Journal of Zhejiang University SCIENCE
ISSN 1009-3095
http://www.zju.edu.cn/jzus
E-mail: jzus@zju.edu.cn

Time domain system identification of unknown initial conditions*

SUNG Wen-pei†1, MATZEN Vernon C.[2], SHIH Ming-hsiang[3]

([1]*Department of Landscape Design and Management, National Chin-Yi Institute of Technology, Taiwan 41111, China*)

([2]*Department of Civil Engineering, North Carolina State University, Raleigh, NC 27695, U.S.A.*)

([3]*Department of Construction Engineering, National Kaoshiang First University of Science and Technology, Taiwan 824, China*)

†E-mail: sung809@chinyi.ncit.edu.tw

Received June 20, 2003; revision accepted Nov. 27, 2003

Abstract: System identification is a method for using measured data to create or improve a mathematical model of the object being tested. From the measured data however, noise is noticed at the beginning of the response. One solution to avoid this noise problem is to skip the noisy data and then use the initial conditions as active parameters, to be found by using the system identification process. This paper describes the development of the equations for setting up the initial conditions as active parameters. The simulated data and response data from actual shear buildings were used to prove the accuracy of both the algorithm and the computer program, which include the initial conditions as active parameters. The numerical and experimental model analysis showed that the value of mass, stiffness and frequency were very reasonable and that the computed acceleration and measured acceleration matched very well.

Key words: Noise, Initial conditions, Gauss-Newton, Active Parameters
doi:10.1631/jzus.2004.1035 **Document code:** A **CLC number:** TP391.4

INTRODUCTION

System identification is a method for using measured data to create or improve a mathematical model of the object being tested. It had been described as the process of selecting the form of the mathematical model and then, using measured test data, systemically adjusting the parameters in this method until, based on a predefined criterion, the best possible correlation is achieved between the predicted and measured response (Matzen and McNiven, 1976). This method is widely used in many research areas, such as civil engineering, electronic engineering, chemical engineering, etc. (Yue and Schlueter, 2002; Bykov *et al.*, 2003; Kruglov *et al.*, 2002). Especially, this method is very useful for analyzing the behavior of structural dynamics. The measured acceleration of free vibration (Hart and Yao, 1977) revealed that there was noise in the high frequency response superimposed on the expected response at the beginning of the vibration. To solve this noise problem, one solution is to skip the noise data, establish a new origin for the time scale, and then use initial conditions as parameters to be found by the system identification process.

Several approaches to system identification had been developed (Natke, 1982; Beck and Jennings, 1980). Many models and approaches had been used, e.g. linear, nonlinear, time domain, frequency domain, modal parameters and physical parameters (McVerry, 1980; Distefano and Rath, 1975; Matzen, 1990; Deng *et al.*, 2003; Fukushima and Sugie, 1999). In this paper, a modified Gauss-Newton minimizing algorithm (Matzen,

* Project (No. NSC-93-2211-E-167-002) supported by the National Science Council of Taiwan, China

1990) is used to find the elements of mass, damping and stiffness matrices of structure based on the time domain method. One of the reasons for using a physical parameter time domain approach is that some modeling problems such as isolated uncertainties may be more easily addressed using this method than the modal parameter frequency domain method. Therefore, in order to demonstrate that this idea of using the initial conditions as active parameters is valid, the following steps are used in this research. The first step of this research is to derive the algorithm including the initial conditions as active parameters. The second step, a pull-back-and-quick-release test is used to generate measured data from actual shear buildings with single-degree of freedom and multi-degree of freedom. These measured data are used to test and verify the usefulness and accuracy of the proposed algorithm.

METHOD OF SYSTEM IDENTIFICATION

Berkey (1970) divided the process of system identification into the following three steps:

(1) Determination of the form of the model and isolation of the unknown parameters.

(2) Selection of a criterion function by means of which the goodness of fit of the model response to the actual system response can be evaluated.

(3) Selection of an algorithm or strategy for adjustment of the parameters in such a way that the differences between model and system responses, as measured by the criterion in Step (2), are minimized.

A good choice for the model is one that can produce good correlation with the measured data, but also contains terms that are directly related to known physical properties. We follow these steps to derive the analysis model using the initial conditions as active parameters.

Form of the mathematical model

The mathematical model for the shear building, tested by free vibration, is assumed to be linear, elastic and viscously damping. The resulting discrete initial value problem takes the following form:

$$[\boldsymbol{M}]\{\ddot{\boldsymbol{X}}\}+[\boldsymbol{C}]\{\dot{\boldsymbol{X}}\}+[\boldsymbol{K}]\{\boldsymbol{X}\}=\{\boldsymbol{P}(t)\} \tag{1}$$

Where: $[\boldsymbol{M}]$: the mass matrix of the structure; $[\boldsymbol{C}]$: the damping matrix of the structure; $[\boldsymbol{K}]$: the stiffness matrix of the structure; $\{\ddot{\boldsymbol{X}}\}$: the acceleration vector; $\{\dot{\boldsymbol{X}}\}$: the velocity vector; $\{\boldsymbol{X}\}$: the displacement vector; $\{\boldsymbol{P}(t)\}$: the load vector; $\{\dot{\boldsymbol{X}}\}=\{\dot{\boldsymbol{X}}_0\}$, $\{\boldsymbol{X}\}=\{\boldsymbol{X}_0\}$: the initial conditions.

