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Stabilization of stochastic nonholonomic systems*

KE Hai-sen†, YE Xu-dong

(*Department of Electrical Engineering, Zhejiang University, Hangzhou 310027, China*)

†E-mail: kehaisen@zju.edu.cn

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Abstract: In this work, we investigate the stabilization control design of nonholonomic stochastic system in strict-feedback form. Under the condition of all states being available for feedback, a state feedback controller was developed via the stochastic Lyapunov-like theorem and backstepping design technique. The controllers guarantee all states of the closed-loop system are bounded in probability, and largely asymptotically stable when the stochastic disturbances equal to zero at the equilibrium point of the open-loop system. Besides, the time-varying technique was introduced to avoid the uncontrollable state of chained system.

Key words: Time-varying technique, Nonholonomic, Backstepping, Stochastic
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INTRODUCTION

In recent years, much progress has been made in the design of control laws for systems subjected to nonholonomic constraint making it impossible to stabilize the system by any time-invariant continuous state feedback control law. This fact stimulates researchers to construct time-varying (Jiang and Nijmeijer, 1999; Jiang, 2001; Samson, 1995; Tian and Li, 2002) or discontinuous (Chang and Chen, 2002; Ge *et al*., 2003; Hu *et al*., 2004; Kim and Tsiotras, 2002; Mnif, 2004; Fukao *et al*., 2000) feedback controllers for the control of nonholonomic systems. For the deterministic system, some controllers have been proposed and tested (Kim and Tsiotras, 2002) on real mobile robot needing complete knowledge of the systems. Because most practical nonhonomic mechanical systems have uncertainties and may be perturbed by unknown disturbances, it is impossible or not exact to obtain the deterministic model. So, an important issue for practical system design is the adaptive or robust consideration against possible modelling errors and external disturbances.

However, studies of stochastic nonholonomic systems were not carried out. To resolve the stabilization of stochastic nonholonomic systems, a state feedback controller was designed in this work via backstepping design technique (Ke and Ye, 2006) and stochastic Lyapunov-like theorem (Deng and Krstic, 1999; Fan and Ge, 2004) under the condition of all states being available for feedback. The remainder of the paper is organized as follows. Section 2 provides some necessary preliminaries. Formulation of stochastic nonholonomic system to be considered is presented in Section 3. A state feedback controller is developed in Section 4 and a simulation example is given in Section 5. Finally, the paper is concluded in Section 6.

PRELIMINARIES ON STABILITY IN PROBABILITY

Consider the nonlinear stochastic system

$$\mathrm{d}\boldsymbol{x}=f(\boldsymbol{x})\mathrm{d}t+g(\boldsymbol{x})\mathrm{d}\boldsymbol{w}, \tag{1}$$

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where $\boldsymbol{x}\in\mathbb{R}^n$ is the state, $\boldsymbol{w}$ is an r-dimensional independent standard Wiener process, f: $\mathbb{R}^n\to\mathbb{R}^n$ and g: $\mathbb{R}^n\to\mathbb{R}^{n\times r}$ are locally Lipschitz.

Definition 1 The equilibrium $\boldsymbol{x}$=0 of Eq.(1) is said to be globally asymptotically stable in probability if for any $t_0\geq 0$ and $\varepsilon>0$, $\lim\limits_{\boldsymbol{x}(t_0)\to 0} P\{\sup_{t\geq t_0}|\boldsymbol{x}(t)|>\varepsilon\}=0$, and for any initial condition $\boldsymbol{x}(t_0)$, $P\{\lim\limits_{t\to\infty}\boldsymbol{x}(t)=0\}=1$.

Theorem 1 (Liu and Zhang, 2004) Consider the stochastic nonlinear system of Eq.(1). If there exists a positive definite, radically unbounded, twice continuously differentiable function $V(\boldsymbol{x})$: $\mathbb{R}^n\to\mathbb{R}$, and constants $c_1>0$, $c_2\geq 0$ such that the infinitesimal generator

$$LV(\boldsymbol{x})=\frac{\partial V}{\partial \boldsymbol{x}}f+\frac{1}{2}Tr\left\{g^{\mathrm{T}}\frac{\partial^2 V}{\partial \boldsymbol{x}^2}g\right\}\leq -c_1V(\boldsymbol{x})+c_2, \tag{2}$$

then

(1) The system almost surely has a unique solution;

(2) The system is bounded in probability;

(3) In addition, if $f(0)=0$, $g(0)=0$, and $LV(\boldsymbol{x})\leq -c_1V(\boldsymbol{x})$, the system is asymptotically stable in the large.

