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Passive control of a class of chaotic dynamical systems with nonlinear observer*

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Abstract: A passive control strategy with nonlinear observer is proposed, which can be used to control a class of chaotic dynamical systems to stabilize at different equilibrium points. If the nonlinear function of chaotic system satisfies Lipschitz condition, the nonlinear observer can observe the state variables of the chaotic systems. An important property of passive system is studied to control chaotic systems, that is passive system can be asymptotically stabilized by state feedback controller whose state variables are presented by nonlinear observer. Simulation results indicated that the proposed chaos control method is very effective in a class of chaotic systems.

Key words: Chaotic dynamical system, Passive theory, Nonlinear observer
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INTRODUCTION

Chaos plays an important role in dynamical systems and is applied in many fields such as physics, chemistry, economics, and so on. The dynamical characters of chaos have been proved to be useful in describing and diagnosing nonlinear systems (Ott *et al*., 1990). However, it is harmful to many systems because of the characters of chaotic systems. People seek for ways to avoid and eliminate it. In general, through modifying parameters or controlling chaotic system, some ways can be found to affect the existing conditions of chaos so that chaos can be avoided (Yang and Liu, 1998; Chen and Chen, 1999).

Many people have begun to give their attention to passive network theory (Hill and Moylan, 1976; Wen, 1999; Byrnes and Isidori, 1991). The character of passive system is one of the network theory concepts, which show characteristics of dissipative network. Passive network theory can be used to analyse the dynamical character of system, such as stabilization, dynamic characteristics. In order to improve the control property of chaotic system, state feedback should be applied to the controlled system when the controller is designed for chaotic system, and the state variables used to complete the state feedback can be detected through sensor. However, the state variables cannot be detected in some conditions (Qi *et al*., 2004; Nam *et al*., 1997; Femat and Alvarez-Ramirez, 1997). Hence, the method of nonlinear systems' observer can be used to design observer of the chaotic system, and realize the control of chaotic system (Li *et al*., 2005).

A new chaos control method useful for avoiding chaos is proposed in this paper. The remainder of the paper is organized as follows. In Section 2, a nonlinear observer of chaotic system is given if the nonlinear function of chaotic system satisfies Lipschitz condition. In Section 3, through giving some definitions and reasoning, we research the properties of passive system. In Section 4, taken the Lorenz system as example, the essential conditions

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are studied, by which Lorenz chaotic system could be equivalent to passive system with nonlinear observer. Section 5 gives the conclusion.

NONLINEAR OBSERVER OF CHAOTIC SYSTEMS

Consider the following chaotic dynamical system described by

$$\begin{cases} \dot{\boldsymbol{z}} = \boldsymbol{A}\boldsymbol{z} + \boldsymbol{B}\boldsymbol{q}(\boldsymbol{z}), \\ \boldsymbol{y} = \boldsymbol{C}^{\mathrm{T}}\boldsymbol{z}, \end{cases} \tag{1}$$

where $\boldsymbol{z} \in \mathbb{R}^n$ are state variables, $\boldsymbol{A} \in \mathbb{R}^{n\times n}$ and $\boldsymbol{B} \in \mathbb{R}^n$ constitute system parameters matrix, and $\boldsymbol{q}:\mathbb{R}^n \to \mathbb{R}$ is a nonlinear function.

Theorem 1 Suppose chaotic system having the form as Eq.(1) and for $\forall z_1, z_2 \in \mathbb{R}^n$, the function $\boldsymbol{q}(\boldsymbol{z})$ satisfies the Lipschitz condition on $\mathbb{R}^n$, that is

$$\|\boldsymbol{q}(z_1) - \boldsymbol{q}(z_2)\| \le r\|z_1 - z_2\|, \tag{2}$$

where r is positive constant and $\|\cdot\|$ is the standard Euclidean norm.

If control parameter k is properly chosen, the state observer of the chaotic system has the form

$$\begin{cases} \dot{\hat{\boldsymbol{z}}} = \boldsymbol{A}\hat{\boldsymbol{z}} + \boldsymbol{B}\boldsymbol{q}(\hat{\boldsymbol{z}}) + \boldsymbol{L}(k)(\boldsymbol{y} - \hat{\boldsymbol{y}}), \\ \hat{\boldsymbol{y}} = \boldsymbol{C}^{\mathrm{T}}\hat{\boldsymbol{z}}, \end{cases} \tag{3}$$

where $\hat{\boldsymbol{z}}$ denotes the dynamic estimate of the state $\boldsymbol{z}$, $\hat{\boldsymbol{y}}$ is the estimated system output, $\boldsymbol{q}(\hat{\boldsymbol{z}})$ represents the estimated function $\boldsymbol{q}(\boldsymbol{z})$.

