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Multi-objective optimization of water supply network rehabilitation with non-dominated sorting Genetic Algorithm-II*

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Abstract: Through the transformation of hydraulic constraints into the objective functions associated with a water supply network rehabilitation problem, a non-dominated sorting Genetic Algorithm-II (NSGA-II) can be used to solve the altered multi-objective optimization model. The introduction of NSGA-II into water supply network optimal rehabilitation problem solves the conflict between one fitness value of standard genetic algorithm (SGA) and multi-objectives of rehabilitation problem. And the uncertainties brought by using weight coefficients or punish functions in conventional methods are controlled. And also by introduction of artificial inducement mutation (AIM) operation, the convergence speed of population is accelerated; this operation not only improves the convergence speed, but also improves the rationality and feasibility of solutions.

Key words: Water supply system, Water supply network, Optimal rehabilitation, Multi-objective, Non-dominated sorting Genetic Algorithm (NSGA)

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INTRODUCTION

Optimal rehabilitation of water supply network has been a main research subject in water supply area for a long time. For the attributes and running characteristics, maintenance and rehabilitation are necessary and long-term tasks for water supply systems. And the most important and hardest part of maintenance and rehabilitation job in water supply system is water supply network rehabilitation.

Water network rehabilitation is a complex problem, and many facets should be concerned in the solving process. It is a discrete variables, non-linear, multi-objective optimization problem (Pu *et al.*, 2003). The concepts used to build decision models for this problem are varied, with the main methods being: general rehabilitation guides, prioritization models and optimization models (Engelhardt, 1999).

The methods of rehabilitation guides and prioritization models have been applied in the early period. Limited by theoretical basis and algorithm level, they cannot evaluate rehabilitation schemes in a scale of whole network. With the development of optimization theory and modeling technology, using a more comprehensive and detailed optimization model to solve rehabilitation problem becomes doable. Therefore optimization models of rehabilitation were generated. Optimization techniques consider the interaction of each main element with the system as a whole, which enable both the performance and the cost of the rehabilitated system to play a role in the formulation of the optimal rehabilitation model (Engelhardt *et al.*, 2000). This approach allows for the trade-off between system performance and cost of rehabilitation. However, such techniques require large numbers of trial evaluations to obtain near-global optimal solutions. By using models of water network system and new optimization methods (SGA, particle

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swarm optimization (PSO), NSGA, NSGA-II), solving these optimization models of rehabilitation becomes possible.

FORMULATION

Currently, the popular optimization model for rehabilitation of water supply network is using minimization of rehabilitation cost and energy cost per year in the investment period as objective and hydraulic performance of network as constraint. With this concept, the general form of rehabilitation optimization model can be expressed by equations below (Zhao, 2003).

Objective

$$\mathrm{Min}W_1=\left(P+\frac{m}{100}\right)\sum_{k\in N}(a+bD_k{}^c)L_k+3.58\sum_{i=1}^{3}(r_iE_it_i)\sum_{j\in n_\mathrm{s}}\frac{H_{ij}Q_{ij}}{\eta_{ij}}, \tag{1}$$

$$P=\frac{I(1+I)^T}{(1+I)^T-1}, \tag{2}$$

where a, b, c are coefficients in formula of pipe construction cost; D_k is diameter (mm) of pipe k; E_i is electricity price in period i (Yuan/(kW·h)); H_{ij} is pressure of pump j in period i (m); I is norm yield rate (%); L_k is length (m) of pipe k; m is lift to save a rate of capital repairs fund (%); N is set of pipes which need be rehabilitated; n_s is set of pump stations; P is equivalent coefficient; Q_{ij} is flow of pump j in period i (L/s); r_i is coefficient of pumping energy in period i; T is repayment period of investment (year); t_i is time in period i (h); η_{ij} is efficiency of pump j in period i (%);

$$i=\begin{cases}1, \text{peak period of electricity consumption;}\\ 2, \text{low period of electricity consumption;}\\ 3, \text{normal period of electricity consumption.}\end{cases}$$

Constraints

Continuous equation:

$$Q_i-\sum_{j\in V_i}q_{ij}=0,\ \ i=1,\ 2,\ ...,\ n. \tag{3}$$

where Q_i is nodal demand (L/s) of node i; q_{ij} is pipe flow (L/s) from node i to node j; V_i is adjacent nodes set of node i.