Young's inequality

$$\boldsymbol{x}^{\mathrm{T}}\boldsymbol{y}\leq\frac{\varepsilon^p}{p}\|\boldsymbol{x}\|^p+\frac{1}{q\varepsilon^q}\|\boldsymbol{y}\|^q,$$

where $\boldsymbol{x}$, $\boldsymbol{y}\in\mathbb{R}^n$, $\varepsilon>0$, $p>1$, $q>1$ and $(p-1)(q-1)=1$.

PROBLEM FORMULATION

Consider the following stochastic nonholonomic system in the chained form

$$\mathrm{d}x_0=u_0\mathrm{d}t, \tag{3.1}$$

$$\left.\begin{aligned}\mathrm{d}x_1&=u_0[x_2\mathrm{d}t+g_1^{\mathrm{T}}(x_1)\mathrm{d}\boldsymbol{w}],\\ \mathrm{d}x_2&=u_0[x_3\mathrm{d}t+g_2^{\mathrm{T}}(x_1,x_2)\mathrm{d}\boldsymbol{w}],\\ &\vdots\\ \mathrm{d}x_n&=u_1\mathrm{d}t+u_0g_n^{\mathrm{T}}(\boldsymbol{x})\mathrm{d}\boldsymbol{w},\end{aligned}\right\} \tag{3.2}$$

where $x_0\in\mathbb{R}$ and $\boldsymbol{x}=[x_1,x_2,\cdots,x_n]^{\mathrm{T}}\in\mathbb{R}^n$ are the states, $u_0\in\mathbb{R}$ and $u_1\in\mathbb{R}$ are control inputs, $\boldsymbol{w}$ is an r-dimensional independent standard Wiener process, nonlinear function $g_i(\bar{x}_i)\in\mathbb{R}^r$ $(1\leq i\leq n)$ are known, smooth and locally Lipschitz.

CONTROLLER DESIGN

In this section, under the condition that the full state of system (3) is available for feedback, we design a state feedback controller via the Lyapunov-like theorem and backstepping technique. First, we design the stabilizing controller of system (3.1). Then, on the basis of the former step, we develop the stabilizing controller of system (3.2). Finally, the stability of the closed-loop system is given.

Controller design of sub-system (3.1)

From system (3), it is obvious that the sub-system (3.2) is stable when $u_0=0$ and is controllable as long as $u_0\neq 0$. This fact leads us to construct the following time-varying control law u_0

$$u_0=-c_0x_0+\lambda\sin t, \tag{4}$$

where $c_0>0$, $\lambda>0$. Clearly, u_0 is a smooth function of time and $\lim\limits_{t\to\infty}u_0(t)\neq 0$.

Proof We prove $\lim\limits_{t\to\infty}u_0(t)\neq 0$ by contradiction. If $\lim\limits_{t\to\infty}u_0(t)\neq 0$, then $u_0(t)=0$, $\forall t\geq T$, from Eq.(3.1), then $\dot{x}_0(t)=0$, $\forall t\geq T$ and therefore $x_0(t)$ is a constant x_0^*, $\forall t\geq T$. This together with Eq.(4) implies that

$$c_0x_0^*=\lambda\sin t. \tag{5}$$

This leads to a contradiction. Therefore, we have $\lim\limits_{t\to\infty}u_0(t)\neq 0$.

Define the candidate Lyapunov function as

$$V_0=x_0^2/2. \tag{6}$$

Differentiating the function V_0 along the solution of Eq.(3.1) yields

$$\dot{V}_0 = x_0(-c_0x_0 + \lambda \sin t) \le -c_0 x_0^* + \lambda \mid x_0 \mid . \qquad (7)$$

From Eq.(7), we can easily conclude that states x_0 and u_0 are bounded on $[0,\infty)$.

