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Pattern formation in mutation of "Game of Life"*

HUANG Wen-gao (黄文高)†, PAN Zhi-geng (潘志庚)

(*State Key Lab of CAD&CG, Zhejiang University, Hangzhou 310027, China*)

†E-mail: ggart@sohu.com

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Abstract: This paper presents pattern formation in generalized cellular automata (GCA) by varying parameters of classic "game of life". Different dynamic behaviors are classified. The influence of remembrance of dynamic behavior of GCA is also studied. Experiments show the emergence of the self-organizing patterns that is analogous with life forms at the edge of chaos, which consist of certain nontrivial structure and go through periods of growth, maturity and death. We describe these experiments and discuss their potential as alternative way for creating artificial life and generative art, and as a new method for pattern genesis.

Key words: Pattern genesis; Artificial life; Generalized cellular automata
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INTRODUCTION

The main idea of artificial life is that functional and aesthetic forms emerge through self-organization from distributed local rules and interactions. Cellular automata (CA) is its classical paradigm, and contributes much to its theory (Langton, 1990). CA and Boolean logic have been utilized for modelling pattern formation in nature (Chua, 1998; Gamow, 1964), meanwhile put forward new methods for generating realistic or interesting images, graphics, animation, music, etc., which constitute new forms of art (Spector and Klein, 2002).

Including CA as a special case, generalized cellular automata (GCA) constitutes an unified paradigm of pattern formation, which is composed of cellular neural network (CNN) with an additional loop (Chua, 1998). CNN is any discrete regular spatial architecture made up of cells (described by a dynamical system) which are coupled to other neighbor cells. In case of two dimensional CNN grid, each cell C_{ij} has variables: cell state: x_{ij}; input: u_{ij}; output: y_{ij}. Let $u_1, u_2, \dots, u_9$ denote input of neighbor cells. Every local Boolean function of $(u_1, u_2, \dots, u_9)$ can be realized by CNN cell. For example, the classic "game of life" Boolean function with 9 inputs can be realized by CNN cell given below (Dogaru, 1999):

$$w=1.5+0.5u_5-|u_1+u_2+u_3+u_4+u_6+u_7+u_8+u_9+2.5|$$
$$\dot{x}_{ij}=-x_{ij}+2f(x_{ij})+w$$
$$y_{ij}=f(x_{ij})=(|x_{ij}+1|-|x_{ij}-1|)/2 \quad (1)$$

GCA is defined by sampling y_{ij} at $t_n=n\Delta T$ (n=1,2,…) and feeding it into the cell input. ΔT is chosen so that all transients have settled down.

In order to explore interesting pattern in GCA, we utilize "life" as "seed" from which mutation in cell parameter is taken. For all the experiments in the next paragraph, we set the initial condition u_0 of all cell to –1 except the middle cell which is +1, and chose periodic CNN boundary condition. The CNN outputs at t_n are displayed as snapshots. The magnitude of cell outputs is coded with grayscale colors.

BASIC CLASSES OF DYNAMIC BEHAVIOR IN MUTATION OF "LIFE"

The mutation of "life" is as given below:

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$$u_{ij}(t_n) = y_{ij}(t_{n-1})$$
$$\sigma = (1+\lambda_1)(u_1+u_4+u_6+u_7)+(1+\lambda_2)(u_2+u_3+u_8+u_9)$$
$$w(u) = 1.5+0.5u_5 - |\sigma+2.5|$$
$$x_{ij}(t_n)=0,\ \dot{x}_{ij} = -x_{ij}+w(u),\ t\in(t_n,t_{n+1})$$
$$y_{ij} = 0.2x_{ij} \quad (2)$$

Two parameters λ_1, λ_2 are introduced as in (Dogaru and Chua, 2000), but with coefficient of $f(x_{ij})$ in the differential equation set to 0; the output is continuous, not binary, giving rise to more rich and colorful output images. The solution of the differential equation is: $x_{ij}=w$.