Energy balance equation:

$$\left.\begin{array}{l}\sum(h_{ij})_1=0\\ \sum(h_{ij})_2=0\\ \quad\cdots\\ \sum(h_{ij})_l=0\end{array}\right\}. \tag{4}$$

where h_{ij} is head loss (m) of pipe which from node i to node j; l is loop number.

Node pressure constraint:

$$H_{\min}\le H_j\le H_{\max},\ \ j\in J. \tag{5}$$

where H_j is pressure (m) of node j; $H_{\min}$ is the minimum service pressure (m); $H_{\max}$ is the maximum service pressure (m); J is node set of network.

Pipe velocity constraint:

$$v_{i\min}\le v_i\le v_{i\max},\ \ i\in P_\mathrm{S}. \tag{6}$$

where v_i is velocity of pipe i (m/s); $v_{i\max}$ is upper boundary of velocity of pipe i (m/s); $v_{i\min}$ is lower boundary of velocity of pipe i (m/s); P_S is pipe set of network.

Diameter standard constraint:

$$D_i\in D_\mathrm{S}=\{D_1,\ D_2,\ ...,\ D_n\} \tag{7}$$

where D_i is diameter (mm) of pipe i; D_S is available standard diameter set.

From the optimization model, we can find that it is a one objective model with minimization of construction cost and energy cost as objective, and network performance and available diameters as constraints. In conventional solving methods, constraints will be transformed as parts of objective function by weighting method or ε-constraint method (Li *et al.*, 1999). These transformations enable the traditional algorithms to be used in solving this optimization model. Unfortunately, the solution obtained by this process largely depends on the values assigned to the weighting factors or the design of ε-constraint function. This approach does not provide a dense spread of the Pareto points. So the best way to solve rehabilitation problem is to abstract the problem as a multi-objective model, and solve the model with a true multi-objective oriented algorithm.

Alteration of optimization model

The concept of alteration is regarding the constraints transformed as parts of objective function with weighting method or ε-constraint function as isolate objective functions. So the constraints which represent performance of water supply network should be expressed as objective functions.

Objective function of pipe velocity can be expressed with expression below:

$$\mathrm{Min}W_2 = \alpha\left(\sum_{i\in P_\mathrm{m}}(\Delta v_i\cdot fl_i)+\sum_{j\in P_\mathrm{x}}(\Delta v_j\cdot fl_j)\right)+ \beta\sum_{k\in P_\mathrm{S}}(|v_k - v_\mathrm{s}|\cdot fl_k), \tag{8}$$

where α, β are weight coefficients; Δv is pipe velocity shortfalls or excesses with velocity boundaries (m/s); fl is pipe flow (L/s); v is pipe velocity (m/s); P_m is set of pipes with velocity below the lower boundary of velocity; P_x is set of pipes with velocity above the upper boundary of velocity; P_S is set of all pipes in the network; v_s is the standard velocity which means the ideal velocity that all pipes want to achieve.

Eq.(8) can be separated as two parts: feasible solution related part and optimal solution related part. Feasible solution related part is calculated as the sum of the velocity shortfalls or excesses (Δv), weighted by the pipe flow (fl): $\sum_{i\in P_\mathrm{m}}(\Delta v_i\cdot fl_i)+\sum_{j\in P_\mathrm{x}}(\Delta v_j\cdot fl_j)$. Optimal solution related part is calculated as the sum of the difference between pipe velocity (v) and standard velocity (v_s), weighted by the pipe flow (fl): $\sum_{k\in P_\mathrm{S}}(|v_k - v_\mathrm{s}|\cdot fl_k)$. And α, β are weights to allow different emphasis on these two parts. This objective function means that we want the schemes with which pipe velocities meet the velocity boundaries and approach to standard velocity most.