Controller design of sub-system (3.2)

When u_0 is considered as a function of time, the sub-system (3.2) looks like a time-varying nonlinear system with a lower triangular form. Therefore, we can proceed with our control design via backstepping method as long as $u_0 \neq 0$. The procedure consists of n steps. At the ith step, $1 \le i \le n-1$, the state variable x_{i+1} is viewed as a fictitious control, for which a "reference" signal α_i is designed. At the nth step, the fictitious control equals the actual control u_1 which completes the design.

As the fact that the sub-system (3.2) is stable when $u_0=0$ and is controllable as long as $u_0 \neq 0$. For simplicity, it is without loss of generality to assume that $u_0 \neq 0$ in the course of design.

To begin with, we can define a smooth function ϕ_i as

$$\begin{cases} \phi_1(u_0) = u_0^{2l+1}, \\ \phi_i(u_0, \dot{u}_0, \cdots, u_0^{(i-1)}) = \dfrac{\dot{\phi}_{i-1}}{u_0}, \; 2 \le i \le n-1, \end{cases} \qquad (8)$$

where l is a nonnegative integer and $l \ge n-1$.

Step 1: Let $z_1=x_1$. From Eq.(3.2) we have

$$\mathrm{d}z_1 = u_0[x_2 \mathrm{d}t + g_1^{\mathrm{T}}(x_1)\mathrm{d}\boldsymbol{w}]. \qquad (9)$$

Since $g_1(x_1)$ is a smooth function of x_1, we can decompose $g_1(x_1)$ as

$$g_1(x_1) = G_1(0) + G_1(x_1)x_1,$$

where $g_1(0)=G_1(0)$ and $G_1(x_1)$ is a known continuous function.

We now view x_2 as a virtual control and design for it the following stabilizing function

$$\alpha_1(\phi_1, x_1) = -c_1\phi_1 z_1 - \left(u_0 + \frac{1}{u_0}\right)\left[\frac{3}{2} + \frac{3}{4}n(n+1) + \frac{3}{2}\left|G(z_1)\right|^4 + \frac{3}{4}\varepsilon_1^{4/3}\right] z_1, \qquad (10)$$

where ϕ_1 is defined as before, c_1, ε_1 are positive constants to be designed. It is clear that $\alpha_1(\phi_1,0)=0$. Define

$$z_2 = x_2 - \alpha_1(x_1), \qquad (11)$$

$$V_1 = z_1^4/4, \qquad (12)$$

By Itô formula, we have

$$\begin{aligned} LV_1 &= [z_1^3\alpha_1 + z_1^3 z_2 + \frac{3}{2}g_1^{\mathrm{T}}g_1 z_1^2]u_0 \\ &= -c_1 u_0^{2l+2} z_1^4 - (1+u_0^2)\left[\frac{3}{2} + \frac{3}{4}n(n+1) + \frac{3}{2}\left|G_1(z_1)\right|^4 + \frac{3}{4}\varepsilon_1^{4/3}\right] z_1^4 + \left[z_1^3 z_2 + \frac{3}{2}g_1^{\mathrm{T}}g_1 z_1^2\right]u_0 \\ &\le -c_1 u_0^{2l+2} z_1^4 - (1+u_0^2)\left[\frac{3}{4}n(n+1)z_1^4 - \frac{1}{4\varepsilon_1^4}z_2^4 - \frac{3}{2}\left|G_1(0)\right|^4 - \frac{3}{2}z_1^4\right], \end{aligned} \qquad (13)$$

where the following inequality was used

$$u_0 z_1^3 z_2 \le \left[\frac{3}{4}\varepsilon_1^{4/3} z_1^4 + \frac{1}{4\varepsilon_1^4} z_2^4\right](1+u_0^2),$$

$$u_0 z_1^2 g_1^{\mathrm{T}} g_1 \le \left[2z_1^4 + \left|G_1(0)\right|^4 + \left|G_1(z_1)\right|^4 z_1^4\right](1+u_0^2).$$