To discriminate different dynamic behaviors quantitatively, the cellular disorder measure $m(t)$ is defined below.

Suppose state can only be +1 or –1, each cell and its neighborhood must belong to one of the 2^9=512 possible configurations; let n_i be the number of cells with configuration i, total number of cells is $N=\sum_{i=0}^{511} n_i$, $p_i=n_i/N$, then

$$m = -\frac{1}{9}\sum_{i=0}^{511} p_i \log_2 p_i \quad (3)$$

When cell state is continuous within [–1,+1], we can transform it into binary pattern with a threshold b, when b=–0.75,–0.5,–0.25,0,0.25,0.5,0.75, calculate the corresponding m_1, m_2, m_3, m_4, m_5, m_6, m_7, the average is defined as hm. Varying λ_1, λ_2, experiments show three classes of different dynamic behavior:

Passive-like class After some time-steps, the output becomes a perfectly homogeneous pattern, all cells are in the same state, and hm decreases toward 0.

Unstable-like class A circular pattern grows from the middle, after reaching the lattice boundary, it implodes with unstable pattern left, the system enters slowly into a chaotic regime. hm increases rapidly during the growing period, then remains unchanged on average. An example is presented in Fig.1 when (λ_1,λ_2)=(–0.18,0). The number in images denotes time-step. CNN grid has 51×81 cells.

Edge of chaos-like class A circular pattern grows from the middle like a "cell", certain membranes emerge, after reaching the lattice boundary, they enter into a "maturity" phase where complex self-organizing patterns are developed. Following that is an "aging" and "dying" phase where the "cell" gradually becomes small and loses its structures, eventually it vanishes and the system enters a chaotic regime. hm increases rapidly during the growing period, then decreases monotonously and finally, after the "cell" vanishes, remains unchanged at a small nonzero value. Two examples are presented in Fig.2 and Fig.3. In two

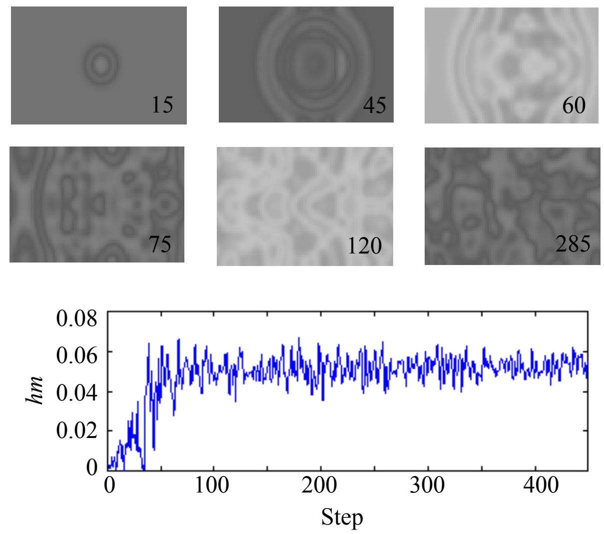

Fig.1 Unstable dynamic developing patterns and *hm*. The number in images denotes time-step