The observability parameter could be chosen as

$$\boldsymbol{L}(k) = \boldsymbol{P}^{-1} \times \boldsymbol{L}_0(k), \tag{4}$$

where $\boldsymbol{L}_0(k)=(c_1k\ c_2k^2\ \dots\ c_nk^n)^{\mathrm{T}}$, control parameter satisfies $k\ge1$, matrix $\boldsymbol{P}$ is constructed with observability matrix $\boldsymbol{O}=(\boldsymbol{C}^{\mathrm{T}}\ \boldsymbol{C}^{\mathrm{T}}\boldsymbol{A}\ \dots\ \boldsymbol{C}^{\mathrm{T}}\boldsymbol{A}^{n-1})^{\mathrm{T}}$, and parameters $\{c_i, i=1, 2, \dots, n\}$ are chosen such that all the roots of $\boldsymbol{p}_n(s)=s^n+c_1s^{n-1}+\dots+c_n$ are contained in the left-hand side of the complex plane.

Proof rank($\boldsymbol{O}$)=$r\le n$ means that there are r rows of linearly independent vectors $\{\boldsymbol{p}_1,\boldsymbol{p}_2,\dots,\boldsymbol{p}_r\}$ in matrix $\boldsymbol{O}$. In order to construct the singularity linear transformation matrix $\boldsymbol{P}$, $n-r$ rows of vectors $\{\boldsymbol{p}_{r+1},\boldsymbol{p}_{r+2},\dots,\boldsymbol{p}_n\}$ are completed besides $\{\boldsymbol{p}_1,\boldsymbol{p}_2,\dots,\boldsymbol{p}_r\}$, which is linearly independent with $\{\boldsymbol{p}_1,\boldsymbol{p}_2,\dots,\boldsymbol{p}_r\}$. So $\boldsymbol{P}=[\boldsymbol{p}_1^{\mathrm{T}}, \boldsymbol{p}_2^{\mathrm{T}}, \dots, \boldsymbol{p}_n^{\mathrm{T}}]^{\mathrm{T}}$. Consider the linear transformation of coordinates $\boldsymbol{w}=\boldsymbol{P}\boldsymbol{z}$. Because matrix $\boldsymbol{P}$ is invertible, system Eq.(1) can be described as

$$\begin{cases} \dot{\boldsymbol{w}} = \boldsymbol{A}_0\boldsymbol{w} + \boldsymbol{B}_0\boldsymbol{f}(\boldsymbol{w}), \\ \boldsymbol{y} = \boldsymbol{C}_0^{\mathrm{T}}\boldsymbol{w}, \end{cases} \tag{5}$$

where $\boldsymbol{f}(\boldsymbol{w})=\boldsymbol{q}(\boldsymbol{P}^{-1}\boldsymbol{w})$, $\boldsymbol{A}_0=\boldsymbol{P}\boldsymbol{A}\boldsymbol{P}^{-1}$, $\boldsymbol{B}_0=\boldsymbol{P}\boldsymbol{B}$, $\boldsymbol{C}_0^{\mathrm{T}}=\boldsymbol{C}^{\mathrm{T}}\boldsymbol{P}^{-1}$. Letting $\boldsymbol{P}\boldsymbol{L}(k)=\boldsymbol{L}_0(k)$, we have

$$\begin{cases} \dot{\hat{\boldsymbol{w}}} = \boldsymbol{A}_0\hat{\boldsymbol{w}} + \boldsymbol{B}_0\boldsymbol{f}(\hat{\boldsymbol{w}}) + \boldsymbol{L}_0(k)(\boldsymbol{y} - \hat{\boldsymbol{y}}), \\ \hat{\boldsymbol{y}} = \boldsymbol{C}_0^{\mathrm{T}}\hat{\boldsymbol{w}}. \end{cases} \tag{6}$$

