Journal of Zhejiang University SCIENCE A
ISSN 1673-565X (Print); ISSN 1862-1775 (Online)
www.zju.edu.cn/jzus; www.springerlink.com
E-mail: jzus@zju.edu.cn

Mathematical modelling of translational motion of rail-guided cart with suspended payload

MITREV Rosen
(*Department of Engineering Logistics, Material handling and Construction Machines, Technical University of Sofia, Sofia 1000, Bulgaria*)
E-mail: rosenm@tu-sofia.bg

Received Mar. 15, 2007; revision accepted Apr. 23, 2007

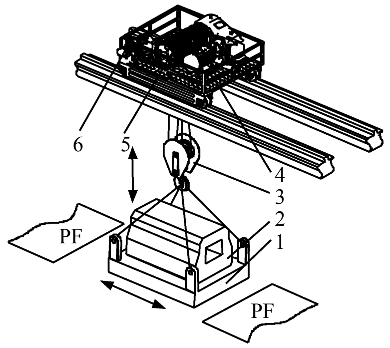
Abstract: In this paper modelling of the translational motion of transportation rail-guided cart with rope suspended payload is considered. The linearly moving cart, driven by a travel mechanism, is modelled as a discrete six degrees of freedom (DOF) dynamic system. The hoisting mechanism for lowering and lifting the payload is considered and is included in the dynamic model as one DOF system. Differential equations of motion of the cart elements are derived using Lagrangian dynamics and are solved for a set of real-life constant parameters of the cart. A two-sided interaction was observed between the swinging payload and the travel mechanism. Results for kinematical and force parameters of the system are obtained. A verification of the proposed model was conducted.

Key words: Transportation rail-guided cart, Travel mechanism, Swinging payload, Discrete dynamical model, Mathematical model
doi:10.1631/jzus.2007.A1395 **Document code:** A **CLC number:** TN929.5

INTRODUCTION

An important element of mass line production of reinforced concrete and steel part is transportation of half-finished product between different production facilities. There are known that many different structural variations of machines used for conducting this operation (Valev, 1988). One widely used solution, especially suitable in case of heavy elements, is a special-purpose transportation rail-guided cart with rectilinear motion and rope suspended payload. A general view of such transportation cart is shown in Fig.1. The operating principles of the transportation cart are as follows: load-handling device 1, which carries the payload 2, is suspended from the transportation cart 4 by polyspast system 3. The cart moves on rails between two adjacent production facilities (PF) and is driven by a travel mechanism 6. The payload is transported vertically by drum type hoisting mechanism 5.

The experimental and theoretical research, as well as simple visual observations, showed that the rectilinear translational motion of the cart is accompanied by vibrational processes, such as payload swinging, cyclic increasing and decreasing of linear velocity of the cart, vibrations in the travel mechanism elements, etc. These processes result from interaction between the cart travel mechanism and swinging payload. There is deep concern for creating a mathematical model that can be used for investigation of such problems like: transient processes in driveline kinematics, kinematical and force parameters of the travel mechanism, maximal dynamical loading of the elements, etc. Particular interest could be paid to the investigation of the influence of the swinging payload on the kinematical and force loading of travel mechanism and conversely—influence of the travel mechanism parameters on the payload swinging. This two-sided interaction between travel mechanism and swinging payload affects positional accuracy of the payload, safety in operation, strength and fatigue of the machine elements, etc.