Objective function of node pressure built with similar concept of pipe velocity objective function, also has feasible solution related part and optimal solution related part:

$$\mathrm{Min}W_3 = \rho\sum_{j\in J_\mathrm{m}}(\Delta h_j\cdot Q_j)+\sigma\sum_{i\in J_\mathrm{x}}(\Delta h_i\cdot Q_i), \tag{9}$$

where ρ, σ are weight coefficients; Δh is node pressure shortfalls or excesses with minimum service pressure (m); Q is nodal demand (L/s); J_m is set of nodes with pressure below the minimum service pressure; J_x is set of nodes with pressure above minimum service pressure. Feasible solution related part is calculated as the sum of the pressure shortfalls (Δh), weighted by the node demand (Q): $\sum_{j\in J_\mathrm{m}}(\Delta h_j\cdot Q_j)$. Optimal solution related part is calculated as the sum of pressure excesses (Δh), weighted by the node demand (Q): $\sum_{i\in J_\mathrm{x}}(\Delta h_i\cdot Q_i)$. And ρ, σ are weights to allow different emphasis on different part of objective function. Node pressure objective function means that we want the schemes of which all node pressure values are above the minimum service pressure but excess least. To determine the nodal pressures and pipe velocities and hence the results of these two objectives of a solution, the steady-state hydraulic network solver, EPANET (Rossman, 1993), is used.

Design of objective functions in two parts is for the reason of doing a dense spread searching in feasible solution space. If there is only feasible solution related part in pipe velocity objective function and node pressure objective function, the multi-objective model becomes one-objective model in feasible solution space, and cannot achieve a dense spread in feasible field. And the algorithm only converges to feasible solutions but has a low efficiency in finding optimal solutions among them.

SOLUTION PROCEDURE

Water supply network design belongs to a group of inherently intractable problems commonly referred to as NP-hard (Templeman, 1982). It is well known that when diameters are assumed as the decision variables (DV), the constraints are implicit functions of the DV, the feasible region is non-convex, and the objective function is multimodal. Hence, conventional optimization methods result in a local optimum which is dependent on the starting point in the search process (Gupta *et al.*, 1999).

In the case of multi-objective optimization, instead of obtaining a unique optimal solution, a set of equally good (non-dominating) optimal solutions is usually obtained (Pareto sets). More precisely, within a Pareto set, when one moves from any one point to another, one objective function improves while the other deteriorates. In absence of any other high level additional information, a decision maker normally cannot choose any one of these non-dominant optimal solutions since all of them are equally competitive and none of them can dominate each other.

Several methods, such as meta-heuristic algorithm (Kim *et al*., 2004), versions of GA (Keedwell and Khu, 2005; Alvisi and Franchini, 2006), NSGA-II (Khu and Keedwell, 2005) and particle swarm optimization algorithm (Liu *et al*., 2007) are available to solve multi-objective optimization problems. NSGA-II (Deb, 2001a; 2001b; Deb *et al*., 2002) is used here to obtain the Pareto set. Use of penalty function is a very popular way of handling constraints. But tuning of the penalty parameter appearing in the penalty function is very time-consuming and normally performed on the basis of trial and error. Unless tuned properly, one may get misdirected totally in the search space. NSGA-II based constraint-handling technique allows one to get rid of the above stated problem of penalty function. Non-dominated sorting based techniques have several advantages over other techniques:

(1) The spread of the Pareto set is excellent;

(2) A single simulation run can yield the entire Pareto set;

(3) They can handle problems with discrete search spaces (Mitra and Gopinath, 2004).

Coding

Real coding method is used in solving this multi-objective rehabilitation model. First, each standard pipe diameter is appointed a real number code, and then code strings were made with all rehabilitated pipes' diameter code. The coding option can be described with Table 1 and Table 2. Suppose that the IDs of rehabilitated pipe are 1, 2, 3, 4, 5.

Table 1 Code of standard pipe diameters

Diameter (mm)	Code	Diameter (mm)	Code
100	0	900	8
200	1	1000	9
300	2	1200	10
400	3	1400	11
500	4	1600	12
600	5	1800	13
700	6	2000	14
800	7	2200	15

Table 2 Code of chromosome of individuals of initial generation

Index	Pipes' diameter scheme (order is pipe 1~pipe 5)	Chromosome code
1	800,1800,1400,800,100	7,13,11,7,0
2	2000,2000,500,300,400	14,14,4,2,3
3	400,1600,1000,700,1400	3,12,9,6,11
4	300,2000,200,200,1800	2,14,1,1,13
5	1400,2200,400,1600,100	11,15,3,12,0
6	2000,1800,900,800,900	14,13,8,7,8
7	700,2000,100,300,1400	6,14,0,2,11
8	2000,200,1200,1000,800	14,1,10,9,7
9	300,1800,2000,1000,100	2,13,14,9,0
10	1200,2200,900,400,100	10,15,8,3,0

Selection operation

1. Disposal of objective values

The three objective functions of rehabilitation model are all minimization functions, but in the process of non-dominated sorting, the individuals that have larger objective values will dominate the ones with smaller objective values, and can be sorted in a preferential rank. So some disposal should be done to objective values and put better individual in the preferential rank. Here reciprocal of the objective values is used in the process of non-dominated sorting. For the sake that the reciprocal of the objective value is still a number in the case that the objective value is zero, a constant was added to objective value before reciprocal disposal.

