}\begin{bmatrix}A_{il}^T\\ F_i^T\end{bmatrix}^T\begin{bmatrix}x(k)\\ \omega(k)\end{bmatrix},
\end{aligned} \tag{16}$$

$$\begin{aligned}
&\mathcal{E}\{\triangle\mathcal{V}(k)\}\\
&= \mathcal{V}\{x(k+1),\theta(k+1)=j|x(k),\theta(k)=i\} - \mathcal{V}(k)\\
&\le \sum_{t=1}^{2^m}\eta_t(k+1)\sum_{l=1}^{2^m}\eta_l(k)\begin{bmatrix}x(k)\\ \omega(k)\end{bmatrix}^T\begin{bmatrix}A_{il}^T\\ F_i^T\end{bmatrix}\sum_{j=1}^{N}\pi_{ij}P_{jt}\\
&\quad \begin{bmatrix}A_{il}^T\\ F_i^T\end{bmatrix}^T\begin{bmatrix}x(k)\\ \omega(k)\end{bmatrix} - x(k)^T\sum_{l=1}^{2^m}\eta_l(k)P_{il}x(k),
\end{aligned} \tag{17}$$

If $\omega(k)\equiv 0$, $\mathcal{E}\{\triangle\mathcal{V}(k)\}<0$ can be made sure by the following inequalities:

$$A_{il}^T\sum_{j=1}^{N}\pi_{ij}P_{jt}A_{il} - P_{il} \le 0. \tag{18}$$

Noting $\sum_{t=1}^{2^m}\eta_t(k+1)=1$, $\sum_{l=1}^{2^m}\eta_l(k)=1$, we can easily get expression (18) from condition (10). Similar to previous research (Wang and Wu, 2022), the mean square stability of the MJSs in Eq. (1) can be satisfied via $\mathcal{E}\{\triangle\mathcal{V}(k)\}<0$ and **Definition 1**.

Under the zero initial condition, the definition

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of $\mathcal{J}$ is considered as follows:

$$\begin{aligned}\mathcal{J} &\triangleq \sum_{s=0}^{k_0}\mathcal{E}\{-2z^T(s)\omega(s)-\gamma\omega(s)^T\omega(s)\}\\ &\le \sum_{s=0}^{k_0}\mathcal{E}\{-2z^T(s)\omega(s)-\gamma\omega(s)^T\omega(s)+\triangle\mathcal{V}(s)\}\\ &\le \sum_{s=0}^{k_0}\{\sum_{t=1}^{2^m}\eta_t(s+1)\sum_{l=1}^{2^m}\eta_l(s)\begin{bmatrix}x(s)\\ \omega(s)\end{bmatrix}^T(\begin{bmatrix}A_{il}^T\\ F_i^T\end{bmatrix}\\ &\quad\sum_{j=1}^{N}\pi_{ij}P_{jt}\begin{bmatrix}A_{il}^T\\ F_i^T\end{bmatrix}^T-\begin{bmatrix}0 & 0\\ 0 & \gamma I\end{bmatrix})\begin{bmatrix}x(s)\\ \omega(s)\end{bmatrix}\\ &\quad-2x(s)^TC_{il}^T\omega(s)-x(s)^T\sum_{l=1}^{2^m}\eta_l(s)P_{il}x(s)\}\\ &= \sum_{s=0}^{k_0}\sum_{t=1}^{2^m}\eta_t(s+1)\sum_{l=1}^{2^m}\eta_l(s)\begin{bmatrix}x(s)\\ \omega(s)\end{bmatrix}^T\Theta_{il}\begin{bmatrix}x(s)\\ \omega(s)\end{bmatrix}\\ &\quad-x(s)^T\sum_{l=1}^{2^m}\eta_l(s)P_{il}x(s)\end{aligned}\tag{19}$$

where

$$\Theta_{il}=\begin{bmatrix}\tilde{A}_{il}^T & \tilde{B}_{il}^T\\ * & \tilde{F}_i^T\end{bmatrix}.$$

The following inequalities can be obtained via expression (11)

$$\begin{bmatrix}\tilde{B}_{il}^T\tilde{F}_i^{-1}\tilde{B}_{il} & \tilde{B}_{il}^T\\ * & \tilde{F}_i^T\end{bmatrix}\le 0,$$

and we can further get the following:

$$\Theta_{il}\le\begin{bmatrix}\tilde{A}_{il}^T-\tilde{B}_{il}^T\tilde{F}_i^{-1}\tilde{B}_{il} & 0\\ * & 0\end{bmatrix}.\tag{20}$$

