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Power duality for multi-antenna OFDM system in broadcast channel with user scheduling*

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Abstract: This paper deals with design and analysis of user scheduling and power allocation for multi-antenna OFDM systems with DPC, ZF-DPC, ZF-BF and TDMA transmit strategies. We consider the general multi-user downlink scheduling problem and power minimization with multi-user rate constraints. According to the channel state, it is shown that there is a power optimal policy which selects a subset of users in each scheduling interval. We present user selection algorithms for DPC, ZF-DPC, ZF-BF and TDMA for multi-antenna OFDM system in broadcast channels, and we also present the practical water-filling solution in this paper. By the selected users with the consideration of fairness, we derive the power optimization algorithm with multi-user rate constraints. We also analyze the power duality of uplink-downlink for the transmit strategies of DPC, ZF-DPC and ZF-BF. Simulation results show that the present user-scheduling algorithm and power minimization algorithm can achieve good power performance, and that the scheduling algorithm can guarantee fairness.

Key words: Multiple-input multiple-output (MIMO), Orthogonal frequency division multiplexing (OFDM), Broadcast channel, Multi-user diversity, Dirty paper coding (DPC), Downlink scheduling
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INTRODUCTION

Multiple-input multiple-output (MIMO) system is well motivated for wireless communication through fading channels because of the potential improvements in transmission rate or diversity gain (Alamouti, 1998). On the other hand, the orthogonal frequency division multiplexing (OFDM) has also emerged as an attractive transmission scheme and is adopted in many existing systems. Therefore, MIMO-OFDM is now a popular topic in both literature and industry. It is well known that multiple antennas can be easily deployed at the base station in cellular systems. However, mobile terminals have a smaller number of antennas than base station due to the size and cost constraints. Thus, it may appear that we do not obtain significant capacity benefit from the multiple transmit antennas. This is true with the transmit strategy of time-division multiple access (TDMA) (Yoo and Goldsmith, 2006; Jagannathan *et al*., 2007). To solve the problem, multi-user must be served simultaneously. One way to accomplish this is called dirty paper coding (DPC), which is a multi-user encoding strategy based on interference presubtraction (Costa, 1983). While DPC is with high complexity, Caire *et al.* present zero forcing dirty paper coding (ZF-DPC) as a suboptimal solution (Caire and Shamai, 2003; Kang and Cho, 2007). As a very simple transmit strategy, zero forcing beamforming (ZF-BF) techniques have been proposed for space division multiple access (SDMA) to remove the co-channel interference in MIMO downlink systems (Spencer *et al*., 2004; Zhang and Letaief, 2007).

Various researchers have investigated the sum capacity gains achievable in the above described system by simultaneously transmitting to several users (Jindal and Goldsmith, 2005). In information

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theoretic terms, multi-user downlink channel may be modeled as a multi-user antenna broadcast channel (BC) (Caire and Shamai, 2003; Viswanath and Tse, 2003; Vishwanath *et al.*, 2003; Viswanathan *et al.*, 2003; Jindal *et al.*, 2004). A broadcast channel is a typical environment encountered in multi-user communication, such as wireless LAN and cellular systems. In BC and MAC, power control has been an important mechanism to solve the near-far problem. In multi-antenna OFDM systems, the optimal power allocation for the desired rate in the achievable rate region is not clearly known until now. Most literature on this topic only address the maximum sum rate problem when each user has his own power budget. In other words, most previous researches are concentrated on the sum capacity, or maximum total throughput, of BC or MAC. In this way, users with strong channels are typically allocated more rates than users with weaker channels, and some users may not be allocated any rate at all. This may be undesirable in certain systems, particularly those with unequal receiver channel qualities, and a more attractive option is to allocate rates more uniformly (Lee and Jindal, 2006).

In most practical cases, each user has a desired date rate and likes to achieve it within minimum power. Thus, it is an important problem to guarantee all the user's desired data rates while consuming minimum power. The motivation of the problem is power saving and reducing the co-channel interference to neighbor cell. Two classes of optimal power allocation are of fundamental interest: one is given power constraints, what is the optimal power and rate allocation to support the boundary surface of the capacity region; the other is given a rate tuple, what is the optimal power and rate allocation to support the boundary of the power region. The first problem can be cast as a weighted sum rate maximization problem, and the second as a weighted sum power minimization problem (Lee and Jindal, 2006).

The focus of this paper is the sum power minimization problem for multi-antenna OFDM system in MAC and BC with user selection. We assume that the channel is frequency-selective. However, it is divided into N_c independent ISI-free subchannels by employing OFDM in the transmission. The channel information is assumed to be perfectly known to the receiver and the transmitter for power control.

In this paper, we study the optimal transmit schemes of DPC, ZF-DPC and ZF-BF techniques with optimal power allocation algorithm by multi-user rate constraints. Oh (2005) has investigated the power duality in Gaussian scalar channel for DPC system, however, the result in (Oh, 2005) is not suitable for Gaussian vector channel; and due to the high complexity of dirty paper coding, the power duality for DPC system is not practical. Motivated by this, we investigate the power duality of BC and MAC, and we also propose the minimization power allocation solution for different transmit strategies.

The contributions of this paper are as follows:

(1) We derived the close form of minimization power allocation for DPC, ZF-DPC and ZF-BF in MIMO-OFDM systems.

(2) We analyzed the power duality between MAC and BC for ZF-DPC and ZF-BF.

(3) We developed the user-scheduling algorithm for DPC, ZF-DPC and ZF-BF with the consideration of fairness.

SYSTEM MODEL AND MULTI-USER TRANSMIT STRATEGIES

In this section, we describe the model of multi-antenna OFDM system in broadcast channel and introduce the transmit schemes such as DPC, ZF-DPC and ZF-BF.

