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Buckling of un-stiffened cylindrical shell under non-uniform axial compressive stress*

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Abstract: This paper provides a review of recent research advances and trends in the area of stability of un-stiffened circular cylindrical shells subjected to general non-uniform axial compressive stresses. Only the more important and interesting aspects of the research, judged from a personal viewpoint, are discussed. They can be crudely classified into four categories: (1) shells subjected to non-uniform loads; (2) shells on discrete supports; (3) shells with intended cutouts/holes; and (4) shells with non-uniform settlements.

Key words: Shell, Buckling, Axial compression, Non-uniform stress
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INTRODUCTION

Over the past half-century, much research has been conducted on the buckling and post-buckling behavior of shells with different geometries and subjected to different loading conditions. Comprehensive surveys of buckling and post-buckling of thin shells were published every 5-10 years (Fung and Sechler, 1960; Budiansky and Hutchinson, 1966; Hoff, 1966; Hutchinson and Koiter, 1970; Sechler, 1974; Babcock, 1983; Simitses, 1986; Noor, 1990). A recent review was carried out more than 5 years ago (Teng, 1996), and shell stability research has advanced further since then. The present paper aims to provide a review of the progress made in the area of buckling of un-stiffened cylindrical shells subjected to general non-uniform compressive stresses.

The paper takes full advantage of previous reviews, especially the review by Teng (1996) which summarized available research on the buckling of shells under non-uniform loading conditions. Emphasis is placed on the achievements that have been made and work that is on-going. The non-uniformity of compressive stresses discussed here results from non-uniform axial loads, non-uniform pressures, discrete supports, intended cutouts/holes, and non-uniform settlements. In view of the diverse sources of non-uniform stresses, a brief description of the physical background is presented in the sections where such descriptions are regarded to be necessary for clarity of understanding. The scope of the paper is confined to shells on the land, with only a limited coverage of shells subjected to external pressures.

The author attempts to provide a thorough, clear and concise review. With so numerous contributions (there have thousands of papers been published on shells in 2001) alone according to Sechler's equation (1974), preparing a paper without any inadvertent omission is very difficult. The author apologizes in advance to all those whose work is not referred to, and to all those who may feel that their contributions in any aspect have not been adequately acknowledged.

SHELLS SUBJECTED TO NON-UNIFORM LOADS

1. Shells subjected to bending

Pure bending seems to be the earliest and simplest case of cylindrical shells subjected to

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non-uniform axial load. Flugge (1960) was the first to investigate the stability problem of medium-length shells under non-uniform axial loads. Based on the developed differential equations, he obtained the results for the pure bending case. Seide and Weingarten (1961) studied the buckling of circular cylindrical shells under pure bending by means of Batdorf's modified Donnell equation and Galerkin's method. They reached the conclusion that the ratio μ (the ratio of the maximum critical stress of a shell loaded by non-uniform axial loads to that of the shell subjected to uniform axial compression) was nearly equal to unity if the maximum critical bending stress was minimized with respect to the longitudinal wavelength. The paper also explains why the ratio μ is 1.3 in the references (Flugge, 1960).

Experimental investigations could be found in the references (Suer et al., 1958; Tho and Spence, 1978; Emmerling, 1984; Otsuka and Koga, 1998). The latter two references further discussed the influence of the length of the cylinders on the buckling mode. For medium-length thin-walled shells, the buckling mode is a diamond pattern similar to the one observed in uniformly compressed shells. When the length of thin-walled cylindrical shells increases, the shells will behavior like tubes. Wave-like ripples were observed after the tube cross-sections are flattened - the Brazier effect (1926), at the center of the compression side of bending. For long thicker tubes, buckling occurs after yielding of the material. Additional bending moments can be applied until the curve of the bending moment versus curvature reaches the limit point (Reddy, 1979). The Brazier effect (1926) is less dominant, but it is still essential in the determination of the bending moment capacity, particularly, in the case of combined bending and external pressure, which often occurs in offshore engineering. Reissner (1961) investigated the elastic buckling of tubes under pure bending. Fabian (1977) extended the load condition to account for the influence of pressure loads. Elastic-plastic buckling of tubes under pure bending or combined bending, pressure and axial loads, was studied by Popov et al. (1974), Reddy (1979), Gellin (1980), Fabian (1981), and Calladine (1983). More comprehensive and recent studies were carried out by Kyriakides and Ju (1992), Ju and Kyriakides (1992), Duban and Ore (1999), Yeh et al. (1999), Bai (2001) and in the references cited there. Bai (2001) also addressed the issue of how to define the design bending moment capacity from moment-curvature relationship for different design requirements.

