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Application of grey relational analysis in sheet metal forming for multi-response quality characteristics*

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Abstract: The theory of grey systems is a new technique for performing prediction, relational analysis and decision making in many areas. In this paper, the use of grey relational analysis for optimizing the square hole flanging process parameters with considerations of the multiple response (the average flanging height, regular flanging and maximum strain) is introduced. Various flanging parameters, such as the blank inner radius r_b, blank inner width B_0, are considered. An orthogonal array is used for the experimental design. Multiple response values are obtained using finite element analysis (FEA). Optimal process parameters are determined by the grey relational grade obtained from the grey relational analysis for multi-performance characteristics (flanging height, regular flanging and maximum strain). Analysis of variance (ANOVA) for the grey relational grade is implemented. The results showed good agreement with the experiment result. Grey relational analysis can be applied in multiple response optimization designs.

Key words: Grey relational analysis, Flanging, Multiple response, Optimization design
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INTRODUCTION

Sheet metal forming is one of the most widely used manufacturing processes in many industries, especially in the automotive industry. However, in the case of complicated sheet metal deformation, improper design of process parameters will lead to defects such as fracture, wrinkle, excessively thinning, etc. In order to improve process efficiency, reduce the machining cost, and improve the quality of processed parts, it is necessary to select the most appropriate process conditions. To avoid consuming time in trial and error tryout procedure and facilitate modification, sheet metal forming simulations were used to evaluate or avoid forming defect. However, it is still a very difficult problem to obtain the optimum result by running simulation code when many parameters need to be determined, because each running of simulation will involve much computation time for a complicated sheet metal process.

In order to minimize these process problems, there is need to develop scientific methods to select forming conditions for sheet metal forming. Most published works focus on optimization of parameters for sheet metal forming applying optimization technology. Huh and Kim (2001) adopted a direct differentiation method and a response methodology to seek for the optimum condition of process parameters. Gantar *et al*.(2002) suggested combining finite element method (FEM) result and visual inspection to optimize process parameters, and applied the method into some complicated parts. Kleiber *et al*.(2002) reported a reliability assessment for sheet metal forming operations. Browne and Hillery (2003) used Taguchi's orthogonal design to investigate the variation and effects of factors. Barlet *et al*.(1996) incorporated an inverse FEM with a sequential quadratic programming (SQP) to perform blank forming optimization. Nakamura *et al*.(1998) developed sweeping

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simplex method and simulated annealing method to optimize die shape. Jakumeit *et al*.(2005) used an iterative parallel Kriging algorithm to the parameter optimization of the sheet metal forming process. Li *et al*.(2006a; 2006b) used the six sigma robust design method and response surface model for sheet metal forming. Huang *et al*.(2004) combined parametric finite element analysis (FEA), artificial neural networks and genetic algorithm to research on the problems of process parameter optimization of sheet metal forming process.

In a system that is complex and multi-variable, the relationship between various factors such as those described above is unclear. Such systems are often called “grey” implying poor, incomplete, and uncertain information. Their analysis by classical statistical procedures may not be acceptable or reliable without large datasets that satisfy certain mathematical criteria. The grey theory, on the other hand, uses relatively small datasets and does not demand strict compliance to certain statistical laws, such as simple or linear relationships among the observables (Tosun, 2006).

The purpose of the present work is to introduce the use of grey relational analysis in selecting optimum forming conditions on multi-performance characteristic. To the best knowledge of the author of this work, there is no published work evaluating the optimization and the effect of forming process by using grey relational analysis in sheet metal. The setting of forming parameters is accomplished using the Taguchi experimental design method. By properly adjusting the control factors, we can improve work efficiency and produce quality parts.

GREY RELATIONAL ANALYSIS

In grey relational analysis, black represents having no information and white represents having all information. A grey system has a level of information between black and white. In other words, in a grey system, some information is known and some information is unknown. In a white system, the relationships among factors in the system are certain; in a grey system, the relationships among factors in the system are uncertain (Deng, 1989).

Grey relational analysis is an impacting measurement method in grey system theory that analyzes uncertain relations between one main factor and all the other factors in a given system. It is actually a measurement of the absolute value of the data difference between sequences, and can be used to measure the approximate correlation between sequences (Tosun, 2006; Ho and Lin, 2003; Chiang and Chang, 2006).

Data pre-processing

Data pre-processing is normally required since the range and unit in one data sequence may differ from the others. Data pre-processing is also necessary when the sequence scatter range is too large, or when the directions of the target in the sequences are different. Data pre-processing is a means of transferring the original sequence to a comparable sequence. Depending on the characteristics of a data sequence, there are various methodologies of data pre-processing available for the grey relational analysis (Tosun, 2006; Deng, 1989; Ho and Lin, 2003; Morán *et al*., 2006).