The disposal of objective values can be expressed with the function below:

$$W_i' = Const/(W_i + Const'), \tag{10}$$

where W_i is original objective value; *Const*, *Const'* are constants varying with different objectives.

2. Individual sorting

Sorting is very important in the selection operation. In NSGA-II, individuals will be sorted according to two parameters: non-dominated rank and crowding distance.

A fast non-dominated sorting approach is used in non-dominated sorting process of NSGA-II. It can reduce the complexity of non-dominated sorting from

$O(MN^3)$ to $O(MN^2)$, where M is objectives and N is individual count of population. And also in NSGA-II, sharing function approach in NSGA is replaced with a crowded-comparison approach that eliminates the difficulties of sharing function approach to some extent, and the new approach does not require any user-defined parameter for maintaining diversity among population members. And also the crowded-comparison approach has a better computational complexity (Deb *et al*., 2002).

After obtaining the non-dominated sorting rank and crowding-distance of individuals, individuals can be sorted by crowded-comparison operator $(\prec_n)$ which guides the selection process at various stages of the algorithm toward a uniformly spread-out Pareto optimal front. The sorting rules can be expressed by the codes below:

if $i_{\text{rank}} < j_{\text{rank}}$ or ($i_{\text{rank}}=j_{\text{rank}}$ and $i_{\text{distance}}>j_{\text{distance}}$)
$i \prec_n j$
end if

where i_{rank} is non-domination rank; i_{distance} is crowding distance.

That is, between two solutions with different non-domination ranks, we prefer the solution with lower (better) rank. Otherwise, if both solutions belong to the same front, then we prefer the solution that is located in a lesser crowded region.

3. Selection

The common selection methods in GA are: roulette wheel selection, stochastic universal sampling, local selection, truncation selection, tournament selection (Wang and Cao, 2002). In this paper roulette wheel selection was used. In the selection process, calculate the fitness value of each individual firstly, and then transfer these fitness values into selection probabilities, whose transfer function is:

$$p_k = f_k \Big/ \sum_{j=1}^{Pop\_size} f_j . \tag{11}$$

By calculating with this function, the fitness value of individual f is transferred to selection probability p. The cumulative probability (p_k') of kth individual can be obtained by adding the individual probabilities starting from top of the list till the kth member. The kth individual is represented by the cumulative probability value between p_{k-1}' and p_k'. In order to choose population size individuals, population size random numbers are chosen between 0 and 1, and the individuals in whose cumulative probability range these numbers lie are selected as the new individuals of the population for further operation.

Crossover operation

Crossover is one of important reasons that the individuals can evolve. Crossover operation is responsible for searching new individuals, which could possibly have better fitness. By crossover, new individuals can be produced, and the searching scope of the problem is expanded. There also are several crossover methods: one point crossover, two points crossover, multi-points crossover and equality crossover. Here, one point crossover method is used as the crossover operation. The one point crossover operation can be expressed by the codes as follows:

$f=rand()\in(0,1)$
if $f>cross\ probability$
$X_{\text{parent1}}=X(rand()\in[1,N])$
$X_{\text{parent2}}=X(rand()\in[1,N])$
$\lambda=rand()\in(0,1)$
$X_{\text{new}}=(1-\lambda)X_{\text{parent1}}+\lambda X_{\text{parent2}}$
end if

where N is the number of individuals in population.

Mutation operation

Mutation operation is carried out to maintain the diversity in the population. It is a local random searching method. Cooperating with selection and crossover operations insures that the population will undergo useful evolution and obtain better individuals. And the mutation operation is also responsible for preserving diversity and preventing population from precocity (Guan, 2004). But since mutation operation has random attribute, although it can achieve the goal of diversity preservation, it also tampers with the constringency of GA. Sometimes the bad gene that mutation operation brought in will be gotten rid of by many generations; in the worst situation, the bad gene will be kept until the end generation. For the sake of bringing in beneficial gene in the mutation operation, a new mutation method: artificial inducement mutation (AIM) was introduced in the NSGA-II in this paper. This operation can steer the population convergence to the field of feasible solutions accelerately, and then use normal mutation operation searching for the best solution in the feasible solutions.