1: load-handling device; 2: payload; 3: polyspast system; 4; transportation cart; 5: drum type hoisting mechanism; 6: travel mechanism

Fig.1 General view of the transportation rail-guided cart with rope suspended payload

LITERATURE REVIEW AND OBJECTIVE OF THE STUDY

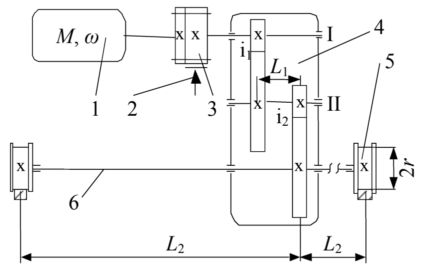
To our best knowledge, the described problem has not received sufficient attention in the accessible literature. Very few studies consider mathematical modelling of rectilinear motion of the cart and envisage the parameters of travel mechanism kinematics and its interaction with payload. Most published studies (Abdel-Rahman *et al*., 2003; Corriga *et al*., 1998; Eksarov and Grigorov, 1981; Al-mousa and Kachroo, 2003; Omar, 2003; Pauluk, 2001) present models suitable for control of movement of different kinds of cranes and crane carts and resultant payload swinging, but without considering detailed modeling of the travel mechanism kinematics. Some works (Petkov *et al*., 1980; Scheffler *et al*., 1977) consider simplified models of the travel mechanism and assumption of small angle of payload swinging, which models are suitable only for qualitative estimation of the two-sided interaction between payload swinging and the travel mechanism. Other works (Jerman, 2006; Ju *et al*., 2006) are devoted to investigation of the influence of the swinging payload on the cranes steel structure behavior.

The method of detailed modelling of the driveline is known (Pettersson, 1997; Rahnejat, 1998) and is used in the present work.

For solving the problems listed above, the main objective of this paper is: to propose a mathematical model of rectilinear translational motion of the rail-guided cart with suspended payload, by which to investigate the two-sided interaction between the swinging payload and kinematical and force loading parameters of the travel mechanism.

KINEMATICS OF THE TRAVEL MECHANISM

The kinematics of the travel mechanism of the cart under consideration and some of its parameters are shown in Fig.2.



1: electric motor; 2: jaw brake; 3: elastic coupling; 4; centrally mounted two stage gearbox; 5: driving wheels; 6: slow-speed shaft; L_1, L_2: lengths of the shafts of the travel mechanism; I, II: gear stages of the two stage gearbox; i_1, i_2: gear ratios of the gearbox I and II stages respectively

Fig.2 Kinematics of the travel mechanism

The electric motor 1 exerts a driving moment, which is transferred to the driving wheels 5 by a centrally mounted two stage gearbox 4 and a slow-speed shaft 6, consisting of two sections with length L_2. The electric motor is connected with gearbox by an elastic coupling 3. Stopping of the cart is realized by jaw brake 2.

MODELLING OF THE CART'S LINEAR MOTION

Dynamical model of the cart

The real cart is a complex mechanical system, consisting of several subsystems: travel mechanism, hoisting mechanism, different auxiliary mechanisms, etc. All subsystems mutually affect each other. The proposed discrete dynamic model of the cart (Fig.3) includes only systems and parts, which affect the translational motion of the cart and has six degrees of freedom (DOF). The bodies are connected by springs and dampers and perform rotational and translational motions under the applied forces and moments. The motion of the bodies is described by the generalized coordinates, shown in Fig.3. The vector of the generalized coordinates of the system has the following form:

$$\{\boldsymbol{q}\}=\{q_1\ q_2\ q_3\ q_4\ q_5\ q_6\}^{\mathrm{T}}.$$

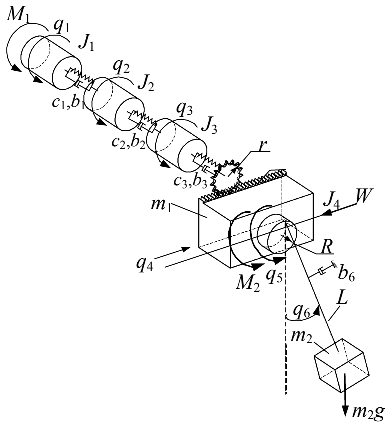


Fig.3 Dynamic model of the linearly moving cart

Generalized coordinates q_1, q_2 and q_3 are associated with motion of the driveline components (Fig.2); q_4 represents the linear motion of the cart; q_5 represents the rotation of the hoisting drum and q_6 represents payload swinging.