Step i ($2 \le i \le n-1$): Assume the smooth function α_j ($1 \le j \le i-1$) has been designed such that

$$z_{j+1} = x_{j+1} - \alpha_j(x_1, \cdots, x_j, \phi_1, \cdots, \phi_j) \qquad (14)$$

and $\alpha_j(0,\ldots,0,\phi_1,\ldots,\phi_j)=0$. The time derivative of $V_{i-1} = \dfrac{1}{4}\sum_{j=1}^{i-1} z_j^4$ satisfies

$$LV_{i-1} \le -\sum_{j=1}^{i-1}\left[c_j u_0^{2l+2} + \frac{3}{4}(n-j+1)(n+j)(1+u_0^2)\right] z_j^4 + \left\{\frac{z_i^4}{4\varepsilon_{i-1}^4} + \sum_{j=1}^{i-1}\left[\frac{3}{2}\left|\bar{G}_j(0)\right|^4 + \frac{3}{4}(i-j)(i-1+j)z_j^4\right]\right\}\cdot(1+u_0^2) \qquad (15)$$

$$z_{i+1} = x_{i+1} - \alpha_i(x_1, \cdots, x_i, \phi_1, \cdots, \phi_i). \qquad (16)$$

Using the definition of z_i, we have

$$\mathrm{d}z_i = u_0\left\{\left[x_{i+1} + F(\bar{x}_i,\bar{\phi}_i)\right]\mathrm{d}t + G_i^{\mathrm{T}}(\bar{x}_i)\mathrm{d}\boldsymbol{w}\right\}, \quad (17)$$

where we define

$$F_i(\bar{x}_i,\bar{\phi}_i) = -\sum_{j=1}^{i-1}\left(\frac{\partial\alpha_{i-1}}{\partial x_j}x_{j+1} + \frac{\partial\alpha_{i-1}}{\partial\phi_j}\phi_{j+1}\right) - \frac{1}{2}\sum_{p,q=1}^{i-1}\frac{\partial^2\alpha_{i-1}}{\partial x_p\partial x_q}g_p^{\mathrm{T}}g_q,$$

$$G_i(\bar{x}_i) = g_i(\bar{x}_i) - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}g_j(\bar{x}_j).$$

Taking the change of coordinates

$$\begin{cases} z_1 = x_1, \\ z_j = x_j - \alpha_{j-1}(x_1,\cdots,x_{j-1}),\ 2\le j\le i. \end{cases} \quad (18)$$

Since the smoothing of $g_j(1\le j\le i)$, $\alpha_{j-1}(2\le j\le i)$ and $\alpha_{j-1}(0,\ \bar{\phi}_{j-1})=0$ $(2\le j\le i)$, $G_i(\bar{x}_i)$ can be decomposed into the following forms

$$G_i(\bar{x}_i) = \bar{G}_i(\bar{z}_i) = \bar{G}_i(0) + \sum_{j=1}^{i}z_j\bar{G}_{ij}(\bar{z}_i), \quad (19)$$

where $\bar{z}_i = [z_1\ \cdots\ z_i]^{\mathrm{T}}$, $\bar{G}_i(0) = G_i(0)$, $\bar{G}_{ij}(\bar{z}_i)$ $(1\le j\le i)$ are known continuous functions.

We now view x_{i+1} as a virtual control and design for it the following stabilizing function

$$\alpha_i = -F_i - c_i\phi_1 z_i - \left(u_0 + \frac{1}{u_0}\right)z_i\cdot\left[\frac{3}{2} + \frac{3}{4}(n-i+1)(n+i) + \frac{3}{2}i\sum_{j=1}^{i}\left|\bar{G}_{ij}\right|^4 + \frac{1}{4\varepsilon_{i-1}^4} + \frac{3}{4}\varepsilon_i^{4/3}\right] \quad (20)$$

Choose Lyapunov candidate function as

$$V_i = V_{i-1} + z_i^4/4. \quad (21)$$

By Itô formula, we have

$$LV_i \le -\sum_{j=1}^{i}\left[c_j u_0^{2l+2} + \frac{3}{4}(n-j+1)(n+j)(1+u_0^2)\right]z_j^4 + (1+u_0^2)\cdot\left\{\frac{z_{i+1}^4}{4\varepsilon_i^4} + \sum_{j=1}^{i}\left[\frac{3}{2}\left|\bar{G}_j(0)\right|^4 + \frac{3}{4}(i-j+1)(i+j)z_j^4\right]\right\}, \quad (22)$$

where we use

$$u_0 z_i^3 z_{i+1} \le \left[\frac{3}{4}\varepsilon_i^{4/3}z_i^4 + \frac{1}{4\varepsilon_i^4}z_{i+1}^4\right](1+u_0^2),$$

$$u_0 z_i^2 G_i^{\mathrm{T}}G_i \le (1+u_0^2)\left\{\left[1 + i\sum_{j=1}^{i}\left|\bar{G}_{ij}(\bar{z}_i)\right|^4\right]z_i^4 + i\sum_{j=1}^{i}z_j^4 + \left|\bar{G}_i(0)\right|^4\right\}.$$