For the optimization model of rehabilitation, the goal of AIM is to make the selected diameters of rehabilitated pipes follow the direction of meeting the constraint of pipe velocity, until the solutions converge to the feasible field. The AIM can be expressed by codes below:

```
If ESpipes≠Φ
// artificial inducement mutation operator
For each pipe in ESpipes
If pipeVelocity>upper limit of velocity and
pipeDia<maxDia
    pipeDia++
  end if
If pipeVelocity<lower limit of velocity and
pipeDia>minDia
    pipeDia—
  end if
else
  Common one point mutation operation
end if
```

where *ESpipes* is the set of pipes that do not meet velocity boundaries.

Fig.1 is the flow chart of NSGA-II with AIM.

CASE STUDY

Case network introduction

In the case of water supply network, with the increase of water consumption, the phenomenon of over fast velocity, high hydraulic slope and lower node pressure has appeared. The chock pipes and lower pressure nodes are shown in Tables 3 and 4, and their positions are shown in Fig.2.

The optimal rehabilitation model of case network was solved by NSGA-II without and with AIM operation, respectively. The parameters in these two algorithms are the same and shown in Fig.3. One thing should be explained is that the mutation probability is the probability of one point random mutation operation. Since AIM is always beneficial to population, so in the algorithm of NSGA-II with AIM operation, its probability is 1, which means use AIM always.

Experimental results and discussions

3D non-dominated Pareto optimal fronts in the initial, 3rd, 6th and 30th generations of NSGA-II

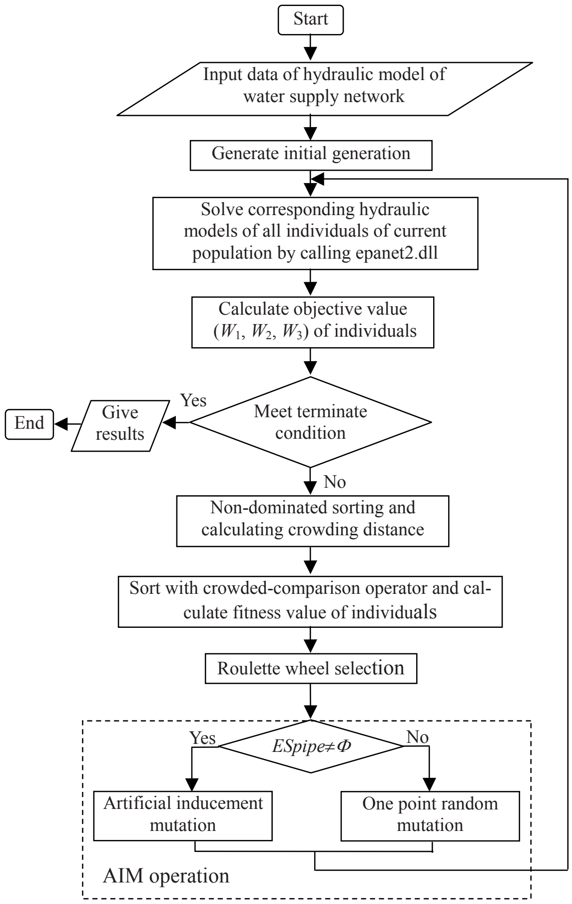

Fig.1 Flow chart of NSGA-II with AIM

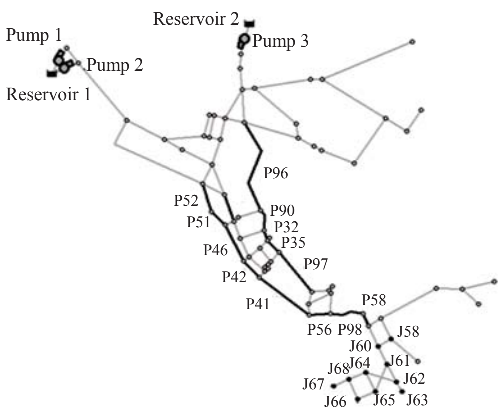

Fig.2 Positions of chock pipes and low pressure nodes in case network

without AIM are shown in Fig.4. The detailed information of individuals of non-dominated Pareto optimal front in the 100th generation is shown in Table 5.