If the target value of the original sequence is infinite, then it has a characteristic of the “higher is better”. The original sequence can be normalized as follows:

$$x_i(k)=\frac{x_i^0(k)-\min_i x_i^0(k)}{\max_i x_i^0(k)-\min_i x_i^0(k)}. \tag{1}$$

When the “lower is better” is a characteristic of the original sequence, then the original sequence should be normalized as follows:

$$x_i(k)=\frac{\max_i x_i^0(k)-x_i^0(k)}{\max_i x_i^0(k)-\min_i x_i^0(k)}. \tag{2}$$

However, if there is a definite target value (desired value) to be achieved, the original sequence will be normalized as:

$$x_i(k)=1-\frac{\left|x_i^0(k)-x^0\right|}{\max_i x_i^0(k)-x^0}, \tag{3}$$

where i=1, …, m; k=1, …, n. m is the number of experimental data items and n is the number of parameters. $x_i^0(k)$ denotes the original sequence, $x_i(k)$ the sequence after the data pre-processing, $\max_i x_i^0(k)$, $\min_i x_i^0(k)$ the largest value and the smallest value of $x_i^0(k)$, x^0 is the desired value of $x_i^0(k)$.

Grey relational coefficient and grey relational grade

In grey relational analysis, the measure of the relevancy between two systems or two sequences is defined as the grey relational grade. When only one sequence $x_0(k)$ is available as the reference sequence, and all other sequences serve as comparison sequences, it is called a local grey relation measurement. After data pre-processing is carried out, the grey relation coefficient $\xi_i(k)$ for the kth performance characteristics in the ith experiment can be expressed as (Tosun, 2006; Deng, 1989; Lo, 2002):

$$\xi_i(k)=\frac{\Delta_{\min}+\rho\Delta_{\max}}{\Delta_{0i}(k)+\rho\Delta_{\max}}, \tag{4}$$

where $\Delta_{0i}(k)$ is the deviation sequence of the reference sequence and the comparability sequence.

$$\Delta_{0i}(k)=\left|x_0(k)-x_i(k)\right|,$$
$$\Delta_{\min}=\min_{\forall j\in i}\min_{\forall k}\left|x_0(k)-x_i(k)\right|,$$
$$\Delta_{\max}=\max_{\forall j\in i}\max_{\forall k}\left|x_0(k)-x_i(k)\right|,$$

where $X_0=\{x_0(k), k=1, 2, \ldots, n\}$ denotes the reference sequence and $X_i=\{x_i(k), k=1, 2, \ldots, n\}$ denotes the comparability sequence. $\rho\in[0,1]$ is the distinguishing or identification coefficient. $\rho=0.5$ is generally used.

After the grey relational coefficient is derived, the grey relational grade is defined as follows (Tosun, 2006; Deng, 1989; Lo, 2002):

$$R_{i0}=\frac{1}{n}\sum_{k=1}^{n}\lambda_k\xi_i(k), \qquad \sum_{k=1}^{n}\lambda_k=1. \tag{5}$$

The grey relational grade represents the level of correlation between the reference sequence and the comparability sequence. If the two sequences are identical, then the value of grey relational grade is equal to 1. The grey relational grade also indicates the degree of influence that the comparability sequence could exert over the reference sequence. Therefore, if a particular comparability sequence is more important than the other comparability sequences to the reference sequence, then the grey relational grade for that comparability sequence and reference sequence will be higher than other grey relational grades (Tosun, 2006; Ho and Lin, 2003; Chan and Tong, 2007).

CASE IMPLEMENTATON

Square hole flanging is applied in sheet metal forming (Huang *et al*., 2004). The forming schematic of flanging is shown in Fig.1. The material is pure aluminum sheet (L_2). The thickness t=1.0 mm, and the hardness value (HV) HV=40. The outer width of the square metal sheet B=1000 mm. B_0 and r_b are the initial square width and inter radius. B_1 and r are the square width and inter radius after flanging. Because the flanging is mainly affected by B_0 and r_b in the above parameters, B_0 and r_b are selected as optimization parameters. B_0 and r_b are subject to the following equations:

$$B_0=B_1-2h+0.86r_d+(3\sim4)t,\ r_b=r/2, \tag{6}$$

where h is the flanging height, r_d is the radius of the flanging die.

