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Nonlinear analysis of consolidation with variable compressibility and permeability*

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Abstract: Terzaghi gave a theory of soil consolidation based on the effective stress principle, which was derived on several ideal assumptions to get a simplified theory. To avoid the limitations involved in Terzaghi's theory, many efforts are being made by scholars to solve the problems in practical engineering situations. This paper presents a generalized theory for one dimensional consolidation of saturated soft clay with variable compressibility and permeability. The semi-analytical solution presented here takes into account the well known empirical e-log$k$ and e-log$p'(\sigma')$ relations under instantaneous loading. Study of the consolidation behaviors showed that the ratio of C_c and C_k (the slope of e-log$p$ and e-log$k$ respectively) govern the ratio of consolidation. A simulative laboratory investigation with GDS advanced consolidation system was made to analyze the clay consolidation process and compare the results with the semi-analytical solution.

Key words: Compressibility, Consolidation, Nonlinear, Permeability
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INTRODUCTION

Since the development of the conventional consolidation theory by Terzaghi (1943) in 1923, many attempts have been made to present some solutions for the problems considering more realistic assumptions on the practical geotechnical engineering. Some efforts have been made to avoid limitation of the assumptions in the Terzaghi's theory. During the consolidation of soil, it is obvious that the coefficient of volume compressibility m_v and the coefficient of permeability k_v are not constant, and decrease with the increase of loading (Schiffman, 1958; Schiffman and Gibson, 1964; Davis and Raymond, 1965). Based on the relationship between e and log$p$, Davis and Raymond (1965) assumed k_v/m_v and c_v to be constant with increase of pressure and developed a nonlinear consolidation theory firstly. Gibson *et al.*(1967) analyzed the consolidation problem assuming large strain and nonlinear compressibility and permeability. Mesri and Rokhsar (1974), Mesri and Tavenas (1983), and Mesri and Choi (1985) solved the governing differential equations numerically considering the effect of variable compressibility and variable permeability. Basak and Madhav (1978) analyzed the problem of sand drain consolidation, incorporating the variation of compressibility and permeability. Li *et al.*(1999a; 1999b; 2000) and Xie *et al.*(2002; 2003a; 2003b) gave some solutions considering the property of the soil's nonlinearity and the characteristics of layered soils to promote the consolidation theory further. In this paper, a semi-analytical solution is presented for the case of void ratio e-log effective stress p and e-log permeability conductivity k_v, especially, the semi-analytical results are also compared with those obtained from experimental investigations with a set of advanced consolidation system. Furthermore, the behavior of nonlinear con-

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solidation of soils is analyzed and the differences between the semi-analytical results and Davis's nonlinear theory are discussed in detail.

THEORETICAL FORMULATIONS

Basic assumptions of the general equation

The problem studied here deals with the vertical consolidation of a compressible medium of finite thickness, assuming

(1) Vertical drainage;

(2) Small strain and no creep;

(3) Validity of Darcy's law;

(4) Saturated and homogeneous soil and incompressible soil grains and pore fluid;

(5) Instantaneous loading;

(6) Change of permeability during consolidation process, in which $e=e_0+C_k\log(k_v/k_{v0})$;

(7) Void ratio-effective stress response of the soil $e=e_0+C_c\log(\sigma'/\sigma_0')$.

Basic formulation of governing equation of the problem

In Fig.1 showing the schematic diagram of the soil stratum, H is the thickness of the soil layer; k_v is the vertical coefficient of permeability; c_v is the coefficient of consolidation; a_v is the coefficient of compressibility; p is the vertical uniform loading.

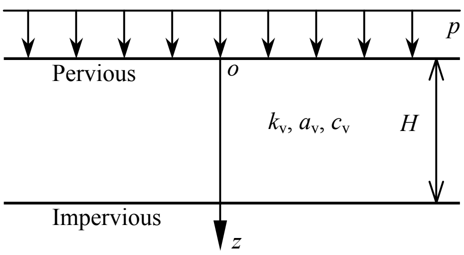

Fig.1 The schematic diagram of soil stratum

From the above assumptions, the experiential relationship equations, that are good representations of the behavior of most natural soft clays for small strain, are following Eqs.(1) and (2), in which e and e_0 is the void ratio corresponding to effective stress σ' and σ_0'; k_v and k_{v0} is the permeability conductivity corresponding to effective stress σ' and σ_0'; C_c is the compressibility index and C_k is the permeability index.

$$e = e_0 - C_c \log(\sigma_0'/\sigma') \tag{1}$$

$$e = e_0 - C_k \log(k_v/k_{v0}) \tag{2}$$

From Eq.(1), the volume compressibility of the soil can be given as

$$m_v = -\frac{1}{1+e_0}\frac{\partial e}{\partial \sigma'} = m_{v0}\frac{\sigma_0'}{\sigma'} \tag{3}$$

where $m_{v0} = \dfrac{C_c}{(1+e_0)\sigma_0' \ln 10}$ is the initial volume compressibility.