The following notations are used:

(1) Mass and inertia parameters of the system: J_1: sum of the reduced mass moments of inertia of the electric motor rotor and the coupling driving disk; J_2: sum of reduced mass moments of inertia of the coupling driven disk, driven and driving gearwheels of the first gear stage; J_3: sum of reduced mass moments of inertia of the driving and driven gearwheels of the second gear stage. All mass moments of inertia are reduced to the driving slow-speed shaft 6 (Fig.2), mass moments of inertia of the gearbox shafts are added to corresponding discrete rotational masses; m_1: sum of mass of the cart and reduced to mass moments of inertia of the driving wheels; m_2: sum of mass of the payload and operating element; J_4: sum of reduced to the driving drum mass moments of inertia of the hoisting mechanism elements;

(2) Geometrical parameters of the system: L: initial length of the suspending rope, measured from the common mass center of the payload and operating element; R: radius of the hoisting mechanism drum; r: radius of the driving wheel of the cart;

(3) Elastic and damping parameters of the system: c_1,b_1: equivalent coefficients of angular stiffness and damping of elastic coupling and electric rotor motor; c_2,b_2: coefficients of angular stiffness and damping of shaft with length L_1 (Fig.2); c_3,b_3: equivalent coefficients of angular stiffness and damping of slow-speed shaft (Fig.2); parameters c_1,b_1,c_2,b_2 are reduced to the slow speed shaft; b_6: coefficient of angular damping of the swinging payload;

(4) Force parameters: $M_1(\dot{q}_1)$: reduced to the slow-speed shaft torque of the travel mechanism electric motor; $M_2(\dot{q}_5)$: torque of the hoisting mechanism electric motor, reduced to drum of the hoisting mechanism; W: resistance of the cart movement.

Damping of the payload oscillations has a complex nature and various origins (Ely, 1997). In the present work we consider that damping of the oscillations is proportional to its speed by damping coefficient b_6. By reason of the insignificant influence of most of the parameters of the hoisting mechanism on the rectilinear motion of the cart and payload swinging, it is represented by a single DOF mechanism. Its purpose in the model is to consider variation of rope length (and accompanying change of the amplitude and the frequency of payload oscillations) when the linear motion of the cart is combined with payload lifting or lowering.

There are several assumptions accepted in the proposed dynamical model: payload and rope behave as a mathematical pendulum; the rope is mass-less and non extensible; there is no slipping between the driving wheels and rails (holonomic constraint); stiffness of the steel frame of the cart is much bigger than stiffness of the elastic joints; the relative oscillations of the bodies are small; the damping forces are proportional to the velocity; gearwheels are considered as absolutely stiff, the shafts are elastic; air resistance is neglected by reason of the low speeds of motion of the cart.

Mathematical model of the cart

The differential equations of motion of the mechanical system are derived using Lagrange's equations of the second kind:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial T}{\partial \dot{q}_i}\right)-\frac{\partial T}{\partial q_i}+\frac{\partial U}{\partial q_i}+\frac{\partial \Phi}{\partial \dot{q}_i}=Q_i,\quad i=1,...,6. \tag{1}$$

$\{\ddot{\boldsymbol{q}}\}$ and $\{\dot{\boldsymbol{q}}\}$ are denoted as the vectors of generalized accelerations and velocities respectively:

$$\{\ddot{\boldsymbol{q}}\}=\{\ddot{q}_1\ \ddot{q}_2\ \ddot{q}_3\ \ddot{q}_4\ \ddot{q}_5\ \ddot{q}_6\}^{\mathrm{T}}, \tag{2}$$

$$\{\dot{\boldsymbol{q}}\} = \{\dot{q}_1\ \dot{q}_2\ \dot{q}_3\ \dot{q}_4\ \dot{q}_5\ \dot{q}_6\}^{\mathrm{T}}. \tag{3}$$