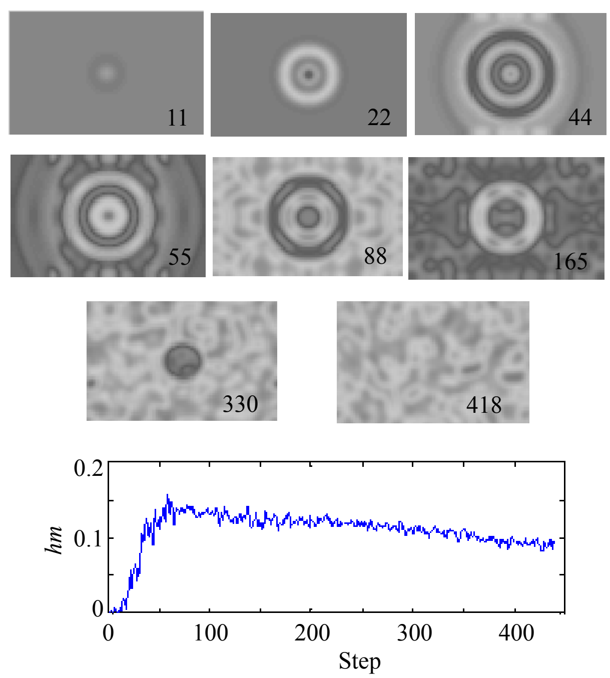

Fig.2 (λ_1,λ_2)=(0,0), the dynamic developing patterns and *hm*. The number in images denotes time-step

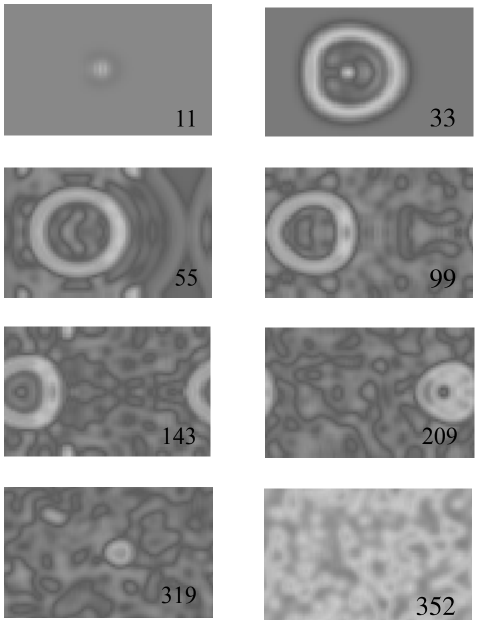

Fig.3 (λ_1,λ_2)=(0.4,–0.4), the dynamic developing patterns. The number in images denotes time-step

cases the evolving process is similar except that the self-organizing cell-like pattern in Fig.3 moves from right to left circularly.

Within the three classes of dynamic behavior, the edge of chaos-like class is the most interesting. It is characterized by emergence of self-organizing unity which discriminates itself clearly from environment and consists of certain nontrivial structure; it has ability of self-maintenance and self-repair in its lifetime; the duration of the transition from beginning to final stable state (homogenous or chaotic) is the longest, which is similar to observation of long transition at the edge of chaos (Langton, 1990). For all the above reasons, we say this class of GCA is working at the edge of chaos.

REMEMBRANCE

Consider another mutation of "life" (Dogaru and Chua, 1998), we add a memory factor r to the model and get the mutation as Eq.(4):

$$u_{ij}(t_n)=\frac{r^2 y_{ij}(t_{n-3})+r y_{ij}(t_{n-2})+y_{ij}(t_{n-1})}{r^2+r+1}$$
$$w=0.9+0.3u_5+0.3u_\sigma-\left|2+0.3u_5+0.6u_\sigma\right|+0.5\left|u_\sigma\right|$$
$$u_\sigma=(u_1+u_2+u_3+u_4+u_6+u_7+u_8+u_9)/2$$
$$x_{ij}(t_n)=x_0,\ \dot{x}_{ij}=-x_{ij}+w,\ t\in(t_n,t_n+\Delta T)$$
$$y_{ij}=f(x_{ij})=x_{ij} \tag{4}$$

Increasing r from 0, dynamic behavior of GCA becomes more and more passive, indicating that memory exerts a reserve effect and reduces activity of cells, as pointed out in (Alonso-Sanz, 1999). Another interesting dynamic behavior of GCA is found near r_0=–0.62720668. When $r<r_0$, the GCA output becomes a perfectly homogeneous pattern after some time steps. When $r>r_0$, the GCA output enters slowly into a stationary chaotic regime, and tends to have longer transition duration when r gets increasingly closer to r_0, somewhat like the edge of chaos. Some appealing pictures were obtained near this edge. An example is given in Fig.4. When r=–0.625, the initial condition of all cell is −1 except five cells with initial condition of +1. The image is interesting in that it resembles the lifetime of an animal: birth, growth, maturation, aging, and eventually death. The magnitude of cell outputs is coded with pseudocolors.

