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Left-handed materials in magnetized metallic magnetic thin films*

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Abstract: The authors' theoretical investigation on the high-frequency response of magnetized metallic magnetic films showed that magnetic films may become left-handed materials (LHMs) near the ferromagnetic resonance frequency of incident waves with right-handed circular polarization (RCP) and linear polarization (LP). The frequency range where LHM exists depends on the waves polarization, the magnetic damping coefficient, and the ferromagnetic characteristic frequency ω_m of the film. There also exists a critical damping coefficient α_c, above which the left-handed properties disappear completely.

Key words: Left-handed materials (LHMs), Magnetic thin film, Ferromagnetic resonance frequency

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INTRODUCTION

Veselago (1968) showed that if both the permittivity ε and the magnetic permeability μ of a material are negative, the propagation direction of an electromagnetic (EM) wave will be opposite to its energy flow direction. Such media are called left-handed materials (LHMs) since the electric field $\boldsymbol{E}$, magnetic field $\boldsymbol{H}$, and wave vector $\boldsymbol{k}$ in these media form a left-hand triplet of vectors, instead of a right-hand triplet observed in conventional materials. For a long time, people have not paid attention to LHMs since the phenomenon has not been observed in real materials. Recently, based on some early ideas of Pendry (2000), Smith *et al*.(2000) constructed metamaterials consisting of metallic strips and split ring resonators (SRRs), which display negative values of effective ε and μ simultaneously, and Shelby *et al*.(2001) demonstrated this material's negative refraction indicative of LHM properties. This discovery reinvigorated the search for LHMs. Many researches also focused on the development of new metamaterials that are potential LHMs (Chui and Hu, 2002; Parimi *et al*., 2004; Grbic and Eleftheriades, 2004; Wu *et al*., 2004). Metallic magnetic materials characterized by magnetic permeability could be potential LHMs, since negative permeability can be achieved in certain magnetic materials above the ferromagnetic resonance frequency, and negative permittivity automatically exists in metals below the plasma frequency which is typically in the ultraviolet frequency range. In this work, we have theoretically demonstrated that ultra-thin metallic ferromagnetic films are indeed LHMs in the vicinity of ferromagnetic resonant frequency for right circularly polarized EM waves and linearly polarized waves, provided that magnetic damping is small enough.

EXISTENCE OF LHM IN METALLIC MAGNETIC THIN FILM

Assuming that the magnetization of a metallic magnetic film is aligned by a static external magnetic field $\boldsymbol{H}_{ext}$ in the direction perpendicular to the film plane, with the z-axis as shown in Fig.1. Then the

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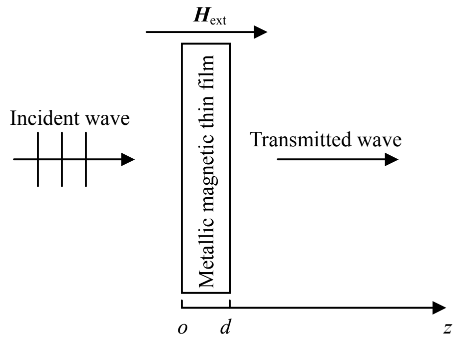


Fig.1 Sketch of a metallic magnetic thin film in an external static magnetic field and incident EM waves

relative permeability of the film is a second-rank tensor (Slichter, 1978), that is,

$$\boldsymbol{\mu}_{\mathrm{r}}=\begin{pmatrix}\mu_1 & \mathrm{j}\mu_2 & 0\\ -\mathrm{j}\mu_2 & \mu_1 & 0\\ 0 & 0 & 1\end{pmatrix} \tag{1}$$

where

$$\begin{cases}\mu_1=1+\dfrac{\omega_{\mathrm{m}}(\omega_0+\mathrm{j}\alpha\omega)}{(\omega_0+\mathrm{j}\alpha\omega)^2-\omega^2},\\ \mu_2=\dfrac{\omega_{\mathrm{m}}\omega}{(\omega_0+\mathrm{j}\alpha\omega)^2-\omega^2},\end{cases} \tag{2}$$

with ω_0 being the ferromagnetic resonance frequency, ω_{m} the characteristic frequency, and α the ferromagnetic damping coefficient. For harmonic EM waves in the film, the magnetic field follows the wave equation (Zhang, 1998):

$$\nabla^2\boldsymbol{H}-\nabla(\nabla\cdot\boldsymbol{H})+\omega^2\varepsilon\,\boldsymbol{\mu}\cdot\boldsymbol{H}=0\,. \tag{3}$$

