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The single-machine scheduling problems with deteriorating jobs and learning effect

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Abstract: In this paper we consider a single-machine scheduling model with deteriorating jobs and simultaneous learning, and we introduce polynomial solutions for single machine makespan minimization, total flow times minimization and maximum lateness minimization corresponding to the first and second special cases of our model under some agreeable conditions. However, corresponding to the third special case of our model, we show that the optimal schedules may be different from those of the classical version for the above objective functions.

Key words: Scheduling, Single-machine, Learning effect, Deteriorating jobs

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INTRODUCTION

Scheduling problems have been studied widely in recent years with many results. In the classical scheduling problems, the processing times of jobs are constant, although, there are many cases where the processing times of jobs are not constant but increasing or decreasing over time so that the processing times of jobs are no longer independent.

For example, a facility is deteriorating because of long time use so that its processing capacity decreases with time, and a job processed later requires longer processing time than that of a job processed earlier. We call such a phenomenon the deterioration of job's processing time. Mosheiov (1992; 1994) was the first one who assumed that jobs' processing times were a linear increasing function of their starting times for processing on a single machine. Namely, for job J_j, $p_j=b_jt$, where $b_j>0$ is its deteriorating rate and $t\geq t_0>0$ is its starting time to be processed. The scheduling measures in his papers are minimization of the makespan, the weighted sum of completion times, the maximum lateness, the total tardiness, the number of tardy jobs, etc. All the problems he studied can be solved in polynomial times. Alidaee and Womer (1999) surveyed the rapidly growing literature on single machine scheduling models with jobs' processing times depending on their starting times. Cheng *et al*.(2004) gave a detailed review of deteriorating jobs' scheduling problems. The reader can find many results in this review.

Biskup (1999) introduced a learning effect into a setting. He assumed a learning process in which the processing time of a given item decreased as the number of similar items processed before it increased, he considered the single machine scheduling problems aimed at minimizing the total flow time, minimizing the weighted sum of completion time deviations from a common due date and minimizing the sum of jobs' completion times. He introduced polynomial time solutions for all these problems. Mosheiov (2001) studied a number of classical objectives, provided polynomial time solutions to some of them and showed for some other cases that the classical solutions did not hold when a learning effect was assumed. Mosheiov and Sidney (2003) considered the case of job-dependent learning effect where the learning of some jobs was faster than that of others in the production process. Bachman and Janiak (2004) considered some single machine scheduling problems where a job's processing time was defined by a decreasing or increasing function which depended on the position of this job in the sequence.

Although deteriorating jobs and learning effect have been well discussed, it is rarely considered simultaneously. It is an interesting issue since it is more realistic, though it becomes complicated. Lee (2004) considered single machine scheduling problems with deteriorating jobs and learning effect simultaneously, i.e. $p_{jr}=(p_0+b_jt)r^{\alpha}$, where p_0 was the common basic processing time, b_j was job J_j's deteriorating rate and α was the learning index. The objective was to minimize the makespan and the total completion time. He introduced the polynomial solutions for both of them. This paper considers also such a single machine model in which the effects of deterioration and learning exist at the same time. The general mathematical model is $p_{jr}=(a_j+b_jt)\alpha^{r-1}$, where $0<\alpha\leq 1$ is the learning effect constant, a_j and b_j denotes the normal processing time and the deteriorating rate, respectively, and t is the starting time of job J_j. First of all, we consider a special case where the deteriorating rates are the same for all jobs. We show that the makespan and the total completion time minimization problems are polynomial solvable. In addition, we prove that the total weighted completion time and the maximum lateness have polynomially time solvable optimal solutions if the weights or the due dates of the jobs are agreeable to their normal processing times. Then, we discuss another special case where there is no normal processing time. Under this model, we show that the makespan and the total completion time minimization problems remain polynomially solvable. The maximum lateness has a polynomial optimal solution if the due dates are agreeable to the deteriorating rate. Finally, we provide four examples to demonstrate that the SDR rule does not give the optimal solution for the makespan and the total completion time minimization problems, the WSDR rule does not give the optimal solution for the total weighted completion time minimization problem and the EDD rule does not give the optimal solution for the maximum lateness minimization problem if the jobs have a common processing time.

The remaining part of this paper is organized as follows: the next section contains the precise formulation of the considered problems. In the third section, we provide polynomial solutions for some special cases and show that the optimal schedule may be very different from that of the classical version for some other special cases. In the last section, remarks are given.