Step n: Using the definition of z_n, we have

$$\mathrm{d}z_n = [u_1 + u_0F_n(\boldsymbol{x})]\mathrm{d}t + u_0G_n^{\mathrm{T}}(\boldsymbol{x})\mathrm{d}\boldsymbol{w}. \quad (23)$$

We now design the actual control as follows

$$u_1 = -(1+u_0^2)z_n\left[\frac{3}{2} + \frac{3}{2}n + \frac{3}{2}n\sum_{j=1}^{n}\left|\bar{G}_{nj}(\boldsymbol{z})\right|^4 + \frac{1}{4\varepsilon_{n-1}^4}\right] - u_0F_n - c_nz_n, \quad (24)$$

and define the Lyapunov candidate function of system (3.2) as

$$V = V_{n-1} + z_n^4/4. \quad (25)$$

By Itô formula, we have

$$LV \le -\sum_{j=1}^{n-1}c_ju_0^{2l+2}z_j^4 - c_nz_n^4 + (1+u_0^2)\sum_{j=1}^{n}\frac{3}{2}\left|\bar{G}_j(0)\right|^4 \quad (26)$$

We now summarize the result of this work in the following theorem.

Theorem 2 Consider the stochastic nonholonomic system (3). A time-varying feedback controller u_0 and a state feedback controller u_1 can be constructively designed so that the closed-loop system is bounded in probability. Besides, when $G_i(0)=0$ $(1\le i\le n)$, the closed-loop system is asymptotically stable in the large.

Proof From the transformation $z_1=x_1$, $z_i=x_i-\alpha_{i-1}$ and $\alpha_{i-1}(0,\ldots,0,\phi_1,\ldots,\phi_{i-1})=0$ ($2\le i\le n$), it can be concluded that $\boldsymbol{x}(t)$ is bounded or asymptotically stable when $\boldsymbol{z}(t)$ is bounded or asymptotically stable. Therefore, we only need to prove $\boldsymbol{z}(t)$ is bounded or asymptotically stable.

When we design the control law as Eqs.(4) and (24), we have

$$LV \le -\sum_{j=1}^{n-1} c_j u_0^{2l+2} z_j^4 - c_n z_n^4 + (1+u_0^2)\sum_{j=1}^{n}\frac{3}{2}\left|\bar{G}_j(0)\right|^4. \quad (27)$$

Noting that V is a positive definite, radically unbounded and twice continuously differentiable function in terms of states of the closed-loop system (3.2) and that u_0 is bounded, then by Theorem 1 we conclude that all states $z_i(t)$ ($1\le i\le n$) in closed-loop system (3.2) is bounded in probability. Furthermore, if $g_i(0)=0$, then $\bar{G}_i(0)=0$ ($1\le i\le n$), which leads to

$$LV \le -\sum_{j=1}^{n-1} c_j u_0^{2l+2} z_j^4 - c_n z_n^4. \quad (28)$$

To prove the convergence of $\boldsymbol{x}(t)$, i.e. $\boldsymbol{z}(t)$, we introduce

$$\psi(t) = \sum_{j=1}^{n-1} c_j u_0^{2l+2} z_j^4 + c_n z_n^4. \quad (29)$$

$\psi(t)$ is uniformly continuous since its derivate is bounded. This together with Eq.(28) implies that $\psi(t)$ is bounded. Therefore, by application of Barbalat's lemma, it can be concluded that $\psi(t)$ converges to zero as $t\to\infty$.

As $\lim_{t\to\infty} u_0(t)\ne 0$ and the nonnegative function V is decreasing and tends to a finite number, it follows by a contradiction argument that $\boldsymbol{z}(t)$ converges to zero in the ultimate.