If the EM wave is linearly polarized and propagates along the external field, from Eq.(3), we can get two eigenvalues k_{RCP} and k_{LCP} for the wave propagation constant, corresponding to right circularly polarized (RCP) and left circularly polarized (LCP) waves, respectively:

$$\begin{cases}k_{\mathrm{RCP}}=\omega\sqrt{\varepsilon\mu_0(\mu_1+\mu_2)}=\dfrac{\omega}{c}\sqrt{\varepsilon_{\mathrm{r}}\mu_{\mathrm{RCP}}},\\ k_{\mathrm{LCP}}=\omega\sqrt{\varepsilon\mu_0(\mu_1-\mu_2)}=\dfrac{\omega}{c}\sqrt{\varepsilon_{\mathrm{r}}\mu_{\mathrm{LCP}}},\end{cases} \tag{4}$$

where $\mu_{\mathrm{RCP}}, \mu_{\mathrm{LCP}}$ is the effective permeability, ε_{r} is the relative permittity of the film, and c is the speed of light. The eigenvalues are complex numbers which can be written as $k=k'-\mathrm{j}k''$.

First, let us consider the simplest case where the damping coefficient α is zero. With Eqs.(2)~(3), the effective permeability $\mu_{\mathrm{RCP}}, \mu_{\mathrm{LCP}}$ become:

$$\mu_{\mathrm{RCP}}=1+\frac{\omega_{\mathrm{m}}}{\omega_0-\omega},\ \mu_{\mathrm{LCP}}=1+\frac{\omega_{\mathrm{m}}}{\omega_0+\omega}\,. \tag{5}$$

Generally, the permittivity of metallic thin films in an external magnetic field should be represented by a tensor. Here for simplicity we assume that metallic magnetic films have low saturation magnetization M_{s} (e.g. NiFe), so that the external field required to align the magnetization perpendicular to the film plane (also the wave propagation direction) is small, resulting in much smaller electron cyclotron frequency compared to the ferromagnetic resonance frequency. The permittivity can thus be written in scalar form as (Zhang, 1998):

$$\varepsilon_{\mathrm{r}}=\varepsilon'-\mathrm{j}\varepsilon''=1-\frac{\omega_{\mathrm{p}}^2}{\omega(\omega-\mathrm{j}\eta)}, \tag{6}$$

where ω_{p} is the plasma frequency, and η the electron damping factor. Since ω_{p} is about 10^{16} Hz for metals, the imaginary part of the permittity ε'' is much larger than the real part ε' in microwave frequencies, and one can use $\varepsilon_{\mathrm{r}}\approx-\mathrm{j}\varepsilon''$. Therefore, the wave propagation constant for the RCP and LCP waves are:

$$\begin{cases}k_{\mathrm{RCP};\pm}=\pm\dfrac{\sqrt{2}}{2}(1-\mathrm{j})\omega\sqrt{\varepsilon''\mu_{\mathrm{RCP}}},\\ k_{\mathrm{LCP};\pm}=\pm\dfrac{\sqrt{2}}{2}(1-\mathrm{j})\omega\sqrt{\varepsilon''\mu_{\mathrm{LCP}}}.\end{cases} \tag{7}$$

There are two choices corresponding to positive and negative k. We assume that the direction of the energy flow is in the positive z-axis, implying the imaginary part of k must be always a negative number. Then the sign of the wave propagation constant is chosen to maintain the sign of its imaginary part negative (Chui and Hu, 2002). For an LCP wave, μ_{LCP} is always positive throughout the frequency range, so the only solution is $k_{\mathrm{LCP},+}$. For an RCP wave, $k_{\mathrm{RCP},+}$ is the solution except the range between ω_0 and $\omega_0+\omega_m$, where μ_{RCP} becomes negative. Therefore, the physically correct solution in the frequencies (ω_0, $\omega_0+\omega_m$) is $k_{RCP,-}=-(1+j)\omega\sqrt{\varepsilon''|\mu_{RCP}|}/\sqrt{2}$, which indicates the phase velocity in the $-z$ direction opposite to the direction of energy flow. Therefore the film will be an LHM for RCP waves in this frequency range.

