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Influence of input acoustic power on regenerator's performance*

TANG Ke†, HUANG Zhong-jie, JIN Tao†‡, BAO Rui, CHEN Guo-bang

(*Institute of Refrigeration and Cryogenics, Zhejiang University, Hangzhou 310027, China*)

†E-mail: ktang@zju.edu.cn; jintao@zju.edu.cn

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Abstract: Performance of a pulse tube cooler significantly depends on the efficient operation of its regenerator. Influence of input acoustic power on regenerator's performance is simulated and analyzed with simple harmonic analysis method. Given regenerator's dimensions and pressure ratio, there is an optimal input acoustic power for achieving a highest coefficient of performance, due to a compromise between relative time-averaged total energy flux in regenerator and relative acoustic power at regenerator's cold end. Additionally, optimal dimensions of regenerator are also estimated and presented for different input acoustic powers. The computed optimal diameter obviously increases with increase of input acoustic power, while the optimal length decreases slightly, and as a result, a larger input acoustic power requires a smaller aspect ratio (length over diameter).

Key words: Regenerator, Pulse tube cooler, Simple harmonic analysis
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INTRODUCTION

As a key component in a pulse tube cooler, a regenerator attracts much attention. Much effort has been focused on its performance improvement, which can be roughly categorized into areas of materials and geometry (Wysokinski *et al*., 2002; Qiu *et al*., 2007), modelling (Swift and Ward, 1996; Zhu and Matsubara, 2004; Pfotenhauer *et al*., 2005; Radebaugh and O'Gallagher, 2006), and measurement (Nam and Jeong, 2003; 2005).

This study concentrates on the influence of input acoustic power on regenerator's performance by means of simulation with simple harmonic analysis method. Optimal dimensions of regenerator operating at different input acoustic powers and fixed input pressure ratio are presented, which may provide some useful information for designing a regenerator.

MODEL

For simple harmonic analysis of regenerator (Swift and Ward, 1996), momentum, continuity and energy equations for screen mesh regenerator can be written as:

$$\frac{\mathrm{d}p_1}{\mathrm{d}x}=-\frac{i\omega\rho_\mathrm{m}}{\phi A}\left[1+\frac{(1-\phi)^2}{2(2\phi-1)}\right]\langle U_1\rangle-\frac{\mu_\mathrm{m}}{\phi A r_\mathrm{h}^2}\left[\frac{c_1(\phi)}{8}+\frac{c_2(\phi)Re_1}{3\pi}\right]\langle U_1\rangle, \tag{1}$$

$$\begin{aligned}\frac{\mathrm{d}\langle U_1\rangle}{\mathrm{d}x}=&-\frac{i\omega\phi A\gamma_\mathrm{m}}{\rho_\mathrm{m}a_\mathrm{m}^2}p_1+\beta_\mathrm{m}\frac{\mathrm{d}T_\mathrm{m}}{\mathrm{d}x}\langle U_1\rangle\\&+i\omega\beta_\mathrm{m}\left[\frac{\phi AT_\mathrm{m}\beta_\mathrm{m}}{\rho_\mathrm{m}c_\mathrm{p,m}}\frac{\varepsilon_\mathrm{s}+(g_\mathrm{c}+\mathrm{e}^{2i\theta_\mathrm{P}}g_\mathrm{v})\varepsilon_\mathrm{h}}{1+\varepsilon_\mathrm{s}+(g_\mathrm{c}+\mathrm{e}^{2i\theta_\mathrm{T}}g_\mathrm{v})\varepsilon_\mathrm{h}}p_1\right.\\&\left.-\frac{1}{i\omega}\frac{\mathrm{d}T_\mathrm{m}}{\mathrm{d}x}\frac{\varepsilon_\mathrm{s}+(g_\mathrm{c}-g_\mathrm{v})\varepsilon_\mathrm{h}}{1+\varepsilon_\mathrm{s}+(g_\mathrm{c}+\mathrm{e}^{2i\theta_\mathrm{T}}g_\mathrm{v})\varepsilon_\mathrm{h}}\langle U_1\rangle\right],\end{aligned} \tag{2}$$