Other recent studies include: the buckling strength of shells subjected to bending moment and high temperature (Murakami et al., 1993, 1995); the localization of collapse in buried pipelines that undergo excessive curvatures due to ground movements (Murray, 1997); elastic-plastic bending buckling strength of tubes with cutouts (Yet et al. 1999); bending moment capacity of corroded tubes (Bai et al., 1998; 1999; Hauch and Bai, 2000); bending buckling of tubes under combined bending, pressure and axial loads (Corona and Kyriakides, 2000; Igland and Moan, 2000). Murakami et al. (1993; 1995) suggested an empirical buckling load reduction factor to determine the allowable buckling stresses for design of shells against buckling. The buckling load reduction factor considered the influence of real geometric imperfection distributions with the amplitude measured by a gauge. The gauge length in the circumferential direction is a little smaller than that specified in the Eurocode 3 (ENV1993-1-6, 1999).

2. Shells subjected to non-uniform axial loads

Libai and Durban (1973) provided a historical sketch of stability studies on shells under non-uniform axial loads prior to early 1970s. The load distributions discussed in the literature available then were roughly grouped into two categories: loads described by harmonic terms (e.g. Bijlaard and Gallagher, 1959) and strip loads: uniformly distributed over a part of the circumference (Hoff et al., 1964). For the former, the conclusion is that the buckling stress ratio $\mu \approx 1$ for a load comprising one low harmonic term. For the latter, Hoff et al. (1964) revealed that the increase of the critical stress contributed by the stress non-uniformity can be ignored, except when the width of the strip is quite narrow (less than $2.5\sqrt{Rt}$).

Durban and Libai (1976), Libai and Durban (1977) studied both load conditions. Closed-form expressions were presented to evaluate the linear buckling strength for the loads expressed in the form of a cosine function with one harmonic index. Approximate interaction formulae are suggested for strip loads and loads of more than one harmonic term. Rigorous numerical analyses were carried out by Ramm and Buchter (1991) to investigate the stability of shells under strip loads distributed on both shell ends. The influence of geometric imperfections and material nonlinearity were taken into consideration. Comparison between the numerical result and the one from design code (DIN18800-4, 1990) showed that: for narrow strip loads, the buckling strength increases dramatically; for wide strip loads, the increase is minor. The design code recommendations are appropriate for the latter case, however, when the width of the strip loads decrease gradually, the design code recommendations become more and more conservative. Teng and Song (2002) studied the problem by assuming the strip loads were located at the shell top edge only. The study showed that the buckling strength for wide strip loads was lower than that under uniform axial compression. The linear buckling mode was analogous to the shear buckling mode. The studies mentioned above dealt with loads varying around the circumference.

Weingarten (1962) considered a shell under longitudinally varying loads (e.g. self-weight effect). His analyses showed that the ratio μ could reach 1.9, when shells are extremely short (with the Batdorf parameter Z not greater than 1.0). The ratio decreased with the increase of Z. For practical shells of medium length (e.g. $Z>100$), the buckling loads of shells under longitudinally varying loads would not exceed the classical buckling strength by 20%. The problem was investigated experimentally by Calladine and Barber (1970). Their test results were revisited by Zhu et al. (1999) in an attempt: (1) to provide the physical background of the empirical formulae for the prediction of the buckling strength of shells subjected to uniform axial compression, and (2) to explain the observed scatter of experimental buckling loads of shells in the literature. The general finite element program ABAQUS was adopted to help understand the mechanics of the self-weight buckling phenomenon. A new series of experiments were carried out to achieve the above-mentioned two objectives (Mandal and Calladine, 2000). The studies led again to the hypothesis that locked-in stresses in statically indeterminate shells (Calladine, 1995) may potentially contribute to the scatter of experimental results reported in the literature.