Multi-antenna OFDM broadcast channel model

Fig.1 shows the multi-user MIMO-OFDM system in broadcast channel. We assume that the transmitter can get perfect knowledge of channel state information. Thus, the transmit schedules a set of users to service and employs power allocation on each subcarrier. We focus on quasi-static channels, where the channel is fixed over the time period of interest (i.e., over the period of the delay constraint). In multi-user MIMO-OFDM system, we consider a wireless communication system with a base station and K users. The base station is with N_t transmit antennas while each of the users only has a single receive antenna. In an OFDM symbol, the number of total subcarriers is N_c. Thus the channel between the base station and user k on subcarrier n can be characterized by $1\times N_t$ matrix $\boldsymbol{h}_{i,n}$. For every user i, the base station transmits data $\boldsymbol{x}_{i,n}$ after an $N_t\times 1$ dimensional beamformer $\boldsymbol{v}_{k,n}$ on subcarrier n. The signal at the ith user's receiver on the nth subcarrier is

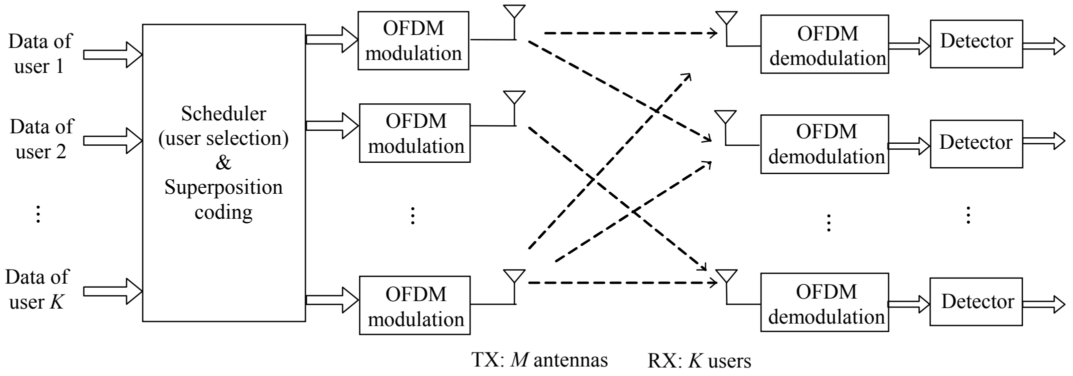

Fig.1 Block diagram of multi-antenna OFDM system in broadcast channel

$$\boldsymbol{y}_{i,n}=\boldsymbol{h}_{i,n}\sum_{j=1}^{K}\sqrt{p_{j,n}}\boldsymbol{v}_{j,n}\boldsymbol{x}_{j,n}+\boldsymbol{w}_{i,n},\tag{1}$$

where $\boldsymbol{w}_{i,n}$ is white Gaussian noise at the ith user on subcarrier n and $p_{j,n}$ is the power allocated to the ith user on the nth subcarrier.

DPC, ZF-DPC and ZF-BF in multi-antenna system

In this subsection, DPC, ZF-DPC and ZF-BF schemes for multi-antenna broadcast channel (BC) are introduced.

1. Dirty paper coding (DPC)

It is now well known that DPC achieves the sum capacity of the multi-antenna BC as well as the full capacity region (Yu and Cioffi, 2004).

DPC is a precoding technique which precancels interference at the transmitter (Caire and Shamai, 2003; Viswanathan *et al.*, 2003; Yu and Cioffi, 2004; Weingarten *et al.*, 2006). We now describe the transmit signal when DPC is utilized. Let $s_k\in\mathbb{C}^{N_\mathrm{t}\times N_\mathrm{r}}$ be its precoding matrix. Then the transmit signal vector $\boldsymbol{x}$ can (roughly) be represented as

$$\boldsymbol{x}=\boldsymbol{v}_1\boldsymbol{s}_1\oplus(\boldsymbol{v}_2\boldsymbol{s}_2\oplus\cdots\oplus(\boldsymbol{v}_{K-2}\boldsymbol{s}_{K-2}\oplus(\boldsymbol{v}_{K-1}\boldsymbol{s}_{K-1}\oplus\boldsymbol{v}_K\boldsymbol{s}_K))\cdots),\tag{2}$$

where ⊕ represents the non-linear dirty paper sum, and v_i is the beamforming vector of user i. We assume that the encoding process is performed in descending order. Dirty-paper decoding at the kth receiver will cancel the interference of $\boldsymbol{v}_{k+1}\boldsymbol{s}_{k+1}$, …, $\boldsymbol{v}_K\boldsymbol{s}_K$, thus the effectively received signal at user k is

$$\boldsymbol{y}_k=\boldsymbol{H}_k\boldsymbol{v}_k\boldsymbol{s}_k+\sum_{j=1}^{k-1}\boldsymbol{H}_k\boldsymbol{v}_j\boldsymbol{s}_j+\boldsymbol{w}_k,\tag{3}$$

where the second term is the multi-user interference which is not cancelled by the DPC principle. If the $\boldsymbol{s}_k$ are chosen Gaussian, the rate of the kth user is expressed as

$$R_k=\log_2\left[\left|\boldsymbol{I}+\boldsymbol{H}_k\left(\sum_{j=1}^{k}\boldsymbol{\Sigma}_j\right)\boldsymbol{H}_k^{\mathrm{H}}\right|\Big/\left|\boldsymbol{I}+\boldsymbol{H}_k\left(\sum_{j=1}^{k-1}\boldsymbol{\Sigma}_j\right)\boldsymbol{H}_k^{\mathrm{H}}\right|\right],\tag{4}$$

where $\boldsymbol{\Sigma}_j=\boldsymbol{v}_jE[\boldsymbol{s}_j\boldsymbol{s}_j^{\mathrm{H}}]\boldsymbol{v}_j^{\mathrm{H}}$ denotes the transmit covariance matrix of user j. The sum capacity of the MIMO BC by DPC can be expressed as

$$\begin{aligned}C_{\mathrm{DPC}}(\boldsymbol{H},\boldsymbol{P})=\sum_{k=1}^{K}\log_2\Bigg[&\left|\boldsymbol{I}+\boldsymbol{H}_k\left(\textstyle\sum_{j=1}^{k}\boldsymbol{\Sigma}_j\right)\boldsymbol{H}_k^{\mathrm{H}}\right|\\&\cdot\left|\boldsymbol{I}+\boldsymbol{H}_k\left(\textstyle\sum_{j=1}^{k-1}\boldsymbol{\Sigma}_j\right)\boldsymbol{H}_k^{\mathrm{H}}\right|^{-1}\Bigg].\end{aligned}\tag{5}$$