The coefficient of permeability can be gotten by subtracting Eq.(1) from Eq.(2)

$$k_v = k_{v0}\left(\frac{\sigma_0'}{\sigma'}\right)^{\frac{C_c}{C_k}} \tag{4}$$

So, the governing equation of one dimensional consolidation is as follows

$$\frac{1}{\gamma_w}\frac{\partial}{\partial z}(k_v\frac{\partial e}{\partial z}) = \frac{1}{1+e_0}\frac{\partial e}{\partial t} \tag{5}$$

According to the principle of effective stress, when the soil is subjected to loading p, the total stress $\sigma = \sigma' + u = \sigma_0' + p$ does not change with time t.

$$\frac{\partial \sigma'}{\partial t} = -\frac{\partial u}{\partial t} \tag{6}$$

Therefore, the governing equation as follows,

$$\frac{\partial}{\partial z}\left[k_{v0}\left(\frac{\sigma_f' - u}{\sigma_0'}\right)^{-\frac{C_c}{C_k}}\frac{1}{\gamma_w}\frac{\partial u}{\partial z}\right] = \frac{1}{\ln 10(1+e_0)}\frac{C_c}{\sigma_f' - u}\frac{\partial u}{\partial t} \tag{7}$$

in which, $\sigma_f' = \sigma_0' + p = \sigma$ is the final effective stress.

The dimensionless form of the above equation is

$$\frac{\partial}{\partial Z}\left[\left(\frac{\sigma_f' - U}{\sigma_0'}\right)^{-\frac{C_c}{C_k}}\frac{\partial U}{\partial Z}\right] = \frac{1}{\frac{\sigma_f'}{\sigma_0'} - U}\frac{\partial U}{\partial T_v} \tag{8}$$

in which, $Z=\dfrac{z}{H}$, $T_{\mathrm{v}}=\dfrac{c_{\mathrm{v0}}t}{H^2}=\dfrac{k_{\mathrm{v0}}\ln 10(1+e_0)\sigma' t}{\gamma_{\mathrm{w}}C_{\mathrm{c}}H^2}$, $U=\dfrac{u}{\sigma'_0}$.

The solution conditions of Eq.(8) are

(1) $T_{\mathrm{v}}=0: U=\dfrac{p}{\sigma'_0}=\dfrac{\sigma'_{\mathrm{f}}}{\sigma'_0}-1$

(2) $Z=0:\ U=0$

(3) $Z=1:\ U=0$ (PTPB) or $\dfrac{\partial U}{\partial Z}=0$ (PTIB).

Solution of the governing differential equation

It is very difficult to solve this differential equation by analytical method, even though the special condition, when $C_{\mathrm{c}}/C_{\mathrm{k}}=1$, was solved by Xie and Leo (1999). A semi-analytical solution similar to the method developed by Li *et al*.(1999a; 1999b; 2000) and Xie *et al*.(2003a; 2003b; 2004) is presented here to deal with the complexity involved. The soil stratum of thickness H is first divided equally into n layers of equal thickness H/n (Fig.2). Let the origin of the coordinate z be at the top of the soil stratum, and denote the distance between the origin and the bottom of each layer as z_i. Thus, $z_0=0$, $z_i=i\cdot H/n$, $i=1, 2, 3, \ldots, n$, and $z_n=H$. Because the thickness of each layer can be very small, at any time, the mean value of each soil parameter of each layer can stand for the value of the soil parameter of this layer, and the error caused by this approximation can be ignored. That is, the soil parameters of layer i can be respectively denoted as permeability coefficient $k_{\mathrm{v}i}$, volume compressibility coefficient $m_{\mathrm{v}i}$, consolidation coefficient $c_{\mathrm{v}i}$ ($i=1, 2, 3, \ldots, n$). Then taking into consideration the varied compressibility and permeability of soil during the process of consolidation, consolidation time is also divided into m small intervals and loading divided into m small loads accordingly. When the time interval is small enough, the coefficient of consolidation at each small layer in each small time interval can be taken as constant.

