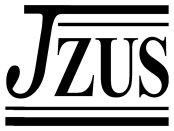
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Surface reconstruction by offset surface filtering

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Abstract: The problem of computing a piecewise linear approximation to a surface from its sample has been a focus of research in geometry modeling and graphics due to its widespread applications in computer aided design. In this paper, we give a new algorithm, to be called offset surface filtering (OSF) algorithm, which computes a piecewise-linear approximation of a smooth surface from a finite set of cloud points. The algorithm has two main stages. First, the surface normal on every point is estimated by the least squares best fitting plane method. Second, we construct a restricted Delaunay triangulation, which is a tubular neighborhood of the surface defined by two offset surfaces. The algorithm is simple and robust. We describe an implementation of it and show example outputs.

Key words: Cloud points, Surface reconstruction, Delaunay triangulation, Offset surface
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INTRODUCTION

Given a set of points that lie on or near a smooth surface, we consider the problem of computing a piecewise linear approximation of this surface. The surface reconstruction is increasingly important in geometric modelling for generating surfaces from cloud points captured from real objects, often by laser range scanners but also by hand-held digitizers, computer vision techniques, edge detection from medical images, or other technologies. Industrial applications include reverse engineering, product design and the construction of personalized medical applications.

The main issues in the surface reconstruction are how to deal with surfaces of arbitrary topology; to allow non-uniform sampling-featureless areas need fewer samples and to produce models with provable guarantees, e.g., smooth manifolds that accurately approximate the actual surface. There is a large body of related work concerning surface reconstruction. The-state-of-the-art report by Mencl and Müller (1998) present a good classification of the existing works. Here, we can distinguish two main approaches.

The first main approach is the implicit shape reconstruction. The surface to be reconstructed is considered as the zero set of an implicit function determined by the cloud points. This surface can be visualized directly using an implicit ray-tracer (Bloomenthal, 1997), or an intermediate explicit representation, such as a mesh of polygons, which can be extracted by the well-known iso-surface algorithms such as Marching Cubes (Lorensen and Cline, 1987). Such methods had been applied to the surface reconstruction by Hoppe *et al*.(1992), Bajaj *et al*. (1995), Bernardini *et al*.(1997), Curless and Levoy (1996), and Bossonnat and Cazals (2000) etc. The main issue in this kind of methods is what implicit function is used to fit the data points. Alexa *et al*.(2001) constructed the implicit function by a projection-based approach (MLS). Carr *et al*.(2001), Turk and Brien (2002) used globally supported radial basis functions (RBFs) to construct smooth surfaces from point cloud data. The level set method (Zhao and

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Osher, 2002) is another good candidate for reconstructing the implicit function. The current implementation of the above methods becomes expensive in time and memory if high accuracy reconstruction is required. The partition of unity approach (Ohtake *et al*., 2003) provides a reconstruction of implicit functions from scatted point data. The main advantages of using implicit functions for surface reconstruction from scatted data are data repairing capabilities and opportunities to edit the resulting objects using standard implicit modelling operations. These approaches are desirable especially in the presence of noise. But they cannot avoid generating extra zero-level sets. This disadvantage makes them difficult for reconstructing the surface with complex boundaries.