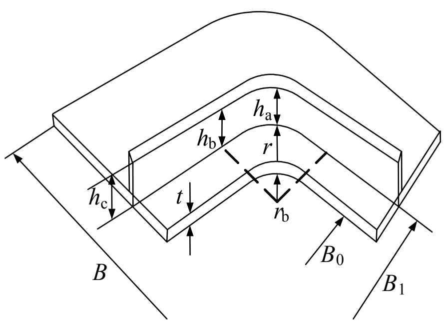

Fig.1 Schematic representation of square hole flanging

Regarding the symmetry, one fourth of the forming is modelled. In this study, the Taguchi method—a powerful tool for parameter design of performance characteristics—is used. According to the Taguchi quality design concept (Lin *et al*., 2006), an orthogonal array with 17 rows (corresponding to the number of simulation experiments) is used for the simulation experiments. The multiple responses in different parameters are summarized in Table 1 (Huang *et al*., 2004).

Table 1 The result of finite element analysis with different parameters

B_0 (mm)	r_b (mm)	h_a (mm)	h_b (mm)	h_c (mm)	ε_{max}
35	4	3.254	3.661	4.039	0.208
33	4	4.420	4.788	5.049	0.336
31	4	5.652	5.854	6.067	0.439
29	4	6.999	6.924	7.062	0.609
35	5	3.569	3.840	4.039	0.216
33	5	4.828	4.895	5.049	0.263
31	5	6.065	5.941	6.065	0.361
29	5	7.554	6.976	7.063	0.492
35	6	3.948	3.946	4.038	0.186
34	6	4.720	4.660	4.740	0.257
33	6	5.170	4.978	5.049	0.266
31	6	6.545	6.007	6.064	0.361
29	6	7.735	7.014	7.067	0.430
35	7	4.291	4.007	4.038	0.206
33	7	5.570	5.029	5.048	0.253
31	7	6.872	6.042	6.061	0.346
29	7	8.338	7.053	7.074	0.511

Multi-response of the square hole flanging

The multi-response quality characteristics of the flanging are defined as follows:

(1) The average flanging height:

$$Y_1 = (h_a + h_b + h_c)/3. \quad (7)$$

(2) The bias from the flanging height:

$$Y_2 = \sqrt{\frac{(h_a - Y_1)^2 + (h_b - Y_1)^2 + (h_c - Y_1)^2}{3}}. \quad (8)$$

(3) Maximum strain:

$$Y_3 = \varepsilon_{max}, \quad (9)$$

where h_a, h_b, h_c are the flanging heights.

At last, the multi-response is summarized in Table 2.

Table 2 Initial multi-response of the flanging

B_0 (mm)	r_b (mm)	Y_1 (mm)	Y_2 (mm)	Y_3
35	4	3.651	0.321	0.208
33	4	4.752	0.258	0.336
31	4	5.858	0.169	0.439
29	4	6.995	0.056	0.609
35	5	3.816	0.193	0.216
33	5	4.924	0.093	0.263
31	5	6.024	0.058	0.361
29	5	7.198	0.254	0.492
35	6	3.977	0.043	0.186
34	6	4.707	0.034	0.257
33	6	5.066	0.079	0.266
31	6	6.205	0.241	0.361
29	6	7.272	0.328	0.430
35	7	4.112	0.127	0.206
33	7	5.216	0.251	0.253
31	7	6.319	0.391	0.346
29	7	7.488	0.601	0.511

Analysis and discussion of experimental result

In the present study, higher flanging height, lower bias and maximum strain are indications of better performance. For data pre-processing in the grey relational analysis process, y_1 is taken as the "higher is better". However, y_2, y_3 are taken as the "lower is better". Let the result of 17 experiments be the comparability sequence $x_i^0(k)$, i=1~17, k=1~3. All the sequences after data pre-processing using Eqs.(1) and (2) are listed in Table 3 and denoted as $x_0(k)$ and $x_i(k)$ for reference sequence and comparability sequence, respectively.

The deviation sequences can be calculated as follows:

$$\Delta_{01}(1) = |x_0(1) - x_1(1)| = |1.000 - 0.000| = 1.000,$$
$$\Delta_{01}(2) = |x_0(2) - x_1(2)| = |1.000 - 0.494| = 0.506,$$
$$\Delta_{01}(3) = |x_0(3) - x_1(3)| = |1.000 - 0.948| = 0.052,$$

So Δ_{01}=(1.000, 0.506, 0.052).

The same calculation method is performed for i=1~17. The grey relational coefficients and grade values for each experiment of the orthogonal array are calculated by applying Eqs.(4) and (5). The grey relational grade is shown in Table 4.

According to the performed experiment design, it is clearly observed from Table 4 that the forming parameters' setting of experiment No. 10 has the highest grey relational grade. Therefore, experiment No. 10 is the optimal machining parameters' setting for minimum bias, strain, and maximum flanging height simultaneously (i.e. the best multi-performance characteristics) among the above experiments.

In addition to the determination of optimum forming parameters, the response table for the Taguchi method is used to calculate the average grey relational grade for each level of the forming parameters. The procedure is: (1) group the grey relational grades by factor level for each column in the orthogonal array; (2) take the average of them.

