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Two-level fuzzy evaluation for classification of credits*

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Abstract: In this paper, classification models are used as tools to make final decision. Fuzzy method provides the mathematical tools for quantitative analysis and dealing with ambiguous concepts. Analytic Hierarchy Process (AHP) is used to obtain the weight of each index and enables examiners to visualize the decision process and obtain more reasonable evaluation values to solve some problems. An example is given at the end of this paper.

Key words: Classification of credits, Analytical Hierarchy Process(AHP), Fuzzy
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INTRODUCTION

A new standard classification of credits based on risk was set up to deepen the financial reform in China in order to improve the capacity to lessen the risk of commercial banks and increase the safety and profit. Compared to the former credit classification method based on time, the quality of credit assets can be evaluated more objectively, scientifically, and correctly. However the classification of credits based on risk faces some problems in practice. Because of the fuzziness and the large number, it is difficult to make a combined decision.

METHOD

Establishment of the indexes system for evaluation

For effective evaluation of credit risk, it is necessary to establish a systematic, comprehensive, and hierarchical indexes system. Different indexes play different roles within the system. We can classify them as shown in Fig.1.

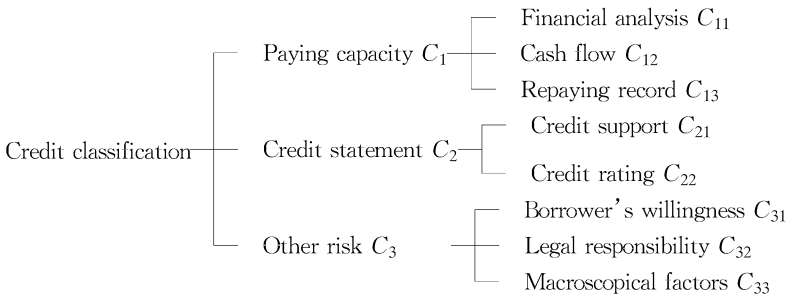

Fig.1 The indexes system of evaluation

The three-layer comprehensive evaluation indexes system proposed here is determined by three major combined indexes: paying capacity (C_1), credit statements (C_2), and other risk (C_3). The original source of repayment should be based on the borrower's paying capacity, measured by financial analysis, cash flow and record of borrower's repay-

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ment. The secondary sources relying on credit support (provided by assigned collateral, or by cosigners, endorsers or guarantors) should influence the final decision on a borrower's credit worthiness. Credit rating is also used to analyze the credit statement. At last, the borrower's willingness to repay, bank management, legal responsibility and other macroscopical factors relevant to the borrower's economic conditions and ability to settle his debts must be also considered.

Determing the weights of each index with AHP

AHP is used to confirm the weight of every index in the evaluation system (Mikko et al., 2000; Kamal et al., 2001).

1. Construct the matrix of pairwise comparisons

Pairwise comparisons are done to determine which element dominates the other. In this matrix, the element $a_{ij}=1/a_{ji}$ and thus, when $i=j$, $a_{ij}=1$. Importance is measured on a 1/9 to 9 scale. 1 indicates that the two alternatives are equally important while 9 indicates alternative i is extremely or absolutely more important than alternative j.

Since the value of a_{ij} of pairwise comparisons is difficult to determine, the Delphi technique can be used. Based on experts judgment, we get the value of a_{ij} and calculate the relative weights of the decision Indexes.

2. Determine whether the input data pass a "consistency test"

Having made all the pair-wise comparisons, the consistency is determined by using the eigenvalue λ_{max}, to calculate the consistency index C_1 as follows: $C_1=(\lambda_{max}-m)/(m-1)$, $CR=(C_1/R_1)$, where m is the matrix size. If the pairwise comparisons do not include any inconsistencies, $\lambda_{max}=n$ (number of rows). The more consistent the comparisons are, the closer the computed λ_{max} is to n. If the matrix is perfectly consistent, the consistency index will be 0. Otherwise, the consistency index will be positive. As a rule of thumb, a CR value of 10% or less is considered to be acceptable. Otherwise, all or some of the comparisons must be repeated in order to resolve the inconsistencies of the parewise comparisons.

3. Aggregate the relative weights of the guideline system(Table 1).

Table 1 Systematic weight of each factor in evaluation system

	C_1	C_2	C_3	w_{ij}(Weight)
	0.637	0.258	0.105	
C_{11}	0.637			0.406
C_{12}	0.258			0.164
C_{13}	0.105			0.067
C_{21}		0.675		0.174
C_{22}		0.325		0.083
C_{31}			0.50	0.053
C_{32}			0.25	0.026
C_{33}			0.25	0.026

Fuzzy multicriteria multilayer evaluation model

1. Establish comments set V and evaluation index set U

$V=\{v_1, v_2, \cdots, v_n\}$ $v_j(j=1,2,\cdots,n)$;

$U=\{u_1, u_2, \cdots, u_m\}$ $u_i(i=1,2,\cdots,m)$.

