



Numerical analysis on car-following traffic flow models with delay time^{*}

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Abstract: Effects of the speed relaxation time on the optimal velocity car-following model (OVM) with delay time due to driver reaction time proposed by Bando *et al.* (1995) were studied by numerical methods. Results showed that the OVM including the delay is not physically sensitive to the speed relaxation times. A modified car-following model is proposed to overcome the deficiency. Analyses of the linear stability of the modified model were conducted. It is shown that coexisting flows appear if the initial homogeneous headway of the traffic flow is between critical values. In addition, phase transitions occur on varying the initially homogeneous headway.

Key words: Car-following models, Delay time, Relaxation time, Phase transitions

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INTRODUCTION

For the past few decades, many researchers have made studies of modelling driver behavior to reproduce the observed features of traffic flow. Recently Kerner and Rehborn (1997) observed that there are three distinct dynamic phases on highways: free traffic flow, coexisting traffic flow and traffic jams. The occurrence of traffic jams without obvious reasons had been explained in terms of the conventional phase transition (Nagatani, 1998). Various traffic flow models, such as car-following models and hydrodynamic models had been studied analytically using the linear stability theory and the nonlinear analysis method to explain these phenomena (Jiang and Wu, 2003). The car-following traffic flow models study movements of a vehicle and the interactions between vehicle pairs in a queuing of traffic flow. In these models, it is easy to consider the local dynamics of cars.

Generally the car-following models are formulated based on the assumption that each driver of a car responds to a stimulus coming from his front car in some special fashion. Usually the response is described in the form of acceleration of the cars. The other factors, for example the stimuli, are described by a function of the speed difference between two consecutive cars, headway (the distance between front bumper of the car ahead and front bumper of the following car), and so on.

The optimal velocity car-following model (OVM) proposed by Bando *et al.* (1995) is

$$\ddot{x}_n(t) = \alpha(V(\Delta x_n(t)) - \dot{x}_n(t)), \quad (1)$$

where $x_n(t)$ denotes the position of the n th vehicle at time t , $n=1, \dots, N$. N is the total number of cars. $\Delta x_n(t) = x_{n-1}(t) - x_n(t)$ is the headway of the n th car, α is the reciprocal of the speed relaxation time T_r . Generally, the values of T_r are different for different traffic flow (Treiber *et al.*, 2000). $V(\Delta x_n(t))$ represents the optimal velocity function of the n th car determined by its headway. The optimal velocity function given by

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Bando *et al.*(1998) increases monotonically but has an upper bound ($v_{\max}$):

$$V(\Delta x_n(t)) = 16.8[\tanh(0.0860(\Delta x_n(t) - 25)) + 0.913]. \quad (2)$$

In car-following models, the introduction of “delay” is necessary and essential to understanding traffic dynamics. According to Bando *et al.*(1998), there are two kinds of delay time: “explicit delay time” and “delay time of car motion”. In this paper delay time refers to the delay due to driver reaction time.

The OVM with delay time (denoted OVMDT) is described as:

$$\ddot{x}_n(t + \tau) = \alpha(V(\Delta x_n(t)) - \dot{x}_n(t)), \quad (3)$$

where τ denotes the delay time. The OVMDT means that the driver acts at time $t + \tau$ by accelerating according to the stimuli (difference of the car speed and the optimal speed at time t) on the right hand side of Eq.(3). In Section 2, we will point out that the model (Eq.(3)) is not physically sensitive to the relaxation time obtained by the method of numerical simulations. Furthermore, we propose the modified model in this section. In Section 3, the linear stability of the new model is discussed. Numerical simulations to study the phase transitions are carried out in Section 4. A summary is given in Section 5.

NUMERICAL SIMULATIONS AND MODIFICATION OF THE OVMDT

Assuming that a caravan of cars is moving along a single lane without any exit and entry for other cars, that there is no bypassing of cars, and that the cars have uniform car characteristics and are numbered 1 to N from the first car downstream. Here we consider $N=100$. It is supposed that the speed of car 1 can be changed controllably.