$$\begin{aligned}\frac{\mathrm{d}T_\mathrm{m}}{\mathrm{d}x}=&\left\{\mathrm{Re}\left[\left(T_\mathrm{m}\beta_\mathrm{m}\frac{\varepsilon_\mathrm{s}+\varepsilon_\mathrm{h}(g_\mathrm{c}+\mathrm{e}^{2i\theta_\mathrm{P}}g_\mathrm{v})}{1+\varepsilon_\mathrm{s}+\varepsilon_\mathrm{h}(g_\mathrm{c}+\mathrm{e}^{2i\theta_\mathrm{T}}g_\mathrm{v})}+1\right.\right.\right.\\&\left.\left.\left.-T_\mathrm{m}\beta_\mathrm{m}\right)p_1\langle\tilde{U}_1\rangle\right]-2\dot{H}_2\right\}\Big/\left\{\frac{\rho_\mathrm{m}c_\mathrm{p,m}}{\omega\phi A}\right.\\&\left.\cdot\mathrm{Im}\left[\frac{\varepsilon_\mathrm{s}+\varepsilon_\mathrm{h}(g_\mathrm{c}-g_\mathrm{v})}{1+\varepsilon_\mathrm{s}+\varepsilon_\mathrm{h}(g_\mathrm{c}+\mathrm{e}^{2i\theta_\mathrm{T}}g_\mathrm{v})}\right]\langle U_1\rangle\langle\tilde{U}_1\rangle\right.\end{aligned}$$

‡ Corresponding author
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$$+2K_{\text{eff}}A(1-\phi)+2K_{\text{w}}\pi D\delta\Big\}, \tag{3}$$

where $\langle U_1\rangle$ denotes volume flow rate. The last term of the denominator in Eq.(3) was introduced by the authors to consider the influence of thermal conduction along the regenerator wall, where D, K_{w} and δ are the regenerator's inner diameter, thermal conductivity and wall thickness, respectively. Detailed introductions of other denotations can be found in (Swift and Ward, 1996).

Eqs.(1) and (2) can be rewritten in the following forms:

$$\frac{\mathrm{d}p_1}{\mathrm{d}x}=-(i\omega l+r_\upsilon)\langle U_1\rangle, \tag{4}$$

$$\frac{\mathrm{d}\langle U_1\rangle}{\mathrm{d}x}=-\left(i\omega c+\frac{1}{r_\kappa}\right)p_1+e\langle U_1\rangle, \tag{5}$$

where l, c, r_υ, r_κ and e are inertance, compliance, viscous resistance, thermal-relaxation resistance and controlled source term per unit length, respectively, which are expressed as:

$$l=\frac{\rho_{\text{m}}}{\phi A}\left[1+\frac{(1-\phi)^2}{2(2\phi-1)}\right], \tag{6}$$

$$c=\frac{\phi A\gamma_{\text{m}}}{\rho_{\text{m}}a_{\text{m}}^2}-\frac{T_{\text{m}}\beta_{\text{m}}^2}{\rho_{\text{m}}c_{\text{p,m}}}\mathrm{Re}(\xi), \tag{7}$$

$$r_\upsilon=\frac{\mu_{\text{m}}}{\phi A r_{\text{h}}^2}\left[\frac{c_1(\phi)}{8}+\frac{c_2(\phi)Re_1}{3\pi}\right], \tag{8}$$

$$r_\kappa=-\frac{\rho_{\text{m}}c_{\text{p,m}}}{\omega\phi A T_{\text{m}}\beta_{\text{m}}^2\,\mathrm{Im}(\xi)}, \tag{9}$$

$$e=\beta_{\text{m}}\frac{1+(1+\mathrm{e}^{2i\theta_{\text{T}}})g_{\text{v}}\varepsilon_{\text{h}}}{1+\varepsilon_{\text{s}}+(g_{\text{c}}+\mathrm{e}^{2i\theta_{\text{T}}}g_{\text{v}})\varepsilon_{\text{h}}}\frac{\mathrm{d}T_{\text{m}}}{\mathrm{d}x}, \tag{10}$$

where

$$\xi=\frac{\varepsilon_{\text{s}}+(g_{\text{c}}+\mathrm{e}^{2i\theta_{\text{P}}}g_{\text{v}})\varepsilon_{\text{h}}}{1+\varepsilon_{\text{s}}+(g_{\text{c}}+\mathrm{e}^{2i\theta_{\text{T}}}g_{\text{v}})\varepsilon_{\text{h}}}. \tag{11}$$