The parameters to be obtained in the system identification process are any combination of the elements of the three coefficient matrices and these are placed in a one-dimensional array, called Active Parameter vector, denoted ***AP***. The algorithm was originally developed to use only zero initial conditions. However, if we conclude that the initial velocity and displacement vectors are included in the Active Parameter vector, we can start from any time. This will prove to be an easy way to avoid the initial noisy experimental data.

Criterion function

A realistic mathematical model must be able to produce a response that matches the structure's response when both the model and the structure are subjected to the same excitation. The error function indicates how well the match is made. In laboratory tests, the acceleration response can be easily measured. Hence, errors in accelerations are only one used in this paper. The function is divided by the duration of the signal, and the acceleration error at each degree of freedom is weighted by a factor W to form a weighted, mean square error function. The final error function for a single degree of freedom (SDOF) system takes the following form:

$$J(\boldsymbol{AP})=\frac{1}{t_d}\int_0^{t_d} W[\ddot{\boldsymbol{X}}_m(t)-\ddot{\boldsymbol{X}}_c(\boldsymbol{AP},t)]^2\,\mathrm{d}t \tag{2}$$

and for number of degrees of freedom (NDOF) system, it takes the following form:

$$J(\boldsymbol{AP})=\frac{1}{t_d}\int_0^{t_d}\langle e(\boldsymbol{AP},t)\rangle\begin{bmatrix}W_1 & 0 & \cdots & 0\\ 0 & W_2 & \cdots & 0\\ \vdots & & \ddots & 0\\ 0 & 0 & \cdots & W_n\end{bmatrix}\{e(\boldsymbol{AP},t)\}\,\mathrm{d}t \tag{3}$$

Where: $\langle e(\boldsymbol{AP},t)\rangle=[\ddot{\boldsymbol{X}}_{mj}(t)-\ddot{\boldsymbol{X}}_{cj}(\boldsymbol{AP},t)]$; n: the number of degrees of freedom; $\ddot{\boldsymbol{X}}_{mj}(t)$: the measured acceleration at the jth DOF; $\ddot{\boldsymbol{X}}_{cj}(\boldsymbol{AP},t)$: the computed acceleration at the jth DOF using the current set of parameters; W_j: the weighting factor for the jth DOF.

Parameter adjustment algorithm

The Gauss-Newton method was selected to systemically adjust the parameters in the mathematical model until the error function was minimized. This method was derived by expanding the error function in Taylor series about the previous set of parameters $\boldsymbol{AP}$.

The error function is as follows:

$$J(\boldsymbol{AP})=\frac{1}{t_d}\int_0^{t_d}\langle e(\boldsymbol{AP},t)\rangle[\boldsymbol{W}]\{e(\boldsymbol{AP},t)\}\,\mathrm{d}t \tag{4}$$

Taylor series expansion of Eq.(4) yields:

$$\begin{aligned}J(\boldsymbol{AP})&=J(\boldsymbol{AP}_i^0)+\langle \boldsymbol{AP}_i-\boldsymbol{AP}_i^0\rangle\left\{\frac{\partial J(\boldsymbol{AP}^0)}{\partial \boldsymbol{AP}_i}\right\}\\&+\frac{1}{2}\langle \boldsymbol{AP}_i-\boldsymbol{AP}_i^0\rangle[H(\boldsymbol{AP}^0)]\{\boldsymbol{AP}_i-\boldsymbol{AP}_i^0\}\\&+HighOrderTerm\end{aligned} \tag{5}$$

Where: $\left\{\frac{\partial J(\boldsymbol{AP}^0)}{\partial \boldsymbol{AP}_i}\right\}$: the Column Gradient vector;

$$[H(\boldsymbol{AP}^0)]=\begin{bmatrix}\frac{\partial^2 J(\boldsymbol{AP}^0)}{\partial \boldsymbol{AP}_1^2} & \frac{\partial^2 J(\boldsymbol{AP}^0)}{\partial \boldsymbol{AP}_1\partial \boldsymbol{AP}_2} & \frac{\partial^2 J(\boldsymbol{AP}^0)}{\partial \boldsymbol{AP}_1\partial \boldsymbol{AP}_3} & \cdots\\ & \frac{\partial^2 J(\boldsymbol{AP}^0)}{\partial \boldsymbol{AP}_2^2} & \frac{\partial^2 J(\boldsymbol{AP}^0)}{\partial \boldsymbol{AP}_2\partial \boldsymbol{AP}_3} & \cdots\\ & & \ddots & \vdots\\ & & & \frac{\partial^2 J(\boldsymbol{AP}^0)}{\partial \boldsymbol{AP}_n^2}\end{bmatrix}$$

: the Hessian Matrix;

$\boldsymbol{AP}_i$: the ith Active Parameter.