We integrate Eq.(3) with a simple Euler scheme using a time discretization of $\Delta t=0.1$ s, and translate the vehicles in each step according to

$$\begin{aligned} \dot{x}_n(t + \Delta t) &= \dot{x}_n(t) + \ddot{x}_n(t)\Delta t, \\ x_n(t + \Delta t) &= x_n(t) + \dot{x}_n(t)\Delta t + \frac{1}{2}\ddot{x}_n(t)(\Delta t)^2. \end{aligned} \quad (4)$$

Initially the headway of the traffic flow is 25 m and the speed is 15.34 m/s while the leading car travels at a steady 14 m/s. In OVM, Bando *et al.*(1998) set τ to be the order of 0.1 s. Therefore we let $\tau=0.1$ s, 0.2 s, 0.3 s and 0.4 s respectively. For the sake of comparison, let $\alpha=0.5 \text{ s}^{-1}$, 1 s^{-1} and 2 s^{-1} , that is, $T_r=2$ s, 1 s and 0.5 s. Results showed that for smaller delay time $\tau=0.1$ s and 0.2 s, only $T_r=0.5$ s can avoid a collision of the traffic flow; while for $\tau=0.3$ s and 0.4 s, collision occurs for all three relaxation times. An example is shown in Fig.1, which is the speed profiles of the 2nd, 8th cars in Fig. 1a and those of the 9th, 10th cars in Fig.1b for $T_r=1$ s, $\tau=0.3$ s. It verifies that only the first 8 cars avoid colliding with each other. The 9th and following cars experience the negative speeds that are impossible in the real traffic. We can also see from Fig.1 that the speeds of most cars in the traffic flow oscillate severely. The above discussions indicate that the OVMDT is unrealistically sensitive to the relaxation time.

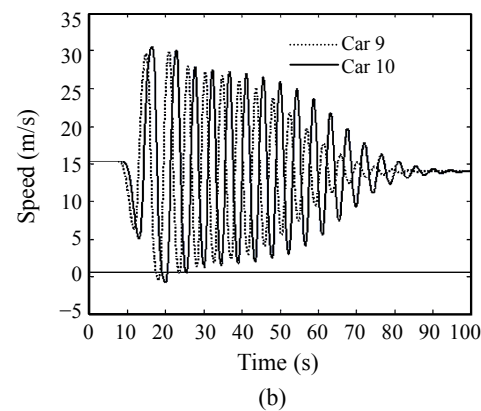
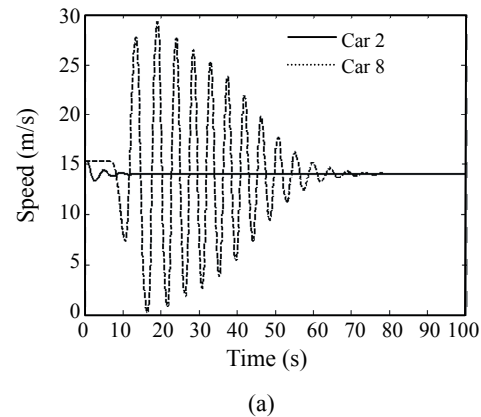


Fig.1 Speed profiles of the 2nd, 8th cars (a) and those of the 9th, 10th cars (b) with $T_r=1$ s, $\tau=0.3$ s for OVMDT

In fact, similar behaviors hold for $\tau=0.1$ s or 0.2 s with both $T_r=2$ s and 1 s and for $\tau=0.3$ s or 0.4 s with all three T_r in the simulations.