Combine expressions (19) and (20), $\mathcal{J}<0$ can be obtained via the following condition

$$\begin{aligned}\sum_{t=1}^{2^m}\eta_t(k+1)\sum_{l=1}^{2^m}\eta_l(k)x(k)^T(\tilde{A}_{il}^T\\ -\tilde{B}_{il}^T\tilde{F}_i^{-1}\tilde{B}_{il}-P_{il})x(k)<0.\end{aligned}\tag{21}$$

Now, it is easy to see that condition (21) is equal to expression (10). It is guaranteed by **Theorem 1** that the MJSs in Eq. (1) are mean square stable and stochastically passive. The proof is completed. □

Theorem 2 If there exist $\bar{P}_{ij}>0$, $\bar{R}_{ij}>0$, $G_i>0$ and scalar $\gamma>0$ satisfying

$$\begin{bmatrix}\bar{R}_{il}-G_i^T-G_i & \mathcal{C}_{il}^T & \hat{\mathcal{A}}_{il}^T\\ * & -\gamma I & \hat{\mathcal{F}}_i^T\\ * & * & \bar{\mathcal{P}}_{it}\end{bmatrix}<0,\tag{22}$$

$$\begin{bmatrix}-\bar{P}_{il} & \bar{P}_{il}^T\\ * & -\bar{R}_{il}\end{bmatrix}<0,\tag{23}$$

$$\begin{bmatrix}\bar{P}_{il}-G_i^T-G_i & y_{iq}^T\\ * & -1\end{bmatrix}\le 0,\tag{24}$$

The MJSs in Eq. (1) are mean square stable and stochastically passive, and the set $\bigcap_{i=1}^{N}\bigcap_{l=1}^{2^m}\Omega(P_{il})$ is referred to as an estimation of DoA-MSS, where

$$\begin{aligned}&\hat{\mathcal{A}}_{il}^T=\begin{bmatrix}\mathcal{A}_{il}^T & \mathcal{A}_{il}^T & \dots & \mathcal{A}_{il}^T\end{bmatrix},\\ &\hat{\mathcal{F}}_i^T=\begin{bmatrix}F_i^T & F_i^T & \dots & F_i^T\end{bmatrix},\\ &\bar{\mathcal{P}}_{il}=-\mathbf{diag}\begin{Bmatrix}\pi_{i1}\bar{P}_{1t} & \pi_{i2}\bar{P}_{2t} & \dots & \pi_{iN}\bar{P}_{Nt}\end{Bmatrix},\\ &\mathcal{X}_i=K_iG_i,\ \mathcal{Y}_i=H_iG_i,\ \bar{P}_{il}=P_{il}^{-1},\ \bar{R}_{il}=R_{il}^{-1},\\ &\mathcal{A}_{il}^T=G_i^TA_i^T+\mathcal{X}_i^TM_l^TB_i^T+\mathcal{Y}_i^T(M_l^-)^TB_i^T,\\ &\mathcal{C}_{il}^T=-G_i^TC_i^T-\mathcal{X}_i^TM_l^TD_i^T-\mathcal{Y}_i^T(M_l^-)^TD_i^T.\end{aligned}$$

Proof. Based on **Theorem 1**, we can get the following condition

$$A_{il}^T\sum_{j=1}^{N}\pi_{ij}P_{jt}A_{il}-\tilde{B}_{il}^T\tilde{F}_i^{-1}\tilde{B}_{il}-P_{il}+\epsilon I<0,\ \epsilon>0.\tag{25}$$

Noting scalar $\epsilon>0$, R_{il} is further considered as

$$R_{il}\triangleq A_{il}^T\sum_{j=1}^{N}\pi_{ij}P_{jt}A_{il}-\tilde{B}_{il}^T\tilde{F}_i^{-1}\tilde{B}_{il}+\epsilon I.$$

Condition (23) (i.e., $R_{il}-P_{il}<0$) is used to ensure ϵ exists.