To minimize $J(\boldsymbol{AP})$, the gradient with respect to $\boldsymbol{AP}_i$ was set equal to zero vector.

$$\frac{\partial J(\boldsymbol{AP})}{\partial \boldsymbol{AP}_i}=0+\langle 0\quad 0\quad \cdots\quad 0\quad 1\quad 0\quad \cdots\rangle\times\left(\left\{\frac{\partial J(\boldsymbol{AP}^0)}{\partial \boldsymbol{AP}_i}\right\}+[H(\boldsymbol{AP}^0)]\{\boldsymbol{AP}_i-\boldsymbol{AP}_i^0\}\right) \tag{6}$$

Where:

$$\begin{aligned}H(\boldsymbol{AP}^0)&=\left.\frac{\partial^2 J(\boldsymbol{AP})}{\partial \boldsymbol{AP}_i\partial \boldsymbol{AP}_j}\right|_{\boldsymbol{AP}=\boldsymbol{AP}^0}\\&=2t_d\int_0^{t_d}[\langle e(\boldsymbol{AP},t)\rangle[\boldsymbol{W}]\left\{\frac{\partial^2\ddot{\boldsymbol{X}}(\boldsymbol{AP},t)}{\partial \boldsymbol{AP}_i\partial \boldsymbol{AP}_j}\right\}\\&+\left\langle\frac{\partial\langle e(\boldsymbol{AP},t)\rangle}{\partial \boldsymbol{AP}_i}\right\rangle[\boldsymbol{W}]\left\{\frac{\partial\ddot{\boldsymbol{X}}(\boldsymbol{AP},t)}{\partial \boldsymbol{AP}_j}\right\}]\mathrm{d}t\end{aligned}$$

In the Gauss-Newton method, the first integral term in the Hessian is eliminated. The terms of the remaining symmetric matrix which is called the Approximate Hessian, denoted $\boldsymbol{AH}$, is given by the following equation:

$$\boldsymbol{AH}_{ij}(\boldsymbol{AP})=2\int_0^{t_d}\left\langle\frac{\partial\ddot{\boldsymbol{X}}_C(\boldsymbol{AP},t)}{\partial \boldsymbol{AP}_i}\right\rangle[\boldsymbol{W}]\left\{\frac{\partial\ddot{\boldsymbol{X}}_C(\boldsymbol{AP},t)}{\partial \boldsymbol{AP}_j}\right\}\mathrm{d}t$$

Eq.(6) can be rearranged to give the following equation:

$$\{\boldsymbol{AP}_i\}=\{\boldsymbol{AP}_i^0\}-a\left[\boldsymbol{AH}(\boldsymbol{AP}^0)\right]^{-1}\left\{\frac{\partial J(\boldsymbol{AP}^0)}{\partial \boldsymbol{AP}_j}\right\} \tag{7}$$

Where: a is a positive scaling factor.

This equation is called the modified Gauss method. It is widely used in optimization and has the advantage of rapid convergence to the minimum without the need to calculate second partial derivatives.

Sensitivity coefficient

To compute the elements of the gradient vector and the Approximate Hessian matrix, it is necessary to evaluate the first partial derivative of the computed acceleration with respect to each parameter. These first partial derivatives, called sensitivity coefficients, are the solutions to partial differential equations. These equations are obtained by partially differentiating Eq.(1) with respect to each parameter.

For example if $\boldsymbol{AP}=\boldsymbol{M}_{ij}$, then partially differentiating Eq.(1) yields:

$$[\boldsymbol{M}]\left\{\frac{\partial\dot{\boldsymbol{X}}}{\partial\boldsymbol{M}_{ij}}\right\}+[\boldsymbol{C}]\left\{\frac{\partial\dot{\boldsymbol{X}}}{\partial\boldsymbol{M}_{ij}}\right\}+[\boldsymbol{K}]\left\{\frac{\partial\boldsymbol{X}}{\partial\boldsymbol{M}_{ij}}\right\}=-\left\{\frac{\partial\boldsymbol{M}}{\partial\boldsymbol{M}_{ij}}\right\}\{\dot{\boldsymbol{X}}\} \tag{8}$$

where: $\left\{\frac{\partial\dot{\boldsymbol{X}}(0)}{\partial\boldsymbol{M}_{ij}}\right\}=\{0\}$; $\left\{\frac{\partial\boldsymbol{X}(0)}{\partial\boldsymbol{M}_{ij}}\right\}=\{0\}$.

If $\boldsymbol{AP}=\boldsymbol{C}_{ij}$, then partially differentiating Eq.(1) yields:

$$[\boldsymbol{M}]\left\{\frac{\partial\ddot{\boldsymbol{X}}}{\partial\boldsymbol{C}_{ij}}\right\}+[\boldsymbol{C}]\left\{\frac{\partial\dot{\boldsymbol{X}}}{\partial\boldsymbol{C}_{ij}}\right\}+[\boldsymbol{K}]\left\{\frac{\partial\boldsymbol{X}}{\partial\boldsymbol{C}_{ij}}\right\}=-\left\{\frac{\partial\boldsymbol{C}}{\partial\boldsymbol{C}_{ij}}\right\}\{\dot{\boldsymbol{X}}\} \tag{9}$$

where: $\left\{\frac{\partial\ddot{\boldsymbol{X}}(0)}{\partial\boldsymbol{C}_{ij}}\right\}=\{0\}$; $\left\{\frac{\partial\boldsymbol{X}(0)}{\partial\boldsymbol{C}_{ij}}\right\}=\{0\}$.

