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Fractal modelling of off-road terrain oriented to vehicle virtual test*

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Abstract: In order to reconstruct typical off-road terrain surface for vehicle performance virtual test, a terrain generation method with controllable roughness was proposed based on fractal dimension. Transverse profile sampling and unevenness characteristics of typical off-road terrain were discussed according to the choices of appropriate wavelength and sampling interval. Since the off-road terrain in virtual environment is self-similar, the method of calculating the discrete fractal Gauss noise and its auto-correlation function were analyzed. The terrain surface fractal dimension was estimated by determining the Hurst coefficient. As typical off-road terrain is rugged terrain, the method of reconstructing it using fractal modelling is presented. The steps include calculating statistical variations in the absolute value of the difference in elevation between two points, plotting the points in log-log space, identifying linear segments and estimating fractal dimension from the linear segments slope. The constructed surface includes information on potholes, bumps, trend and unevenness of terrain, and can be used as the excitation of vehicle performance virtual test.

Key words: Off-road terrain, Transverse profile of terrain, Terrain surface reconstruction, Fractal dimension
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INTRODUCTION

Vehicle performance virtual test is an effective mode comparing to road test. The system modelling of each component, including vehicle, terrain, environment, plays an important role for test results. Some scholars have put forward efficient methods in this domain. For example, Schmeitz *et al*.(2004) presented both tyre and vehicle models over arbitrary road profiles, and investigated how the vehicle system behaved and how the enveloping model that generated an effective road surface contributed to this behavior. Sun (2001) adopted computer simulation and field measurement to investigate dynamic pavement loading generated by vehicle-pavement interaction and established an approximate relationship between road surface roughness and the measurement instrument coefficient of variation and vehicle speed. Some metrics have been put forward and used for measuring handling and ride comfort considering terrain conditions and tyre types (Uys *et al.*, 2006).

It is necessary for vehicle performance virtual test to model off-road terrain. Mathematical models for terrain or tyre displacement can provide high accuracy data for the simulation system, including vehicle positions, dynamic parameters and their real-time variables (Fukami *et al.*, 2006), and can also improve the fidelity and immersion of the system. Two approaches are usually adopted in terrain modelling: (1) Generating stochastic elevation curves along the vehicle's longitudinal direction according to the statistical indicators such as power spectrum density (PSD), variances, etc. This means is commonly used in the modelling of level roads; (2) Reconstructing the terrain surface using computer aided geometry design (CAGD) and computer graphics

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(CG) technology (Luo, 2005). The input terrain data are the unevenness of typical off-road terrain, including soft soil, sandy soil, quarry, and so on.

These works have contributed much to the technology for measuring and quantifying the road roughness in a meaningful way. The purpose of off-road terrain modelling includes: (1) describing the mechanics of tyres enveloping off-road profile features (Siddharthan *et al.*, 2005); (2) characterizing the 3D aspects of roughness for virtual world rather than using a 2D line along the wheel track.

Many problems in the reconstruction of terrain surface remain unsolved as yet, because: (1) since the sample number is finite, and many simplifications are adopted, the constructed terrain is usually so smooth that the model does not accord with reality; (2) there are some difficulties for capturing well the 3D aspects of terrain unevenness and the irregular and less structured shapes found on off-road terrain, such as ubiquitous potholes or bumps.

Mandelbrot (1982) proposed fractals as a family of mathematical functions for describing natural phenomena such as coastlines, mountains, etc. Fractal sets and functions have been found useful for describing many other environmental properties. Provided that the off-road terrain in virtual environment is self-similar, it is in accord with the condition for employing the fractal geometry theory. The benefits for terrain model reconstruction using fractal theory are clear, because it is possible for finite sampling data to regenerate complex natural terrain model in simulation system, and the model is not the same as smooth surfaces, but with roughness.