The average grey relational grade values are shown in Table 5. Since the grey relational grade represents the level of correlation between the reference sequence and the comparability sequence, the greater value of the grey relational grade means that the comparability sequence has a stronger correlation to the reference sequence at this level of the factor (Tosun, 2006; Ho and Lin, 2003). In other words, regardless of category of the performance characteristics, a greater grey relational grade value corresponds to better performance (Tosun, 2006; Lin *et al.*, 2006). Therefore, the optimal level of the process parameters is the level with the greatest grey relational grade value. An asterisk (*) indicates that the level value results in a better forming performance. Based on the grey relational grade values given in Table 5, the optimal forming performance for multiple response is obtained for B_0 (level 5), r_b (level 3) combination. Fig.2 shows the effect of forming parameters on the multi-performance characteristics and the response graph of each level of the forming parameters for the performance.

As listed in Table 5, the difference between the maximum and the minimum value of the grey relational grade of the forming parameters indicates the significance of the role that every controllable factor plays over the multi-performance characteristics. It is shown that the forming performance is mainly affected by the B_0 in B_0 and r_b.

Table 3 The multi-response sequences after data pre-processing

B_0 (mm)	r_b (mm)	Y_1 (mm)	Y_2 (mm)	Y_3
35	4	0.000	0.494	0.948
33	4	0.287	0.605	0.645
31	4	0.575	0.761	0.402
29	4	0.871	0.960	0.000
35	5	0.043	0.720	0.929
33	5	0.332	0.897	0.818
31	5	0.618	0.957	0.586
29	5	0.924	0.611	0.277
35	6	0.085	0.984	1.000
34	6	0.275	1.000	0.832
33	6	0.369	0.920	0.811
31	6	0.666	0.634	0.586
29	6	0.944	0.481	0.423
35	7	0.120	0.836	0.953
33	7	0.408	0.618	0.842
31	7	0.695	0.370	0.622
29	7	1.000	0.000	0.232

Table 4 The calculated grey relational grades and their order

Exp. No.	Relational grade	Order
1	0.168	15
2	0.177	14
3	0.209	8
4	0.280	3
5	0.202	10
6	0.246	6
7	0.271	4
8	0.203	9
9	0.283	2
10	0.286	1
11	0.255	5
12	0.193	11
13	0.190	12
14	0.230	7
15	0.188	13
16	0.164	16
17	0.158	17

Table 5 Response table for the grey relational grade of the factors at different levels

Parameter	Average grey relational grade by factor level					
	Level 1	Level 2	Level 3	Level 4	Level 5	Max-min
B_0	0.221	0.217	0.209	0.208	0.286*	0.078
r_b	0.209	0.231	0.241*	0.185	–	0.056

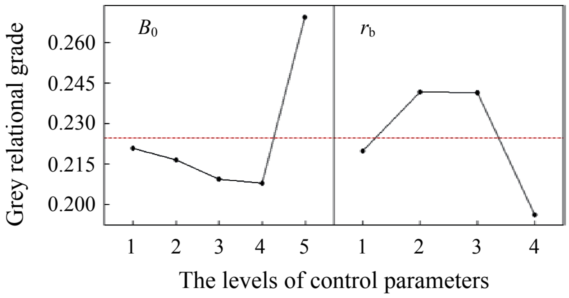

Fig.2 Effect of control parameter levels on the multi-performance

Verify the optimization result

From the above analysis, the optimum values are 34 mm (level 5) for B_0, 6 mm (level 3) for r_b. The square hole flanging is simulated making use of the above optimum parameters. The average height from this simulation is 4.7 mm, which is consistent with the result in (Huang *et al*., 2004) and consistent with reported experiment result in (Lu *et al*., 2000).

CONCLUSION

In this paper, the optimal forming parameters are determined for the multi-performance characteristics (minimum flanging bias, strain and maximum flanging height) in the flanging process by using the grey relational analysis.

The grey relational analysis, based on the Taguchi method's response table, is proposed as a way of studying the optimization of forming process parameters in sheet metal. Flanging bias, strain and maximum flanging height are selected to be the quality targets. From the response table of the average grey relational grade, the largest value of grey relational grade for the forming parameters is found. These corresponding values are the recommended levels of controllable forming parameters for the multi-performance characteristics. It is found that B_0 has stronger effect than r_b on the multi-performance characteristics. The optimum values are 34 mm (level 5) for B_0, 6 mm (level 3) for r_b. The optimization result is consistent with the reported experiment result.

This study indicates that grey relational analysis approach can be applied successfully to other researches in which performance is determined by many parameters at multiple quality requests.

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