If $\boldsymbol{AP}=\boldsymbol{K}_{ij}$, then partially differentiating Eq.(1) yields:

$$[\boldsymbol{M}]\left\{\frac{\partial\ddot{\boldsymbol{X}}}{\partial\boldsymbol{K}_{ij}}\right\}+[\boldsymbol{C}]\left\{\frac{\partial\dot{\boldsymbol{X}}}{\partial\boldsymbol{K}_{ij}}\right\}+[\boldsymbol{K}]\left\{\frac{\partial\boldsymbol{X}}{\partial\boldsymbol{K}_{ij}}\right\}=-\left\{\frac{\partial\boldsymbol{K}}{\partial\boldsymbol{K}_{ij}}\right\}\{\boldsymbol{X}\} \tag{10}$$

where: $\left\{\frac{\partial\ddot{\boldsymbol{X}}(0)}{\partial\boldsymbol{K}_{ij}}\right\}=\{0\}$; $\left\{\frac{\partial\dot{\boldsymbol{X}}(0)}{\partial\boldsymbol{K}_{ij}}\right\}=\{0\}$.

Eqs.(8), (9) and (10) can only be valid at the beginning of the response because the initial conditions for Eq.(1) are only known at time=0. As mentioned previously, some experimental data have noise at the beginning and it would be convenient to skip the noisy data and use data with known initial conditions from a non-zero starting time. However, they are not known. One solution to this dilemma is to make the elements active parameters (***AP***). Typical equations for finding sensitivity coefficients for initial displacements and initial velocities are given below:

If $\boldsymbol{AP}=\dot{\boldsymbol{X}}_3(t')$ and time starts from t'

$$[\boldsymbol{M}]\left\{\frac{\partial\ddot{\boldsymbol{X}}(t)}{\partial\dot{\boldsymbol{X}}_3(t')}\right\}+[\boldsymbol{C}]\left\{\frac{\partial\dot{\boldsymbol{X}}(t)}{\partial\dot{\boldsymbol{X}}_3(t')}\right\}+[\boldsymbol{K}]\left\{\frac{\partial\boldsymbol{X}(t)}{\partial\dot{\boldsymbol{X}}_3(t')}\right\}=\{0\} \tag{11}$$

where: $\left\{\frac{\partial\dot{\boldsymbol{X}}(t')}{\partial\dot{\boldsymbol{X}}_3(t')}\right\}=\begin{Bmatrix}0\\0\\1\\0\\\vdots\end{Bmatrix}$; $\left\{\frac{\partial\boldsymbol{X}(t')}{\partial\dot{\boldsymbol{X}}_3(t')}\right\}=\{0\}$.

Then

$$\left\{\frac{\partial\ddot{\boldsymbol{X}}(t)}{\partial\dot{\boldsymbol{X}}_3(t')}\right\}=[\boldsymbol{M}]^{-1}\left[-[\boldsymbol{C}]\left\{\frac{\partial\dot{\boldsymbol{X}}(t)}{\partial\dot{\boldsymbol{X}}_3(t')}\right\}\right] \tag{12}$$

If $\boldsymbol{AP}=\boldsymbol{X}_3(t')$and time starts from t'

$$[\boldsymbol{M}]\left\{\frac{\partial\ddot{\boldsymbol{X}}(t)}{\partial\boldsymbol{X}_3(t')}\right\}+[\boldsymbol{C}]\left\{\frac{\partial\dot{\boldsymbol{X}}(t)}{\partial\boldsymbol{X}_3(t')}\right\}+[\boldsymbol{K}]\left\{\frac{\partial\boldsymbol{X}(t)}{\partial\boldsymbol{X}_3(t')}\right\}=\{0\} \tag{13}$$

where: $\left\{\frac{\partial\boldsymbol{X}(t')}{\partial\boldsymbol{X}_3(t')}\right\}=\begin{Bmatrix}0\\0\\1\\0\\\vdots\end{Bmatrix}$; $\left\{\frac{\partial\dot{\boldsymbol{X}}(t')}{\partial\boldsymbol{X}_3(t')}\right\}=\{0\}$.

Then

$$\left\{\frac{\partial\ddot{\boldsymbol{X}}(t)}{\partial\boldsymbol{X}_3(t')}\right\}=[\boldsymbol{M}]^{-1}\left[-[\boldsymbol{K}]\left\{\frac{\partial\boldsymbol{X}(t)}{\partial\boldsymbol{X}_3(t')}\right\}\right] \tag{14}$$

VERIFICATION USING SIMULATED DATA

The program was tested to ensure that the modified algorithm, using the initial conditions as active parameters and the reduced sensitivity array, were correctly implemented. This was accomplished using simulated data. In these experiments, assigning values to all of the parameters numerically simulated measured data. Then the program was run using initial set parameters different from the assigned set and a non-zero starting time. The parameters used were similar to those expected from the laboratory models. The simulated accelerations were used as input file, with a varied set of values for active parameters, including mass, damping, stiffness and initial conditions. The numerical experiments used noise-free data and an initial set of parameters that are though to be typical of the initial estimates that could be made when real data are employed. To check the non-zero initial condition modification, four experiments were performed, each starting from a different time. The first try, without skipping any point, started from time equal to zero. The second try, skipping twenty-five points, one time step equal to 0.07 second, started from the time of 1.75 seconds. The third and fourth tries, skipping fifty and seventy-five points individually, started from time equal 3.5 seconds and 5.25 seconds respectively. All results are shown in Table 1 and Table 2. In each case, the parameters converged to the assigned values and the correct initial displacements and velocities were obtained. Moreover, the value of the error was very nearly zero. These simulated data experiments demonstrated that the algorithms proposed in this paper are correct.