The second main approach is to construct a triangle mesh directly from the point cloud data. In this kind of approach, the methods may fall into two categories: sculpting-based approaches and region-growing approaches. Sculpting-based approaches are inspired by computational geometry. They output a set of facets from a geometric data structure such as the Delaunay triangulation of the points. Early result in this direction are the sculpting method of Bossonnat (1984) and the α-shapes of Edelsbrunner and Mücke (1994). Later on, Amenta *et al*.(1998) proposed a new Voronoi-based surface reconstruction algorithm with correctness guarantees under a given sampling condition. Efficient and robust codes are now available for computing Voronoi diagrams and Delaunay triangulations (Devillers, 1998). These methods are fast. Other algorithms in this category are Gamma-graphs (Veltkamp, 1991), A-shapes (Melkemi, 1997), crust algorithm (Amenta and Bern, 1999; Amenta *et al*., 2000; 2001), Gabriel graphs (Attene and Spagnuolo, 2000), Umbrella Filter algorithm (Adamy *et al*., 2002), and Geometric convection approach (Chaine, 2003) etc. Region-growing approaches construct the mesh starting with a seed triangle patch, and progressively adding new triangles attached to the partially constructed mesh. The contributions of Bernardini *et al*.(1999), Huang and Menq (2002), and Lin *et al*.(2004) fall into this category. The advantages of the algorithms in this category are that they are very fast and can handle the processing of huge data sets, but suffer from the sampling deficiencies and the bad seed triangle. Another algorithm is that of Mencl (1995) which produces a triangulated surface by filling the contours of an extension of the Euclidean minimum spanning tree of the points. Floater and Reimers (2001) did well in parameterizing the scattered points and then compute the 2D Delaunay triangulations. The algorithm of Amenta *et al*.(1998) is the first one with provable guarantee in 3 dimensions. These theoretical results hold when the sampling is sufficiently dense. However the restrictive sampling conditions are rarely met in practical applications. Often, in practice, the reconstructed surface may not be a manifold, may have additional holes, triangles. To ensure correct reconstruction, a post-processing of the reconstructed surface is often needed. Adamy *et al*.(2002) suggested a nice way to achieve topology correctness.

In this paper, we present a new method for reconstructing a piecewise linear surface from the point cloud. The sampling criteria presented here is admittedly implicit. Namely, if one wishes to reconstruct a surface from sampled data, then the sampling of the surface must be sufficiently fine. The main idea in our method is to construct a restricted Delaunay triangulation with the help of two offset surfaces. A primary advantage in using offsets versus existing work (Amenta and Bern, 1999; Amenta *et al*., 2000) relying upon the medial axis as approximating the medial axis is a difficult task (Amenta *et al*., 2001) whereas the method here requires no such computation.

OUTLINE OF THE RECONSTRUCTION ALGORITHM

The inputted point cloud $\boldsymbol{P}$ is a finite set of points scattered in three-dimensional Euclidean space sampled from a "compact, connected, orientable 2D manifold surface, embedded in $\mathbb{R}^3$" (Hoppe *et al*., 1992). And we assume that the sample is sufficiently dense. The outputted result is structural information in terms of a triangular mesh or complex connecting the scattered points. There are two main steps in our algorithm:

(1) Normal estimation. Here we use the least squares best fitting plane to estimate the unit normal vector at every point in the inputted data. The main idea comes from Hoppe *et al*.(1992). First, a discrete local neighborhood of a point can be defined through the spatial relations of sampled points. Given a point

$\boldsymbol{p}\in\boldsymbol{P}$, we gather together the k points of $\boldsymbol{P}$ nearest to $\boldsymbol{p}$. The obtained points set is denoted by $Nbhd(p)$ and is called the k-neighborhood of $\boldsymbol{p}$ (We currently assume k to be a user-specified parameter). The centroid of $Nbhd(\boldsymbol{p})$ is represented as a point $\boldsymbol{o}$. Then we can obtain a symmetric 3×3 positive semi-definite matrix

$$\boldsymbol{CV}=\sum_{y\in Nbhd(\boldsymbol{p})}(\boldsymbol{y}-\boldsymbol{o})^{\mathrm{T}}(\boldsymbol{y}-\boldsymbol{o})$$

If $\lambda_1\geq\lambda_2\geq\lambda_3$ denote the eigenvalues of $\boldsymbol{CV}$ associated with unit eigenvectors v_1, v_2, v_3, respectively. It follows that the plane

$$\boldsymbol{T}(x):(\boldsymbol{x}-\boldsymbol{o})^{\mathrm{T}}\boldsymbol{v}_3=0$$

through $\boldsymbol{o}$ minimizes the sum of squared distances to the set $Nbhd(\boldsymbol{p})$. Thus $\boldsymbol{v}_3$ approximates the surface normal at point $\boldsymbol{p}$.

(2) The restricted Delaunay triangulation. We use the offsets to remove the triangles from Delaunay triangulation. Let $\boldsymbol{T}$ be the set of the triangles that remained. $\boldsymbol{T}$ contains only those triangles in which all three vertices are sample points. And then an acceptable piecewise linear manifold can be selected from $\boldsymbol{T}$ by a manifold extraction step.