This paper addresses two issues in terrain fractal modelling. As a typical off-road terrain, the measuring technology for rugged terrain is discussed, and the terrain surface reconstruction algorithm for longitudinal and transverse profile data is presented.

SAMPLING AND CHARACTERIZING OF TERRAIN TRANSVERSE PROFILE

Decision on sampling wavelength and minimal sampling interval

Sample elevation data on the longitudinal section (Fig.1), vehicle velocity and measuring wavelength are listed in Table 1, where f_L and f_H represent the minimal and maximal frequency, and λ_{max} and λ_{min} represent wavelength of long and short wave, respectively. The measuring instrument and the sample size ensure the wavelength of long wave, while the sampling interval ensures the wavelength of short wave. Provided sampling interval is Δx, the wavelength λ_{min} equals $2\Delta x$, and $\Delta x=\lambda_{min}/2=208/2=104$ mm. And Δx varies from 50 mm to 150 mm when measuring.

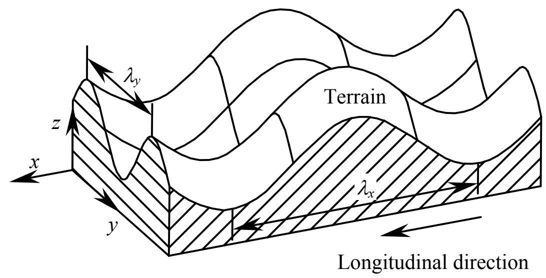


Fig.1 Terrain profile along longitudinal and transverse direction

Table 1 Vehicle velocity and the wavelength in longitudinal sampling

Terrain condition	v_{max} (km/h)	v_{min} (km/h)	f_L (s^{-1})	f_H (s^{-1})	λ_{max} (m)	λ_{min} (m)
Good	150	60	0.5	−	83.33	−
Moderate	60	30	−	−	−	−
Bad	30	15	−	40	−	0.208

Comparing with the longitudinal profile, the transverse profile unevenness affects mainly the front axle on wheel shimmy and the passenger on horizontal vibration. Since the transverse width of off-road terrain is narrow, the sampling interval should less than the value shown in Table 1, and the sampling frequency should larger for avoiding frequency aliasing. The choice criterion of sampling interval and frequency is that the obtained data can characterize the rugged micro profile. On the other hand, the maximal vehicle velocity is limited to 30 km/h for safety, and the minimal 15 km/h for efficiency. In our sampling work, the transverse interval Δy varies from 10 mm to 20 mm, and the sample size is determined by the terrain width, but it should be 2^m commonly, where m is integer. The sample number along x direction (shown in Fig.1) is no less than 10 for the assurance of total accuracy. Fig.2 shows the terrain photos we sampled at a quarry as typical off-road terrain. Fig.3 shows a sample of transverse terrain profile, in which some points are deleted for clarity.

Fig.2 Longitudinal surface profile of quarry terrain

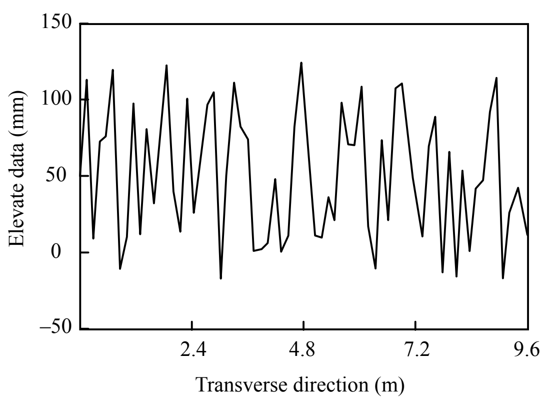


Fig.3 A sample of transverse terrain profile

Characterizing of transverse unevenness

For the reconstruction of smooth terrain, the longitudinal and transverse sampling points sets are enough for surface interpolation or fitness. But the surface should be of fidelity and can excite the running vehicle along vertical and lateral directions, so the variance, mean, PSD of sampling data are employed for characterizing transverse unevenness, and the input energy of saltation points on rugged terrain is determined by transient signal test technology. Fig.4 shows the PSD characterizing the transverse profile unevenness.