2. Zero forcing beamforming (ZF-BF) in a MISO BC

Multiple transmit antennas can potentially yield an N_t-fold increase in the sum capacity, where N_t is the number of transmit antennas. Yoo and Goldsmith (2006) showed that employing ZF-BF to a set of N_t nearly orthogonal users with large channel norms is asymptotically optimal as the number of users grows large.

In ZF-BF, the beamforming matrix takes a pseudo inverse of the channel matrix. In multi-user MISO case, we first select a user subset S to be served together where $|S|\le N_\mathrm{t}$ ($|S|$ denotes the size of user set) and then build the corresponding channel matrix $\boldsymbol{H}(S)$. The beamforming matrix $\boldsymbol{v}(S)$ is written as

$$\boldsymbol{v}(S)=\boldsymbol{H}^{\mathrm{H}}(S)[\boldsymbol{H}(S)\boldsymbol{H}^{\mathrm{H}}(S)]^{-1}.\tag{6}$$

As a result, the achievable throughput of ZF-BF for a given user set S is given by

$$R_{\text{ZF-BF}}(S)=\max\sum_{i\in S}\log_2(1+d_iP_i), \tag{7}$$

where d_i denotes the channel gain of user i, and P_i is obtained by water-filling.

3. Zero forcing dirty paper coding (ZF-DPC) in a MISO BC

Finding simple ways to achieve performance close to capacity is still of great interest for the MIMO enhanced broadcast channel (Viswanath and Tse, 2003). In (Caire and Shamai, 2003), a simple approach based on QR decomposition is proposed to optimize the sum rate for channel matrices with full row rank. It is referred to as zero forcing dirty paper coding (ZF-DPC) (Viswanath and Tse, 2003).

As in ZF-BF, let S be a user subset and $\boldsymbol{H}(S)$ the corresponding channel matrix. Then, by taking QR decomposition, the channel matrix can be expressed as $\boldsymbol{H}(S)=\boldsymbol{R}^{\mathrm{H}}\boldsymbol{Q}^{\mathrm{H}}$, where $\boldsymbol{Q}$ is a unitary matrix and $\boldsymbol{R}$ is an upper triangular matrix. Let $\{r_{i,i}\}$ denote the diagonal elements of $\boldsymbol{R}^{\mathrm{H}}$, then the sum rate of ZF-DPC is

$$R_{\text{ZF-DPC}}(S)=\sum_{i\in S}\log_2(1+|r_{i,i}|^2\,P_{i,i}), \tag{8}$$

where the allocated power $P_{i,i}$ is obtained by water-filling.

POWER DUALITY OF BC AND MAC

This section analyzes the duality of Gaussian BC and MAC. For a given BC, there exists a dual MAC whose channel response is the same with the transmitters and the receivers swapped. This relationship is reciprocal. Thus, the duality is an important notation that gives insight into fundamental relationships in multi-user information theory. Jindal *et al.*(2004) showed that the capacity region of the Gaussian scalar broadcast channel is equal to that of the dual Gaussian multiple-access channel under the same sum-power constraints. We deduced the power duality of vector channels for ZF-DPC and ZF-BF in this section.

The uplink-downlink duality results in precoding and beamforming are similar to the duality relationship between the capacity (DPC) region of MIMO-MAC and MIMO-BC. Here, we present the analysis of duality of precoding system and the relationship of BC powers and MAC powers.

Power duality of precoding for ZF-DPC

We first consider the uplink with the user order as 1,…,K. We assume that the linear receiver vectors are $\boldsymbol{v}_1,\ldots,\boldsymbol{v}_K$, and use successive cancellation with the order as 1,…,K. The SIR (signal interference ratio) of user k is (Viswanath and Tse, 2003)

$$SIR_k=p_k^{\mathrm{M}}\,|\boldsymbol{h}_k\boldsymbol{v}_k|^2\Big/\Big(1+\sum\nolimits_{j=k+1}^{K}p_j^{\mathrm{M}}\,|\boldsymbol{h}_j\boldsymbol{v}_k|^2\Big), \tag{9}$$

with the signals from users 1,…,j−1 being decoded and perfectly cancelled, where p_k^{M} is the power allocated to user k and $\boldsymbol{h}_k\in\mathbb{C}^{1\times N_{\mathrm{t}}}$ is the channel vector of user k.

In BC, we employ linear precoder but use a transmission strategy based on known interference at the receiver. Thus, the transmit signal is $\boldsymbol{x}=\sum\nolimits_{k=1}^{K}\boldsymbol{x}_k\boldsymbol{v}_k$, and the received signal at user k is

$$\boldsymbol{y}_k=\boldsymbol{x}_k\boldsymbol{h}_k\boldsymbol{v}_k+\sum_{j>k}\boldsymbol{x}_j\boldsymbol{h}_k\boldsymbol{v}_j+\sum_{j<k}\boldsymbol{x}_j\boldsymbol{h}_k\boldsymbol{v}_j+\boldsymbol{h}_k\boldsymbol{w}. \tag{10}$$

We use Gaussian independent inputs for $\boldsymbol{x}_1,\ldots,\boldsymbol{x}_K$ with variances $p_1^{\mathrm{B}},\ldots,p_K^{\mathrm{B}}$ and perform precoding for each user. Hence, we obtain the rates of user k

$$R_k=\log_2(1+SIR_k), \tag{11}$$

where

$$SIR_k=p_k^{\mathrm{B}}\,|\boldsymbol{h}_k\boldsymbol{v}_k|^2\Big/\Big(1+\sum\nolimits_{j=1}^{k-1}p_j^{\mathrm{B}}\,|\boldsymbol{h}_j\boldsymbol{v}_k|^2\Big). \tag{12}$$