PROBLEM FORMULATION

In this paper, we focus on studying the effects of deteriorating and learning simultaneously. The model is described as follows.

There are n jobs to be processed on a single machine. All jobs are available for processing at $t=t_0>0$. The machine can handle one job at a time and preemptions are not allowed. Each job J_j has a deterioration rate b_j and a normal processing time a_j. If job J_j is scheduled in the rth position in a sequence, then its actual processing time is

$$p_{jr}=(a_j+b_jt)\alpha^{r-1}, \quad (1)$$

where $0<\alpha\leq 1$ is the learning index.

In the following, we examine several classical single-machine scheduling problems corresponding to three different special cases of our model.

THREE SPECIAL CASES OF OUR MODEL

The first special case: $b_j \equiv b$

Theorem 1 For the problem $1|p_{jr}=(a_j+bt)\alpha^{r-1}|C_{\max}$, an optimal schedule can be obtained by sequencing the jobs in a non-decreasing order of $\{a_j\}$ (SPT rule).

Proof Suppose $a_i<a_k$. Let σ and σ' be two schedules where the different between σ and σ' is only a pairwise interchange of two adjacent jobs J_i and J_k, that is, $\sigma=[S_1,J_i,J_k,S_2]$ and $\sigma'=[S_1,J_k,J_i,S_2]$, where S_1,S_2 are partial sequences. Furthermore, we assume that there are $(r-1)$ jobs in S_1. Thus, J_i and J_k are the rth and $(r+1)$th jobs, respectively, in σ. Let t be the completion time of the last job in S_1. Then, the actual processing time of job J_i is $p_{ir}=(a_i+bt)\alpha^{r-1}$ in σ and its completion time is $C_i(\sigma)=t+p_{ir}=a_i\alpha^{r-1}+(1+b\alpha^{r-1})t$, while, the actual processing time of job J_k is $p_{kr+1}=(a_k+bC_i(\sigma))\alpha^{r}$ in σ and its completion time is

$$C_k(\sigma)=C_i(\sigma)+p_{kr+1}$$
$$=a_k\alpha^r+a_i\alpha^{r-1}(1+b\alpha^r)+t(1+b\alpha^{r-1})(1+b\alpha^r). \quad (2)$$

Similarly, it is easy to derive that the completion times of jobs J_k and J_i in σ' are

$$C_k(\sigma')=t+p_{ir}=a_k\alpha^{r-1}+(1+b\alpha^{r-1})t,$$
$$C_i(\sigma')=a_i\alpha^r+a_k\alpha^{r-1}(1+b\alpha^r)+t(1+b\alpha^{r-1})(1+b\alpha^r), \quad (3)$$

respectively.

Based on $a_i<a_k$, $0<\alpha\leq 1$, Eqs.(2) and (3), we have $C_k(\sigma)-C_i(\sigma')=(a_k-a_i)(\alpha-1-b\alpha^r)\alpha^{r-1}<0$.

Thus, we know that the schedule σ dominates σ' and complete the proof.

For the problem $1|p_{jr}=(a_j+bt)\alpha^{r-1}|\Sigma C_j$, by the standard pairwise interchange argument as in Theorem 1, we can show that the optimal schedule for it can be obtained by the SPT rule. Thus, we have the following theorem.

Theorem 2 For the problem $1|p_{jr}=(a_j+bt)\alpha^{r-1}|\Sigma C_j$, an optimal schedule can be obtained by sequencing the jobs in a non-decreasing order of $\{a_j\}$ (SPT rule).

In the following, we consider the objective functions of minimizing the weighted total completion times and maximum lateness. We show that these two objective functions can be minimized in polynomial time under some agreeable conditions.

Theorem 3 For the problem $1|p_{jr}=(a_j+bt)\alpha^{r-1}|\Sigma w_jC_j$, if jobs have agreeable weighted normal processing times, i.e. $a_i\leq a_k$ implies $w_i\geq w_k$, for all jobs J_i and J_k, then an optimal schedule can be obtained by sequencing the jobs in a non-decreasing order of $\{a_j/w_j\}$ (WSPT rule).

Proof Consider a schedule σ, suppose it is not a WSPT sequence and is optimal. In this schedule, there must be at least two adjacent jobs, say job J_j followed by job J_k, such that $a_j/w_j>a_k/w_k$.