3. Shells subjected to wind load

Extensive experiments and theoretical analyses were conducted to study the stability of cylindrical shells subjected to wind loads (Resinger and Greiner, 1982; Jones, 1983; Uematsu and Uchiyama, 1985), because wind loads are of practical importance to thin-walled shell structures, e.g. silos, tanks, chimneys. The distributions of external pressures on the shell surface vary from code to code (AS1170.2, 1989; DIN18800-4, 1990; GBJ9-87, 1987), but they have common characteristics: only a part of the circumference, the so-called stagnation zone, is under circumferential compression, while the rest is under suction. It is common knowledge that a shell buckles due to circumferential stresses. The buckling pattern along the meridian depends on the boundary conditions at the upper end, and on the stiffness of edge stiffeners or stabilizing girders (Schmidt et al., 1998). A comprehensive review and descriptions of the buckling of shells under wind loads can be found in books (Brown and Nielsen, 1998; Teng and Rotter, 2002)

The common approach for calculating the buckling strength of cylindrical shells subject to wind loads employs the concept of 'equivalent uniform external pressure' (DIN18800-4, 1990; ENV1993-1-6, 1999). Reccent research (Greiner and Derler, 1995; Schneider and Thiele, 2001) showed that the behavior of long, slender cylindrical shells under wind loads is quite different from that of short, stocky ones. The failure mode of long slender shells under wind loads is due to axial compressive stresses. Large axial compressive stresses exist near the base of the shell and approximately near the mid-height of the shell. Buckling is observed on these areas. Systematic non-linear analyses were carried out by Greiner and Derler (1995) to investigate the imperfection sensitivity of this buckling problem. Design recommendations for long, slender shells subject to wind loads are under development (Schneider and Thiele, 2001).

4. Shells (silos) under eccentric discharging

Eccentric discharging is known to be a cause of many observed failures in silos (Ross et al., 1980; Wood, 1980; Bushnell, 1981; Ravenet, 1981, 1983; Rotter et al., 1989; Jenkyn and Goodwill, 1987; Allen, 1989; Clercq, 1990; Guggenberger, 1997; Pavlovic, 1997; Wood, 1997). During the eccentric discharging, a flow channel forms around the eccentric outlet and propagates to the upper surface of the stored materials. The flow channel may be more or less convergent, which depends on the properties of the stored materials. The pressures within the flow channel are lower than those in the surrounding material. This leads to the non-uniform normal wall pressures around the circumference. Up to date, there have been dozens of expressions and descriptions of the non-uniform normal pressure distribution alternatively based on analytical models, numerical calculations and experimental results (Jenike, 1967; Colijn and Peschi, 1981; Wood, 1983; McLean and Bravin, 1985; Rotter, 1986; Ooi et al., 1990; Borcz and Rahim, 1991; Horabik et al., 1992; Blight and Gohnert, 1993; Ooi and She, 1997; Nielsen, 1998).

For a circular cylindrical shell, the non-uniformity of normal pressures has a deleterious effect. It produces circumferential and meridional bending moments. The circumferential bending moments cause the flexible shells to deform greatly and can also cause yielding of the shell wall (e.g. Jenike, 1967; Roberts and Ooms, 1983), while the meridional bending moments induce non-uoiform axial stresses on the shells. The size of non-uniform axial stresses on the shells depends on the boundary conditions of the shells, but they can easily exceed the buckling strength of shells without considering the effects of eccentric discharge (Buchert, 1967; Rotter, 1986). The latter reference also contains a historical review of the investigations on eccentric discharging.

Qualitative explanation of the buckling failures is that they result from a combination of (1) a decrease of internal pressures in the shell wall, (2) a decrease of restraints from the stored granular materials, and (3) a flattening of a shell wall near the flow channel (Rotter, 1985; Kemp, 1990a).

To capture the essence of non-uniform axial compressive stresses caused by non-uniform wall pressures, the concept of patch loads was adopted by researchers and introduced into the silo loading codes (e.g. AS3774-1996, 1996, DIN1055 Part 6, 1987; ENV1991-4, 1995; ISO11697, 1995). However, the specifications on the size, the position and the location of the patch loads vary notoriously from one code to another, this suggests there is a long road for theory to be applied in practice.