VERIFICATION USING EXPERIMENTAL DATA

The laboratory tests aimed at examining the actual behavior of the shear building in order to demonstrate that the algorithms proposed in this paper are correct. A single story model building and a multi-story shear building were tested; both structures were tested with an added mass and without it. Then, we skipped the noisy data and find all the parameters to demonstrate the accuracy of the proposed expressions. As most of the mass of the shear building was concentrated at the girder level, it was assumed for this research that the masses are lumped at each degree of freedom. The mass matrix generated was diagonal. The stiffness matrix was formed by inducing a unit displacement to a degree of freedom while keeping all other degrees of freedom equal to zero; then the elastic forces at the degree of freedom required to maintain the deflected shape was used to compute the stiffness matrix. The shear building had non-zero off-diagonal terms in the stiffness matrix. Thereafter, the analytical frequencies were computed. The Rayleigh damping matrix (Rapid System, 1999) was used to find the damping matrix.

Experimental test

Table 1 Numerical investigation of SDOF (Unit: non-dimensional)

Mass=1.0	Initial parameters	Skip numbers	Computed ***AP***	Actual ***AP***	Error (%)
Damping	0.016		0.00604	0.006	0.67
Stiffness	28		30	30	0
Velocity	–0.1	0	–0.3	–0.3	0
Displacement	0.2		–0.0000307	0	0.0037
Damping	0.016		0.006	0.006	0
Stiffness	28		30	30	0
Velocity	–0.1	25	–0.271	–0.27	0.37
Displacement	0.2		0.446	0.45	0.89
Damping	0.016		0.00606	0.006	1
Stiffness	28		30	30	0
Velocity	–0.1	50	–0.201	–0.201	0
Displacement	0.2		0.807	0.81	0.37
Damping	0.016		0.00606	0.006	1
Stiffness	28		0	30	0
Velocity	–0.1	75	–0.1	–0.1	0
Displacement	0.2		1.03	1.03	0

Table 2 Numerical investigation of MDOF (Unit: non-dimensional)

$Mass_{11}$=1.0	Initial parameters	Skip numbers	Computed ***AP***	Actual ***AP***	Error (%)
$Mass_{22}$	1.02		1.0	1.0	0
Damping C_{11}	0.0025		0.000492	0.0005	1.6
C_{12}=C_{21}	–0.0009		–0.000303	–0.0003	1.0
C_{22}	0.0013		0.00089	0.00086	3.49
Stiffness K_{11}	5.2		5.0	5.0	0
K_{12}=K_{21}	–2.01	0	–2.0	–2.0	0
K_{22}	3.8		4.0	4.0	0
Displacement X_1	–0.13		–0.13	–0.13	0
X_2	–0.2		–0.333	–0.33	0.11
Velocity $\dot{X}_1$	0		0.0000045	0	0
$\dot{X}_2$	0		0.000002239	0	0
$Mass_{22}$	1.02		1.0	1.0	0
Damping C_{11}	0.0025		0.000491	0.0005	1.8
C_{12}=C_{21}	–0.0009		–0.00028	–0.0003	6.67
C_{22}	0.0013		0.00865	0.00086	0.58
Stiffness K_{11}	5.2		5.0	5.0	0
K_{12}=K_{21}	–2.01	25	–2.0	–2.0	0
K_{22}	3.8		3.98	4.0	0.5
Displacement X_1	–0.13		–0.0713	–0.07122	0.11
X_2	–0.2		–0.229	–0.22863	0.16
Velocity $\dot{X}_1$	0		0.295	0.29515	0.05
$\dot{X}_2$	0		–0.0567	–0.05692	0.40
$Mass_{22}$	1.02		1.0	1.0	0
Damping C_{11}	0.0025		0.000491	0.0005	1.8
C_{12}=C_{21}	–0.0009		–0.000307	–0.0003	2.23
C_{22}	0.0013		0.00886	0.00086	3.02
Stiffness K_{11}	5.2		5.0	5.0	0
K_{12}=K_{21}	–2.01	50	–2.0	–2.0	0
K_{22}	3.8		4.0	4.0	0
Displacement X_1	–0.13		0.0564	0.056358	0.07
X_2	–0.2		0.00903	0.0090	0.33
Velocity $\dot{X}_1$	0		–0.395	–0.3953	0.08
$\dot{X}_2$	0		0.135	0.13525	0.18
$Mass_{22}$	1.02		1.0	1.0	0
Damping C_{11}	0.0025		0.000493	0.0005	1.4
C_{12}=C_{21}	–0.0009		–0.000293	–0.0003	2.33
C_{22}	0.0013		0.00851	0.00086	1.05
Stiffness K_{11}	5.2		5.0	5.0	0
K_{12}=K_{21}	–2.01	75	–2.0	–2.0	0
K_{22}	3.8		4.0	4.0	0
Displacement X_1	–0.13		0.00415	0.0049	1.5
X_2	–0.2		0.301	0.30158	0.19
Velocity $\dot{X}_1$	0		0.579	0.5799	0.16
$\dot{X}_2$	0		0.217	0.21683	0.08

The structure and test setup are shown in Fig.1 for the single degree of freedom and Fig.2 for the multi-degree of freedom structure. They were constructed of flexible aluminum columns and rigid steel girders. The joints of the structures were fixed by placing a bolt through the columns and into the girder. Then the accelerometers are connected to the end of the girders. These structures were fixed to the base to the nearly rigid supporting structure so that it lined up with the pull-back force. For this research, static test and dynamic test were used to test the shear building models.