LHM FOR RCP INCIDENT WAVES

For real metallic ferromagnetic films, the damping coefficient is non-zero. So the permeability for RCP waves is:

$$\mu_{RCP}=1+\frac{\bar{\omega}_m}{(1+j\alpha\bar{\omega})-\bar{\omega}}, \tag{8}$$

where $\bar{\omega}_m=\omega_m/\omega_0$, $\bar{\omega}=\omega/\omega_0$. The bias field H_0 must be larger than $4\pi M_s$ (demagnetization factor) in order to align the magnetization perpendicular to the film plane. Consequently, $\bar{\omega}_m$ must be equal to or greater than one. The dependence of $\mu_{RCP}=\mu'_{RCP}-j\mu''_{RCP}$ on α is shown in Fig.2, where the frequencies with negative μ'_{RCP} decreases with increasing α. The dependence reflects on the complex wave propagation constant k_{RCP}. What is more important is the sign change of the loss constant k''_{RCP}. As we have mentioned, when k''_{RCP} changes from positive to negative, the solution of the wave propagation constant k_{RCP} should change to $-k_{RCP}$ in order to keep the imaginary part of the wave propagation constant negative. Fig.3 shows the calculated real and imaginary part of the propagation constant $k_{RCP}=(\omega/c)\sqrt{\varepsilon_r\mu_{RCP}}$, where ε_r follows Eq.(6). It can be seen that there are sudden sign changes of the loss constant $k''_{RCP}=-\mathrm{Im}(k_{RCP})$ from positive to negative when the ferromagnetic damping coefficient α is below a critical value of α_c since μ_{RCP} becomes negative. At the same frequency, the phase constant $k'_{RCP}=\mathrm{Re}(k_{RCP})$ is zero. Just as described above, when k''_{RCP} changes from positive to negative, one should use $k_{RCP,-}$ in Eq.(8) instead of $k_{RCP,+}$. Combining these effects, the solutions are indicated as circles in Fig.3a in the whole frequency spectrum.

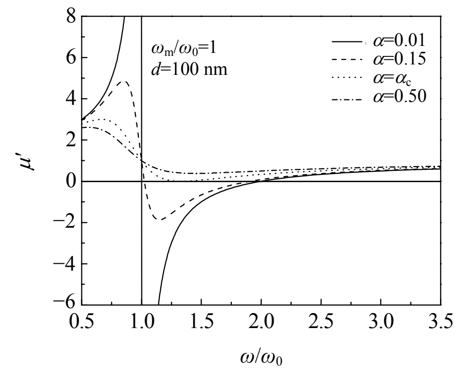
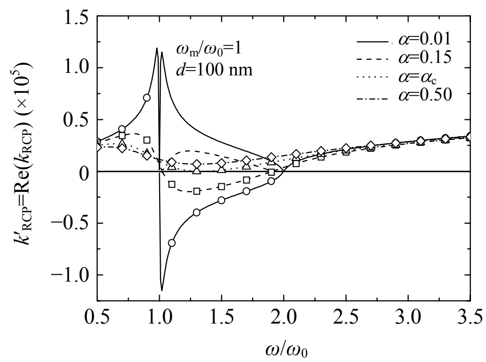
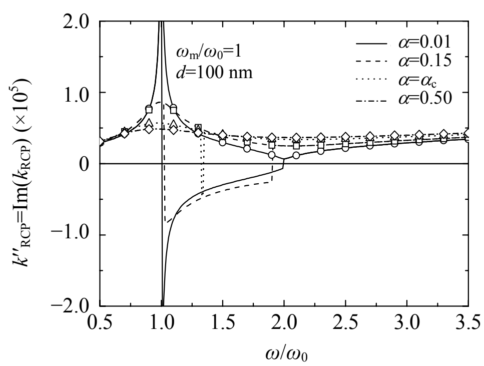


Fig.2 The frequency dependence of the real part of the permeability with different values of α



(a)



(b)

Fig.3 Complex wave number as a function of normalized frequency. (a) When $\alpha<\alpha_c$, k'_{RCP} has two zeros points between which LHM exist. The lines represent the positive solution for various values of α. These solutions become non-physical at frequency higher than the ferromagnetic resonance frequency $\omega=\omega_0$. The circles represent the true solution which indicates wave direction reversal. (b) The loss constant k''_{RCP}. The absolute value of $-k''_{RCP}$. should always be positive. The lines and circles have the same convention as in (a)