Comments set is the linguistic description of the conclusion drawn for the estimated project. The number of the indexes can be more or less, but the key indexes reflecting the evaluation have to be included, which is the precondition guaranteeing the validity of the evaluation conclusion (Shan et al., 1999).

2. Form the evaluation matrix R

The following matrix represents evaluation matrix R:

$$R=\begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ r_{m1} & r_{m2} & \cdots & r_{mn} \end{bmatrix}$$

Where $r_{ij}(i=1, 2, \cdots, m; j=1, 2, \cdots, n)$ represents the satisfying degree of each criterion to comment u_i, that is the row of the matrix R.

U, V, and R compose a fuzzy comprehensive model. We get the function value P, which is the estimated value of the evaluation and shows the satisfaction of the comment on quantity. P is the result of the fuzzy comprehensive evaluation set of the u_i.

$$P = W \circ R = \{p_1, p_2, \cdots, p_n\}$$

For the weighting set of the evaluation index u_i normalized to sum to one,

$W = \{w_1, w_2, \cdots, w_m\}$, where $\sum_{i=1}^{m} w_i = 1$.

If $\sum_{i=1}^{n} p_j \neq 1$, then $\tilde{P} = (\tilde{p}_1, \tilde{p}_2, \cdots, \tilde{p}_n)$, where $\tilde{p}_j = p_j / \sum_{j=1}^{n} p_j \quad j = 1, 2, \cdots, n$.

3. Determine with the AHP the weights of each index.

4. Select fuzzy operators.

Since in most cases a variety of criteria are used to evaluate classification of credit, we further extend the techniques of combining fuzzy operators. Four different models are generalized as follows: $M(\wedge, \vee)$ considers only important factors; $M(\bullet, \vee)$ and $M(\wedge, \oplus)$ emphasizes important factors; $M(\bullet, \oplus)$ considers every single factor. The calculation model is shown in Table 2 (Cao et al., 2000).

Table 2 Fuzzy operators

Model	Fuzzy operator	Rule of calculation
$M(\wedge, \vee)$	$\wedge$, $\vee$	$p_j = \bigvee_{i=1}^{n} (w_i \wedge r_{ij})$
$M(\bullet, \vee)$	$\bullet$, $\vee$	$p_j = \bigvee_{i=1}^{n} w_i r_{ij}$
$M(\wedge, \oplus)$	$\wedge$, $\oplus$	$p_j = \sum_{i=1}^{n} (w_i \wedge r_{ij})$
$M(\bullet, \oplus)$	$\bullet$, $\oplus$	$p_j = \sum_{i=1}^{n} (w_i r_{ij})$

Use of different fuzzy operator can yield different results $\tilde{P}_1$, $\tilde{P}_2$, $\tilde{P}_3$, $\tilde{P}_4$, for construction of another fuzzy matrix $\tilde{R}'$. The weight of fuzzy operators is defined as $W' = w'_1, w'_2, \cdots, w'_m$. A two-level fuzzy evaluation model $M(\bullet, \oplus)$ is used to aggregate the vector $\tilde{P}' = W' \bullet \tilde{R}' = \{p'_1, p'_2, \cdots, p'_n\}$.

AN EXAMPLE

Set comments set and the evaluation index set

Comments set $V = \{$pass, specially, mentioned, substandard, doubtful, loss$\}$.

Evaluation index set $U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8\}$.

Where u_1 is the financial capacity of borrower; u_2 is the cash flow; u_3 is comprised of other relevant factors affecting the ability of the borrower to pay back what he borrowed; u_4 is credit rating; u_5 is credit support; u_6 is bank management; u_7 is borrower's willingness to repay; u_8 is legal responsibility.

Calculating the weighting set of each evaluation index

According to the above AHP method, the weight is determined as follows:

$W = \{w_1, w_2, \cdots, w_8\} = \{0.406, 0.164, 0.067, 0.174, 0.083, 0.053, 0.026, 0.026\}$.

Format the evaluation matrix *R*

Superior institutions, other financial institutions, experts, etc. can implement ideal classification of credit. Their ability to make sound decisions on the basis of R is very important for the success of the evaluation. They can use their experience, values and knowledge to improve the veracity of the classification. In this case, five experts were invited to give the satisfying degree of the indexes.