Given the sampling elevation data x_n, n=1, 2, …, N, and the mean μ, the estimated mean square is $\hat{\psi}^2 = \frac{1}{N}\sum_{n=1}^{N} x_n^2$, and the estimated variance is $\hat{\sigma}^2 = \frac{1}{N}\sum_{n=1}^{N}(x_n - \mu)$.

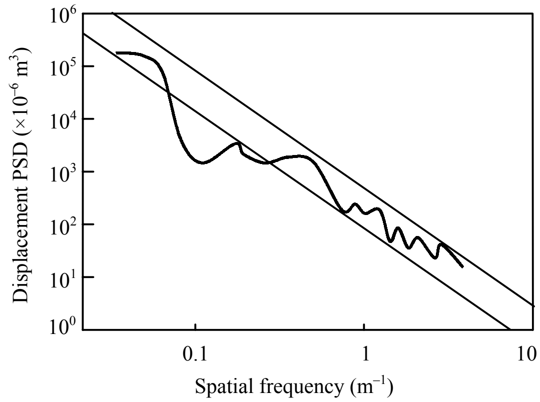


Fig.4 PSD of the sampled quarry

According to identical equation of Parseval, the PSD of terrain equals the sum of variances in time domain, and can be calculated by finite discrete Fourier transformation.

FRACTAL DIMENSION ESTIMATION OF RUGGED TERRAIN

Definitions

Fractal Brownian motion is a stochastic process defined as follows:

(1) $X(0)$=constant;

(2) $[X(t+\Delta r)-X(t)]\sim N(0, \Delta r\sigma^{2H})$, for $t\geq 0$ and $\Delta r>0$, i.e.

$$P[X(t+\Delta r)-X(t)\leq x] = \sqrt{(2\pi)}(\Delta r)^{-H}\int_{-\infty}^{x}\exp\left(\frac{-u^2}{2(\Delta r)^{-2H}}\right)\mathrm{d}u. \tag{1}$$

Note that $X(t+\Delta r)-X(t)$ and $P(t+\Delta s)-P(t+\Delta r)$ are statistically independent, $\Delta s>\Delta r>0$.

It can be concluded that the increment $X(t+\Delta r)-X(t)$ is steady, and its variance is in direct proportion to Δr^{2H}, i.e.

$$E[X(t+\Delta r)-X(t)]^2 \propto \Delta r^{2H}, \tag{2}$$

where H is Hurst coefficient. When H=0.5, it turns into typical Brownian motion. The fractal dimension of Brownian motion curve is $2-H$, while the surface is $3-H$. So the fractal dimension of off-road terrain in virtual environment can be estimated by the calculation of H.

Discrete fractal Gauss noise and its auto-correlation function

Provided that $B(n)$ and $B(n-m)$ are probability values at two points within normal distribution, we have

$$E[B(n)-B(n-m)]^2=c[n-(n-m)]^{2H}, \quad (3)$$

where c is constant. Define discrete fractal Gauss noise (DFGN) $I_m(n)=B(n)-B(n-m)$, then its variance is

$$\sigma^2=E[I_m(n)]^2=c[n-(n-m)]^{2H}=cm^{2H}. \quad (4)$$

The DFGN $I_m(n)$ is a stochastic process whose mean is zero and variance is $\sigma^2 m^{2H}$. Let m=1, then $E[I_m(n)]^2=c$, and $c=\sigma^2$.