By defining A_k and B_k as

$$A_k=1+\sum_{j=1}^{k-1}p_j^{\mathrm{B}}\,|\boldsymbol{h}_k\boldsymbol{v}_j|^2,\ B_k=1+\sum_{j=k+1}^{K}p_j^{\mathrm{M}}\,|\boldsymbol{h}_j\boldsymbol{v}_k|^2, \tag{13}$$

we rewrite the rates in the MAC and BC as

$$R_k^{\mathrm{M}}=\log_2\Big(1+p_k^{\mathrm{M}}\big|\boldsymbol{h}_k\boldsymbol{v}_k\big|^2\big/B_k\Big), \tag{14}$$

$$R_k^{\mathrm{B}}=\log_2\Big(1+p_k^{\mathrm{B}}\big|\boldsymbol{h}_k\boldsymbol{v}_k\big|^2\big/A_k\Big). \tag{15}$$

Thus, if the power satisfies

$$p_k^{\mathrm{B}}/A_k=p_k^{\mathrm{M}}/B_k,\quad k=1,\ldots,K \tag{16}$$

we can get

$$\sum_{k=1}^{K} p_k^{\mathrm{M}} = \sum_{k=1}^{K} p_k^{\mathrm{B}}. \tag{17}$$

We now show that given a set of MAC powers and a MAC decoding order, there exists a set of BC powers:

$$p_1^{\mathrm{B}} = p_1^{\mathrm{M}} \Big/ \left(1+\sum\nolimits_{j=2}^{K} p_j^{\mathrm{M}} \,|\, \boldsymbol{h}_j \boldsymbol{v}_1 \,|^2\right), \tag{18}$$

$$p_2^{\mathrm{B}} = p_2^{\mathrm{M}} \frac{1+p_1^{\mathrm{B}} \,|\, \boldsymbol{h}_2 \boldsymbol{v}_1 \,|^2}{1+\sum\nolimits_{j=3}^{K} p_j^{\mathrm{M}} \,|\, \boldsymbol{h}_j \boldsymbol{v}_1 \,|^2}, \tag{19}$$

$$\cdots$$

$$p_K^{\mathrm{B}} = p_K^{\mathrm{M}} \left(1+\sum\nolimits_{j=1}^{K-1} p_j^{\mathrm{B}} \,|\, \boldsymbol{h}_K \boldsymbol{v}_j \,|^2\right). \tag{20}$$

Similarly, MAC powers can be derived from BC powers:

$$p_K^{\mathrm{M}} = p_K^{\mathrm{B}} \Big/ \left(1+\sum\nolimits_{j=1}^{K-1} p_j^{\mathrm{B}} \,|\, \boldsymbol{h}_K \boldsymbol{v}_j \,|^2\right), \tag{21}$$

$$p_{K-1}^{\mathrm{M}} = p_{K-1}^{\mathrm{B}} \frac{1+p_K^{\mathrm{M}} \,|\, \boldsymbol{h}_K \boldsymbol{v}_{K-1} \,|^2}{1+\sum\nolimits_{j=1}^{K-2} p_j^{\mathrm{B}} \left|\boldsymbol{h}_{K-1} \boldsymbol{v}_j\right|^2}, \tag{22}$$

$$\cdots$$

$$p_1^{\mathrm{M}} = p_1^{\mathrm{B}} \left(1+\sum\nolimits_{j=2}^{K} p_j^{\mathrm{M}} \,|\, \boldsymbol{h}_j \boldsymbol{v}_1 \,|^2\right). \tag{23}$$

Power duality of precoding for ZF-BF

Note that in ZF-DPC, user k only sees interference from users $1,\ldots,k-1$, in contrast to the linear beamforming strategy where it sees interference from all other users.

Thus, for each $k=1,\ldots,K$, the uplink SIR of user k is

$$SIR_k = p_k^{\mathrm{M}} \left|\boldsymbol{h}_k \boldsymbol{v}_k\right|^2 \Big/ \left(1+\sum\nolimits_{j=1,j\neq k}^{K} p_j^{\mathrm{M}} \,|\, \boldsymbol{h}_j \boldsymbol{v}_k \,|^2\right), \tag{24}$$

and the uplink SIR of user k is

$$SIR_k = p_{\mathrm{k}}^{\mathrm{B}} \,|\, \boldsymbol{h}_k \boldsymbol{v}_k \,|^2 \Big/ \left(1+\sum\nolimits_{j=1,j\neq k}^{K} p_j^{\mathrm{B}} \,|\, \boldsymbol{h}_j \boldsymbol{v}_k \,|^2\right). \tag{25}$$

By defining $\tilde{A}_j$ and $\tilde{B}_j$ as

$$\begin{cases} \tilde{A}_j = 1+\sum\nolimits_{j=1,j\neq k}^{K} p_j^{\mathrm{B}} \,|\, \boldsymbol{h}_j \boldsymbol{v}_k \,|^2, \\ \tilde{B}_j = 1+\sum\nolimits_{j=1,j\neq k}^{K} p_j^{\mathrm{M}} \,|\, \boldsymbol{h}_j \boldsymbol{v}_k \,|^2, \end{cases} \tag{26}$$

we can rewrite the rates in the MAC and BC as

$$R_k^{\mathrm{M}} = \log_2\left(1+p_k^{\mathrm{M}} \,|\, \boldsymbol{h}_k \boldsymbol{v}_k \,|^2 \big/ \tilde{B}_k\right), \tag{27}$$

$$R_k^{\mathrm{B}} = \log_2\left(1+p_k^{\mathrm{B}} \,|\, \boldsymbol{h}_k \boldsymbol{v}_k \,|^2 \big/ \tilde{A}_k\right). \tag{28}$$