The evaluation matrix is obtained as follows:

$$R = \begin{bmatrix} 0 & 2/5 & 2/5 & 1/5 & 0 \\ 0 & 2/5 & 2/5 & 1/5 & 0 \\ 0 & 1/5 & 3/5 & 1/5 & 0 \\ 2/5 & 2/5 & 1/5 & 0 & 0 \\ 2/5 & 3/5 & 0 & 0 & 0 \\ 0 & 0 & 2/5 & 2/5 & 1/5 \\ 2/5 & 2/5 & 1/5 & 0 & 0 \\ 0 & 2/5 & 2/5 & 1/5 & 0 \end{bmatrix}$$

Calculate the result of fuzzy evaluation

Four different fuzzy operators were used

to calculate the result of fuzzy evaluation.

$P = A \circ R = (0.406, 0.164, 0.067, 0.174, 0.083, 0.053, 0.026, 0.026)$

$$\circ \begin{bmatrix} 0 & 2/5 & 2/5 & 1/5 & 0 \\ 0 & 2/5 & 2/5 & 1/5 & 0 \\ 0 & 1/5 & 3/5 & 1/5 & 0 \\ 2/5 & 2/5 & 1/5 & 0 & 0 \\ 2/5 & 3/5 & 0 & 0 & 0 \\ 0 & 0 & 2/5 & 2/5 & 1/5 \\ 2/5 & 2/5 & 1/5 & 0 & 0 \\ 0 & 2/5 & 2/5 & 1/5 & 0 \end{bmatrix}$$

From the model $M(\wedge, \vee)$, we can obtain the outcome:

$P_1 = (0.174, 0.4, 0.4, 0.2, 0.053)$.

$\widetilde{P}_1 = (0.142, 0.326, 0.326, 0.163, 0.043)$.

For model $M(\bullet, \vee)$, $P_2 = (0.070, 0.162, 0.162, 0.081, 0.011)$.

$\widetilde{P}_2 = \{0.144, 0.333, 0.333, 0.167, 0.023\}$.

For model $M(\wedge, \oplus)$, $P_3 = (0.283, 0.94, 0.91, 0.51, 0.053)$.

$\widetilde{P}_3 = \{0.105, 0.349, 0.338, 0.189, 0.020\}$.

For model $M(\bullet, \oplus)$, $P_4 = \widetilde{P}_4 = (0.113, 0.382, 0.340, 0.154, 0.011)$.

Integrate the fuzzy evaluation

According to the vectors $\widetilde{P}_1, \widetilde{P}_2, \widetilde{P}_3, \widetilde{P}_4$, we can establish the evaluation result matrix $\widetilde{R}'$.

$$\widetilde{R}' = \begin{bmatrix} 0.142 & 0.326 & 0.326 & 0.163 & 0.043 \\ 0.144 & 0.333 & 0.333 & 0.167 & 0.023 \\ 0.105 & 0.349 & 0.338 & 0.189 & 0.020 \\ 0.113 & 0.382 & 0.340 & 0.154 & 0.011 \end{bmatrix}$$

The weights of the fuzzy operators W were taken as 0.2, 0.25, 0.25, 0.3.

A two-level fuzzy evaluation model $M(\bullet, \oplus)$ was used to integrate the evaluation.

$\widetilde{P}' = W' \bullet \widetilde{R}' = (0.125, 0.350, 0.335, 0.168, 0.023)$

From the final evaluation, p'_2 equal to 0.350 was the maximum.

CONCLUSIONS

In this article, the AHP and fuzzy mathematics integrated method is used to synthetically evaluate the credit classification. The advantages of the model are its ability to make both qualitative and quantitative decisions, and its being easy to apply and to understand. The two-level fuzzy evaluation model emphasizes important factors as well as considers all other possibly affecting factors. Optimum application of the model yields accurate evaluation of credit classification.

The classification of credit is a practical job. The model in this paper can help examiners evaluate and judge in a more scientific way. The whole evaluation procedure was implemented with the use of microcomputer using C language and a related database. This approach has feasibility and practical value. The result of the evaluation can be important reference for analysis and judgment. Moreover, the method can be applied to typical enterprises loans. As for other special loan forms such as consumptions loan, policy loan and so on, the model has to be adjusted thought modifying the index or changing the weighting according to different characteristics. For example, for consumption loan, the family income, family wealth and other financial indicators can determine the paying capacity of the consumptions loan. For short-term loan, the debtor's short-term liquidity should be emphasized. As to long-term loan, the debtor's long-term payment ability should be gauged on the basis of indicators such as financing structure, profit efficiency, and operation capability.

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