Static test

The assembled shear buildings were tested statically to provide the information need to create a stiffness matrix. Stiffness coefficients cannot easily be computed directly, so flexibility coefficients were found instead. After each of the terms in the flexibility matrix had been found, the matrix could be inverted to give the stiffness matrix. Displacements at each story were produced by hanging a weight on the structure at the first one of the stories and by hanging a string on the structures of the other stories. The displacements were measured using an LVDT.

Dynamic test

For dynamic testing, the accelerometers were attached at the end of girders and the lead wires

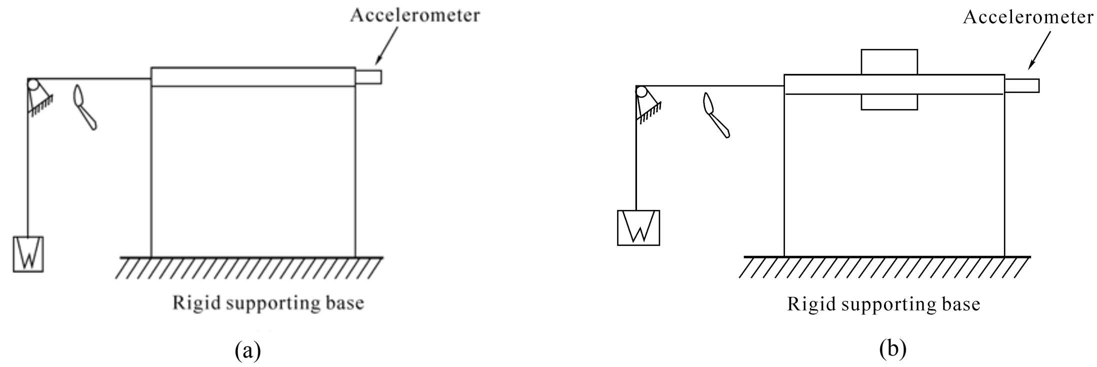


Fig.1 The structure and test setup for SDOF
(a) test setup w/o added mass; (b) test setup w/ added mass

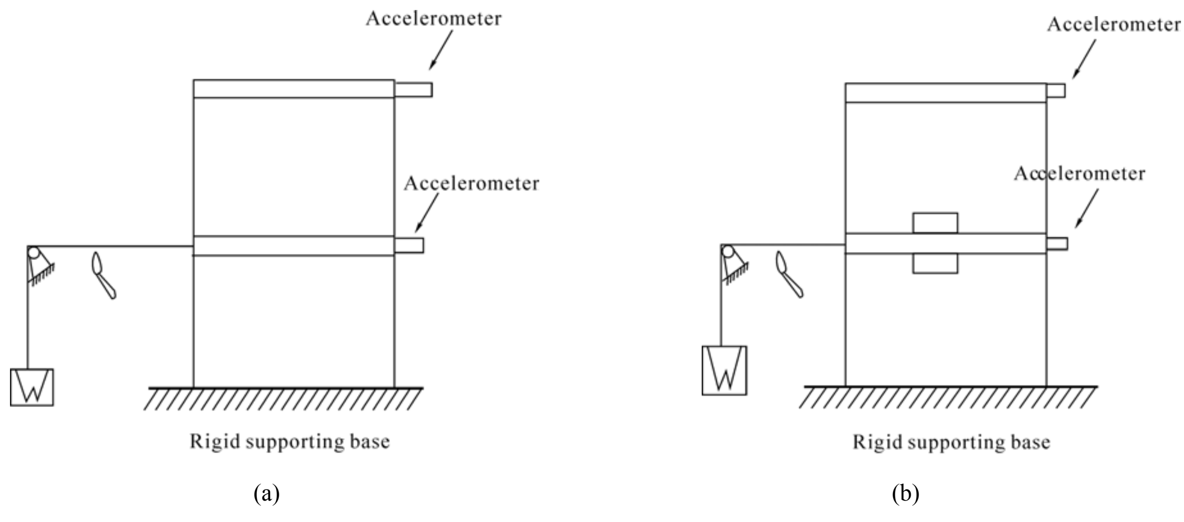


Fig.2 The structure and test setup for MDOF
(a) test setup w/o added mass; (b) test setup w/ added mass at 1st floor

were taped to the columns. Then, we set data acquisition system gains. The equipment used in the free vibration were (1) Rapid system 4×4 digital storage oscilloscope, (2) PCB accelerometers Model 308B02, and (3) PCB I.C.P. 4-channel power supply model 482A04. DC coupling was used to record the experimental data. In order to conduct the free vibration tests, the structure being tested would be loaded statically as described in the previous section. The string would then be cut using a torch allowing the structure to vibrate freely. The data recorded from rapid system were voltages. Then, the following equation was used to convert the voltage to acceleration.

$$Response = \frac{Datum - 127.5}{16\,points / Div} \times \frac{OscilloscopeGain(V / Div)}{Accel.Sensitivity(V / g)} \times \frac{386.4\ \text{in}/\text{s}^2}{g} \tag{15}$$

where: *Datum*: the voltage read from the rapid system; *Response*: the acceleration (unit: in/s^2).