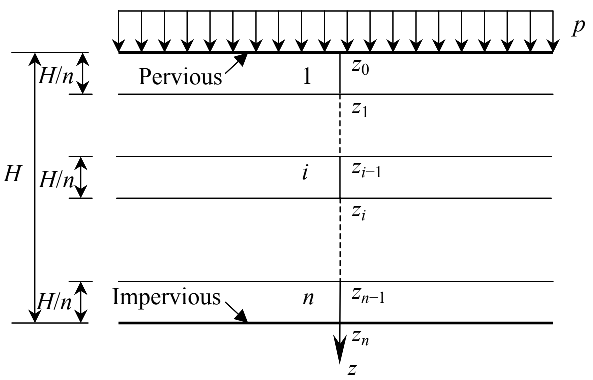

Fig.2 Divided layers of subsoil

So, the differential equation governing the consolidation process of each small layer can now be written as:

$$c_{\mathrm{v}i}\frac{\partial^2 u_i}{\partial z^2}=\frac{\partial u_i}{\partial t}\quad (z_{i-1}\le z\le z_i,\ i=1, 2, 3, \ldots, n) \tag{9}$$

where, $u_i=u_i(z,t)$ and $c_{\mathrm{v}i}=k_{\mathrm{v}i}/(m_{\mathrm{v}i}/\gamma_{\mathrm{w}})$, and γ_{w} is the unit weight of water, the excess pore water pressure and consolidation coefficient of small layer i respectively ($t_{k-1}\le t\le t_k$; $k=1, 2, 3, \ldots, m$). $c_{\mathrm{v}i}$ changes with time t and depth z, and can be deduced from mean excess porewater pressure u'_i at time t_{k-1} ($u'_i=q_0$ at $t=0$),

$$k_{\mathrm{v}i}=k_{\mathrm{v0}}\left(\frac{\sigma'_0}{\sigma'_0-u'_i}\right)^{\frac{C_{\mathrm{c}}}{C_{\mathrm{k}}}} \tag{10}$$

$$m_{\mathrm{v}i}=m_{\mathrm{v0}}\frac{\sigma'_0}{\sigma'_0-u'_i} \tag{11}$$

$$c_{\mathrm{v}i}=c_{\mathrm{v0}}\left(\frac{\sigma'_0}{\sigma'_0-u'_i}\right)^{\frac{C_{\mathrm{c}}}{C_{\mathrm{k}}}-1} \text{ and } c_{\mathrm{v0}}=\frac{k_{\mathrm{v0}}}{\gamma_{\mathrm{w}}m_{\mathrm{v0}}} \tag{12}$$

The boundary conditions for Eq.(9) are:

$z=0:\ u_1=0$

$z=H$: $\left.\dfrac{\partial u_n}{\partial z}\right|_{z=H}=0$ (impervious);

$$\left.u_n\right|_{z=H}=0 \text{ (pervious)} \tag{13a}$$

$z=z_i$: $u_i=u_{i+1}$;

$$k_{\mathrm{v}i}\frac{\partial u_i}{\partial z}=k_{\mathrm{v}(i+1)}\frac{\partial u_{i+1}}{\partial z},\ i=1, 2, 3, \ldots, n-1 \tag{13b}$$

and the initial condition of each time period is $u_i=u'_i$.

Thus, the problem of one-dimensional consolidation of soil with varied compressibility and permeability under the instantaneous loading has been reduced to the problem of one-dimensional linear consolidation of layered soils. According to the general solutions by Xie and Pan (1995), the corresponding solution of Eq.(9) can be given as

$$u_i=\sum_{m=1}^{\infty}C_m g_{mi}(z)\mathrm{e}^{-\beta_m t},\ i=1, 2, 3, \ldots, n \tag{14}$$

where,

$\beta_m=\lambda_m^2 c_{\mathrm{v}1}/H^2$;