Define error $\boldsymbol{e} = \boldsymbol{w} - \hat{\boldsymbol{w}}$. Since $k\ge1$ and matrix $\boldsymbol{P}$ is an invertible matrix, $\boldsymbol{e}(t)\to0$ implies that $\boldsymbol{z}(t) \to \hat{\boldsymbol{z}}(t)$. Hence, it suffices to prove that $\boldsymbol{e}(t)\to0$. According to Eqs.(5) and (6)

$$\dot{\boldsymbol{e}} = \boldsymbol{M}_0\boldsymbol{e} + \boldsymbol{B}_0[\boldsymbol{f}(\boldsymbol{w}) - \boldsymbol{f}(\hat{\boldsymbol{w}})], \tag{7}$$

where $\boldsymbol{M}_0 = \boldsymbol{A}_0 - \boldsymbol{L}_0(k)\boldsymbol{C}_0^{\mathrm{T}}$, whose eigenfunction is $\boldsymbol{p}_n(s)=s^n+c_1s^{n-1}+\dots+c_n$.

Through properly chosen parameter c_i, all roots of $\boldsymbol{p}_n(s)$ can be contained in the left-hand side of the complex plane. Lyapunov equation $\boldsymbol{P}_0\boldsymbol{M}_0 + \boldsymbol{M}_0^{\mathrm{T}}\boldsymbol{P}_0 = -\boldsymbol{I}_n$ has $\boldsymbol{P}_0>0$.

Choosing Lyapunov function $\boldsymbol{V}=\boldsymbol{e}^{\mathrm{T}}\boldsymbol{P}_0\boldsymbol{e}$, we have

$$\begin{aligned} \dot{\boldsymbol{V}} &= \dot{\boldsymbol{e}}^{\mathrm{T}}\boldsymbol{P}_0\boldsymbol{e} + \boldsymbol{e}^{\mathrm{T}}\boldsymbol{P}_0\dot{\boldsymbol{e}} \\ &\le -\|\boldsymbol{e}\|^2 + 2\|\boldsymbol{e}^{\mathrm{T}}\boldsymbol{P}_0(\boldsymbol{B}_0[\boldsymbol{f}(\boldsymbol{w}) - \boldsymbol{f}(\hat{\boldsymbol{w}})])\| \\ &\le -\|\boldsymbol{e}\|^2 + 2\|\boldsymbol{e}^{\mathrm{T}}\|\cdot\|\boldsymbol{P}_0\|\cdot\|\boldsymbol{B}_0\|\cdot|\boldsymbol{f}(\boldsymbol{w}) - \boldsymbol{f}(\hat{\boldsymbol{w}})|. \end{aligned} \tag{8}$$

Since $\boldsymbol{q}(\boldsymbol{w})$ is a globally Lipschitz function on $\mathbb{R}^n$, $\boldsymbol{f}(\boldsymbol{w})$ is also a globally Lipschitz function. In this case, there exists $r>0$, such that

$$\|\boldsymbol{f}(\boldsymbol{w}) - \boldsymbol{f}(\hat{\boldsymbol{w}})\| \le r\|\boldsymbol{w} - \hat{\boldsymbol{w}}\|. \tag{9}$$

From Eqs.(8) and (9), we have

$$\begin{aligned}\dot{V} &\le -\|\boldsymbol{e}\|^2 + 2r\lambda_{\max}(\boldsymbol{P}_0)\cdot\|\boldsymbol{B}_0\|\cdot\|\boldsymbol{e}\|^2 \\ &\le -(1-k_{\mathrm{f}}\lambda_{\max}(\boldsymbol{P}_0)\|\boldsymbol{B}_0\|)\cdot\|\boldsymbol{e}\|^2,\end{aligned} \tag{10}$$

where $k_{\mathrm{f}}=2r$.

If $k_{\mathrm{f}}\lambda_{\max}(\boldsymbol{P}_0)\|\boldsymbol{B}_0\|<1$, there should be $\dot{V}<0$. The error system is globally stable, that is, the state variables of observer can approximate the state variables of the chaotic system.