In addition to recording the accelerations, we measured the frequencies of each shear building with a real time spectrum analyzer using the rapid system 4×4 to acquire four simultaneous channels of data. It calculated the frequency spectra using the digital signal processor.

Test results

All of the tests results recorded by the rapid system indicated that the problem of noise at the beginning of the measured acceleration datum was resolved by skipping the noise points. The parameters in the active parameter vector must be independent in order for the minimization algorithm to work correctly. Since the equations of motion for free vibration were homogeneous, each equation could be multiplied by a non-zero constant without affecting its behavior. Therefore, we always fixed the entire first row of the mass matrix. Using the measured accelerations and initial parameters estimated from the structure, the proposed algorithms were able to determine the minimizing set of parameters. The active parameters were the initial conditions, all elements of damping and stiffness matrices, and all elements except the first row of mass matrix. All of the active parameters are shown in Eqs.(6) to (14).

The parameters in the active parameter vector must be independent in order for the minimization algorithm to work correctly. Since the equations of motion for free vibration were homogeneous, each equation can be multiplied by a nonzero constant without affecting its behavior. Therefore, to acquire the actual parameters of the tested shear building, the idea of using a known added mass was extended here to structure of more than one degree of freedom using system identification (Matzen, 1988). The basic procedure was that the original structure was tested using pull-back and quick release test to find an appropriate subset of the parameters which were not unique. Therefore, the test was repeated with a known mass added at one of the degrees of freedom and new values were obtained for the subsets of parameters. By comparing the two subsets of parameters and calculating the value of a scaling factor, the correct set of parameters could be recorded. After the parameters were found, Eqs.(16) and (17) were used to find the following actual parameters:

$$\begin{aligned} & SF_{OM} \times M_{pb-OM} + \Delta M \\ & = SF_{OM} \times AVG\left(\frac{PB-OM}{PB-AM}\right) \times M_{pb-AM} \end{aligned} \tag{16}$$

$$SF_{AM} = SF_{OM} \times AVG\left(\frac{PB-OM}{PB-AM}\right) \tag{17}$$

where: SF_{OM}: Scaling factor of the pull-back test with original mass; SF_{AM}: Scaling factor of the pull-back test with added mass; *PB–OM*: the convergent values of the pull-back test with original mass; *PB–AM*: the convergent values of the pull-back test with added mass.

As for the actual parameters, the weight of the added mass was the same as that measured after multiplying the scaling factor. The stiffness very closely correlated to the stiffness obtained experimentally. Table 3 compares experimental, analytical and computed results of SDOF with stiffness and frequency. Table 4 compares experimental, analytical and computed results of SDOF with frequency. Table 5 compares the experimental, analytical and computed values of mass and stiffness. Figs.3 and 4 reveal that the measured response and the simulated response using the final parameters matched very well.

CONCULSION

The completed simulated and experimental data for testing the algorithms were used in the initial conditions as active parameters for the pull-back and quick release test. It was clear that using the initial conditions as element of active parameters (***AP***) or skipping the high frequencies points at the beginning of response were good ideas. From the tables, the results of mass, stiffness and frequencies from the proposed algorithms are fairly reasonable. The figures show that the measured and simulated responses match very well using the final

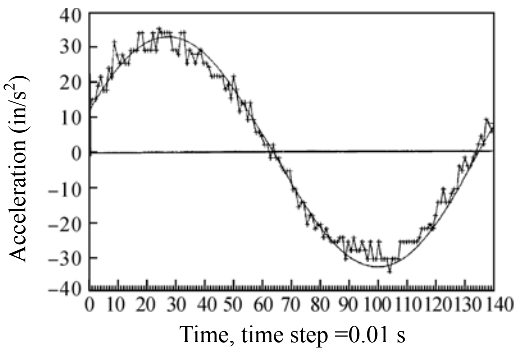


Fig.3 Comparison of measured and computed acceleration

Table 3 Comparison of experimental, analytical and computed results-SDOF with stiffness and frequency