It can be seen from Fig.4 that in initial generation only 12 non-dominated Pareto optimal solutions were obtained. With successive evolution the dominated solutions were eliminated and replaced by better solutions so that the number of non-dominated Pareto optimal solutions increased to 43 (there are cases that some individuals are superpositioned) at the 30th generation. It is also seen that with the increase of the number of generations, better solutions are obtained.

From Table 5 it can be seen that the rehabilitation schemes were improved to some extent. But with further analysis, we can find that although the node pressure constraint was met, there are still some pipes which do not satisfy the velocity constraint. So all these solutions are not feasible solutions, and cannot be adopted. This phenomenon is mainly because although new diameters can be brought in by mutation

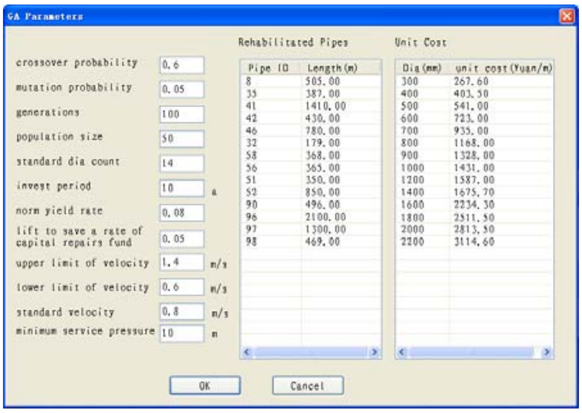

Fig.3 Parameters setting dialog of NSGA-II

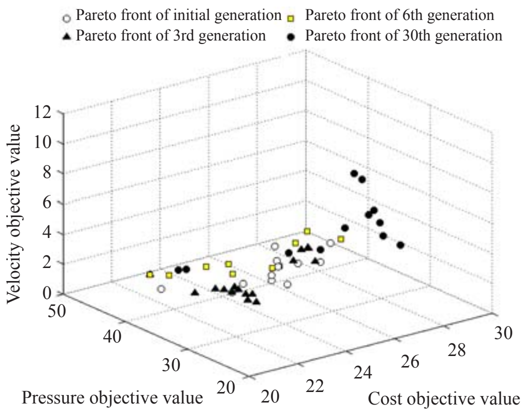

Fig.4 3D Pareto optimal fronts of initial, 3rd, 6th and 30th generations of NSGA-II without AIM

Table 3 Data of chock pipes in case network

Pipe ID	Diameter (mm)	Length (m)	Hydraulic slope (m/km)	Velocity (m/s)
8	300	505.96	7.77	1.57
32	750	179.83	2.58	1.67
35	750	387.09	2.07	1.48
41	600	1410.74	3.28	1.52
42	600	430.40	3.31	1.53
46	600	780.28	3.58	1.60
51	600	350.52	3.77	1.64
52	600	850.39	3.81	1.65
56	400	365.76	5.64	1.58
58	400	368.80	5.51	1.56
90	750	496.30	3.35	1.77
96	800	2100.00	2.95	1.59
97	600	1300.00	5.19	1.80
98	400	469.00	9.12	1.89

Table 4 Data of lower pressure nodes in case network

Node ID	Elevation (m)	Demand (L/s)	Pressure (m)	Head (m)
58	3.20	9.86	9.68	12.88
60	4.27	9.74	8.64	12.91
61	3.96	6.85	8.78	12.74
62	3.96	14.61	8.77	12.73
63	4.27	6.09	8.44	12.71
64	5.49	10.81	7.17	12.66
65	5.49	19.47	7.17	12.66
66	4.60	9.74	8.01	12.61
67	5.24	8.37	7.31	12.55
68	4.59	6.20	8.01	12.60

Table 5 Information of individuals of Pareto optimal front in the 100th generation of NSGA-II without AIM