Current research on shells under eccentric discharging falls into two categories. One is prediction of wall pressure distributions. Comprehensive information on numerical simulations of flow patterns and experimental measurements of pressure distributions can be found in papers (Rotter et al., 1997; Chen et al., 1998), in the book (Brown and Nielson, 1998) and in the special journal issue (Journal of Engineering Mechanics, Vol 127, No. 10, 2001). The predictive capacity of two numerical methods: discrete element method and finite element method, for wall pressure distributions are reported in Rotter et al. (1998) and Holst et al. (1999a; 1999b). The second is investigation of the structural behavior of shells (Brown, 1996; Guggenberger, 1996; Rotter, 1996; Aguado et al., 1999; Briassoulis, 2000; Rotter, 2001a, 2001b; Song, 2002). Brown (1996) studied the influence of patch loads on rectangular shells, while Guggenberger (1996) investigated circular cylindrical shells. Rotter (2001a) studied a full-scale shell (R = 6 m, H = 36 m) by using a linear finite element analysis.

Song (2002) carried out systematic investigations on the influence of patch loads on the stability of shells of different sizes. The hierarchical analyses include LA - linear elastic analysis; GNA - geometrically nonlinear elastic analysis; GMNA - geometrically and materially nonlinear analysis; GMNIA - geometrically and materially nonlinear analysis with imperfections. The studies demonstrate the importance of patch loads in the designs of silos according to linear elastic analysis. However, their roles gradually decrease in the hierarchical analyses. Song (2002) also evaluates the buckling strength of shells subjected to non-uniform axial compressive stresses with the procedures recommended in a newly published shell design code/book (ENV1993-4-1, 1999; Rotter, 2001b).

Comprehensive studies on patch loads (Brown, 1996; Guggenberger, 1996; Rotter, 1996; Song, 2002) provided much information on the eccentric discharging of silos. However, a satisfactory solution of the problem, especially the wall pressure distributions and the assessment of the buckling strengths of shells under non-uniform compressive stresses need further work.

SHELLS ON DISCRETE SUPPORTS

1. Column-supported shells (silos)

Many steel shells (silos) are supported on columns to permit ease of access beneath the vessels. The discrete column supports induce high axial compression stresses in silos just above the supports. The distribution of axial compression stresses is very complex. Such stresses have a peak value at the end of the columns, and decrease quickly along the silo height and around the circumference. Research on the behavior of column-supported shells demonstrated the complexity of the problem. Rigorous theoretical studies began in the early nineties. Linear and nonlinear bifurcation buckling analyses of perfect silos were taken as a starting point (Teng and Rotter, 1990, 1991a, 1992). The following intensive analyses considered the effects of material non-linearity and the influences of geometric imperfections (Guggenberger, 1991; Teng and Rotter, 1991b; She and Rotter, 1993; Dhanens et al., 1993; Greiner and Guggenberger, 1996; Guggenberger, 1998). Guggenberger (1998) discussed the influences of high strength steels with the yielding stresses up to 355 N/mm^2.

Greiner and Guggenberger (1998) studied the beneficial influences of internal pressure and recommended a design procedure for unstiffened silos with constant wall thickness. Guggenberger et al. (2000) presented a similar buckling assessment criterion that is suitable for daily engineering work. The influences of an edge-ring stiffener at the bottom edge and a stepped wall thickness were addressed by Guggenberger et al. (2002). The silo was assumed to consist of only two regions of different wall thickness, the lower part being fixed to be 50% thicker than the upper one. For other practically stiffened column-supported silos (Guggenberger, 1998), corresponding research results are not yet available.

2. Saddle-supported shells

Horizontal shells, e. g. liquid storages and transportation elements in the chemical, petrochemical and energy industries, are often supported on two saddles. When designing these shells, the critical stress region is located either at mid-span or at the saddle-shell interaction area. Similar to column-supported silos, a stress concentration is inevitable and complex stress states are expected near the vicinity of supports. The shells may lose load carrying capacity due to one of the following modes of failure: fatigue (Duthie and Tooth, 1977; Krupka, 1991a); plastic squeeze/punching (Tooth and Jones, 1982; Krupka, 1994) and elastic/elastic-plastic buckling (Krupka, 1991a, 1991b, 1994; Chan et al., 1998).