Recently, several researchers (Helbing and Tilch, 1998; Davis, 2003) proposed that the effect of the speed difference between two consecutive cars should be considered in the OVM and OVMDT. We find that the deficiency of the OVMDT aforementioned can be modified if we add an adjusting term as follows:

$$\ddot{x}_n(t) = \alpha(V(\Delta x_n(t-\tau)) - \dot{x}_n(t-\tau)) + \beta G(\Delta \dot{x}_n(t-\tau_1), \Delta x_n(t-\tau_1)), \quad (5)$$

where $\Delta \dot{x}_n(t) = \dot{x}_{n-1}(t) - \dot{x}_n(t)$ is the speed difference between the $(n-1)$ th and n th cars. $\beta \geq 0$ is a constant that denotes the force of the adjusting term. Further study showed that β is related to τ (to be discussed in future work). G is a function of speed difference and headway that is used to adjust the car acceleration at time t . τ_1 denotes the time needed to adjust the car acceleration. In this paper we let $\tau_1=0.1\tau$. Eq.(5) means that the driver acts at time t by accelerating according to both the stimuli at time $t-\tau$ and the adjustment at time $t-\tau_1$. We can choose the function G according to the following conditions:

(1) The sign of function $G(x, y)$ is consistent with that of x .

(2) $G(x, y)$ is a monotonously increasing function of x and y respectively.

In fact, the function G in Eq.(5) indicates that if the speed of the front car is larger than that of the following car at time $t-\tau_1$, G is positive to achieve larger acceleration or smaller deceleration at time t . Moreover, for larger headway at time $t-\tau_1$, the G value should be higher. On the contrary, if the speed of the front car is smaller than that of the following car at time $t-\tau_1$, G is negative to achieve smaller acceleration or larger deceleration at time t . Also, for larger headway at time $t-\tau_1$, the G value should be higher.

According to these findings, we choose G as:

$$G(\Delta \dot{x}_n(t-\tau_1), \Delta x_n(t-\tau_1)) = \begin{cases} \Delta \dot{x}_n(t-\tau_1)(1 + (\tanh(\Delta x_n(t-\tau_1) - 25))^3), & \Delta \dot{x}_n(t-\tau_1) \geq 0, \\ \Delta \dot{x}_n(t-\tau_1)(1 - (\tanh(\Delta x_n(t-\tau_1) - 25))^3), & \Delta \dot{x}_n(t-\tau_1) < 0. \end{cases} \quad (6)$$

In the following, we will carry out the same simulation for the new model (Eq.(5)) with Eq.(6). Let $\tau=0.5$ s, 1.1 s (Treiber *et al.*, 2004) and $T_r=10$ s, 40 s (Treiber *et al.*, 2000), which are closer to the real traffic. Accordingly, we use the update time step $\Delta t=0.05$ s and 0.11 s in the simulations and let τ and τ_1 be a multiple of Δt .

Results showed that for both τ and corresponding T_r (usually the larger τ corresponds to a larger T_r), the collisions of cars are avoided and the oscillations of the car speed are weakened. This can be seen clearly in Fig.2. The speed profiles of the 10th, 100th and 500th cars with $\tau=0.5$ s, $\beta=1.5$ s⁻¹ and $T_r=10$ s in Fig.2a and $\tau=1.1$ s, $\beta=3.5$ s⁻¹ and $T_r=40$ s in Fig.2b are shown. In Fig.2b, the car speed is in an acceptable range even for a very enormous number $N=500$.