Index	Rehabilitation cost per year (Yuan)	Energy cost per year (Yuan)	Number of nodes that do not meet pressure constraint	Number of pipes that do not meet velocity constraint
0	2071630.50	1594264.25	0	8
1	2223924.00	1548276.87	0	2
5	1787864.62	1577684.50	28	4
6	1819977.62	1587648.62	19	5
8	2202550.75	1564433.75	0	3
9	1967257.87	1645459.37	0	5
10	2150773.25	1561423.50	0	2
16	1957207.12	1647132.12	0	4
18	2041618.12	1480649.00	0	6
22	1822743.37	1635729.25	0	6
28	1777813.87	1588044.37	19	4
31	1745701.00	1578103.37	28	4
34	1906633.75	1641831.87	0	4
39	2073730.87	1499369.37	0	4

operation, the mutation probability is low and mutation operation has a random attribute, so the searching scope was limited, and the constringency of the algorithm was not good. If a larger solution scope searching must be achieved, a larger population size and generation size should be assigned, and also the quality of individuals of initial generation should be improved. But all these operations will result in the very time consuming situation. This conflict can be solved by introduction of AIM.

3D non-dominated Pareto optimal fronts in the initial, 3rd, 6th, and 30th generations of NSGA-II with AIM are shown in Fig.5. The detailed information of individuals of non-dominated Pareto optimal front in the 100th generation is shown in Table 6.

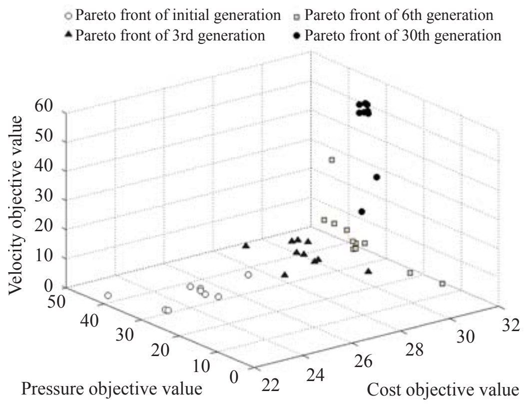

Fig.5 3D Pareto optimal fronts of the initial, 3rd, 6th and 30th generations of NSGA-II with AIM

Table 6 Information of individuals of Pareto optimal front of the 100th generation of NSGA-II with AIM

Index	Rehabilitation cost per year (Yuan)	Energy cost per year (Yuan)	Number of nodes that do not meet pressure constraint	Number of pipes that do not meet velocity constraint
1	1760528.75	1479173.25	0	0
2	1855309.12	1477399.87	0	0
3	1833963.25	1467635.00	0	0
5	1803437.25	1489670.87	0	0
10	1857008.62	1471633.00	0	0
11	1847293.50	1467635.00	0	0
12	1852434.75	1470633.62	0	0
13	1831088.87	1462651.25	0	0
15	1868639.37	1477399.87	0	0
16	1777517.50	1479173.25	0	0
18	1748898.00	1475196.75	0	0
33	1759431.25	1458747.12	0	2
35	1765886.75	1475196.75	0	0
39	1818361.37	1493464.50	0	0
40	1838885.25	1494793.75	0	0
43	1843678.50	1471633.00	0	0
46	1798863.37	1488790.25	0	0

It can be seen from Fig.5 that the Pareto fronts of these generations were separated more obviously than fronts shown in Fig.4, which means that the evolution is accelerated by AIM. Table 6 shows that individuals which meet both constraints of node pressure and pipe velocity were generated. This indicates that AIM does guide the population convergence to feasible solutions field accelerately. And optimal solutions in feasible solution field can be searched with the rest of the evolution process.

Comparison of Pareto fronts of NSGA-II with and without AIM in different generations is illustrated in Fig.6. From this figure the function of AIM can be shown more clearly. As the initial generations of each algorithm were generated randomly, Pareto fronts in initial generation of two algorithms were mixed up. With the evolution process going on, Pareto fronts of NSGA-II with AIM were pushed to better position rapidly, and the distance between Pareto fronts of two algorithms became larger.

The hydraulic information of one of the best solutions of NSGA-II with AIM is listed in Table 7. The pressure information of lower pressure nodes in the rehabilitated network is listed in Table 8. Data of Tables 7 and 8 show that the chock pipes have been eliminated and the pressure of the lower pressure nodes has been improved to an acceptable level. In this rehabilitation scheme, the diameters of some chock pipes have not been modified; this is because