The total kinetic energy of the system T in terms of generalized coordinates and velocities is

$$\begin{aligned} T &= \frac{1}{2}J_1\dot{q}_1^2 + \frac{1}{2}J_2\dot{q}_2^2 + \frac{1}{2}J_3\dot{q}_3^2 + \frac{1}{2}J_4\dot{q}_5^2 + \frac{1}{2}m_1\dot{q}_4^2 + \\ &\dots + \frac{1}{2}m_2[\dot{q}_6(L-Rq_5)\cos q_6 - \dot{q}_5 R\sin q_6 + \dot{q}_4]^2 + \\ &\dots + \frac{1}{2}m_2[\dot{q}_6(L-Rq_5)\sin q_6 - \dot{q}_5 R\cos q_6]^2. \end{aligned} \tag{4}$$

The total potential energy of the system U is

$$\begin{aligned} U &= \frac{1}{2}c_1(q_1-q_2)^2 + \frac{1}{2}c_2(q_2-q_3)^2 + \dots \\ &+ \frac{1}{2}c_3(q_3-q_4/r)^2 - m_2 g(L-Rq_5)\cos q_6. \end{aligned} \tag{5}$$

The total dissipation energy of the system Φ is

$$\begin{aligned} \Phi &= \frac{1}{2}b_1(\dot{q}_1-\dot{q}_2)^2 + \frac{1}{2}b_2(\dot{q}_2-\dot{q}_3)^2 + \\ &\dots + \frac{1}{2}b_3(\dot{q}_3-\dot{q}_4/r)^2 + b_6\dot{q}_6. \end{aligned} \tag{6}$$

The generalized forces Q_i corresponding to the generalized coordinates Eq.(1) are derived by means of virtual work method.

The obtained six second order nonlinear differential equations are complex and for convenience are represented in matrix form

$$[\boldsymbol{A}]\{\ddot{\boldsymbol{q}}\} + [\boldsymbol{B}]\{\dot{\boldsymbol{q}}\} + [\boldsymbol{C}]\{\boldsymbol{q}\} + \{\boldsymbol{N}\} = \{\boldsymbol{Q}\}. \tag{7}$$

The following notations apply in Eq.(7):

$[\boldsymbol{A}]_{6\times6}$: mass-inertia matrix of the system with variable coefficients, which have the following form:

$$[\boldsymbol{A}]_{6\times6} = \begin{bmatrix} [\boldsymbol{A}_1]_{3\times3} & [\boldsymbol{0}]_{3\times3} \\ [\boldsymbol{0}]_{3\times3} & [\boldsymbol{A}_2]_{3\times3} \end{bmatrix}, \tag{8}$$

where $[\boldsymbol{A}_1]_{3\times3}=\mathrm{diag}(J_1, J_2, J_3)$; $[\boldsymbol{A}_2]_{3\times3}=\lfloor a_{ij}\rfloor$, i=1,…,3, j=1,...,3; $a_{11}=m_1+m_2$; $a_{21}=a_{12}=-m_2R\sin q_6$; $a_{22}=J_4+m_2R^2$; $a_{31}=a_{13}=m_2(L-Rq_5)\cos q_6$; $a_{32}=a_{23}=0$; $a_{33}=m_2(L-Rq_5)^2$.

$[\boldsymbol{0}]_{3\times3}$: zero matrix; $[\boldsymbol{C}]_{6\times6}$: matrix of elasticity, which has the following notation:

$$[\boldsymbol{C}]_{6\times6}=\lfloor c_{ij}\rfloor_{6\times6},\quad i=1,\dots,6;\, j=1,\dots,6, \tag{9}$$

where $c_{11}=c_1$; $c_{21}=c_{12}=-c_1$; $c_{22}=c_1+c_2$; $c_{32}=c_{23}=-c_2$; $c_{33}=c_2+c_3$; $c_{43}=c_{34}=-c_3/r$; $c_{44}=c_3/r^2$.