RESTRICTED DELAUNAY TRIANGULATION

At first, we will briefly review some elementary facts about offsets and the restricted Delaunay triangulation, taken from (Wallner *et al*., 2001; Sakkalis *et al*., 2004; Edelsbrunner and Shah, 1994). Then we present a simple algorithm for constructing the restricted Delaunay triangulation.

Offset surfaces

Offset surface is useful in geometric modelling, such as in the construction of tolerance zones, the generation of tool paths for numerical control machining, and application to interval solids, etc. Let $\boldsymbol{F}$ be an orientable dimensional manifold surface embedded in $\mathbb{R}^3$, which is C^2 (at each point of the manifold where the second derivative exists and is continuous). Let $\lambda\in\mathbb{R}$, then the offsets $\boldsymbol{F}_0(\lambda)$ of $\boldsymbol{F}$ can be defined as

$$\boldsymbol{F}_0(\lambda)=\{x+\lambda\boldsymbol{n}_x|x\in\boldsymbol{F}\},$$

where $\boldsymbol{n}_x$ is the unit surface normal of $\boldsymbol{F}$ at x.

An offset is in general more complex than its progenitor (the initial surface $\boldsymbol{F}$). It may self-intersect locally when the absolute value of the offset distance exceeds the minimum radius of curvature in a concave region. Also, an offset may intersect globally when the distance between two distinct points on the progenitor reaches a local minimum (Wallner *et al*., 2001). Furthermore, Wallner *et al*.(2001) showed a method for determining the maximum offset distance such that the offset does not self-intersect. Sakkalis *et al*.(2004) proved that the offset $\boldsymbol{F}_0(\lambda)$ is ambient isotopic to its progenitor $\boldsymbol{F}$ when the offset distance does not exceed a tolerance threshold calculated from $\boldsymbol{F}$.

In this paper we use the offset to construct the restrict Delaunay triangulation. Let $\boldsymbol{P}$ be the point cloud sampled from $\boldsymbol{F}$, we have the point cloud sampled from two offsets of $\boldsymbol{F}$ as follows

$$\boldsymbol{Q}=\{\boldsymbol{p}\pm\lambda\boldsymbol{n}_{\boldsymbol{p}}|\boldsymbol{p}\in\boldsymbol{P}\},\tag{1}$$

where $\boldsymbol{n}_{\boldsymbol{p}}$ is the estimated unit surface normal of $\boldsymbol{F}$ at $\boldsymbol{p}$, and $\lambda=\alpha\rho$. α is a positive user-defined value and less than 1. Usually, we use α=0.8. We use the threshold value ρ to approximate the maximum offset distance such that the offset does not self-intersect. In fact, calculating the maximum offset distance exactly is very complex work. And we do not need the precise maximum. We offer a practical way to determine the value ρ.

$$\rho=\left(\sum_{i=1}^{m}d(\boldsymbol{p}_i)\right)\Big/m,$$

where $d(\boldsymbol{p}_i)=\min_{0<j<m+1,\,j\neq i}\left|\boldsymbol{p}_i-\boldsymbol{p}_j\right|$, $\boldsymbol{p}_i\in\boldsymbol{P}$.

The value ρ is just an experimental result. Examples show that it is available from our algorithm.

Construction of the restricted Delaunay triangulation

A Voronoi cell of $\boldsymbol{p}\in\boldsymbol{P}$ is defined as the set of points $\boldsymbol{x}\in\mathbb{R}^3$ such that $|\boldsymbol{p}-\boldsymbol{x}|\leq|\boldsymbol{q}-\boldsymbol{x}|$ for any $\boldsymbol{q}\in\boldsymbol{P}$ and $\boldsymbol{p}\neq\boldsymbol{q}$. The collection of Voronoi cells is $\boldsymbol{V}_{\boldsymbol{P}}=\{\boldsymbol{V}_{\boldsymbol{p}}|\,\boldsymbol{p}\in\boldsymbol{P}\}$. $\boldsymbol{V}_{\boldsymbol{P}}$ is called the Voronoi diagram of $\boldsymbol{P}$. Let $\boldsymbol{D}_{\boldsymbol{P}}$ denote

the Delaunay triangulation of $\boldsymbol{P}$. The Delaunay triangulation has an edge $\boldsymbol{pq}$ if and only if $\boldsymbol{V_p}$, $\boldsymbol{V_q}$ share a face, has a triangle $\boldsymbol{pqr}$ if and only if $\boldsymbol{V_p}$, $\boldsymbol{V_q}$, $\boldsymbol{V_r}$ share an edge, and has a tetrahedron $\boldsymbol{pqrs}$ if and only if $\boldsymbol{V_p}$,$\boldsymbol{V_q}$,$\boldsymbol{V_r}$,$\boldsymbol{V_s}$ share a Voronoi vertex.