Then, inequalities (26) can be given

$$-R_{il}+A_{il}^T\sum_{j=1}^{N}\pi_{ij}P_{jt}A_{il}-\tilde{B}_{il}^T\tilde{F}_i^{-1}\tilde{B}_{il}<0.\tag{26}$$

Further, the proposition that inequalities (26) and (27) are equivalent can be proven using the Schur complement:

$$\begin{bmatrix}-R_{il} & -C_{il}^T & \begin{bmatrix}A_{il}^T & A_{il}^T & \dots & A_{il}^T\end{bmatrix}\\ * & -\gamma I & \begin{bmatrix}F_i^T & F_i^T & \dots & F_i^T\end{bmatrix}\\ * & * & \bar{\mathcal{P}}_{il}\end{bmatrix}<0.\tag{27}$$

Letting auxiliary matrix $\mathbf{diag}\begin{Bmatrix}G_i & I & I\end{Bmatrix}$ and G_i be the undetermined matrix variable pre- and post-multiplying the inequality (27) by matrix $\mathbf{diag}\begin{Bmatrix}G_i & I & I\end{Bmatrix}$ and its transpose, we can get

$$\begin{bmatrix}-G_i^TR_{il}G_i & -G_i^TC_{il}^T & \hat{\mathcal{A}}_{il}^T\\ * & -\gamma I & \hat{\mathcal{F}}_i^T\\ * & * & \bar{\mathcal{P}}_{il}\end{bmatrix}<0.\tag{28}$$

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According to $-G_i^T R_{il} G_i \leq \bar{R}_{il} - G_i^T - G_i$, (22) is obtained.

According to the definitions of the symmetric polyhedron set $\mathcal{L}(H_i)$ and the ellipsoid set $\Omega(P_{il})$, expression (29) can be satisfied via expression (12)

$$|h_{iq}x(k)| \leq 1, \quad \forall x(k) \in \bigcap_{i=1}^{N} \bigcap_{l=1}^{2^m} \Omega\left(P_{il}\right), \tag{29}$$

where h_{iq} denotes the qth row of H_i. Furthermore, we have

$$h_{iq}^T h_{iq} - P_{il} \leq 0. \tag{30}$$

According to the Schur complement theorem, inequality (30) is equivalent to

$$\begin{bmatrix} -P_{il} & {h_{iq}}^T \\ * & -1 \end{bmatrix} \leq 0. \tag{31}$$

By means of congruent transformation via matrix $\mathbf{diag}\left\{G_i \quad 1\right\}$ and $-G_i^T P_{il} G_i \leq \bar{P}_{il} - G_i^T - G_i$, if inequality (31) holds, we have

$$\begin{bmatrix} -G_i^T P_{il} G_i & G_i^T h_{iq}^T \\ * & -1 \end{bmatrix} \leq \begin{bmatrix} \bar{P}_{il} - G_i^T - G_i & y_{iq}^T \\ * & -1 \end{bmatrix} \leq 0, \tag{32}$$

where y_{iq} denotes the qth row of $\mathcal{Y}_i$. The proof is completed. □

Remark: The problem of LMI-coupled variables is overcome by introducing G_i and by means of matrix congruence transformation. Moreover, it can be judged using **Theorem 2** that $x(0)$ is in the DoA-MSS. To intuitively ascertain whether $x(0)$ is encompassed within the DoA-MSS, the optimal control problem is introduced.

OP1:

$$\mathbf{sup} \quad \tau$$

$$\bar{P}_{il} > 0,\ \bar{R}_{il} > 0,\ G_i > 0,\ \mathcal{X}_i > 0,\ \mathcal{Y}_i > 0$$

$$\mathbf{s.t.}\ (a) \qquad \tau x(0) = \bigcap_{i=1}^{N} \bigcap_{l=1}^{2^m} \Omega(P_{il}),$$
$$(b) \qquad \text{LMIs (22)} - \text{(24)}.$$

Combining the definition of $\Omega(P_{il})$, condition (a) is equivalent to the expression $\tau^2 x(0)^T P_{il} x(0) \leq 1$. Furthermore, expression (33) can be obtained via the Schur complement; it yields the following:

$$\begin{bmatrix} -\rho & x(0)^T \\ * & -\bar{P}_{il} \end{bmatrix} \leq 0. \tag{33}$$

Here, the symbol $\rho = \tau^{-2}$.