Buckling failures dominate the design when the ratio of shell radius to thickness (R/t) exceeds 150 (Chan et al., 1998). Whether there is elastic or elastic-plastic buckling depends on the yield stress of the materials. Buckling failure modes also have close connection with (1) the arrangement of the saddles, welded to the shell or the shells loosely resting on the saddles, and (2) if the shells are stiffened or not. Buckling failures may result from longitudinal compressive stresses (Krupka, 1991a, 1991b, 1994; Chan et al. 1998), circumferential compressive stress (Kendrick and Tooth, 1986; Kemp, 1990b; Krupka, 1991b; Chan et al., 1998) and shear stresses (Krupka, 1994).

Two types of buckling modes, which are directly caused by longitudinal compressive stresses, identified from experimental tests were: (1) diamond-shaped buckling (Krupka, 1991a, 1991b, 1994; Chan et al., 1998), and (2) crumpled shape buckling (Chan et al., 1998). The latter buckling mode spreads over a much wider region and occurs more suddenly than diamond-shaped buckling.

Despite the many studies as mentioned above, only limited research is available on the assessment of the buckling loads. Krupka (1991a; 1991b; 1994) presented an empirical formula for calculating the maximum longitudinal compressive membrane stress, which was in turn used to compare with the allowable buckling stress calculated according to design codes (e. g. ECCS 1988). Much attention has been paid to how to calculate the stress distributions accurately and how to reduce the maximum stresses (Chien and Tu, 1988; Ong, 1995; Ong and Lu, 1995; Baniotopoulos, 1996; Nash et al., 1998). The problem of buckling is still open and waiting for a deeper theoretical solution and a clearer explanation (Krupka, 1994). Chan et al. (1998) compared experimental buckling stresses with the allowable buckling stresses predicted under the recommendations of British Standard BS5500 (1997) and European recommendations ECCS (1988). Underestimation and overestimation of the buckling loads had been observed among similar series of experiments.

SHELLS WITH CUTOUTS/HOLES

Cutouts commonly appear in cylindrical shells as access ports, doors, or windows. Small-sized cutouts are also devised to connect pipelines. They induce stress concentrations near the cutouts (Van Dyke, 1965; Savin, 1968). Local stiffeners, e. g. door frames, are employed to strengthen the weakened shell when the cutout size is relatively large. If the shells are subjected to axial compression, the buckling and post-buckling behavior are important considerations for the design of such shells (Tennyson, 1968; Almroth and Holmes, 1972; Starnes, 1974; Toda, 1980a; Eggwertz and Samuelson, 1991; Jullien and Limam, 1998; Yeh et al., 1999).

Tennyson (1968) was the first to investigate the effects of non-reinforced circular cutouts on the buckling behavior of nearly 'perfect' circular cylindrical shells. The shells were made of photoelasic plastic by spin-casting technology, followed by imposing cutouts onto the shell. Almroth and Holmes (1972) investigated the influence of rectangular-shaped cutouts in imperfect shells. Toda (1980b) carried out experimental investigations on the effect of elliptic cutouts.

Starnes (1974) carried out a series of experiments to investigate the influence of the shape and size of the cutouts. Circular, rectangular and square cutouts were imposed onto Mylar shells. A non-dimensional geometric parameter $\bar{r}(=r/\sqrt{Rt})$ was finally identified as a key parameter to measure the effects of various cutouts. Here, r, R and t represent the cutout sizes, the radius and the thickness of the shell respectively. Excellent descriptions of the buckling behavior of the shells with cutouts under axial compression were provided. The experimental collapse loads of shells were compared with the linear bifurcation loads calculated by the NASTRAN computer program. The comparison demonstrated that the linear bifurcation buckling analysis had no practical use for designs, as it provided upper bound buckling loads for small $\bar{r}$ and lower bound for large $\bar{r}$. Similar phenomena were also observed by Jullien and Limam (1998).

Jullien and Limam (1998) conducted much more detailed studies in that they considered more factors: the cutout shape – square, rectangular, and circular; the cutout position; the cutout dimensions – axial and/or circumferential sizes; and the interaction of multiple cutouts in the shell. The research also identified that the key characteristic parameter to evaluate the effect of the cutouts on the buckling loads was the non-dimensional value $\bar{r}$ (Starnes, 1974), with the definition of r for the rectangular cutout being somewhat different from the one suggested by Starnes (1974). Recommendations are provided for the design of shells with cutouts. The reduction factor for the load carrying capacity, as in the classical buckling analysis of imperfect shells without cutouts (ECCS, 1988), is a linear piece-wise function of the value $\bar{r}$, consisting of three segments.