$g_{mi}(z)=A_{mi}\sin(\mu_i\lambda_m \frac{z}{H})+B_{mi}\cos(\mu_i\lambda_m \frac{z}{H})$

$$C_m=\frac{\sum_{i=1}^{n}b_i\int_{z_{i-1}}^{z_i}[u_i'+(q_k-q_{k-1})]g_{mi}(z)\mathrm{d}z}{\sum_{i=1}^{n}b_i\int_{z_{i-1}}^{z_i}g_{mi}^2(z)\mathrm{d}z}$$

$$=\frac{2\sum_{i=1}^{n}u_i'\sqrt{a_ib_i}[A_{mi}(C_i-D_{i+1})}{\sum_{i=1}^{n}\sqrt{a_ib_i}\left[\frac{\mu_i\lambda_m}{n}(A_{mi}^2+B_{mi}^2)+(B_{mi}^2-A_{mi}^2)\right.}\rightarrow$$

$$\leftarrow\frac{+B_{mi}(B_{i+1}-A)]}{\left.(D_{i+1}B_{i+1}-C_iA_i)+2A_{mi}B_{mi}(C_i^2-D_{i+1}^2)\right]} \tag{15}$$

the definitions of parameters λ_m, μ_i, a_i, b_i, A_{mi}, B_{mi}, C_i, D_i can be found in Xie and Pan (1995).

From the above solution of excess porewater pressure, the average degree of consolidation of layer i defined in terms of settlement can be given by:

$$U_i=\frac{1}{q_0h_i}\int_{z_{i-1}}^{z_i}[q(t)-u_i]\mathrm{d}z$$

$$=1-\sum_{m=1}^{\infty}\frac{A_{mi}(C_i-D_{i+1})+B_{mi}(B_{i+1}-A_i)}{\mu_i\lambda_m}C_m\mathrm{e}^{-\beta_m t} \tag{16}$$

The total average degree of consolidation of the whole clay stratum defined in terms of settlement, U_{s} can be given by:

$$U_{\mathrm{s}}=\frac{\int_0^H(e_0-e)\mathrm{d}z}{\int_0^H(e_0-e_{\mathrm{f}})\mathrm{d}z}=\frac{\sum_{i=1}^{n}\log\left[\frac{\sigma_{\mathrm{f}}'}{\sigma_0'}-n\left(\frac{\sigma_{\mathrm{f}}'}{\sigma_0'}-1\right)U_i\right]}{n\log\frac{\sigma_{\mathrm{f}}'}{\sigma_0'}} \tag{17}$$

The total average degree of consolidation defined in terms of effective stress, U_{p}, can be derived as follows:

$$U_{\mathrm{p}}=1-\frac{\sum_{i=1}^{n}u_i\mathrm{d}z}{\sigma_{\mathrm{f}}'-\sigma_0'}=1-\sum_{i=1}^{n}U_i \tag{18}$$

As can be seen from Eq.(17) and Eq.(18), in which $h_i=z_i$, unlike that indicated by Terzaghi's 1-D consolidation theory, the total average degree of consolidation defined in terms of settlement and that defined in terms of effective stress are not the same, i.e., $U_{\mathrm{s}}\neq U_{\mathrm{p}}$, and will be the same only if the value of b_i=1, i.e. soil compressibility at each layer is equal. However, as mentioned above, soil compressibility and permeability of a layer are variable.

Analysis of the results obtained in the present theory

Berry and Wilkinson (1969) pointed out that the ratio $C_{\mathrm{c}}/C_{\mathrm{k}}$ is about 0.5~2, and mostly between 0.5 and 1, so the value of $C_{\mathrm{c}}/C_{\mathrm{k}}$ is selected to be 0.5 and 1 here.

Table 1 comprises the degree of consolidation by Davis and Raymond (1965)'s solution with that in the presented solution, in which $\sigma_{\mathrm{f}}'/\sigma_0'$=1.0. Comparison of the presented solution with Davis's theory proves the correctness of this semi-analytical method; the difference between them is about 2%. Additionally, when $C_{\mathrm{c}}/C_{\mathrm{k}}$<1, the actual pore pressure obtained by the present theory is less than that in the correspon-

Table 1 Percentage deviation of U_{p} in Davis's theory compared to present theory

Time factor T_{v}	U_{p}		Result from Davis's theory	Percentage deviation of Davis's theory	
	$C_{\mathrm{c}}/C_{\mathrm{k}}$=0.5	$C_{\mathrm{c}}/C_{\mathrm{k}}$=1	$C_{\mathrm{c}}/C_{\mathrm{k}}$=1	$C_{\mathrm{c}}/C_{\mathrm{k}}$=0.5	$C_{\mathrm{c}}/C_{\mathrm{k}}$=1
0.01	13.85%	11.44%	11.78%	−8.9%	−2.9%
0.02	19.60%	16.12%	16.32%	−6.8%	−1.2%
0.1	43.90%	35.94%	36.21%	−5.7%	−0.8%
0.2	61.65%	50.55%	51.12%	−3.3%	−1.1%
1	98.85%	93.13%	93.36%	5.0%	−0.2%

ding Davis's theory. That is to say, the actual consolidation process takes place faster than expected by Davis's theory. In fact, when σ'_f/σ'_0=1.0, k_v, m_v and c_v is constant, the solutions can become the traditional Terzaghi's one dimensional linear theory, as can be proved by the calculation.