The Voronoi cell restricted to the surface $\boldsymbol{F}$ is $\boldsymbol{V_{p,F}}=\boldsymbol{V_p}\cap\boldsymbol{F}$. The restricted Voronoi diagram $\boldsymbol{V_{P,F}}$ is defined as the collection of restricted Voronoi cells. The dual of the restricted Voronoi diagram defines the restricted Delaunay triangulation $\boldsymbol{D_{P,F}}$ (Edelsbrunner and Shah, 1994). Edelsbrunner and Shah (1994) showed that the underlying space of $\boldsymbol{D_{P,F}}$ is homeomorphic to $\boldsymbol{F}$ if a closed ball property holds. Amenta and Bern (1999) used the above result to prove that if $\boldsymbol{P}$ is a ε-sample of $\boldsymbol{F}$ with $\varepsilon\lesssim 0.1$, then $\boldsymbol{D_{P,F}}$ contains triangles forming a piecewise-linear manifold homeomorphic to $\boldsymbol{F}$.

Similar to the method in Amenta and Bern (1999), we also used restricted Delaunay triangulation to reconstruct the underlying surface. However we obtain $\boldsymbol{D_{P,F}}$ in an easier way. We calculate the Delaunay triangulation of $\boldsymbol{P}\cup\boldsymbol{Q}$ (defined in Eq.(1)), and then collect the triangles whose vertices all belong to $\boldsymbol{P}$.

$$\boldsymbol{T}=\{\boldsymbol{t}|\boldsymbol{t}\in\boldsymbol{D_{P\cup Q}}, \text{ all vertices of } \boldsymbol{t} \text{ belong to } \boldsymbol{P}\},$$

where $\boldsymbol{Q}$ is defined in Eq.(1).

We assume again that the sample is sufficiently dense. Then we use the set $\boldsymbol{T}$ to approximate the restricted Delaunay triangulation $\boldsymbol{D_{P,F}}$. A simple geometric interpretation of the above method is as follows (See Fig.1~Fig.5). Every Voronoi cell $\boldsymbol{V_p}$, $\boldsymbol{p}\in\boldsymbol{P}$, in the Voronoi diagram $\boldsymbol{V_{P\cup Q}}$ is bounded by two offsets. Furthermore, the set contains all Voronoi cells, which intersect the surface $\boldsymbol{F}$, just is the collection of cells

$$\boldsymbol{V}^1=\{\boldsymbol{V_p}|\boldsymbol{p}\in\boldsymbol{P}, \boldsymbol{V_p}\in\boldsymbol{V_{P\cup Q}}\}.$$

And thus the dual of $\boldsymbol{V}^1$ (See Fig.3 and Fig.4 for the illustration in two-dimensional space) contains the restricted Delaunay triangulation. Because the set $\boldsymbol{T}$ belong to the dual of $\boldsymbol{V}^1$, the triangle in T is bounded by two offsets.

From Fig.1 to Fig.5, we illustrate our method in two-dimensional space. In Fig.1, the point cloud $\boldsymbol{P}$ and its two offsets $\boldsymbol{Q}$ are shown. In Fig.2, the Delaun-

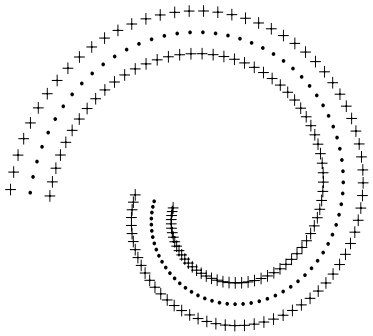


Fig.1 Illustration of the point cloud (·) and its two offsets (+) in two-dimensional space