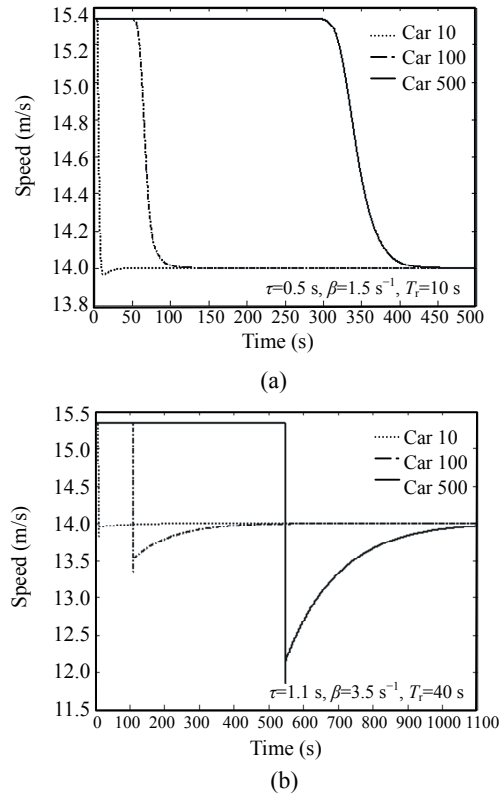


Fig.2 Speed profile of the 10th, 100th and 500th cars with $\tau=0.5$ s, $\beta=1.5$ s⁻¹, and $T_r=10$ s in (a) and $\tau=1.1$ s, $\beta=3.5$ s⁻¹, and $T_r=40$ s in (b) for the modified OVMDT (Eq.(5), denoted “MOVMDT”)

Fig.3 compares the OVMDT and the MOVMDT. The speed profiles of the 100th car with $\tau=1$ s, $\beta=3.5$ s⁻¹ and $T_r=10$ s for these two models are shown. We can see that the MOVMDT eliminates the serious speed overshoot that occurs in the OVMDT.

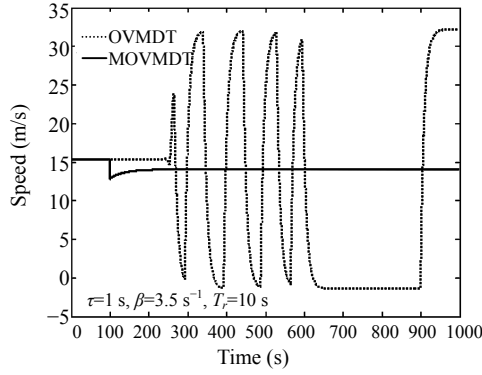


Fig.3 Comparison of speed of the 100th car with $\tau=1$ s, $\beta=3.5$ s⁻¹, and $T_r=10$ s for the OVMDT and the MOVMDT

ANALYSIS OF THE LINEAR STABILITY OF THE MOVMDT

In this section we will discuss the linear stability of the MOVMDT. Let $x_n^0(t) = V(h)t - hn$, $n=1, 2, \dots$, N be a uniform solution of Eq.(5), $h=L/N$ is the homogeneous headway, L is the length of the road. $y_n(t)=e^{ikn+zt}$ is a small disturbance to this equilibrium state, i.e.,

$$x_n(t) = x_n^0(t) + y_n(t). \quad (7)$$

Substituting Eq.(7) into Eq.(5) and neglecting higher order terms of y_n , we have:

$$(z\tau)^2 e^{z\tau} + (a\tau - \beta F \tau e^{0.9z\tau} (e^{-ik-1})) (z\tau) - a\tau^2 V'(h) (e^{-ik} - 1) = 0, \quad (8)$$

$$\text{here } F = \begin{cases} 1 + (\tanh(0.0860(h-25)))^3, & h \leq 25, \\ 1 - (\tanh(0.0860(h-25)))^3, & h > 25. \end{cases}$$

The expansion of $z\tau$ about $ik \sim 0$ is: $(z\tau) = (z\tau)_1 ik + (z\tau)_2 (ik)^2 + \dots$, here

$$\begin{aligned} (z\tau)_1 &= -V'(h)\tau, \\ (z\tau)_2 &= \frac{\alpha V'(h)\tau/2 - (V'(h))^2 \tau + \beta F V'(h)\tau}{\alpha}. \end{aligned} \quad (9)$$

It is known that the equilibrium solutions are linear stable when $(z\tau)_2 > 0$, i.e.,

$$\alpha > 2(V'(h) - \beta F), \quad (10)$$

or

$$1/T_r > 2(V'(h) - \beta F). \quad (11)$$

This linear stable condition (Eq.(10)) indicates that when a small disturbance is added into a homogeneous traffic, the instability of the initial homogeneous flow occurs in some interval of h that is decided by α and β .