According to (Swift, 2002), acoustic power loss in regenerator is mainly contributed by three parts: viscous dissipation $\dot{W}_\nu$, thermal-relaxation dissipation $\dot{W}_\kappa$ and influence of source term $\dot{W}_e$. For a short segment dx of regenerator, we have

$$\mathrm{d}\dot{W}_\nu=\frac{r_\upsilon}{2}\left|\langle U_1\rangle\right|^2\mathrm{d}x, \tag{12}$$

$$\mathrm{d}\dot{W}_\kappa=\frac{1}{2r_\kappa}\left|p_1\right|^2\mathrm{d}x, \tag{13}$$

$$\mathrm{d}\dot{W}_e=-\frac{1}{2}\mathrm{Re}(e\tilde{p}_1\langle U_1\rangle)\,\mathrm{d}x. \tag{14}$$

A simulation program for regenerator has been written with the above equations. Cooling power $\dot{Q}_{\text{c}}$ is calculated by:

$$\dot{Q}_{\text{c}}=\dot{H}_{\text{PT}}-\dot{H}_{\text{R}}=FOM\cdot\dot{W}_{\text{PT}}-\dot{H}_{\text{R}}, \tag{15}$$

where $\dot{H}_{\text{PT}}$ and $\dot{H}_{\text{R}}$ are time-averaged total energy flux in pulse tube and in regenerator, respectively. $\dot{W}_{\text{PT}}$ is acoustic power at the cold end of regenerator. Figure of Merit (FOM) is the ratio of time-averaged total energy flux to acoustic power flow in pulse tube, which is given as 0.7 in our computation (Rawlins *et al*., 1994; Marquardt and Radebaugh, 2000).

RESULT AND ANALYSIS

Influence of input acoustic power

The operating conditions, such as working gas of helium, operating frequency of 40 Hz, mean pressure of 3.0 MPa, warm and cold end temperatures of 300 K and 80 K, and various input acoustic powers to the warm end of the regenerator with a fixed pressure ratio of 1.3 are considered in the simulation. The analyzed regenerator has structural dimension of 25 mm×60 mm×0.2 mm (inner diameter×length×wall thickness), packed with 400# stainless steel screen discs.

Fig.1 shows the curves of relative time-averaged total energy flux (ratio of the time-averaged total energy flux in regenerator to input acoustic power at its warm end), relative acoustic power at cold end (ratio of the acoustic power at regenerator's cold end to the input acoustic power) and coefficient of performance (COP=cooling power/input acoustic power). It is seen that there exists an optimal input acoustic power at which the regenerator COP reaches maximum. For the above conditions, the optimal value is about 80 W. The increase of COP with input acoustic power up to 80 W is mostly attributed to the decrease of relative time-averaged total energy flux, which includes not only enthalpy flow but also thermal conductance through the packings and the regenerator wall. And then, it is the decrease of relative acoustic power at the cold end that mainly causes the drop of COP in the case of input acoustic power larger than 80 W.

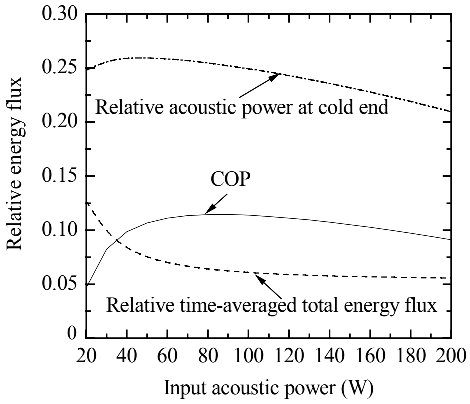

Fig.1 Relative time-averaged total energy flux, relative acoustic power at cold end and COP vs input acoustic power

In order to analyze the variation of relative acoustic power at the regenerator's cold end, relative viscous, thermal-relaxation and source term dissipations changing with input acoustic power are also computed and presented in Fig.2. Comparison of the curves in Fig.2 indicates that source term dissipation, which is much larger than viscous and thermal-relaxation dissipations, is the primary factor for acoustic power loss, however, the marked increase of relative viscous dissipation with the addition of input acoustic power dominates the variation of total