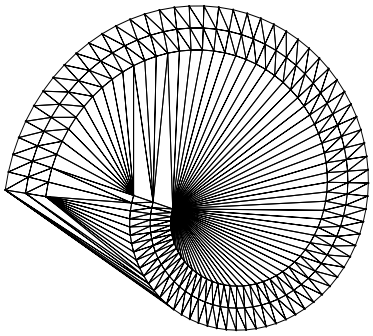


Fig.2 Illustration of $\boldsymbol{D_{P\cup Q}}$ in two-dimensional space

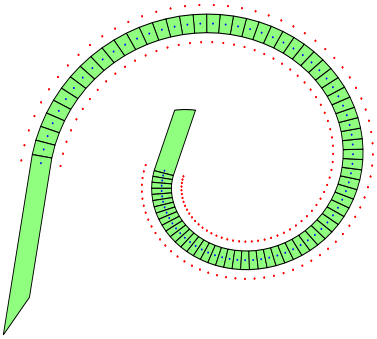


Fig.3 Illustration of $\boldsymbol{V}^1$ in two-dimensional space

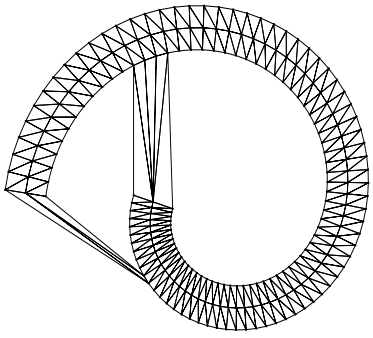


Fig.4 The dual triangulation of $\boldsymbol{V}^1$

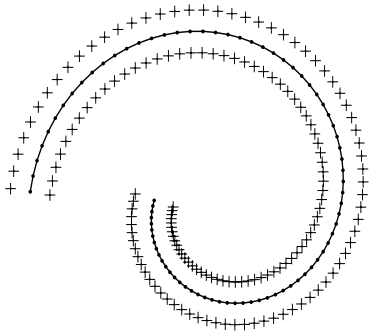


Fig.5 Illustration of T in two-dimensional space. And T is just the piecewise approximation to the curve

ay triangulation is implemented on the points set. In Fig.3 and Fig.4, we draw the collection of Voronoi cells V^1, containing all the restricted Voronoi cells. The dual of V^1 contains the piecewise approximation to the curve underlying the inputted point cloud. Fig.5 is the reconstruction curve obtained by connecting the line segments in the set T.

Now, we can present the pipeline of our algorithm in detail:

```
Complex T OSF (Points_set P)
{
To estimate the unit normal vector n_p at p∈P;
Q={p±λn_p|p∈P};   //(λ is calculated in Section 3.1)
T={t|t∈D_{P∪Q}, all vertices of t belong to P};
 return T;
}
```

IMPLEMENTATION

During implementation of the algorithm, we faced some difficulty with the manifold extraction step (connecting the triangles to obtain a piecewise linear manifold). Treating the complex T as a triangular mesh depends heavily on the assumptions that the surface is smooth, has no boundaries, and that the sampling is sufficiently dense. In practice the inputted data do not satisfy these assumptions. So a post processing of T is usually needed. Similar to the idea in Adamy *et al*.(2002), we also use the umbrella condition to mark the unwanted triangle. A vertex v satisfies the umbrella condition if there exists a set of triangles incident to v which form a topological disk. A triangle in T is marked as a 'good' triangle if its three vertices all satisfy the umbrella condition. A triangle in T is marked as a 'bad' triangle if one of its vertices does not satisfy the umbrella condition. All the 'good' triangles form an initial triangular mesh M_1. Then we use region-growing approaches (Lin *et al*., 2004) to construct the final mesh starting with M_1, and progressively adding 'bad' triangles in T attached to the partially construction mesh.

RESULTS

We implemented our algorithm using C++ programming language. Our implementation is based on the Computational Geometry Algorithms Library CGAL (available on http://www.cgal.org), which includes fast and robust Delaunay triangulations for two and three dimensions. We test our algorithm on several examples (See Fig.6~Fig.11) on a PC with Intel Pentium IV CPU 2.8 GHz and 1 G RAM memory. Additional information is listed in Table 1, which summarizes more detailed information about the runtimes and the number of output triangles for our algorithm.