Assume job J_j is scheduled in the rth position and start to be processed at time t. Perform an adjacent pairwise interchange of job J_j and job J_k, denote the new schedule as σ', then, job J_k is scheduled in the rth position in σ'. The completion times of all the jobs processed before J_k in σ' are not affected by this interchange, and we have, under σ,

$$C_j(\sigma)=t+(a_j+bt)\alpha^{r-1}=a_j\alpha^{r-1}+t(1+b\alpha^{r-1}),$$
$$C_k(\sigma)=C_j(\sigma)+(a_k+bC_j(\sigma))\alpha^r$$
$$=a_k\alpha^r+a_j\alpha^{r-1}(1+b\alpha^r)+t(1+b\alpha^{r-1})(1+b\alpha^r).$$

Similarly, for σ', we have

$$C_k(\sigma')=a_k\alpha^{r-1}+t(1+b\alpha^{r-1}),$$
$$C_j(\sigma')=a_j\alpha^r+a_k\alpha^{r-1}(1+b\alpha^r)+t(1+b\alpha^{r-1})(1+b\alpha^r).$$

So, we have

$$C_k(\sigma)-C_j(\sigma')=(a_k-a_j)(\alpha-1-b\alpha^r)\alpha^{r-1}>0. \quad (4)$$

At the same time, the difference between the total weighted completion time of the first $(r+1)$ jobs in σ and σ' is

$$(a_j-a_k)\alpha^{r-1}(1-\alpha)(w_j+w_k)+(a_jw_k-a_kw_j)\alpha^r(1+b\alpha^{r-1})$$
$$+tb\alpha^r(1+b\alpha^{r-1})(w_k-w_j).$$

Since $a_j\geq a_k$, $w_i\geq w_k$ and $a_j/w_j>a_k/w_k$, then we have

$$\Sigma w_jC_j(\sigma)>\Sigma w_jC_j(\sigma'). \quad (5)$$

From Eqs.(4) and (5), we have $\Sigma w_jC_j(\sigma)>\Sigma w_jC_j(\sigma')$.

It follows that the weighted sum of completion times under σ' is strictly less than that under σ. This contradicts the optimality of σ and proves the theorem.

Theorem 4 For the problem $1|p_{jr}=(a_j+bt)\alpha^{r-1}|L_{\max}$, if jobs have agreeable due dates and normal processing times, i.e. $a_j\leq a_k$ implies $d_j\leq d_k$, then an optimal schedule can be obtained by sequencing the jobs in a non-decreasing order of $\{d_j\}$ (EDD rule).

Proof Suppose there exists an optimal schedule σ, but it is not an EDD sequence. In this schedule, there must be at least two adjacent jobs, say J_j and J_k, such that $d_j>d_k$ (it implies $a_j>a_k$). Suppose also that job J_j and J_k are the rth and $(r+1)$th jobs in σ. Schedule σ' is obtained from σ by interchanging jobs in the rth and $(r+1)$th position of σ. Then, for σ, we have

$$L_j(\sigma)=a_j\alpha^{r-1}+t(1+b\alpha^{r-1})-d_j\,,$$
$$L_k(\sigma)=a_k\alpha^r+a_j\alpha^{r-1}(1+b\alpha^r)+t(1+b\alpha^{r-1})(1+b\alpha^r)-d_k\,.$$

Similarly, for σ', we have

$$L_k(\sigma')=a_k\alpha^{r-1}+t(1+b\alpha^{r-1})-d_k\,,$$
$$L_j(\sigma')=a_j\alpha^r+a_k\alpha^{r-1}(1+b\alpha^r)+t(1+b\alpha^{r-1})(1+b\alpha^r)-d_j\,.$$

Since $d_j>d_k$ and $a_j>a_k$, then we have

$$\max\{L_j(\sigma'), L_k(\sigma')\}\le\max\{L_j(\sigma), L_k(\sigma)\}.$$

Hence, interchanging the jobs J_j and J_k will not increase the value of $L_{\max}$. So we have shown that EDD sequence is an optimal schedule.

The second special case: a_j≡0

Theorem 5 For the problem $1|p_{jr}=b_jt\alpha^{r-1}|C_{\max}$, an optimal schedule can be obtained by sequencing the jobs in a non-decreasing order of $\{b_j\}$ (SDR rule).