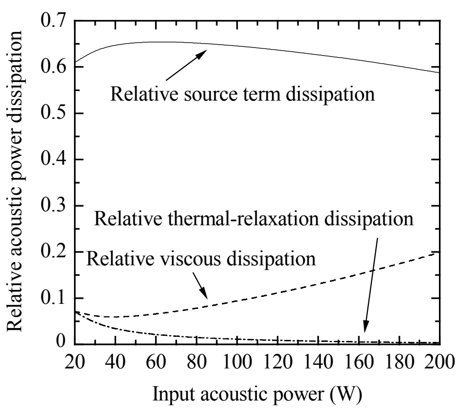

Fig.2 Relative viscous, thermal-relaxation and source term dissipations in regenerator vs input acoustic power

acoustic power loss in regenerator and then the variation of acoustic power at the regenerator's cold end. Viscous dissipation is significantly related to space-averaged velocity amplitude on flow area ($\langle u_1\rangle=\langle U_1\rangle/(\phi A)$). For a given regenerator, the increase of space-averaged velocity amplitude, due to the rise of input acoustic power at fixed pressure ratio, results in rapid increase of viscous dissipation, and finally affects the COP. On the other hand, the dimensions of regenerator need to be optimized at a given input acoustic power.

Dimension optimization of regenerator

Under the above-mentioned conditions, dimension optimization of the regenerator for maximum cooling power at 80 K was done at different input acoustic powers. Figs.3a~3d show the results at input acoustic power of 50 W, 100 W, 150 W and 200 W, respectively. Apparently, there is an optimal dimension for each given input acoustic power. In the simulation, the COP of regenerator with optimal dimension is roughly 0.115, and is weakly influenced by different input acoustic powers.

The variation of optimal diameter and length with input acoustic power is presented in Fig.4, according to which a larger input acoustic power at a fixed pressure ratio requires a regenerator with larger diameter and smaller length. The computed values of optimal diameter could be fitted by a power-law function of input acoustic power with nonlinear least squares fitting method, i.e. $D=2.86\dot{W}_{\text{in}}^{0.5}$, while a function of $L=42.22\dot{W}_{\text{in}}^{-0.4}+53.34$ could be used to represent the relation of optimal length to input acoustic power. Optimal aspect ratios (length over diameter) are also calculated and plotted in Fig.5. The optimal aspect ratio markedly decreases with the increase of the input acoustic power, that is, a "fatter" regenerator may work better at a larger input acoustic power. The variation of optimal aspect ratio with input acoustic power could be fitted by $r=30.95\dot{W}_{\text{in}}^{-0.6}+0.15$.

CONCLUSION

A simulation program for regenerator has been written with simple harmonic analysis method to