Therefore, **OP2** can be given as follows:

OP2:

$$\mathbf{inf} \quad \rho$$

$$\bar{P}_{il} > 0,\ \bar{R}_{il} > 0,\ G_i > 0,\ \mathcal{X}_i > 0,\ \mathcal{Y}_i > 0$$

$$\mathbf{s.t.}\ (a) \qquad \begin{bmatrix} -\rho & x(0)^T \\ * & -\bar{P}_{il} \end{bmatrix} \leq 0,$$
$$(b) \qquad \text{LMIs (22)} - \text{(24)}.$$

It means that $x(0)$ is in the DoA-MSS, if the optimal value $\rho_{min} < 1$. Moreover, the smaller value of ρ_{min} means that the DoA-MSS is closer to the real one. Subsequently, **OP2** is applied within the simulation.

In practical engineering projects, real-time acquisition of internal mode information becomes highly costly due to time-delay phenomena caused by physical characteristics of sensor components and limited network channel resources. The MID control strategy is a classical approach to cope with asynchrony between the system and controller (Goncalves et al., 2009; Nie et al., 2024; Oliveira et al., 2014). The following corollary gives the conditions when the controller's mode is totally unavailable.

3.1 Corollary

The MJSs in Eq. (1) are mean square stable and stochastically passive, and the set $\bigcap_{i=1}^{N} \bigcap_{l=1}^{2^m} \Omega(P_{il})$ is called an estimation of the DoA-MSS, if there exist $\bar{P}_{ij} > 0$, $\bar{R}_{ij} > 0$, $G_i > 0$, and scalar $\gamma > 0$ satisfying expressions (23), and (24) and

$$\begin{bmatrix} \bar{R}_{il} - G_i^T - G_i & \hat{\Psi}_{il}^T & \hat{\Xi}_{il}^T \\ * & -\gamma I & \hat{\mathcal{F}}_i^T \\ * & * & \bar{\mathcal{P}}_{it} \end{bmatrix} < 0, \tag{34}$$

where

$$\hat{\Xi}_{il}^T = \begin{bmatrix} \Xi_{il}^T & \Xi_{il}^T & \dots & \Xi_{il}^T \end{bmatrix}, \hat{\mathcal{X}}_i = KG_i,$$
$$\hat{\mathcal{F}}_i^T = \begin{bmatrix} F_i^T & F_i^T & \dots & F_i^T \end{bmatrix}, \hat{\mathcal{Y}}_i = HG_i,$$
$$\Xi_{il}^T = G_i^T A_i^T + \hat{\mathcal{X}}_i^T M_l^T B_i^T + \hat{\mathcal{Y}}_i^T (M_l^-)^T B_i^T,$$
$$\hat{\Psi}_{il}^T = -G_i^T C_i^T - \hat{\mathcal{X}}_i^T M_l^T D_i^T - \hat{\mathcal{Y}}_i^T (M_l^-)^T D_i^T.$$

Remark: The definition of an MID controller is considered as $u(k) = Kx(k)$. Moreover, function (35) can be satisfied via *Lemma* 1:

$$\sigma(Kx(k)) = \sum_{l=1}^{2^m} \eta_l(k)(M_l K + M_l^- H)x(k). \tag{35}$$

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As a special case, MID control is considered as a conservative control strategy. Similar to the above process, condition (34) can be obtained based on Eq. (35).

4 Simulations

In this section, we verify the validity of the sufficient conditions. Utilizing the MATLAB LMI Toolbox, we solve for the parameters of the MD controller. The outcomes of the simulation are then visually presented through graphical representations.

Example 1 We consider a four-mode discrete-time MJS, whose parameters are as follows (Wang et al., 2012):

$$A_1 = \begin{bmatrix} 1.72 & -1.40 \\ 0.80 & -0.80 \end{bmatrix}, A_2 = \begin{bmatrix} 1.58 & -0.36 \\ 0.80 & -1.12 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} 0.76 & -0.28 \\ 0.80 & -0.96 \end{bmatrix}, A_4 = \begin{bmatrix} 1.28 & -0.38 \\ 0.80 & -0.88 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, B_2 = \begin{bmatrix} 5 \\ -6 \end{bmatrix}, B_3 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, B_4 = \begin{bmatrix} 0.8 \\ -1 \end{bmatrix},$$

$$C_1 = C_2 = C_3 = C_4 = \begin{bmatrix} 0.1 & 0 \end{bmatrix},$$

$$D_1 = D_2 = D_3 = D_4 = 0.2,$$

$$F_1 = F_2 = F_3 = F_4 = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix},$$

$$\Pi = \begin{bmatrix} 0.3 & 0.3 & 0.1 & 0.3 \\ 0.25 & 0.25 & 0.3 & 0.2 \\ 0.3 & 0.1 & 0.3 & 0.3 \\ 0.2 & 0.3 & 0.2 & 0.3 \end{bmatrix}, x(0) = \begin{bmatrix} -2 \\ 1.5 \end{bmatrix}.$$