LABORATORY SIMULATION

Laboratory consolidation tests were made to compare the experimental results with the semi-analytical solutions obtained above and to prove their correctness.

Test setup

A new set of PC-controlled advanced one-dimensional laboratory consolidation system (Fig.3), including consolidation cell (Rowe and Barden, 1966) of 63.5 mm inner diameter and 20 mm height were prepared to make the test according to the standard of BS1377 (BSI, 1990).

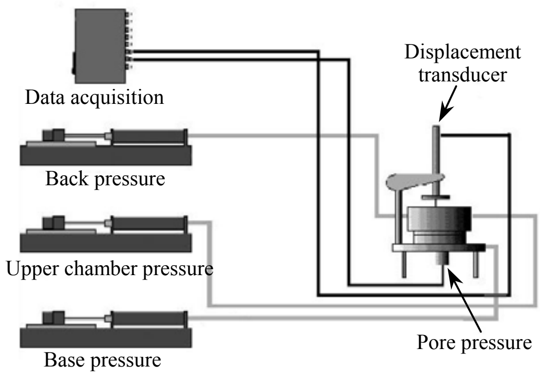

Fig.3 Test setup

Loading forced automatically by three GDS digital hydraulic controllers as desired can supply the maximum pressure of up to 2 MPa. One controller was used for supplying the diaphragm pressure, one for supplying back pressure, and one for supplying base pressure when permeability conductivity was being measured. Each controller was driven by a stepper motor consisting of a pressure cylinder in which a piston can move smoothly to reach any pressure. In stand alone mode, the instruments were in turn a general purpose constant source, a volume change gauge, a pore pressure measuring system, a flow pump and a digital pipette, which can also be programmed through its own control panel to ramp and cycle pressure and volume change linearly with respect to time.

All the original and calculated data during the test could be measured by the transducer connected to GDS 8 channel serial data acquisition pad, and saved by the technical software of GDSLAB version II. The volume change was logged by hydraulic controller measuring diaphragm pressure, back pressure and base pressure. Porewater pressure could be measured by a pressure transducer at the base of the sample, and settlement was measured by the displacement transducer with a travel of 25 mm mounted atop the cell. A difference from conventional consolidation system is that the porewater pressure of the sample could be measured in time. All pressure control had a resolution of 1 kPa.

The permeability test was conducted in the GDS system at the end of each incremental loading for consolidation test and the same effective stress was maintained to get the k_v corresponding to the e changes with the effective stress.

Table 2 The physical parameters of samples

Sample	Density ρ (g/cm^3)	Water ratio w (%)	Density of soil granule d_s
XS	1.70	53.34	2.756
YY	1.81	42.43	2.723

Test procedure

Before the test, the samples were saturated by back pressure in the Rowe and Barden consolidation cell till the ratio of saturation was larger than 93%. Fig.3 shows the arrangement of consolidation test with vertical drainage in GDS system. After setting of the Rowe and Barden cell, two permeable bronze discs were placed on the top and bottom of the sample, which was then saturated with back pressure. According to the British Standard 1377 (BSI, 1990), 50, 100 and 200 kPa was applied until the desired saturation was reached. The upper chamber pressure should be smaller than 70, 120 and 220 kPa respectively to prevent swelling.

The first step was consolidation at effective stress of 50 kPa, and the permeability test is implemented under the same consolidation stress. The rest of the consolidation pressures, such as 100, 200, 400 and 800 kPa, and permeability test may be deduced by analogy. The back pressure valve should be closed until the pore pressure reaches a steady value, when it is opened, and consolidation starts. The time required for porewater pressure to reach a steady value depended on the samples characteristics, but 24 h can be considered to achieve consolidation. Additionally, with volume change or settlement, pore pressure dissipation of sample due to secondary compression may develop if the test loading is not removed for a prolonged period (Lee and Xie, 1993). All the original data and calculated data from the consolidation test can be found in the data file with suffix of .gds.

Determination of C_c/C_k

Permeability measurements were taken at the end of each load increment, so that values of permeability and effective stress available for the same value of void ratio. From Fig.4 and Fig.5, the C_c and C_k values for YY clay are obtained as 0.467 and 0.901, respectively. A similar test on XS clay yielded C_c value of 0.360 and C_k value of 0.387. These two figures showed the linear trend of these two clays, with the C_c/C_k value of YY clay being 0.518 and that of XS clay being 0.930.