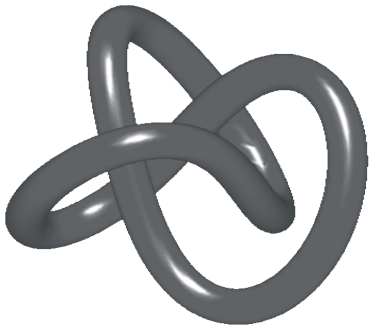


Fig.6 Knot



Fig.7 Fandisk



Fig.8 Knee



Fig.9 Rabit

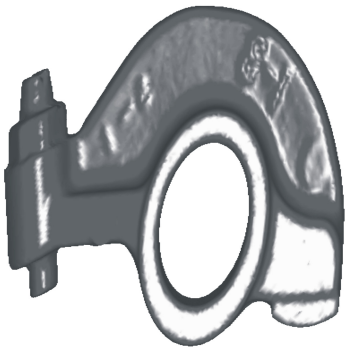


Fig.10 RockerArm



Fig.11 Club

Table 1 Performance of our algorithm for different objects, statistic in the table: the number of points in the point clouds, the number of triangles in the reconstructed mesh, the time of processing

Objects	Points No.	Triangles No.	Time (s)
Knot	5759	11213	9.157
Fandisk	25893	50378	47.052
Knee	37888	75224	74.743
Rabit	67038	134029	125.047
RocherArm	40177	79729	80.414
Club	209779	419319	480.538

CONCLUSIONS AND FUTURE WORK

In this paper, we present a new surface reconstruction algorithm, called OSF algorithm. This algorithm is suitable for dealing with arbitrary topology surface, to allow non-uniform sampling and especially for the sampled points from smooth surface. Our method can be treated as an extension to the Voronoi filtering by Amenta and Bern (1999). By adding poles, the Voronoi filtering introduced points in the normal direction. However, they used the farthest Voronoi point to estimate the normal direction. This will fail when the farthest Voronoi points reach to infinity, and so will cause big errors when the point clouds are sampled from open surfaces, especially when the point clouds are nearly on a plane. In our algorithm, we use the least squares best fitting plane method to approximate the normal vector. The computation becomes local and more robust. Planned future work includes decreasing the runtime of our algorithm, studying the sampling according to the normal information, providing provable guarantees.

References

Adamy, U., Giesen, J., John, M., 2002. Surface reconstruction using umbrella filters. *Comput. Geom*, **21**(1-2):63-86.

Alexa, M., Behr, J., Cohen-Or, D., Fleishman, S., Levein, D., Silva, C.T., 2001. Point Set Surfaces. IEEE Visualization 2001, p.21-28.

Amenta, N., Bern, M., 1999. Surface reconstruction by Voronoi filtering. *Discrete Comput. Geo*., **22**(4):481-504.

Amenta, N., Bern, M., Kamvysselis, M., 1998. A New Voronoi-based Surface Reconstruction Algorithm. Proc. SIGGRAPH 1998, p.412-415.

Amenta, N., Choi, S., Dey, T.K., Leekha, N., 2000. A Simple Algorithm for Homeomorphic Surface Reconstruction. Proceedings of the 16th Annual ACM Symposium on Computational Geometry (SCG'2000), p.213-222.

Amenta, N., Choi, S., Kolluri, R.K., 2001. The power crust, unions of balls, and the medial axis transform. *Comput. Geom. Theory Appl.*, **19**:127-153.

Attene, M., Spagnuolo, M., 2000. Automatic surface reconstruction from point sets in space. *Computer Graphics Forum*, **19**(3):457-465.

Bajaj, C., Bernardini, F., Xu, G., 1995. Automatic Reconstruction of Surfaces and Scalar Fields from 3D Scans. Proc. SIGGRAPH '95.

Bernardini, F., Bajaj, C., Chen, J., Schikore, D., 1997. Triangulation-based Object Reconstruction Methods. Proc. 13th Annu. ACM Symp. Comput. Geom., p.481-484.

Bernardini, F., Mittleman, J., Rushmeier, H., Silva, C., Taubin, G., 1999. The ball-pivoting algorithm for surface reconstruction. *IEEE Trans. Visualization Compute Graphics*, **5**(4):349-359.