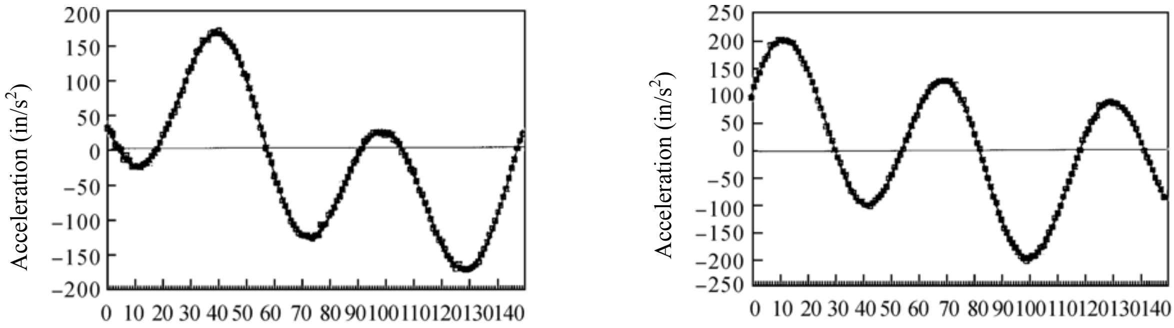
	Experimental results		Analytical results		Computed results	
	Added mass	W/o added mass	Added mass	W/o added mass	Added mass	W/o added mass
Stiffness (in^4)	18.32	18.32	18.40	18.40	18.35	18.31
Frequency (Hz)	6.77	8.78	6.68	8.59	6.80	8.82

Table 4 Comparison of experimental, analytical and computed results-SDOF with frequency (Unit: Hz)

Original structure	Experimental result	Analytical result	Computed result
Low frequency	5.6	5.696	5.594
High frequency	17.1	17.198	17.161
	Added mass on the first floor of structure		
Low frequency	4.9	5.062	4.832
High frequency	14.7	14.856	14.818
	Added mass on the second floor of structure		
Low frequency	4.5	4.7097	4.673
High frequency	15.6	15.88	15.709

Table 5 Comparison of experimental, analytical and computed values of mass and stiffness (added mass=0.00411 lb×s^2/in, stiffness=in^4)

	Experimental results	Analytical results	Computed results		
			Original structure	Added mass on the 1st floor	Added mass on the 2nd floor
$Mass_{11}$	0.00575	N/A	0.00574	0.00986	0.00575
$Mass_{22}$	0.00594	N/A	0.00588	0.00588	0.0100
K_{11}	28.932	30.198	28.629	29.091	29.508
K_{12}	−30.071	−30.198	−29.245	−29.275	−28.531
K_{21}	−29.087	−30.198	−29.245	−29.274	−29.531
K_{22}	48.979	51.905	48.933	48.159	48.418



– Computed acceleration; + Measured acceleration

(a)

– Computed acceleration; + Measured acceleration

(b)

Fig.4 Comparison of measured and computed acceleration-MDOF
(a) Comparison for the 2nd floor; (b) Comparison for the 1st floor

parameters. These results indicate that the proposed algorithms are useful for actual experiment. Thus, the system identification for using initial conditions as active parameters is truly practical.

References

Beck, J.L., Jennings, P.C., 1980. Structural identification using linear model and earthquake engineering. *Journal of Earthquake Engineering and Structural Dynamics*, **8**(1):145-160.

Berkey, G.A., 1970. System identification–An introduction and a survey. *Simulation*, **5**(4):151-166.

Bykov, A., Naumenko, O.V., Pshenichnikov, A.M., Sinitsa, L.N., Scherbakov, A.P., 2003. An expert system for identification of line in vibrational-rotational spectra. *Optics and Spectroscopy*, **94**(4):528-537.

Deng, R., Davies, P., Bajaj, A.K., 2003. Flexible polyurethane foam modeling identification of viscoelastic parameters for automotive seating applications. *Journal of Sound and Vibration*, **262**(3):391-417.

Distefano, N., Rath, A., 1975. System identification in nonlinear structural seismic dynamics. *Journal of Computer Method in Applied Mechanics and Engineering*, **5**(3):353-372.

Fukushima, H., Sugie, T., 1999. Modeling set Identification in Frequency-Domain and its application to joint design with robust control. *Asian Journal of Control*, **1**(2):66-74.

Hart, G.C., Yao, J.T.P., 1977. System identification in structural dynamics. *Journal of the Engineering Mechanics Division*, *ASCE*, **103**(6):578-585.

Kruglov, S.P., Zalozhnev, A.Y., Novikov, D.A., 2002. Problem of center assignment in linear active system. *Avtomatika I Telemekhanika*, **12**:78-91.

Matzen, V.C., 1988. Time Domain Identification of Element Matrices. Proceeding of Workshop on Model Determination for Large Space System, Jet Propulsion Laboratory D-5574.

Matzen, V.C., 1990. Time Domain Identification of Linear Structure. Research Report of North Carolina State University, Raleigh, NC, U.S.A.

Matzen, V.C., McNiven, H.D., 1976. Investigation of the Inelastic Characteristics of a Single Story Steel Structure Using System Identification and Shaking Table Experiments. Report No. EERC 76-20, Earthquake Engineering Research Center, University of California, Berkeley.

McVerry, G.H., 1980. Structural Identification in the frequency domain from earthquake records. *Journal of Earthquake Engineering and Structural Dynamics*, **8**:161-180.

Natke, H.G., 1982. Multi-Degree-of-Freedom-System-A Review Identification of Vibration Structures Courses and Lectures–No. 272. International Center for Mechanics Science, Udine, Italy, Springer-Verlay, NY, U.S.A.

Rapid System Inc., 1999. 433N, 34th St., Seattle, WA 98103.

Yue, M., Schlueter, R., 2002. Bifurcation Subsystem Identification. Proceedings of the IEEE Power Engineering Society Transmission and Distribution Conference, **3**(Summer):1599-1604.