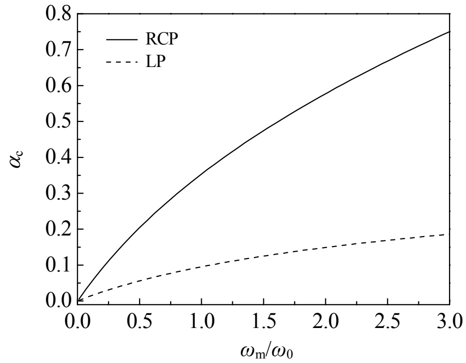


Fig.4 The critical magnetic damping coefficients for RCP and LP incident waves

It should be noticed that the frequency range for the negative $k'_{\rm RCP}$ decreases with increasing α. Over certain value, $k'_{\rm RCP}$ is always positive. The critical value α_c is determined by setting $\mu'_{\rm RCP}$ and its derivative zero, and has the form of

$$\alpha_{\rm c}=\frac{\bar{\omega}_{\rm m}}{2}\sqrt{1/(1+\bar{\omega}_{\rm m})}\ . \tag{9}$$

It is clear that the existence of LHM depends on the polarization of the incident wave, the damping coefficient α, as well as the ferromagnetic characteristic frequency ω_m.

LHM FOR LINEARLY POLARIZED WAVES

Fully magnetized magnetic film is a gyromagnetic medium, in which one can observe the Faraday effect of an LP wave propagating along the magnetization direction. An LP wave can be decomposed into RCP and LCP waves (Pozar, 1998). If the incident wave at z=0 is $\boldsymbol{E}(0)=\hat{x}\boldsymbol{E}$, then

$$\boldsymbol{E}(0)=\boldsymbol{E}^{\rm RCP}(0)+\boldsymbol{E}^{\rm LCP}(0), \tag{10}$$

where $\boldsymbol{E}^{\rm RCP}(0)=(\hat{x}-{\rm j}\hat{y})\boldsymbol{E}/2$, $\boldsymbol{E}^{\rm LCP}(0)=(\hat{x}+{\rm j}\hat{y})\boldsymbol{E}/2$. Since the RCP and LCP waves have different wave propagation constants, the total field of the wave at distances z is

$$\boldsymbol{E}(z)=[\hat{x}E_x(z)+\hat{y}E_y(z)]{\rm e}^{-{\rm j}(k_{\rm RCP}+k_{\rm LCP})z/2}, \tag{11}$$

where $E_x(z)=\boldsymbol{E}\cos[(k_{\rm RCP}-k_{\rm LCP})z/2]$ and $E_y(z)=-\boldsymbol{E}\cdot\sin[(k_{\rm RCP}-k_{\rm LCP})z/2]$. For thin films with thickness much smaller than the wavelength, Eq.(11) indicates that the LP wave propagates in an effective medium with propagation constant $k_{\rm eff}=(k_{\rm LCP}+k_{\rm RCP})/2$. This in turn gives an effective relative permeability of the medium $\mu_{\rm r}^{\rm eff}$, which, if the permittivity of the medium is constant, can be expressed as

$$\mu_{\rm r}^{\rm eff}=\frac{1}{4}\left(\sqrt{\mu_{\rm LCP}}+\sqrt{\mu_{\rm RCP}}\right)^2. \tag{12}$$

It is evident that LHMs will appear at a much narrower frequency region for LP incident waves since the effective permeability is a kind of average of $\mu_{\rm RCP}$ and $\mu_{\rm LCP}$, where only $\mu_{\rm RCP}$ exhibits negative value at certain frequencies. In addition, the critical damping coefficient α_c also becomes smaller for LP waves as shown in Fig.4, where α_c for RCP waves is calculated from Eq.(12), and α_c for LP waves is calculated numerically. Consequently, for some materials, one may observe LHM for RCP incident waves but not for LP incident waves.

CONCLUSION

In conclusion, theoretical investigation of the possibility of realizing LHM in metallic magnetic thin films revealed that these magnetic films may exhibit left-handed behaviors near the vicinity of the ferromagnetic resonant frequency ω_0. The frequency range where LHM exists depends on the polarizations of the incident waves, magnetic damping coefficient α, and the ferromagnetic characteristic frequency ω_m of the film. For RCP incidence, the upper frequency limit extends to $\omega_0+\omega_m$ for a small α, but decreases with increasing α. In another words, the bandwidth for observing left-handed properties is inversely proportional to the damping coefficient. There also exists a critical value of α_c, above which the left-handed properties disappear completely. In addition to RCP EM waves, left-handed properties may also exist for LP waves. But the bandwidth is much narrower and the critical value of α_c is much smaller compared to the case with RCP EM waves.

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