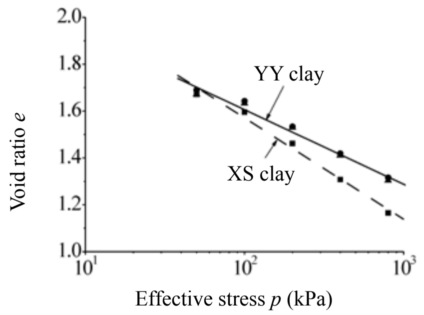

Fig.4 e-log$p$ curve of saturated clay

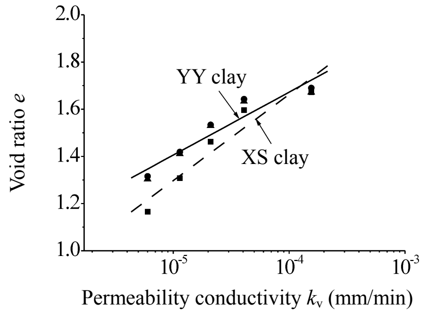

Fig.5 e-logk_v curve of saturated clay

Analysis of experimental results and comparison with theoretical results

The saturated samples were subjected to an initial pressure at the end of the saturation period. The value of displacement of samples was read from the instant of application of the load to the end of 24 h, as per the conventional procedure of Standard for Soil test method (Chinese National Standard GB/T50123-1999). A plot of the degree of consolidation (U) vs nondimensional time factor T_v was obtained for each pressure range under σ'_f/σ'_0 =1.0. Because the pressure 50 kPa was not very consistent with the whole shape of these curves, so the effective stress σ' was selected for 100, 200, 400, 800 kPa. Time-settlement curves obtained from the laboratory tests were compared with the results from the presented theory and also from Davis's theory. Fig.6 and Fig.7 comparing theoretical and experimental U_s-T_v curve for these two clays respectively shows fairly good agreement between them.

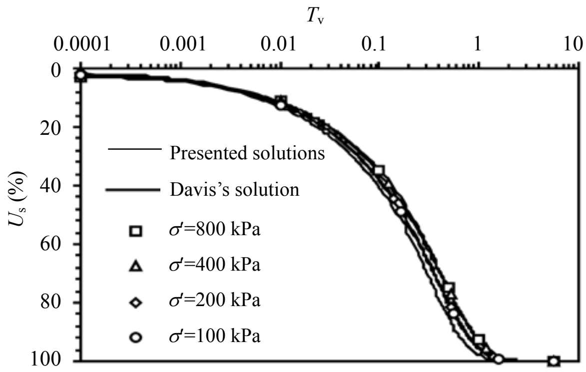

Fig.6 Comparison of semi-analytical solution and experimental U_s-T_v curve for XS clay

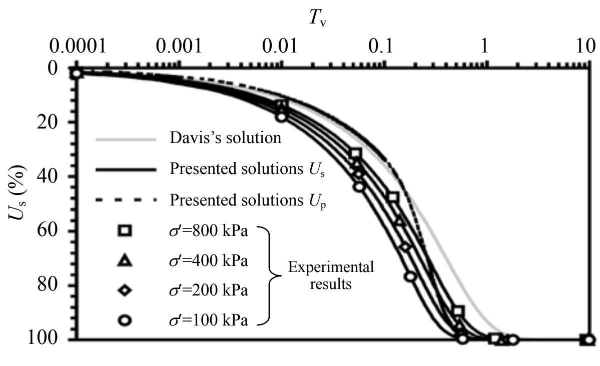

Fig.7 Comparison of semi-analytical solution and experimental U_s-T_v curve for YY clay

CONCLUSION

The following conclusions were obtained from this study:

1. The value of C_c/C_k decides whether it is necessary to take under consideration the effect of nonlinear property. When C_c/C_k<1, such as when C_c/C_k=0.518 as revealed by tests, the actual consolidation degree is less than that by the conventional theory without consideration of nonlinear characteristics. When C_c/C_k=1, the difference between them is not very obvious.

2. The semi-analytical solution is a very effective method for solving the difficult consolidation problems taking varied compressibility and permeability into account. The degree of consolidation defined by effective stress and by settlement is different in this method.

3. The GDS advanced consolidation system with back pressure is an effective method for analyzing the consolidation behavior of clay. Fairly good agreement exists between theoretical results and the consolidation test results.

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