PROPERTIES OF PASSIVE SYSTEM

Consider a continuous chaotic system given by difference equation as follows

$$\begin{cases}\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{g}(\boldsymbol{x})\cdot\boldsymbol{u}, \\ \boldsymbol{y} = \boldsymbol{h}(\boldsymbol{x}),\end{cases} \tag{11}$$

where, $\boldsymbol{x}\in\mathbb{R}^n$ are the state variables, $\boldsymbol{u}\in\mathbb{R}^m$ is the input, $\boldsymbol{y}\in\mathbb{R}^p$ is the output, $\boldsymbol{f}$ and $\boldsymbol{g}$ are smooth vector fields. Function $\boldsymbol{h}$ is smooth mapping. We suppose that the vector field $\boldsymbol{f}$ has at least one equilibrium point, without loss of generality, we can assume that the equilibrium point is $\boldsymbol{x}=0$.

Theorem 2 (Byrnes and Isidori, 1991) A system of the form Eq.(11) is said to be passive if function $\boldsymbol{f}(\boldsymbol{x})$ and $\boldsymbol{g}(\boldsymbol{x})$ exist and there is a real-valued constant d, such that $\forall t\ge0$

$$\int_0^t \boldsymbol{u}^{\mathrm{T}}(s)\boldsymbol{y}(s)\mathrm{d}s \ge d, \tag{12}$$

or if there is a constant $k_{\mathrm{p}}>0$ and real-valued constant d, such that $\forall t\ge0$

$$\int_0^t \boldsymbol{u}^{\mathrm{T}}(s)\boldsymbol{y}(s)\mathrm{d}s + d \ge \int_0^t k_{\mathrm{p}}\cdot\boldsymbol{y}^{\mathrm{T}}(s)\boldsymbol{y}(s)\mathrm{d}s. \tag{13}$$

In other words, a system of the form Eq.(11) is said to be passive if there is a nonnegative function $\boldsymbol{V}(\boldsymbol{x}):\boldsymbol{X}\to\mathbb{R}$, $\boldsymbol{V}(0)=0$, named storage function, such that $\forall \boldsymbol{x}\in\boldsymbol{X}$, $\forall t\ge0$

$$\boldsymbol{V}(\boldsymbol{x}) - \boldsymbol{V}(\boldsymbol{x}_0) \le \int_0^t \boldsymbol{y}^{\mathrm{T}}(s)\boldsymbol{u}(s)\mathrm{d}s. \tag{14}$$

A passive system with storage function $\boldsymbol{V}(\boldsymbol{x})$ is said to be strictly passive if there exists a positive definite function $\boldsymbol{S}(\boldsymbol{x})$, such that $\forall \boldsymbol{x}\in\boldsymbol{X}$, $\forall t\ge0$

$$\boldsymbol{V}(\boldsymbol{x}) - \boldsymbol{V}(\boldsymbol{x}_0) = \int_0^t \boldsymbol{y}^{\mathrm{T}}(s)\boldsymbol{u}(s)\mathrm{d}s - \int_0^t \boldsymbol{S}(\boldsymbol{x}(s))\mathrm{d}s. \tag{15}$$

According to Kalman-Yacubovitch-Popov lemma and the definition of zero-state detectable property discussed in (Byrnes and Isidori, 1991; Qi *et al*., 2005), Theorem 3 can be drawn. More details are omitted there.

Theorem 3 (Byrnes and Isidori, 1991) Suppose a system is passive with storage function $\boldsymbol{V}(\boldsymbol{x})$ which is positive definite and suppose the system is locally zero-state detectable. Let $\boldsymbol{w}$ be any smooth function such that $\boldsymbol{y}^{\mathrm{T}}\boldsymbol{h}(\boldsymbol{y})>0$ and $\boldsymbol{h}(0)=0$ for each nonzero $\boldsymbol{y}$. The control law $\boldsymbol{u}(t)=-\boldsymbol{h}(\boldsymbol{y})$ asymptotically stabilizes at the equilibrium point $\boldsymbol{x}=0$.

According to Theorem 3, if the system is passive, there must be a controller $\boldsymbol{u}(t)=-\boldsymbol{h}(\boldsymbol{y})$, which makes the passive system Lyapunov stable at the equilibrium point $\boldsymbol{x}=0$. Therefore, non-passive chaotic systems can be equivalent to passive systems through designing system controllers, and then can be stabilized at the equilibrium point $\boldsymbol{x}=0$.