$[\boldsymbol{B}]_{6\times6}$: matrix of damping, which has the following notation:

$$[\boldsymbol{B}]_{6\times6}=\lfloor b_{ij}\rfloor_{6\times6},\ i=1,\dots,6;\, j=1,\dots,6, \tag{10}$$

where: $b_{11}=b_1$; $b_{21}=b_{12}=-b_1$; $b_{22}=b_1+b_2$; $b_{32}=b_{23}=-b_2$; $b_{33}=b_2+b_3$; $b_{43}=b_{34}=-b_3/r$; $b_{44}=c_3/r^2$; $b_{66}=b_6$.

$\{\boldsymbol{N}\}_{6\times1}$: vector consisting of Coriolis and centrifugal terms

$$\{\boldsymbol{N}\}_{6\times1}=\{n_i\}_{6\times1},\quad i=1,\dots,6, \tag{11}$$

where

$$\begin{aligned} n_4 &= m_2\dot{q}_6^2(L-Rq_5)\sin q_6 - 2m_2R\dot{q}_6\dot{q}_5\cos q_6, \\ n_5 &= m_2\dot{q}_6^2R(L-Rq_5), \\ n_6 &= -2m_2\dot{q}_6\dot{q}_5R(L-Rq_5). \end{aligned}$$

All elements in Eqs.(8)~(11) with indexes different from those pointed above are equal to zero.

The vector of generalized forces $\{\boldsymbol{Q}\}$ has the following form:

(1) For starting period of the cart, combined with lifting or lowering of the payload:

$$\{\boldsymbol{Q}\}=\{Q_1\ Q_2\ Q_3\ Q_4\ Q_5\ Q_6\}^{\mathrm{T}}, \tag{12}$$

where $Q_1 = M_1(\dot{q}_1)$; $Q_2=Q_3=0$; $Q_4=-W$; $Q_5 = M_2(\dot{q}_5) - m_2gR\cos q_6$; $Q_6=m_2g(L-Rq_5)\sin q_6$.

(2) For stopping period of the cart, combined with lifting or lowering of the payload, vector $\{\boldsymbol{Q}\}$ has the same form, except that $Q_1=-M_{1\mathrm{st}}$, where $M_{1\mathrm{st}}$ is denoted reduced to low-speed shaft stopping moment of the brake.

The system of differential Eq.(7) is suitable for investigation of the mechanical system parameters in case of large payload swinging. If the angle of payload swinging is small, system Eq.(7) can be simplified. In this case we can suppose that $\sin q_i \approx q_i$, $\cos q_i \approx 1$ and vector $\{\boldsymbol{N}\}$, consisting of higher order terms, can be dropped.

NUMERICAL EXAMPLE AND DISCUSSIONS

The proposed system of differential equations is nonlinear and suitable as its solving is a numerical method. Solution is realized by fourth-order Runge-Kutta fixed-step method and all initial conditions are set to zero. There is considered a case of starting of the cart with lifting of the payload. Calculations are performed for a set of constant parameters of the real-life cart and linear laws of motion of the travel and hoisting mechanism are considered.

As results of realized solutions, there are obtained changes in time of the force and kinematical parameters of the mechanical system and they are shown in Figs.4~6.

It is obvious that the force loading in the elastic coupling at the starting period of the cart is considerably larger than its static value—about 2.5 times (Fig.4).