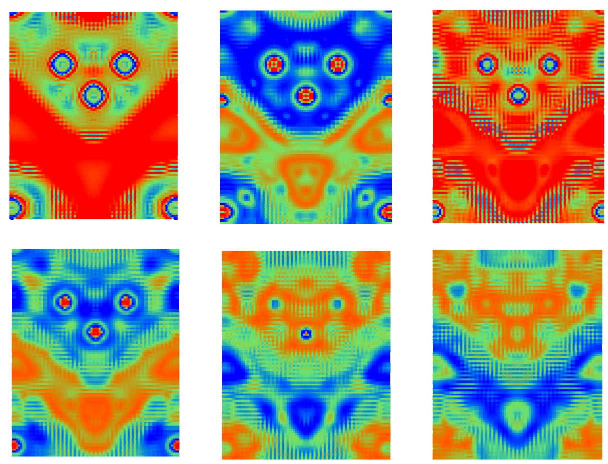

Fig.4 Dynamic developing animal-like patterns. CNN grid has 101×81 cells

CONCLUSION

This paper presents the pattern formation in GCA by mutations of classic "game of life". Experiments show the emergence of the self-organizing patterns similar to those of life forms at the edge of chaos, and potential for generating aesthetic patterns.

The mathematical theory behind cell mutations and its relation with biology may be further explored in the future (Peter, 1974; 1978). Other CNN topologies and cell models may be explored, which may lead to patterns with more aesthetic appeal or more novel function, e.g. self-reproduction. The effect of evolution may also be studied by allowing populations of GCA to evolve and then have the resulting patterns evaluated by certain aesthetic selection. This exploration may lead to alternative way for creating artificial life and artificial art, and new pattern genesis methods for CAD and CG.

References

Alonso-Sanz, R., 1999. The historic prisoner's dilemma. *Int. J. Bifurcation and Chaos*, **9**(6):1197-1210.

Chua, L.O., 1998. CNN: A Paradigm for Complexity. World Scientific Series on Nonlinear Science, Series A, Vol. 31.

Dogaru, R., 1999. Universal CNN cells. *Int. J. Bifurcation and Chaos*, **9**(1):1-48.

Dogaru, R., Chua, L.O., 1998. Rectification Neural Networks: A Novel Adaptive Architecture and Its Application for Implementing the Local Logic of Cellular Neural Networks. Electronics Research Laboratory Memorandum, No. UCB/ERL M98/4, UC Berkeley.

Dogaru, R., Chua, L.O., 2000. Mutation of the "game of life": a generalized cellular automata perspective of complex adaptive systems. *Int. J. Bifurcation and Chaos* **10**(8):1821-1866.

Gamow, G., 1964. Combinatorial Principles of Genetics. *In*: Beckenbach, E.F.(Ed.), Applied Combinatorial Mathematics. John Wiley and Sons Inc., New York, p.515-535.

Langton, C.G., 1990. Computation at the edge of chaos: phase transitions and emergent computation. *Physica D*, **42**:12-37.

Peter, S.H., 1974. A algorithm for the distance between two finite sequence. *Journal of Combinatorial Theory (A)*, **16**:253-258.

Peter, S.H., 1978. A algorithm for compositions of a multiset. *Journal of Combinatorial Theory (A)*, **24**:339-347.

Spector, L., Klein, J., 2002. Complex Adaptive Music Systems in the BREVE Simulation Environment. Workshop Proceedings of the 8th International Conference on the Simulation and Synthesis of Living Systems, Sydney, Australia, p.17-23.