Provided that $R_m(n+k, n)$ is auto-correlation function, we have

$$R_m(n+k, n)=E[I_m(n+k)I_m(n)] =(\sigma^2/2)[(k+m)^{2H}-2k^{2H}+(k-m)^{2H}]. \quad (5)$$

The DFGN is steady, and it has no relation to the positions, but has relation to the distance k between two points. Therefore, Eq.(5) becomes

$$R_m(k)=(\sigma^2/2)[(k+m)^{2H}-2k^{2H}+(k-m)^{2H}]. \quad (6)$$

Estimation of fractal dimension

It is non-linear between the auto-correlation function $R_m(k)$ and Hust coefficient H. Given $k=m$, we have

$$R(m)=(\sigma^2/2)[(2m)^{2H}-2m^{2H}]=(\sigma^2/2)(2^{2H}-2)m^{2H}. \quad (7)$$

Let m=1, then $R(1)=(\sigma^2/2)(2^{2H}-2)$ and $R(m)=R(1)m^{2H}$, so

$$\frac{1}{2}\ln\frac{R(m)}{R(1)}=H\cdot\ln(m). \quad (8)$$

Eq.(8) shows the linear relation between $\ln(m)$ and $\ln\frac{R(m)}{R(1)}$. Therefore, the fractal dimension can be obtained by $2-H$ for curve or $3-H$ for surface.

RECONSTRUCTION OF RUGGED TERRAIN USING FACTAL MODELLING

The basic task to reconstruct off-road terrain is to generate a spatial surface for the longitudinal and transverse profile data. The surface is obtained by the interpolation of those points, and it is not smooth but rough. The sampling interval of rugged terrain should not affect the surface roughness on the whole. Therefore, the reconstruction of terrain using fractal theory is related to the CAGD method, and is different from the smoothing algorithm.

In previous sections, we discussed the method of fractal dimension estimation; here, we consider elevation data $z(p)$ (with $p=(x,y)$). The procedure for estimating fractal dimension from a set of irregularly terrain sampled elevations $z(p)$ is described in the following three steps.

Computation of statistics of $\Delta z_{\Delta d}$

Let us consider two points on the xOy plane: (x, y) and $(x+dx, y+dy)$, as shown in Fig.5. The distance between them is $\Delta d=[(dx)^2+(dy)^2]^{1/2}$. We should obtain statistical variations in the absolute value of the difference in elevation between these two points: $\Delta z_{\Delta d}=|z_{x,y}-z_{x+dx, y+dy}|$.

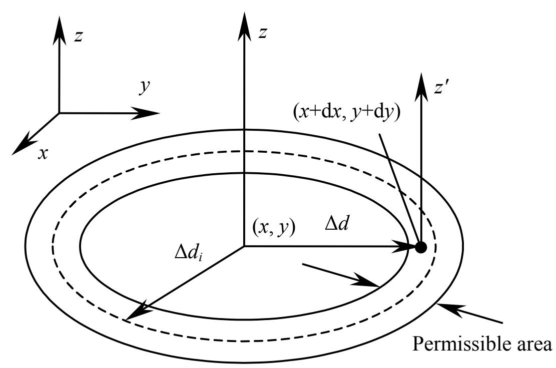


Fig.5 Accommodating irregular sampling intervals

Since the sample sizes and the sampling interval may be different from each other, and the corresponding points are distributed irregularly on the xOy plane, we should make the data distribute regularly. For i=0, 1, ..., m, and $\Delta d_k<\Delta d_{k+1}$, Let us prepare counters a_i, b_i and c_i to correspond to distance Δd_i. They are used to compute expected values, standard deviations, and numbers of sample pairs, respectively.

Let ε be a small distance that satisfies $0<\varepsilon<\Delta d_i$, for any i. It represents the width of a circular permissible area including a circle of radius Δd_j. Suppose there is a data point at (x+dx, y+dy) with elevation z'. If $|\Delta d_j-\Delta d|$ is less than ε, the point lies in the permissible area shown in Fig.5, and the counters a_j, b_j, and c_j should be updated as $a_j+|z-z'|$, $b_j+(z-z')^2$, and c_j+1, respectively.