Proof Suppose $b_i\le b_k$. Let σ and σ' be two schedules where the different between σ and σ' is only a pairwise interchange of two adjacent jobs J_i and J_k, that is, $\sigma=[S_1,J_i,J_k,S_2]$ and $\sigma'=[S_1,J_k,J_i,S_2]$, where S_1, S_2 are partial sequences. Furthermore, we assume that there are $(r-1)$ jobs in S_1. Thus, J_i and J_k are the rth and $(r+1)$th jobs in σ, respectively. Let t be the completion time of the last job in S_1. Then, the actual processing time of job J_i is $p_{ir}=b_it\alpha^{r-1}$ and its completion time is

$$C_i(\sigma)=t+p_{ir}=(1+b_j\alpha^{r-1})t.$$

While, the actual processing time of job J_k is $p_{kr+1}=b_kC_i(\sigma)\alpha^{r-1}$ and its completion time is

$$C_k(\sigma)=C_i(\sigma)+p_{kr+1}=t(1+b_i\alpha^{r-1})(1+b_k\alpha^r). \quad (6)$$

Similarly, it is easy to derive that the completion times of J_k and J_i in σ' are

$$C_k(\sigma')=t+p_{ir}=(1+b_k\alpha^{r-1})t,$$
$$C_i(\sigma')=t(1+b_k\alpha^{r-1})(1+b_i\alpha^r), \quad (7)$$

respectively.

Based on Eqs.(6) and (7), we have

$$C_k(\sigma)-C_i(\sigma')=t(b_i-b_k)(\alpha^{r-1}-\alpha^r)<0.$$

Thus, we know that the schedule σ dominates σ' and complete the proof.

Theorem 6 For the problem $1|p_{jr}=b_jt\alpha^{r-1}|\Sigma C_j$, an optimal schedule can be obtained by sequencing the jobs in a non-decreasing order of $\{b_j\}$ (SDR rule).

Proof By the standard pairwise interchange argument as in Theorem 5, we can prove it.

But for the problem $1|p_{jr}=b_jt\alpha^{r-1}|\Sigma w_jC_j$, the WSDR (Weighted Smallest Deteriorating Rate) rule is not an optimal one.

Example 1 Suppose n=2, t_0=1, b_1=2, b_2=2.5, w_1=3 and w_2=3.5, α=0.85. The sequence obtained by the WSDR rule is (1,2) and the total weighted flow time is 41.85, however, the total weighted flow time of sequence (2,1) is 40.6.

Theorem 7 For the problem $1|p_{jr}=b_jt\alpha^{r-1}|L_{\max}$, if jobs have agreeable due dates and deteriorating rates, i.e. $b_i\le b_k$ implies $d_i\le d_k$, then an optimal schedule can be obtained by sequencing the jobs in a non-decreasing order of $\{d_j\}$ (EDD rule).

Proof By the standard pairwise interchange argument as in Theorem 4, we can prove it.

The third special case: a_j≡a_0≠0

For this special case, $p_{jr}=(a_0+b_jt)\alpha^{r-1}$, we can show by examples that the SDR policy no longer provides the optimal solutions for the makespan minimization and the total flow time minimization, the WSDR rule does not provide the optimal solution for the total weighted flow time minimization and the EDD rule is no longer true.

Example 2 (Makespan minimization) Suppose n=2, t_0=1, a_0=1, b_1=0.5, b_2=1, α=0.85. The makespan of the SDR sequence is 5.3, while the makespan of the alternative sequence is 5.1.

Example 3 (Total flow-time minimization) Suppose n=3, t_0=1, a_0=1, b_1=0.5, b_2=1.0, b_3=1.2, α=0.85. The total flow time obtained by the SDR sequence (1,2,3) is 19.2, but the optimal sequence is (2,1,3) with the total flow time 18.4.

Example 4 (Total weighted flow-time minimization) Suppose n=2, t_0=1, a_0=1, b_1=0.4, b_2=0.6, w_1=2, w_2=2.5, α=0.85. The total weighted flow time obtained by the WSDR sequence is 14.7, but the optimal sequence is (2,1) with the total weighted flow time 14.5.

Example 5 (Maximum lateness minimization) Suppose n=2, t_0=1, a_0=1, b_1=0.5, b_2=1, d_1=2.0, d_2=2.1, α=0.85. The maximum lateness of the sequence obtained by the EDD rule is 3.2, but the optimal sequence is (2,1) with the maximum lateness 3.1.

CONCLUSION

In this paper, we consider some single machine scheduling problems with deteriorating jobs and a learning effect simultaneously, and introduce polynomial solutions for single machine makespan minimization, total flow time minimization and maximum lateness minimization corresponding to the first and second special cases of our model under some agreeable conditions. However, corresponding to the third special case of our model, we show that the optimal schedules may be different from those of the classical version for the above objective functions.

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