The external disturbance is considered as $\omega(k) = 1/k$. Setting performance index $\gamma = 1$, we can get $\rho_{\min} = 0.3438$ by solving the optimal problem **OP 2** (Ma and Zhang, 2012; Wang et al., 2010). Since $\rho_{\min} < 1$, this indicates that the initial state of the system is within the DoA-MSS, thereby guaranteeing the robustness of the system. Plugging $\rho_{\min}$ we can obtain the performance index $\gamma = 1.6935$. And, we can get the following MD controller gain matrices:

$$K_1 = \begin{bmatrix} -0.8608 & 0.6954 \end{bmatrix}, K_2 = \begin{bmatrix} -0.0862 & -0.0603 \end{bmatrix},$$
$$K_3 = \begin{bmatrix} -0.7719 & 0.5563 \end{bmatrix}, K_4 = \begin{bmatrix} -1.1391 & 0.6846 \end{bmatrix},$$
$$H_1 = \begin{bmatrix} -0.5318 & 0.4317 \end{bmatrix}, H_2 = \begin{bmatrix} -0.0858 & -0.0590 \end{bmatrix},$$
$$H_3 = \begin{bmatrix} -0.3512 & 0.3086 \end{bmatrix}, H_4 = \begin{bmatrix} -0.5345 & 0.2241 \end{bmatrix}.$$

With $u(k) = 0$, we simulate the open-loop system (1) under the aforementioned parameters to observe its state evolution without a controller. The entire state trajectory was recorded and visualized (Fig. 1), which clearly demonstrates the system's inherent instability in the absence of control. When implementing the proposed MD control law to form a closed-loop system undergoing identical modal transitions, the system states exhibit rapid convergence as shown in Fig. 2. Comparative analysis with Fig. 1 confirms that the designed MD controller successfully stabilizes the system. The complete modal evolution trajectories experienced by the system under both conditions (Figs 1 and 2) are chronologically documented in Fig. 3.

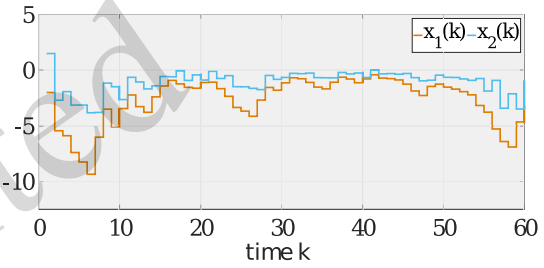

Fig. 1 System state without controller

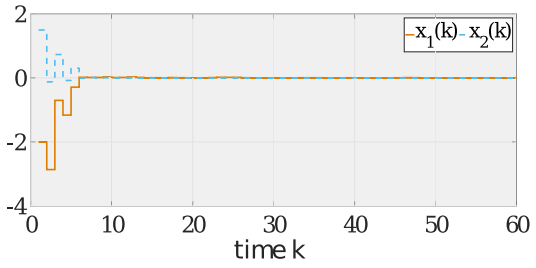

Fig. 2 System state under MD controller

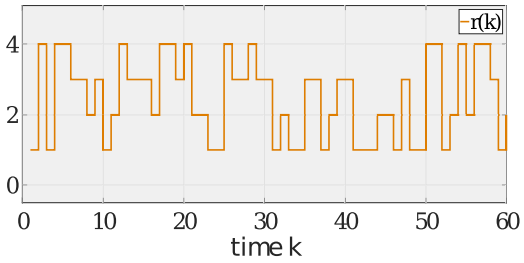

Fig. 3 System's mode evolution

Example 2 This example uses a direct current (DC) motor device to demonstrate the superiority of our MD controller design technology. The DC