ILLUSTRATIVE EXAMPLE

Consider the following stochastic chained systems

$$\left.\begin{aligned} \mathrm{d}x_0 &= \mathrm{d}u_0, \\ \mathrm{d}x_1 &= u_0[x_2\mathrm{d}t + \frac{1}{2}x_1^2\mathrm{d}\boldsymbol{w}], \\ \mathrm{d}x_2 &= u_1\mathrm{d}t. \end{aligned}\right\} \quad (30)$$

For this system, the virtual control α_1 and control u_0 and u_1 are designed as

$$u_0=-c_0z_0+\lambda\sin t, \quad (31)$$

$$\alpha_1(x_1,\phi_1) = -\left[c_1\phi_1 + \left(u_0+\frac{1}{u_0}\right)\left(\frac{3}{64}z_1^4+\frac{3}{4}\varepsilon_1^{4/3}\right)\right]z_1, \quad (32)$$

$$\begin{aligned} u_1 = &-c_2z_2 - (1+u_0^2)\left[\frac{3}{64}\left(-\frac{\partial\alpha_1}{\partial x_1}x_1\right)^4 + \frac{1}{4\varepsilon_1^4}\right]z_2 \\ &+u_0\left[\frac{\partial\alpha_1}{\partial x_1}x_2 + \frac{\partial\alpha_1}{\partial\phi_1}\phi_2 + \frac{1}{8}\frac{\partial^2\alpha_1}{\partial x_1^2}x_1^4\right], \end{aligned} \quad (33)$$

where

$$\phi_1 = u_0^{2l+1},$$

$$\phi_2(u_0,\dot{u}_0) = \frac{\dot{\phi}_1}{u_0} = (2l+1)\frac{u_0^{2l}\dot{u}_0}{u_0} = (2l+1)\dot{u}_0u_0^{2l-1}.$$

We choose $\varepsilon_1=1$, $c_0=0.3$, $c_1=2.5$, $c_2=1$, $l=1$ and set the initial condition at $x_0(0)=1.5$, $x_1(0)=1$, $x_2(0)=2$. Fig.1 depicts the simulation results.

These simulation results clearly showed that the robust controllers presented in this work guarantee the boundedness and convergence of all the states in the closed-loop system.

CONCLUSION

In this paper, a state feedback controller was constructively designed by employing the stochastic Lyapunov-like theorem and backstepping design technique. The controllers guarantee all states of the closed-loop system bounded in probability, and moreover are asymptotically stable in the large when the stochastic disturbances equal to zero at the equilibrium point of the open-loop system. The time-varying technique was introduced to avoid the uncontrollability of the chained system. The simulation results illustrate the feasibility of the design procedure.

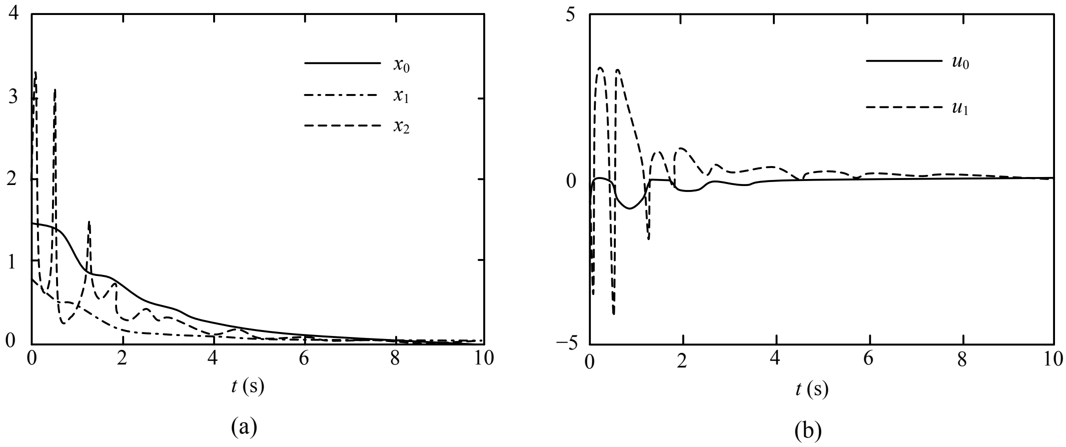

Fig.1 Evolution of the states (a) and the controls (b)

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