Thus, if the power satisfies

$$p_k^{\mathrm{B}} / \tilde{A}_k = p_k^{\mathrm{M}} / \tilde{B}_k, \quad k=1,\cdots,K \tag{29}$$

then

$$\sum_{k=1}^{K} p_k^{\mathrm{M}} = \sum_{k=1}^{K} p_k^{\mathrm{B}}. \tag{30}$$

We now show that given a set of MAC powers and a MAC decoding order, there exists a set of BC powers:

$$p_1^{\mathrm{B}} = p_1^{\mathrm{M}} \frac{1+\sum\nolimits_{j=2}^{K} p_j^{\mathrm{B}} \,|\, \boldsymbol{h}_1 \boldsymbol{v}_j \,|^2}{1+\sum\nolimits_{j=2}^{K} p_j^{\mathrm{M}} \,|\, \boldsymbol{h}_j \boldsymbol{v}_1 \,|^2}, \tag{31}$$

$$p_2^{\mathrm{B}} = p_2^{\mathrm{M}} \frac{1+\sum\nolimits_{j=1,j\neq 2}^{K} p_j^{\mathrm{B}} \,|\, \boldsymbol{h}_2 \boldsymbol{v}_j \,|^2}{1+\sum\nolimits_{j=1,j\neq 2}^{K} p_j^{\mathrm{M}} \,|\, \boldsymbol{h}_j \boldsymbol{v}_2 \,|^2}, \tag{32}$$

$$\cdots$$

$$p_K^{\mathrm{B}} = p_K^{\mathrm{M}} \frac{1+\sum\nolimits_{j=1}^{K-1} p_j^{\mathrm{B}} \,|\, \boldsymbol{h}_K \boldsymbol{v}_j \,|^2}{1+\sum\nolimits_{j=1}^{K-1} p_j^{\mathrm{M}} \,|\, \boldsymbol{h}_j \boldsymbol{v}_K \,|^2}. \tag{33}$$

Similarly, MAC powers can be derived from BC powers:

$$p_K^{\mathrm{M}} = p_K^{\mathrm{B}} \frac{1+\sum\nolimits_{j=1}^{K-1} p_j^{\mathrm{M}} \,|\, \boldsymbol{h}_j \boldsymbol{v}_K \,|^2}{1+\sum\nolimits_{j=1}^{K-1} p_j^{\mathrm{B}} \,|\, \boldsymbol{h}_K \boldsymbol{v}_j \,|^2}, \tag{34}$$

$$p_{K-1}^{\mathrm{M}} = p_{K-1}^{\mathrm{B}} \frac{1+\sum\nolimits_{j=1,j\neq K-1}^{K} p_j^{\mathrm{M}} \,|\, \boldsymbol{h}_j \boldsymbol{v}_{K-1} \,|^2}{1+\sum\nolimits_{j=1,j\neq K-1}^{K} p_j^{\mathrm{B}} \,|\, \boldsymbol{h}_{K-1} \boldsymbol{v}_j \,|^2}, \tag{35}$$

$$\cdots$$

$$p_1^{\mathrm{M}} = p_1^{\mathrm{B}} \frac{1+\sum\nolimits_{j=2}^{K} p_j^{\mathrm{M}} \,|\, \boldsymbol{h}_j \boldsymbol{v}_1 \,|^2}{1+\sum\nolimits_{j=2}^{K} p_j^{\mathrm{B}} \,|\, \boldsymbol{h}_1 \boldsymbol{v}_j \,|^2}. \tag{36}$$

OPTIMAL PROBLEM OF MULTI-USER POWER ALLOCATION

The work of DPC power allocation has been well done in (Oh, 2005). Therefore, we focus our work on the power allocation optimization for ZF-DPC, ZF-BF BC, and the results can be extended to MAC by power duality.

Optimal problem

We consider the case that each receiver has only one antenna. The optimal power allocation problem is determining the minimum required to transmit power to achieve a given rate vector $\boldsymbol{R}$, which is an optimization problem as follows:

$$\min \sum_{j=1}^{K}\sum_{n=1}^{N_c} p_{j,n}, \quad \text{s.t.} \ \sum_{n=1}^{N_c} \log_2(1+d_{j,n}p_{j,n}) \geq R_j, \quad j=1,\ 2,\ \ldots,\ K \tag{37}$$

where $p_{j,n}$ is the power on the nth subcarrier of user j, $d_{j,n}$ denotes the channel gain of user j on the nth subcarrier.

Lemma 1 For Gaussian BC with channel gain $\boldsymbol{d}$=[d_1 d_2 … d_K] and noise variance 1, the optimal power of $p_{j,n}$ can be noted as

$$p_{j,n} = \left[\frac{2^X}{\ln 2} - \frac{1}{d_{j,n}}\right]^+, \tag{38}$$

$$X = \frac{R_j - \sum_{n=1}^{\tilde{N}(j)} \log_2(d_{j,n}/\ln 2)}{\tilde{N}(j)},$$

where $[x]^+$ denotes $\max\{x,\ 0\}$, and $\tilde{N}(j)$ is the number of allocated subcarriers to user j, and $d_{j,n}$ denotes the channel gain of user j on the nth subcarrier.