PASSIVE CONTROL OF LORENZ SYSTEM

The differential equations of Lorenz chaotic system can be described as follows (Lorenz, 1963)

$$\begin{cases}\dot{x}_1 = 10x_2 - 10x_1, \\ \dot{x}_2 = 28x_1 - x_2 - x_1x_3, \\ \dot{x}_3 = x_1x_2 - (8/3)x_3.\end{cases} \tag{16}$$

Add the controller u to the second equation of Eq.(16). Suppose $z_1=x_1$, $z_2=x_3$, $y=x_2$, then the system can be expressed in normal form

$$\begin{cases}\dot{z}_1 = -10z_1 + 10y, \\ \dot{z}_2 = -(8/3)z_2 + z_1y, \\ \dot{y} = 28z_1 - y - z_1z_2 + u.\end{cases} \tag{17}$$

We have

$$\begin{cases}\dot{\boldsymbol{z}} = \boldsymbol{f}_0(\boldsymbol{z}) + \boldsymbol{p}(\boldsymbol{z},\boldsymbol{y})\boldsymbol{y}, \\ \dot{\boldsymbol{y}} = \boldsymbol{b}(\boldsymbol{z},\boldsymbol{y}) + \boldsymbol{a}(\boldsymbol{z},\boldsymbol{y})\boldsymbol{u},\end{cases} \tag{18}$$

where $\boldsymbol{f}_0(\boldsymbol{z})=[-10z_1\ (-8/3)z_2]^{\mathrm{T}}$, $\boldsymbol{p}(\boldsymbol{z},\boldsymbol{y})=[10\ z_1]^{\mathrm{T}}$, $\boldsymbol{a}(\boldsymbol{z},\boldsymbol{y})=1$, $\boldsymbol{b}(\boldsymbol{z},\boldsymbol{y})=28z_1-y-z_1z_2$.

Choose a storage function

$$\boldsymbol{V}(\boldsymbol{z},\boldsymbol{y})=\boldsymbol{W}(\boldsymbol{z})+0.5\boldsymbol{y}^2, \tag{19}$$

where $\boldsymbol{W}(\boldsymbol{z})=0.5z_1^2+0.5z_2^2$, which is the Lyapunov function of $\boldsymbol{f}_0(\boldsymbol{z})$, and $\boldsymbol{W}(0)=0$. We have

$$\dot{\boldsymbol{W}}=\frac{\mathrm{d}\boldsymbol{W}}{\mathrm{d}t}=[z_1\ z_2]\begin{bmatrix}-10z_1\\-(8/3)z_2\end{bmatrix}. \tag{20}$$

Because $\dot{\boldsymbol{W}}\le 0$, $\boldsymbol{W}(\boldsymbol{z})$ is the Lyapunov function of $\boldsymbol{f}_0(\boldsymbol{z})$, and $\boldsymbol{f}_0(\boldsymbol{z})$ is globally asymptotically stable. We choose

$$\boldsymbol{u}=(1-k_{\mathrm{a}})\boldsymbol{y}+v-38z_1, \tag{21}$$

where k_{a} is a positive real constant, v is the external input signal. Enter Eq.(21) into Eq.(16),

$$\begin{cases}\dot{x}_1=10(x_2-x_1),\\ \dot{x}_2=-10x_1-k_{\mathrm{a}}x_2-x_1x_3+v,\\ \dot{x}_3=x_1x_2-(8/3)x_3.\end{cases} \tag{22}$$

According to the designing method of nonlinear observer, the observer of Lorenz system has the following form

$$\begin{cases}\dot{\hat{x}}_1=-10(\hat{x}_1-\hat{x}_2)+L_1(k)(y-\hat{y}),\\ \dot{\hat{x}}_2=28\hat{x}_1-\hat{x}_2-\hat{x}_1\hat{x}_3+L_2(k)(y-\hat{y}),\\ \dot{\hat{x}}_3=\hat{x}_1\hat{x}_2-(8/3)\hat{x}_3+L_3(k)(y-\hat{y}),\\ \qquad\qquad \hat{y}=\hat{x}_1.\end{cases}$$

Let the initial value of observability parameter be $\boldsymbol{L}_0(k)=(6k\ 12k^2\ 8k^3)^{\mathrm{T}}$ so that the eigenvalues of $\boldsymbol{p}_n(s)$ are all -2, then $\boldsymbol{L}(k)=(6k\ 6k+1.2k^2\ 8k^3)^{\mathrm{T}}$.