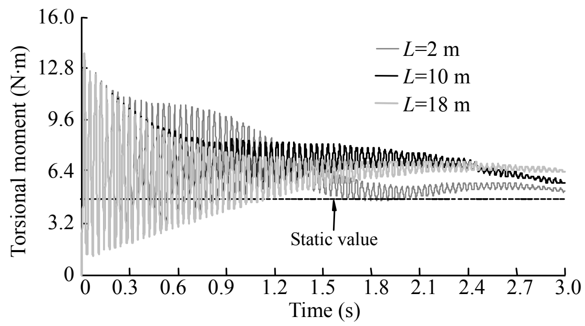


Fig.4 Torsional moment in the elastic coupling

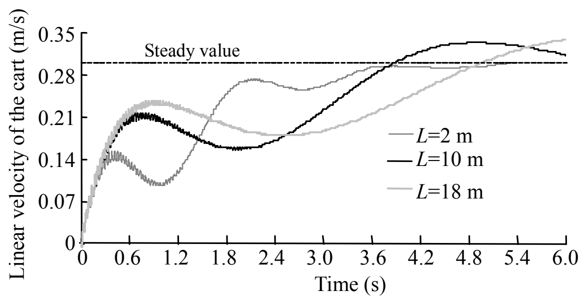


Fig.5 Linear velocity of the cart

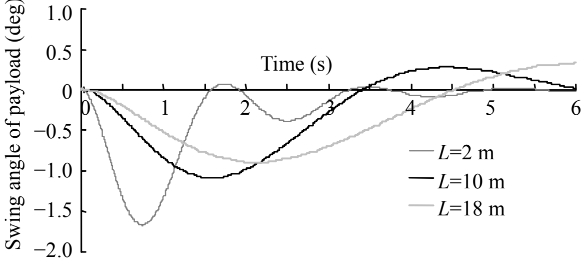


Fig.6 Swinging angle of the payload

This fact is a result of the elasticity of the travel mechanism parts, which introduces high frequency vibrations, and payload swinging, which introduces additional low-frequency force component in the travel mechanism. This fact must be considered in design, life duration and reliability calculations. The swinging payload (Fig.5) has noticeable influence on the linear velocity of the cart, which increases and decreases cyclically dependent on the frequency of the payload swinging. The amplitude and frequency of the payload swinging (Fig.6) depend on the current rope length and law of motion of the rotor of the electric motor and they have damped character.

VERIFICATION OF THE PROPOSED MATHEMATICAL MODEL

The verification of the derived mathematical model is realized by a comparison of obtained results for parameters with those obtained by solving the well-known classical linearised 2 DOF model (Fig.7). Differential equations of motion of the bodies of 2 DOF model are (Eksarov and Grigorov, 1981):

$$\begin{aligned} &(m_1+m_2)\ddot{x}+m_2L\ddot{\varphi}=F-W, \\ &\ddot{x}+L\ddot{\varphi}=-g\varphi, \end{aligned} \tag{13}$$

where F denotes the driving force.

Direct comparison of the results is incorrect because of the different structures of the models—in the 2 DOF model, kinematics of the travel mechanism are not considered and the rope has constant length. As both models have constant parameters and obey the laws of motion of the electric motor rotor, the

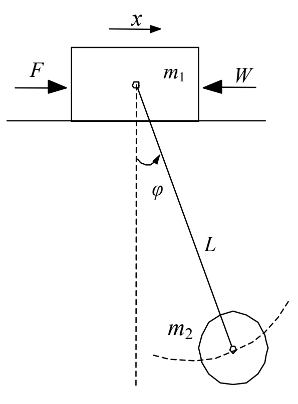


Fig.7 Classical 2 DOF model

behavior of both models is similar. This is achieved by setting in the proposed model negligible values of the mass moments of inertia of rotational components of the travel and hoisting mechanisms. The results of comparison of the models are shown in Figs.8 and 9.

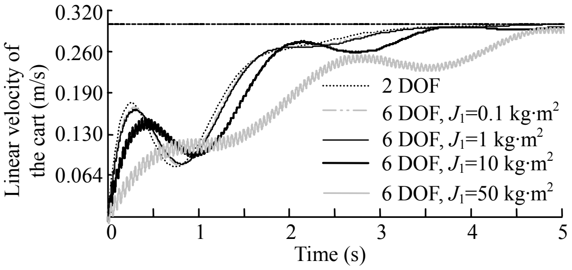


Fig.8 Results from verification of the proposed model—linear velocity of the cart