Proof The cost function can be written as

$$L = \sum_{j=1}^{K}\sum_{n=1}^{N_c} p_{j,n} - \sum_{j=1}^{K}\left(\lambda_j \sum_{n=1}^{N_c} \log_2(1+d_{j,n}p_{j,n}) - R_j\right). \tag{39}$$

The gradient provides us with the following condition:

$$\frac{\partial L}{\partial p_{j,n}} = 1 - \lambda_j \cdot \frac{d_{j,n}\log_2 \mathrm{e}}{1+p_{j,n}d_{j,n}} = 0, \tag{40}$$

$$\frac{\partial L}{\partial \lambda_j} = -\left[\sum_{n=1}^{N_c} \log_2(1+d_{j,n}p_{j,n}) - R_j\right] = 0. \tag{41}$$

From Eq.(40), we get

$$p_{j,n} = \frac{\lambda_j}{\ln 2} - \frac{1}{d_{j,n}}. \tag{42}$$

From Eq.(41), we get

$$\lambda_j = 2^{\frac{R_j - \sum_{n=1}^{N_c} \log_2(d_{j,n}\log_2 \mathrm{e})}{N_c}}. \tag{43}$$

Substituting Eq.(43) into Eq.(42), the result will be

$$p_{j,n} = \left[\frac{2^{\frac{R_j - \sum_{n=1}^{N_c} \log_2(d_{j,n}\log_2 \mathrm{e})}{N_c}}}{\ln 2} - \frac{1}{d_{j,n}}\right]^+. \tag{44}$$

In practical solution of power allocation, some subcarriers will not be allocated any power, thus, in Eq.(44), N_c will be substituted as $\tilde{N}(j)$. The determination of $\tilde{N}(j)$ will be done in the next section.

MULTI-USER SCHEDULE FOR MULTI-ANTENNA OFDM BC

In this section, we develop user group selection algorithms for ZF-DPC, ZF-BF for MIMO-OFDM system in broadcast channel.

As mentioned above, ZF-DPC is a suboptimal solution to DPC based on QR decomposition. For multi-subcarrier DPC or ZF-DPC systems, we have to select user across subcarriers. Thus, we first select the users for each subcarrier, and then carry out the selection work on other subcarriers. The user selection only occurs among those not-selected users, unless all the users are selected. This work extends Yoo (Yoo and Goldsmith, 2006) and Dimic's (Dimic and Sidiropoulos, 2005) work to multi time slot and multi-subcarrier with consideration of fairness. Let the number of transmit antennas be N_t. The number of selected users must be less than N_t. In general, different selection yields different value of needed power. Furthermore, different ordering with the same set of users yields different needed power. Here, we use greedy algorithm to determine the user order and minimize the consumption power. Let T denote the user set, and N the subcarrier set. $\boldsymbol{h}_k(N_s)$ is the channel vector of the kth user on the N_s-th subcarrier, $S(N_s)$ the selected user set on the N_s-th subcarrier and N_c the total number of subcarriers.

Algorithm 1 (for DPC, ZF-DPC)

Step 1: Initialization.

- Set $T=\{1,\ldots,N_K\}$, $N=\{1,\ldots,N_\mathrm{c}\}$.
- $n=1$.
- Let $S(N_\mathrm{s})=\varnothing$ (empty set), $N_\mathrm{s}=\{1,\ldots,N_\mathrm{c}\}$.

Step 2: If $T=\varnothing$, then set $T=\{1,\ldots,N_K\}$.

- Let $r_{1,k}(N_\mathrm{s})=\boldsymbol{h}_k(N_\mathrm{s})\boldsymbol{h}_k^\mathrm{H}(N_\mathrm{s})$, where $k\in T$ and $N_\mathrm{s}\in N$. Find a user $s_1(N_\mathrm{s})$ such that

$$s_1=\arg\max_{\{k\in T,N_\mathrm{s}\in N\}} r_{1,k}(N_\mathrm{s}). \tag{45}$$

- Set $S(N_\mathrm{s})=S(N_\mathrm{s})\cup\{s_1\}$, $T=T-\{s_1\}$.

Step 3: while $n\leq N_\mathrm{t}$

- Increase n by 1.
- Project each remaining channel vector onto the orthogonal complement of the sub-space spanned by the channels of the selected users. The projector matrix is (Dimic and Sidiropoulos, 2005)

$$\begin{aligned}\boldsymbol{P}_n^{\perp}(N_\mathrm{s})=\boldsymbol{I}_{N_\mathrm{t}}-\boldsymbol{H}^\mathrm{H}[S_{n-1}(N_\mathrm{s})]\{\boldsymbol{H}[S_{n-1}(N_\mathrm{s})]\\ \cdot\boldsymbol{H}^\mathrm{H}[S_{n-1}(N_\mathrm{s})]\}^{-1}\boldsymbol{H}[S_{n-1}(N_\mathrm{s})],\end{aligned} \tag{46}$$

where $\boldsymbol{I}_{N_\mathrm{t}}$ is the $N_\mathrm{t}\times N_\mathrm{t}$ identity matrix, and $\boldsymbol{H}(S_{n-1})$ denotes the channel vectors of the users selected in the first $n-1$ steps on the N_s-th subcarrier.

$$\boldsymbol{H}(S_{n-1})=[\boldsymbol{h}_{s_1}^\mathrm{H}\quad \boldsymbol{h}_{s_2}^\mathrm{H}\quad\cdots\quad \boldsymbol{h}_{s_{n-1}}^\mathrm{H}]. \tag{47}$$

Let $r_{n,k}(N_\mathrm{s})=|\boldsymbol{h}_k(N_\mathrm{s})\boldsymbol{P}_n^{\perp}(N_\mathrm{s})|^2$. Due to the idempotence of $\boldsymbol{P}_n^{\perp}(N_\mathrm{s})$, we have

$$\boldsymbol{r}_{n,k}(N_\mathrm{s})=\boldsymbol{h}_k(N_\mathrm{s})\boldsymbol{P}_n^{\perp}(N_\mathrm{s})\boldsymbol{h}_k^\mathrm{H}(N_\mathrm{s}). \tag{48}$$

- Finding a user s_n on the N_s-th subcarrier such that

$$s_n=\arg\max r_{n,k}(N_\mathrm{s}). \tag{49}$$

- Set $S(N_\mathrm{s})=S(N_\mathrm{s})\cup\{s_n\}$, $T=T-\{s_n\}$.

Step 4: $N=N-\{N_\mathrm{s}\}$, $n=1$, while $N\neq\varnothing$,

- go to Step 2.