Samuelson and Eggwertz (1992) suggested a reduction factor curve for the analysis of shell stability after they made use of the experimental results produced by a number of researchers (Montague and Horne, 1981; Miller, 1982, 1983). The beneficial effects of the stiffeners around the cutouts and the deleterious effects of yielding of the materials in the vicinity of the cutouts were included. The curve consists of two parts, linear segment and curved segment. The linear segment corresponds to small-sized cutout, which has no effect on the buckling loads. The curved segment indicates that the cutout has significant effects on the buckling loads.

The recommendations provided by Samuelson and Eggwertz (1992), Jullien and Limam (1998) were for isotropic, homogeneous shells according to ECCS (1988). Transfer them into newly edited code formats (ENV1993-1-6, 1999) is straightforward. Recent research interests in this area are compressed composite cylindrical shells/panels (Starnes and Rose, 1997; Hilburger et al., 2001).

SHELLS SUBJECTED TO NON-UNIFORM SETTLEMENT

Uniform shell base settlements do not critically affect the shell strength, although a possible aftermath is the invalidity of the connection between the shells and the accessories. Non-uniform deformations will cause shell distortion, induce additional non-uniform axial compressive stresses and cause the shells to buckle (Malik et al., 1977; Kamyab and Palmer, 1989; D'Orazio et al., 1989; Palmer, 1992, 1994; Jonaidi and Ansourian, 1998).

The measured settlement values have an irregular shape that can be expressed as a Fourier series in n harmonics (Malik et al., 1977; Marr et al., 1982). The most frequently used expression for the foundation settlements during the study of the shell strength is to assume the settlements in a cosine curve with harmonic index (n) varying from 2 to high values (Palmer, 1994). The stability of shells with non-uniform settlements has close connection with the harmonic index n, boundary conditions-open-topped or closed-topped, and whether the internal pressures were taken into consideration.

Jonaidi and Ansourian (1998) revealed that the buckling loads increased almost linearly with the harmonic index n. The buckling modes also changed with n. For a lower n, the linear bifurcation mode was in the form of shear buckling and extends over the entire height, which means that shear stresses in the shell are also important. For a higher n, the deformations are more localized. The transition harmonic index n, which distinguishes the failure mode from shear buckling to local axial buckling, is about 5 for shells with a constant wall thickness. For shells with a varied thickness (tapered shells), the problem is more complex. It varies with the tapering ratio of maximum wall thickness to minimum wall thickness (t_{max}/t_{min}) and the ratio of height to radius (H/R) greatly (Jonaidi and Ansourian, 2000).

REMARKS

Stability assessment of shell structures has long been, and continues to be, a concern for researchers. Not only are shell structures used in many fields, but also their analyses bring one to the forefront of nonlinear analyses and sophisticated constitutive equations. National/international design codes have been available for decades. However, they are mainly confined to shells with simple geometry and under simple load conditions, e. g. cylindrical shells under uniform axial compression, conical shells under uniform external pressure. There is little information on shells with complex geometry and subjected to non-uniform loads (or stresses). However, there exist nearly no shell structures in practical engineering that do not belong, more or less, to the latter category, especially when various imperfections (material, boundary conditions, loads and geometry) are taken into consideration.

This paper provides a review of recent research advances and trends in the area of stability of un-stiffened circular cylindrical shells subjected to general non-uniform axial compressive stresses The paper is not claimed to be complete. Over one hundred cited references have demonstrated the complexity of the problems and present valuable information, but the problems are far from being solved. It can be expected that the design of shell structures will continue to depend on empirical or semi-empirical formulae in the foreseeable future. Satisfactory solutions to the problems need the combined use of theoretical analyses, experimental investigations and numerical simulations. The present paper does not cover this area, as they were discussed by Knight and Starnes (1997), Singer (1997). Knight and Starnes (1997) emphasized recent advances in structural analysis methods and computer technologies. Singer (1997) reviewed the historical development of shell buckling tests and emphasized the importance of the interaction of theories, experiments and numerical studies.

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