Table 7 Data of chock pipes after rehabilitation

Pipe ID	Diameter (mm)	Length (m)	Hydraulic slope (m/km)	Velocity (m/s)
8	300	505.96	7.77	1.57
32	750	179.83	2.58	1.67
35	750	387.09	2.07	1.48
41	600	1410.74	3.28	1.52
42	600	430.40	3.31	1.53
46	600	780.28	3.58	1.60
51	600	350.52	3.77	1.64
52	600	850.39	3.81	1.65
56	400	365.76	5.64	1.58
58	400	368.80	5.51	1.56
90	750	496.30	3.35	1.77
96	800	2100.00	2.95	1.59
97	600	1300.00	5.19	1.80
98	400	469.00	9.12	1.89

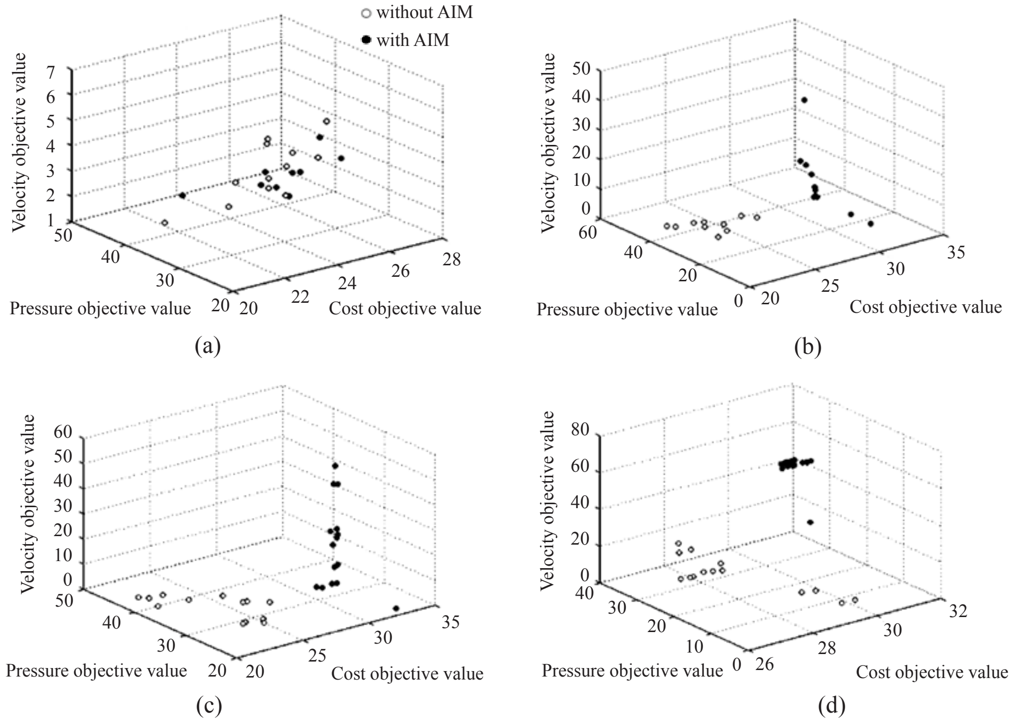

Fig.6 Comparison of Pareto fronts of NSGA-II with and without AIM in the initial (a), 6th (b), 10th (c) and 100th (d) generations

Table 8 Data of lower pressure nodes after rehabilitation

Node ID	Pressure (m)	Node ID	Pressure (m)
58	23.98	64	21.48
60	22.95	65	21.48
61	23.08	66	22.32
62	23.08	67	21.62
63	22.75	68	22.32

that with the magnifying of diameters of some rehabilitated pipes, the transportation capacity of these pipes was increased, more water was transported by these pipes, and velocities of other supposed rehabilitated pipes became slower consequently and met the velocity constraint without diameter modification.

CONCLUSION

By introduction of NSGA-II, the problem of multi-objective of optimal rehabilitation model with one fitness value of conventional GA is solved. The shortcomings that brought in by weighting method or ε-constraint method have been eliminated.

By introduction of AIM, the population is directed to feasible solutions field rapidly, and can search the best solution in the feasible field. So the convergence of algorithm has been improved, and can give more feasible and better solutions.

By comparing the results of two NSGA-IIs with and without AIM in the case study, the advantage and feasibility of AIM are shown and evaluated.

In this paper one type of multi-objective optimal model of water supply network rehabilitation was solved with NSGA-II and AIM. In fact there are several different concepts in building multi-objective optimal model for water supply network rehabilitation. NSGA-II and AIM can still work in solving other multi-objective optimal models built in other concepts.

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