Step 5: Beamforming on each subcarrier:

- on each subcarrier, let $\boldsymbol{v}=\boldsymbol{Q}^\mathrm{H}$, where $\boldsymbol{H}[S(N_\mathrm{s})]=\boldsymbol{LQ}$ is QR-type decomposition of $\boldsymbol{H}[S(N_\mathrm{s})]$.

Step 6: Dirty paper coding processing: applied on the rows of $\boldsymbol{L}$.

Step 7: Power allocation: water-filling across subcarriers and data streams.

Algorithm 2 (for ZF-BF)

Step 1: Initialization.

- Set $T=\{1,\ldots,N_K\}$, $N=\{1,\ldots,N_\mathrm{c}\}$.
- $n=1$.
- Let $S(N_\mathrm{s})=\varnothing$.

Step 2: If $T=\varnothing$, then set $T=\{1,\ldots,N_K\}$.

- Let $\boldsymbol{r}_{1,k}(N_\mathrm{s})=\boldsymbol{h}_k(N_\mathrm{s})\boldsymbol{h}_k^\mathrm{H}(N_\mathrm{s})$, where $k\in T$ and $N_\mathrm{s}\in N$. Find a user $s_1(N_\mathrm{s})$ such that

$$s_1=\arg\max_{\{k\in T,N_\mathrm{s}\in N\}} \boldsymbol{r}_{1,k}(N_\mathrm{s}). \tag{50}$$

- Set $S(N_\mathrm{s})=S(N_\mathrm{s})\cup\{s_1\}$, $T=T-\{s_1\}$.

Step 3: while $n\leq N_\mathrm{t}$,

- Increase n by 1.
- Denote the achievable rate $R_\mathrm{ZF}[S(N_\mathrm{s})]$, find a user s_n, such that

$$s_n=\arg\max_{s_n\in T} R_\mathrm{ZF}[S(N_\mathrm{s})\cup\{s_n\}]. \tag{51}$$

- Set $S(N_\mathrm{s})=S(N_\mathrm{s})\cup\{s_n\}$, $T=T-\{s_n\}$.

Step 4: If $R_\mathrm{ZF}(S_n)_\mathrm{max}<R_\mathrm{ZF}(S_{n-1})_\mathrm{max}$, $S(N_\mathrm{s})=S(N_\mathrm{s})-\{s_n\}$.

Step 5: $N=N-\{N_\mathrm{s}\}$.

Step 6: If $N\neq\varnothing$, set $n=1$, go to Step 2.

Step 7: Beamforming on each subcarrier:

- on each subcarrier, let

$$\boldsymbol{v}=\boldsymbol{H}^\mathrm{H}[S(N_\mathrm{s})]\{\boldsymbol{H}[S(N_\mathrm{s})]\boldsymbol{H}^\mathrm{H}[S(N_\mathrm{s})]\}^{-1}, \tag{52}$$

where $\boldsymbol{H}[S(N_\mathrm{s})]=\boldsymbol{LQ}$ is QR type decomposition of $\boldsymbol{H}[S(N_\mathrm{s})]$.

Step 8: Power allocation: water-filling across subcarriers and data streams.

SIMULATION AND DISCUSSION

In this section, we provide some numerical examples to illustrate the performance of the power allocation and scheduling algorithms in this paper. In the multi-user MIMO-OFDM system, we assume the number of transmit antenna is N_t, and each user with a single receive antenna. With spatial multiplexing, the maximum number of active users on each subcarrier is less than N_t.

We assumed that the discrete-time channel impulse response was generated according to the Hiperlan2 Channel Model C in (Medbo and Schramm, 1998). The channels between different transmit and receive antennas are assumed to be independent. The transmitter power was allocated across subcarriers by water-filling principle presented in Section 3. The receiver used linear decoder with perfect channel knowledge. It is also assumed that the transmitter has perfect knowledge of the channel state information.

Experiment 1 The first experiment compares the needed sum-power of DPC, TDMA, ZF-DPC and ZF-BF. The MIMO-OFDM system in this experiment is configured as: the number of transmitter antennas is 4, and each user is equipped with one receive antenna. The number of subcarriers is 64 and the cyclic prefix length is 16. The *Rate0* in Fig.2 is randomly created which ranges from 0.01 to 0.16. The rate of each user is the multiple of *Rate0*, and the number of users in this experiment is 80. Fig.2 shows the needed power of the power allocation algorithms. Compared with classical power allocation for TDMA, the transmitter schemes of ZF-DPC, DPC and ZF-BF only need much less power to reach the same rate.

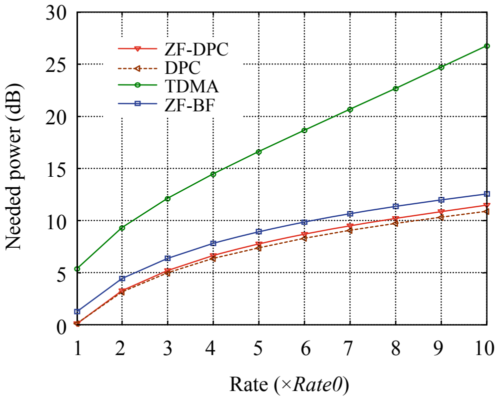

Fig.2 Sum power of different rates requirement with 4 transmit antennas

Experiment 2 The second experiment is about the needed power of selection without consideration of fairness. The configuration of multi-antenna OFDM system is the same as that of Experiment 1. The rate of each user is randomly created by *Rate0*, which is defined in Experiment 1. In this experiment, only those users with better channel condition is selected on each subcarrier. By 10000 times of channel realization, we can see from Fig.3 the needed power of TDMA, DPC, ZF-DPC and ZF-BF. Compared with Fig.2, we can see that the needed power of algorithm without fairness consideration is less than that with consideration of fairness. This is because that Algorithm 1 and Algorithm 2 have to consider the fairness among users. Thus the decrement of needed power can be considered as the loss of fairness.