Suppose the initial point of Lorenz system is (1,−1,1), and the initial point of the observer is (6,6,6). The controller $u=(1-k_{\mathrm{a}})\hat{x}_2+v-38\hat{x}_1$. Enter v=0 into the controller. Fig.1 is the state error output between the chaotic system and the observer when k_{a}=1, k=1. Fig.2 is the Lorenz system outputs when k_{a}=1, k=1. Fig.2 shows that it takes only very short time for the system to be rapidly stabilized at the equilibrium point (0,0,0) while the outputs of the observer approximate the state of the chaotic system.

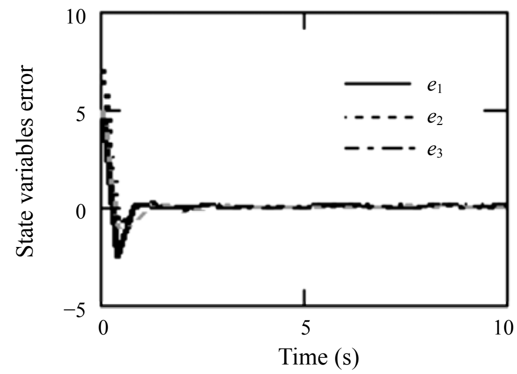

Fig.1 State variable error output between Lorenz system and the observer when k_{a}=1, k=1

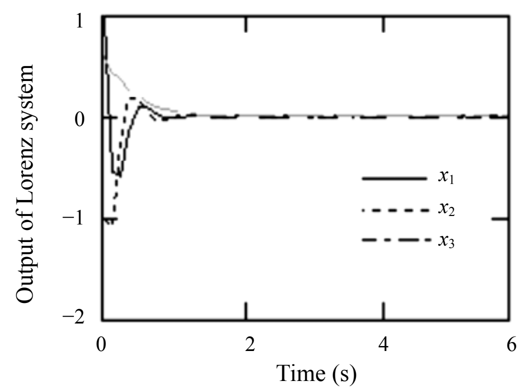

Fig.2 Output of Lorenz system when the equilibrium point is (0,0,0) and the control parameters k_{a}=1, k=1, v=0

We will study how to design the controller to stabilize the chaotic system at any equilibrium point (x_1^*,x_2^*,x_3^*). Let $\dot{x}=0$ and entering (x_1^*,x_2^*,x_3^*) into Eq.(21)

$$x_1^*=x_2^*,\ x_3^*=(3/8)x_1^*x_2^*,\ v=10x_1^*+k_{\mathrm{a}}x_2^*+x_1^*x_2^*.$$

Choose the new equilibrium point (1,1,0.375) and put $v=11+k_{\mathrm{a}}$ into Eq.(22). Fig.3 is the chaotic system outputs when k_{a}=0.02, k=1. Fig.3 shows that it takes only very short time for the system to be rapidly stabilized at the equilibrium point (1,1,0.375) while the outputs of the nonlinear observer approximate the state variables of the chaotic system. Therefore we can stabilize the chaotic system at any desired point through varying external input v.

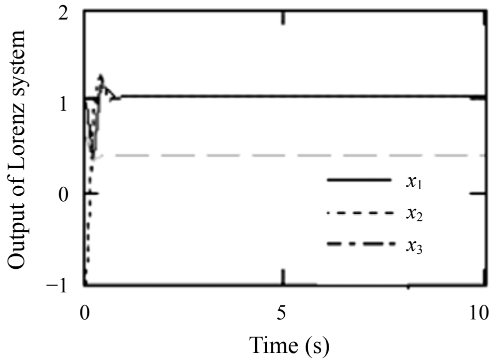

Fig.3 Output of Lorenz system when the equilibrium point is (1,1,0.375) and the control parameters k_a=0.02, k=1, v=0

CONCLUSION

A class of chaotic dynamical systems can be controlled with passive control theory. This control method combines chaotic systems with passive systems. Based on the nonlinear observer theory, weakly minimum phase nonlinear systems and minimum phase nonlinear systems transformed by chaotic systems can be globally asymptotically stabilized at the desired point by state feedback, whose state can be obtained by the nonlinear observer, and then implement control of the closed-loop system stabilization.

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