The stability condition also presents the jamming transition curve among the free traffic, the co-existing traffic and the traffic jams. Fig.4 shows the phase diagrams for this curve with $\beta=1$ s⁻¹ and $\beta=1.4$ s⁻¹. The point A (B) means that for $\beta=1$ s⁻¹ ($\beta=1.4$ s⁻¹), when the value of $1/T_r$ is larger than that of A (B), the traffic flow is always at the steady state; but that on the contrary, below these values, the traffic flow is unstable and will evolve in time into an inhomogeneous coexisting phase of both jammed and free states. Under these situations, phase transitions occur if the initial homogeneous headway varies. This will be discussed in the next section.

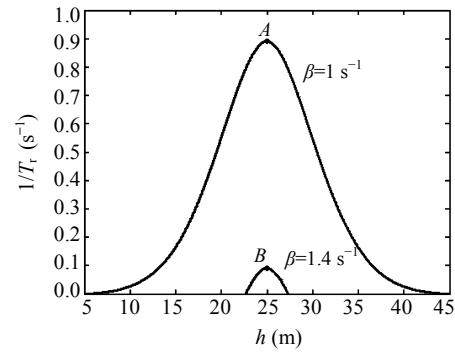


Fig.4 The phase diagrams in the space of $1/T_r$ against h with $\beta=1$ s⁻¹ and $\beta=1.4$ s⁻¹

PHASE TRANSITIONS OF THE MOVMDT

Phase transitions of the MOVMDT will be studied with numerical simulations in this section. Considering the periodic boundary condition, the initial condition is given as follows:

$$x_1(0)=2, x_n(0)=-h(n-1), n=2, \dots, N,$$

and

$$\dot{x}_n(0) = 0, \quad \text{for } n=1, \dots, N, \quad (12)$$

with $N=100$, $h=L/N$ and L is changed to get various h . In the simulations we let $\beta=1$ s⁻¹ and $\beta=1.4$ s⁻¹ respectively. Generally traffic instabilities increase with the reaction times of the drivers (This will be seen in

Fig.7). For simplification we let $\tau=1$ s and the updated time step $\Delta t=0.1$ s.

As Fig.4 shows, if the value of $1/T_r$ is larger than $A=0.89 \text{ s}^{-1}$ ($B=0.09 \text{ s}^{-1}$), the traffic flow is homogeneous and there is no jamming transition occurs. In these situations, for any initial h , the plots of speed against headway for all cars are consistent with the optimal velocity function, as shown in Fig.5.

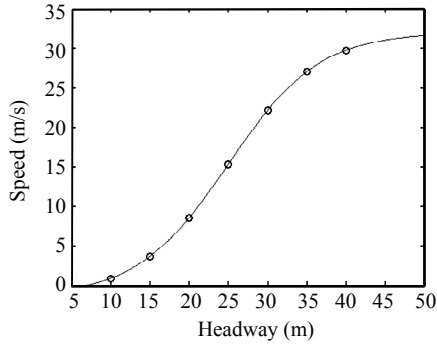


Fig.5 Plots of speed against headway for all cars at time step 10000 with $\tau=1$ s, $\beta=1.4 \text{ s}^{-1}$, and $T_r=10$ s. A single circle represents the homogeneous traffic flow for an initial h and the line is the optimal velocity function (Eq.(2))

If $1/T_r < 0.89 \text{ s}^{-1}$ ($1/T_r < 0.09 \text{ s}^{-1}$) for $\beta=1 \text{ s}^{-1}$ ($\beta=1.4 \text{ s}^{-1}$) and the initial homogeneous headway h is between two critical values depending on both β and T_r , the traffic flow evolves in time to the inhomogeneous flow with kink waves and only two distinct headways stay. The high-headway region is associated with the free traffic states and the low-headway region is associated with the jammed traffic states. Therefore, the coexisting phase of both free and jammed states appears in the unstable region of Fig.4. Phase transitions occur among the free traffic, the coexisting traffic and the traffic jam on varying the initial homogeneous headway h as shown in Fig.6. The plots of the headway Δx_n for all the cars n with $h=10$ m, 25 m, 40 m for $\tau=1$ s, $\beta=1 \text{ s}^{-1}$ and $T_r=10$ s at time step 10000 are shown in Fig.6a and the corresponding plots of speed $\dot{x}_n$ against the headway Δx_n for all cars are shown in Fig.6b where the homogeneous traffic is represented by a single circle and the inhomogeneous traffic is represented by a loop.