Bloomenthal, J.(Ed.), 1997. Introduction to Implicit Surfaces. Morgan Kaufmann, San Francisco, California.

Bossonnat, J.D., 1984. Geometric structures of three-dimensional shape reconstruction. *ACM Trans. Graphics*, **3**(4):266-286.

Bossonnat, J.D., Cazals, F., 2000. Smooth Surface Reconstruction via Natural Neighbor Interpolation of Distance Functions. Proceedings of SCG'2000 (ACM Symposium on Computational Geometry), p.223-232.

Carr, J., Beatson, R.K., Cherrie, J.B., Mitchell, T.J., Fright, W.R., Mccallum, B.C., Evans, T.R., 2001. Reconstruction and Representation of 3D Objects with Radial Basis Functions. Proc. SIGGRAPH 2001, p.67-76.

Chaine, R., 2003. A Geometric Convection Approach of 3-D Reconstruction. Proc. of Eurographics and ACM SIGGRAPH Symp. on Geometry Processing, p.218-229

Curless, B., Levoy, M., 1996. A Volumetric Method for Building Complex Models from Range Images. Proc. SIGGRAPH '96, p.303-312.

Devillers, O., 1998. Improved Incremental Randomized Delaunay Triangulation. Proc. 14th Annual ACM Symp. Comput. Geom., p.106-115.

Edelsbrunner, H., Mücke, E., 1994. 3D alpha shapes. *ACM Trans. Graphics*, **13**(1):43-72.

Edelsbrunner, H., Shah, N., 1994. Triangulating Topological spaces. Proc. 10th ACM Symp. Comput. Geom., p.285-292.

Floater, M.S., Reimers, M., 2001. Meshless parameterization and surface reconstruction. *Computer Aided Geometry Design*, **18**(2):77-92.

Hoppe, H., DeRose, T., Duchamp, T., McDonald, J., Stuetzle, W., 1992. Surface reconstruction from unorganized points. *Comput. Graphics*, **26**(2):71-78.

Huang, J., Menq, C.H., 2002. Combinational manifold mesh reconstruction and optimization from unorganized points with arbitrary meshes. *Computer-Aided Design*, **34**(2):149-165.

Lorensen, W.E., Cline, H.E., 1987. Marching cubes: A high resolution 3D surface construction algorithm. *Computer Graphics*, **21**(4):163-169.

Lin, H.W., Tai, C.L., Wang, G.J., 2004. A mesh reconstruction algorithm driven by an intrinsic property of a point cloud. *Computer-Aided Design*, **36**(1):1-9.

Melkemi, M., 1997. A-shapes and Their Derivatives. Proc. 13th Annual ACM Symp. Comput. Geom., p.367-369.

Mencl, R., 1995. Surface Reconstruction from Unorganized Points in Space. Abstracts 11th European Workshop Comput. Geom., p.67-70.

Mencl, R., Müller, H., 1998. Interpolation and Approximation of Surfaces from Three-dimensional Scatted Data Points. State of the Art Reports, Eurographics1998, p.51-67.

Ohtake, Y., Belyaev, A., Alexa, M., Turk, G., Seidel, H.P., 2003. Multi-level Partition of Unity Implicits. Proc. SIGGRAPH 2003.

Sakkalis, T., Peter, T.J., Bisceglio, J., 2004. Isotopic approximations and interval solids. *Computer-Aided Design*, **36**:1089-1100.

Turk, G., Brien, O.J., 2002. Modeling with implicit surfaces that interpolate. *ACM Trans. on Graphics*, **21**(4):855-873.

Veltkamp, R.C., 1991. The gamma-neighborhood graph. *Comput. Geom.*, **1**:227-246.

Wallner, J., Sakkalis, T., Maekawa, T., Pottman, H., Yu, G., 2001. Selfintersections of offset curves and surfaces. *Int. J. Shape Modeling*, **7**(1):1-21.

Zhao, H., Osher, S., 2002. Visualization, Analysis and Shape Reconstruction of Unorganized Data Sets. *In*: Osher, S., Paragios, N.(Eds.), Geometric Level Set Methods in Imaging, Vision and Graphics. Springer.