After considering all pairs of data points, we ensure that c_i is larger than a threshold number of pairs. If c_i is small, we should check whether the samples number was sufficient. Otherwise, the sample standard deviation can be computed by (Pentland, 1984; Arakawa and Krotkov, 1996):

$$S_{\Delta d_i}=S[|z_{x,y}-z_{x+\mathrm{d}x,y+\mathrm{d}y}|]=\sqrt{\frac{B_i-(A_i^2/C_i)}{C_i-1}},$$

and the sample mean by

$$E_{\Delta di}=E[|z_{x,y}-z_{x+\mathrm{d}x,y+\mathrm{d}y}|]=A_i/C_i.$$

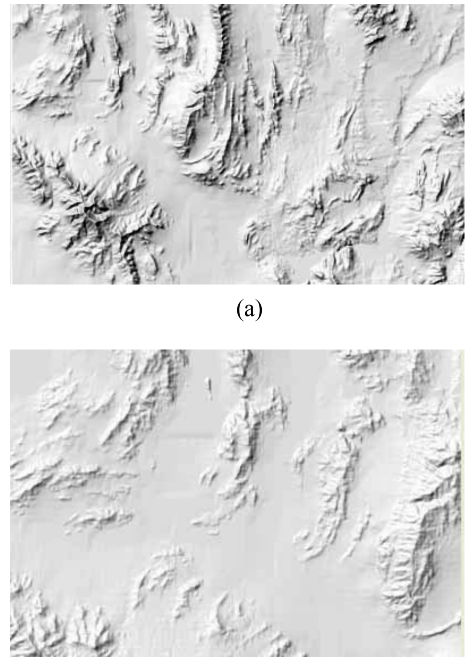
Plotting the points in log-log space and identifying linear segments

The point coordinates are ($\log\Delta d_i$, $\log S_{\Delta d_i}$) or ($\log\Delta d_i$, $\log E_{\Delta d_i}$). Because most terrains exhibit self-similarity only over certain scales, and not over all scales, it is necessary to segment sets of points that are linear. Therefore, iterative least-square line-fitting is employed. A set of points that lie within a specified distance of the line can be constructed according to this approach. This technique is not sensitive to the variation of the threshold value. In the case where several terrain surface patterns exist, each with a different fractal dimension, multiple linear segments appear in the log-log plot.

Fractal dimension estimation of terrain surface

Using the points on a line in the log-log space, the fractal dimension of the pattern can be estimated by the difference between the dimension of the pattern and the slope of the line formed by the points.

Changing the value of H can modify different three dimension shapes. And the roughness of the reconstructed terrain surface is modified, too. Fig.6 shows two terrain models with fractal dimension 2.50 and 2.25, respectively.



(a)

(b)

Fig.6 Terrain model with different fractal dimension. (a) Fractal dimension is 2.50; (b) Fractal dimension is 2.25

CONCLUSION

In this paper, the elevation data sampling technology of off-road transverse profile is discussed. The terrain is modelled using fractal geometry theory. Since the fractal dimension is controlled by modifying the Hurst coefficient, the reconstructed terrain surface with different roughness can be obtained conveniently. From the whole process of reconstruction, we can draw a conclusion that the surface combines the unevenness information with terrain model, while the unevenness is the excitation of the vehicle test. Therefore this terrain construction method provides the basic input to the simulation system for vehicle performance virtual test.

Although the constructed terrain surface is of fidelity and immersion, it is necessary for the terrain model to transform scenes continually in dynamic simulation. On the other hand, the application of terrain model in combination with tyre simulation should be investigated. The objective of this investigation is to analyze the 3D moving load considering the terrain response. The investigation addresses both the contact stress distribution of off-road vehicle tires and the responses of flexible off-road terrain, representing different environmental and material conditions. So the development of algorithms for terrain locomotion in virtual environment with efficiency is an important issue for continuing work.

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