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motor device (Oliveira et al., 2014) can be modeled as system (1) with the following parameters:

$$A_1 = \begin{bmatrix} -0.4799 & 5.1546 & 0 \\ -3.8162 & 14.4732 & 0 \\ 0.1399 & 0 & -0.9255 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -1.6026 & 9.1632 & 0 \\ -0.5918 & 3.0317 & 0 \\ 0.0740 & 0 & -0.4338 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} 0.6346 & 0.9178 & 0 \\ -0.5056 & 2.4811 & 0 \\ 0.3865 & 0 & 0.0982 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 5.8705 \\ 15.5010 \\ 0 \end{bmatrix}, B_2 = \begin{bmatrix} 10.2951 \\ 2.2282 \\ 0 \end{bmatrix}, B_3 = \begin{bmatrix} 0.7874 \\ 1.5302 \\ 0 \end{bmatrix},$$

$$C_i = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1.5 & 0 \\ 0 & 0 & 2.5 \end{bmatrix}, D_i = \begin{bmatrix} 0 \\ 0 \\ 0.5 \end{bmatrix},$$

$$F_i = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}.$$

The TPM is given as follows:

$$\Pi = \begin{bmatrix} 0.9 & 0.05 & 0.05 \\ 0.36 & 0.6 & 0.04 \\ 0.1 & 0.1 & 0.8 \end{bmatrix}.$$

The external disturbance input is considered as follows:

$$\omega(k) = \begin{bmatrix} v(k) & v(k) & v(k) \end{bmatrix}^T,$$

$$v(k) = \begin{cases} 0.9^k \tilde{v}(k), & 0 \le k \le 20 \\ -0.5, & 50 \le k \le 55 \\ 0, & else \end{cases}$$

where $\tilde{v}(k)$ is a random signal between '-1' and '1'.

Under initial state $x(0) = \begin{bmatrix} -5 & 3 & -2 \end{bmatrix}^T$, both the MID and MD control strategies are implemented for comparative simulation analysis. The state trajectories of the DC motor system under the same modal evolution process were recorded and are presented in Figs 4 and 5 when applying MID and MD controllers, respectively. The comparative results demonstrate that the MD control strategy achieves superior control performance compared to the MID strategy. This outcome aligns with our theoretical predictions, as the MD control paradigm effectively utilizes system information to enhance closed-loop stability.

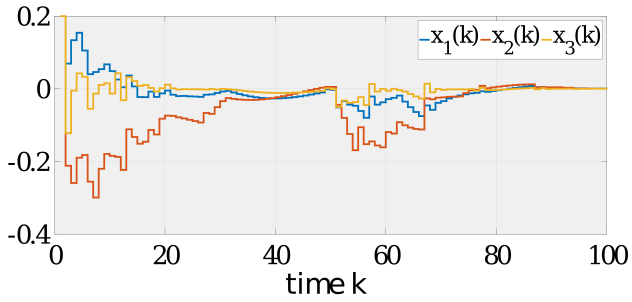

Fig. 4 System state under MID controller

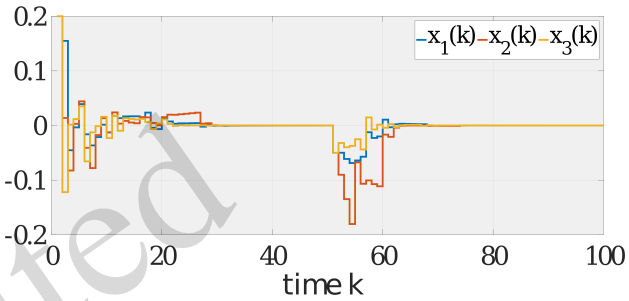

Fig. 5 System state under MD controller

5 Conclusions

An MD controller design methodology has been proposed to guarantee the mean square stability and stochastic passivity for discrete-time MJSs with actuator saturation. By introducing a saturation-dependent Lyapunov function, we have derived relatively less-conservative sufficient conditions. An MID control corollary has been introduced as a special case to complement our theoretical findings. At the end of this paper, the superiority of our theoretical result has been validated through two numerical examples. Notably, with the advancement of communication network technologies, engineering requirements for networked system stability and security have become increasingly stringent. In scenarios involving unobservable mode information and constrained network bandwidth resources, we will further investigate robust control strategies under denial-of-service (DOS) attacks using asynchronous event-triggered control mechanisms, and we think it is a challenging research direction.

Contributors

Liqing Wang guided the research. Jiaming Tang designed the research and drafted the paper. Mingkun Wang and Feiyu Yang conducted and analyzed the experiments. All the authors had participated in writing the paper.

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Conflict of interest

All the authors declare that they have no conflict of interest.

Data availability

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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