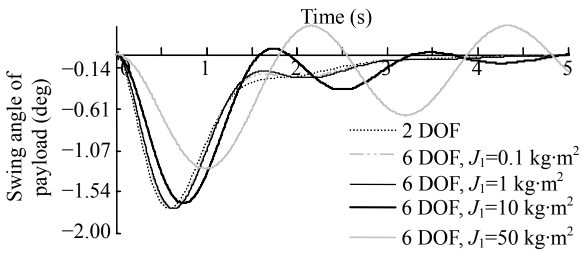


Fig.9 Results from verification of the proposed model—swinging angle of the payload

Apparently, results of the 6 DOF model with small values of mass moments of inertia of the travel mechanism components agreed well with the results obtained from the classical model. Increasing of the mass moments of inertia of the travel mechanism components has noticeable influence on the parameters of motion of the cart and swinging payload.

CONCLUSION

The study proposed and verified the mathematical model of rectilinear translational motion of the rail-guided cart with suspended payload by which we can investigate the two-sided interaction between the swinging payload and the travel mechanism.

Theoretical and numerical investigations yielded the following conclusions:

(1) The proposed 6 DOF model of the translational motion of the cart is suitable for investigation of the two-sided interaction of the swinging payload and kinematical, force and other parameters of the travel mechanism;

(2) The proposed mathematical model can be used for analysis, synthesis and optimization of machines with similar kinematics and structure.

(3) Parameters of the travel mechanism noticeably affect the motion of the cart and swinging payload, which also introduces additional low-frequency force component in the travel mechanism and affects the cart motion.

References

Abdel-Rahman, E.M., Nayfeh, A.H., Masoud, Z.N., 2003. Dynamics and control of cranes: A review. *Journal of Vibration and Control*, **9**(7):863-908. [doi:10.1177/1077546303009007007]

Al-mousa, A., Kachroo, P., 2003. Control of rotary cranes using fuzzy logic. *Shock and Vibration*, **10**:81-95.

Corriga, G., Giua, A., Usai, G., 1998. An implicit gain-scheduling controller for cranes. *IEEE Transactions on Control Systems Technology*, **6**(1):15-20. [doi:10.1109/87.654873]

Eksarov, P., Grigorov, B., 1981. About possibility for damping of payload swinging in transient motion of overhead crane. *Machinebuilding*, **3**:114-117 (in Bulgarian).

Ely, P.T., 1997. Internal Damping Rates of Construction Cranes. Master Thesis, Virginia Polytechnic Institute and State University, USA.

Jerman, B., 2006. An enhanced mathematical model for investigating the dynamic loading of a slewing crane. *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science*, **220**(4):421-433. [doi:10.1243/09544062C08205]

Ju, F., Char, Y.S., Cui, F.S., 2006. Dynamic response of tower crane induced by the pendulum motion of the payload. *International Journal of Solids and Structures*, **43**(2):376-389. [doi:10.1016/j.ijsolstr.2005.03.078]

Omar, H.M., 2003. Control of Gantry and Tower Cranes. Ph.D Thesis, Virginia Polytechnic Institute and State University, USA, p.12-14.

Pauluk, M., 2001. Time Optimal Control of 3D Crane. MMAR 2001: Proceedings of the 7th IEEE International Conference on Methods and Models in Automation and Robotics, p.122-128.

Petkov, G., Karaivanov, P., Kamburov, D., 1980. Experimental Research of Material Handling Machines. Tehnika, Sofia, p.191-192 (in Bulgarian).

Pettersson, M., 1997. Driveline Modeling and Control. Ph.D Thesis, Linköpings University, Sweden, p.23-48.

Rahnejat, H., 1998. Multibody Dynamics: Vehicles, Machines and Mechanisms. Professional Engineering Publishing, London, p.275-332.

Scheffler, M., Dresig, H., Kurth, F., 1977. Unstetigförder 2. VEB Verlagtechnik, Berlin, p.45-48.

Valev, V., 1988. Technology of Reinforced Concrete Constructions. Tehnika, Sofia, p.122-163.