Fig.7 shows the plots of the headway Δx_n against the car n for $h=25$ m for $\tau=0.6$ s, $\Delta t=0.06$ s and $\tau=1$ s, $\Delta t=0.1$ s with $\beta=1 \text{ s}^{-1}$ and $T_r=10$ s at time step 10000. From this figure we can see that for given values of β

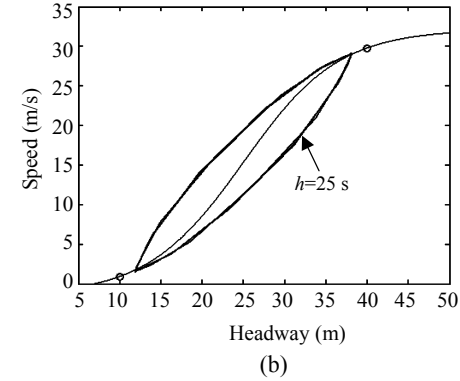
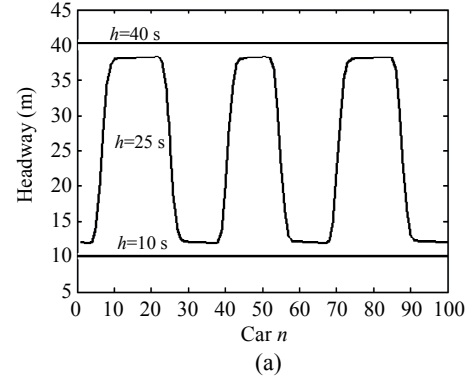


Fig.6 (a) Plots of the headway Δx_n for all the cars n for $h=10$ m, 25 m, 40 m with $\tau=1$ s, $\beta=1 \text{ s}^{-1}$ and $T_r=10$ s at time step 10000; (b) The corresponding plots of speed $\dot{x}_n$ against the headway Δx_n for all cars. In (b), the homogeneous traffic is represented by a single circle and the inhomogeneous traffic is represented by a loop. The lines in (b) are the optimal velocity function (Eq.(2))

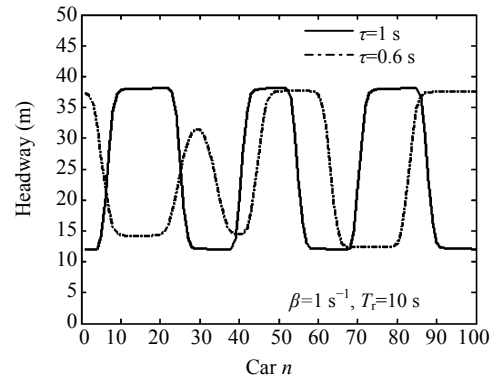


Fig.7 Plots of the headway Δx_n against the car n for $\tau=0.6$ s, $\Delta t=0.06$ s and $\tau=1$ s, $\Delta t=0.1$ s with $\beta=1 \text{ s}^{-1}$, $T_r=10$ s at time step 10000. $h=25$ m

and T_r , the amplitude of the oscillating inhomogeneous headway with larger τ is greater than that with smaller τ , which means the traffic instabilities increase with τ .

SUMMARY

In this work, we study the linear stability and phase transitions of a new car-following traffic flow model considering the delay time due to driver reaction time by numerical methods with appropriate relaxation times. If the initial homogeneous headway of the traffic flow is between two critical values depending on the model parameter β and the relaxation time T_r , the homogeneous traffic flow is unstable and will evolve in time into an inhomogeneous coexisting phase of both jammed and free states as Fig.6 shows. Under these situations, phase transitions among the free traffic, coexisting traffic and the traffic jams occur when the initial headway varies.

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