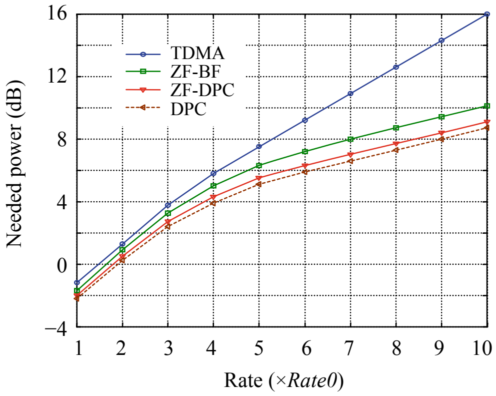

Fig.3 Sum power of different rates requirement without consideration of fairness among users

Experiment 3 The third experiment is about the fairness of user selection. In this experiment, we consider the number of subcarriers is 1024, and the user-scheduling scheme is as Algorithm 2. We consider 100 users in this experiment. Without loss of generality, we only consider the scenario of ZF-BF, and the results of ZF-DPC would be similar to that of ZF-BF. We test the scheduling Algorithm 2 for 10000 times. Fig.4 shows that by Algorithm 2, the selected frequency of each user is very close. Thus, by Algorithm 2, the system can achieve considerable fairness, and we can also look at this experiment as the duality of Experiment 2.

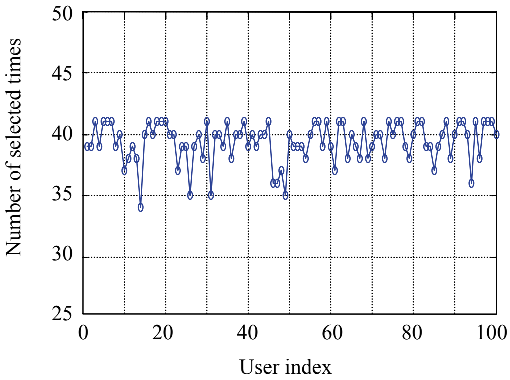

Fig.4 Effect of fairness consideration by Algorithm 2 of ZF-BF

Experiment 4 The influence of multi-user diversity is tested in this experiment. The rate of each user in this experiment is randomly created which ranges from 0.0156 to 0.25. Without loss of generality, the number of users in each simulation scenario is the multiple of 64. We also assume that there are 64 users that would be served at one time slot. Fig.5 shows that with the increment of the number of users, the needed power will decrease. This is because that multi-user gain is related to the size of user set. When the user number is large, the probability of selecting users with good channel state is higher than that of small number.

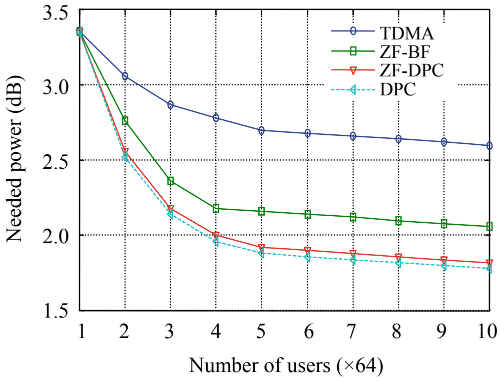

Fig.5 Effect of multi-user diversity

Experiment 5 The influence of the number of transmit antennas is tested in this experiment. The OFDM configuration is the same as that of Experiment 1, and the rate of each user is randomly created which ranges from 1 to 8. Here, we assume that the number of users is 64. We compared the total power consumption of TDMA, ZF-BF and ZF-DPC. Fig.6 shows that the total power of the three scenarios is the same when the number of the transmit antennas is one. The reason is that ZF-DPC and ZF-BF are actually TDMA when the transmitter only has a single antenna. We can also find that with the increment of the number of transmit antennas, the needed power will decrease. While Fig.6 also tells us that the number of transmit antennas does not need to be large, because the power saving is not obvious with a large number of transmit antennas.

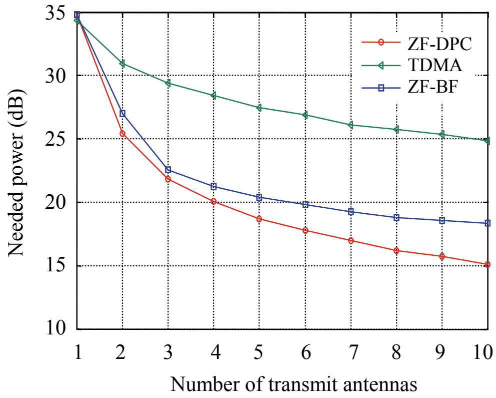

Fig.6 Effect of the number of transmit antennas

CONCLUSION

In this paper, we consider the user scheduling and power allocation for MIMO-OFDM system in broadcast channel. The objective of user scheduling is to achieve the fairness among user selection, and the power allocation is to minimize the transmit power. We propose user-scheduling algorithms for ZF-DPC, DPC and ZF-BF in multi-antenna OFDM system. We also present the general solution of minimization of transmitter power. We have examined the transmit strategies for multi-antenna BC with a large number of users. We have shown that ZF-BF can achieve considerable performance but at much lower complexity. The proposed user-scheduling algorithms indeed has good fairness performance for different transmit strategies. We also show that the needed power will be reduced with the increment of the number of users. This is because with a large number of users, the transmitter can choose users with good channel condition. Here, good channel condition means good channel quality and good orthogonality among multi-user channel vectors.

Although we focus on BC, we also mention the duality between BC and MAC. By duality of MAC and BC, all the results in this paper are directly applicable to a MAC scenario. In other words, we can also minimize the sum-power of all the terminals in MAC.

For the most of this paper, we only investigate users with a single receive antenna. The case of users with multiple receive antennas would be an important extension of the paper. Moreover, this paper is based on the assumption that the base station has perfect channel state information. When the transmitter only has partial channel knowledge, the problem in this paper is a new practical problem. The encoding order of DPC is very important in power minimization, thus, more work is needed in this area.

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