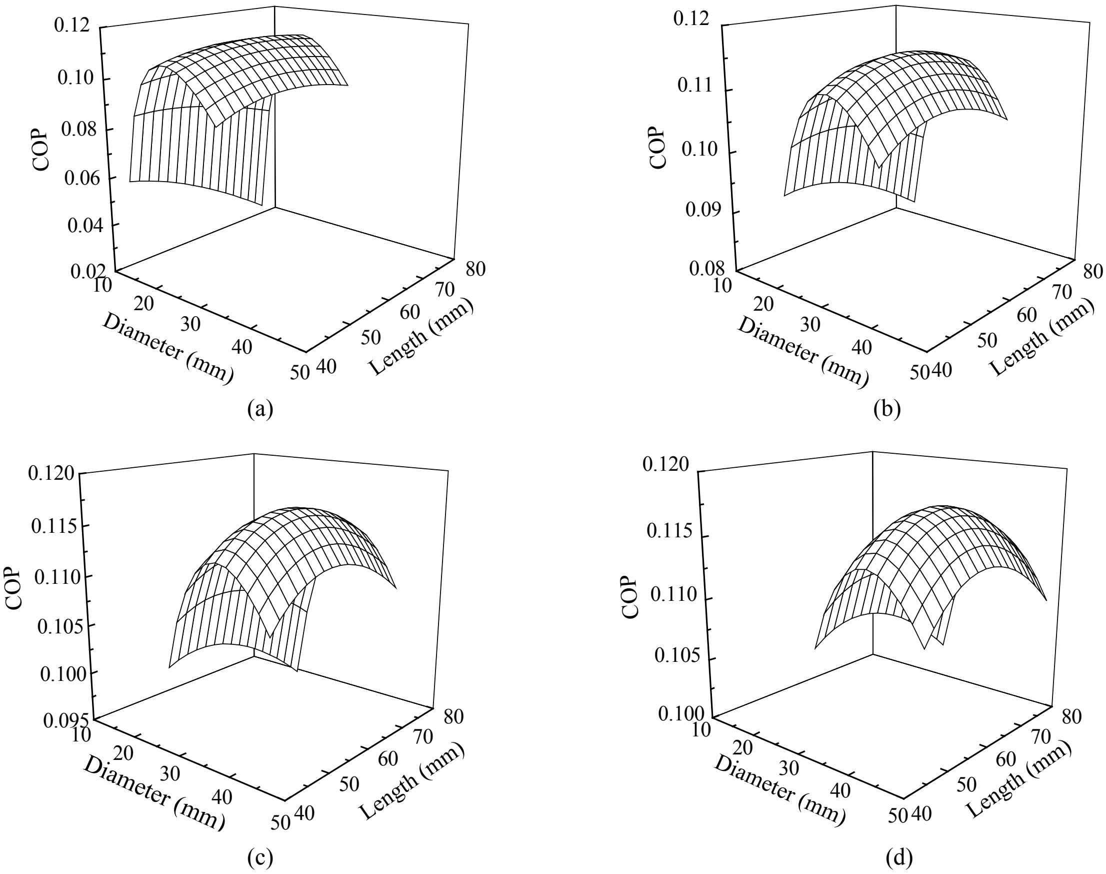

Fig.3 COP at input acoustic power of (a) 50 W, (b) 100 W, (c) 150 W, (d) 200 W

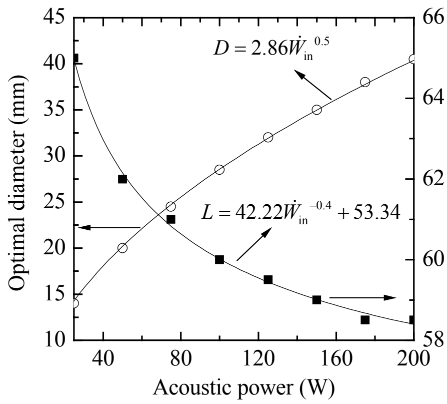

Fig.4 Optimal diameter and length vs input acoustic power

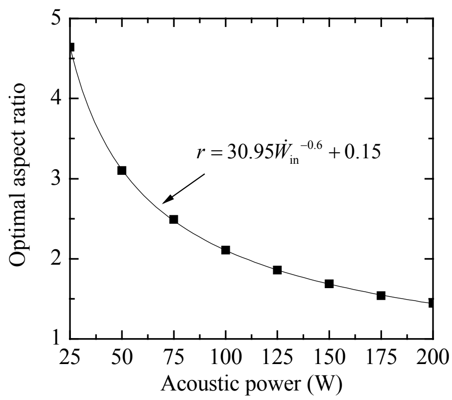

Fig.5 Optimal regenerator's aspect ratio vs input acoustic power

analyze the influence of input acoustic power on regenerator performance. Simulation results indicated that for a given dimension and pressure ratio, in the case of relatively small input acoustic power, an increase of input acoustic power leads to a decrease of relative time-averaged total energy flux in the regenerator, which is helpful for raising the COP. However, the marked increase of viscous dissipation, due to further increase of input acoustic power, causes the deterioration of COP. Thus, there is an optimal input acoustic power for maximum COP. In addition, optimal regenerator dimensions for given input acoustic powers are computed. The optimal diameter evidently increases, while the optimal length slightly decreases, with the increase of input acoustic power, and as a result, a larger input acoustic power requires a smaller aspect ratio. Under the conditions considered in our simulation, the variation of optimal diameter, length and aspect ratio with input acoustic power can be fitted by $D=2.86\dot{W}_{\mathrm{in}}^{\ 0.5}$, $L=42.22\dot{W}_{\mathrm{in}}^{\ -0.4}+53.34$, and $r=30.95\dot{W}_{\mathrm{in}}^{\ -0.6}+0.15$, respectively.

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