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Mechanical analysis and reasonable design for Ti-Al alloy liner wound with carbon fiber resin composite high pressure vessel

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Abstract: To consider the internal pressure loaded by both the cylindrical Ti-Al alloy liner and the carbon fiber resin composite (CFRC) wound layers, two models are built. The first one is a cylinder loaded with the internal pressure in the hoop direction only. In this model, the total hoop direction load is distributed over all layers under the internal pressure. The second one is a cylinder loaded with the internal pressure in the axial direction only. In this model, the total axial load is distributed over all cylinders under the internal pressure. Taking the boundary conditions of the continuous displacement between layers into account, a group of equations are built. From these equations, we get the solutions of stresses in both hoop direction and axial direction loaded by every layer under internal pressures. After the stresses are obtained, a reasonable design can be done. An example is given in the final section of this study.

Key words: Mechanical analysis, Composite, Pressure vessel, Models
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INTRODUCTION

The light weight high pressure composite vessels are commonly used in modern industry. Since higher pressure results in higher stress of vessel and needs higher energy for its operation, it is important to well understand the mechanics of this type of vessel. Due to the anisotropic performance of carbon fiber resin composite (CFRC) wound layer and the complicate layer arrangement, it is difficult to analyze a cylinder mechanically. More and more light weight high pressure composite vessels are used in modern advanced industry, such as spaceflight and aeronautic structures (Bi, 2006), hydrogen storage vessels in hydrogen fuel cell vehicles (Zheng and Liu, 2006), and others. Since weight is important to the mobile vessels in these fields, the higher pressure and lighter weight are required, and the Ti-Al alloy liner wound with CFRC vessels can satisfy this requirements. Because the mechanical property of Ti-Al alloy is different from that of CFRC, that is, the Ti-Al alloy is isotropic while the CFRC is anisotropic, the mechanical property of carbon fiber wound vessel is different from the traditional metal vessel. Therefore, it is difficult to use the traditional formula to calculate stresses. The traditional computation is suitable for classic CFRC laminate plate but not for CFRC wound high pressure vessels (Agarwal and Broutman, 1990; Wild and Vickers, 1997; Perreux and Lazuardi, 2001), because of several factors such as fiber winding angle, pretension force and solidifying method. This study tries to find the stress formula for the cylinders of this type of vessel which will be useful in designing the carbon fiber wound Ti-Al alloy liner vessels (Paul and Timothy, 2003; Zheng, 2006b).

STRUCTURE OF VESSEL

The cross section of a carbon fiber wound Ti-Al alloy liner vessel is shown in Fig.1. The inner shell is Ti-Al alloy liner (assumed to be the 1st layer) and the exterior layers are CFRC (assumed to be (m−1) layers in all), which are wound on the liner (Zheng and Cao, 2005). The two ends of the cylinder are formed heads, such as hemispherical and elliptical heads, and all fibers are wound in a balanced way on end-closure surfaces (Zheng, 2006a). The vessel is subjected to the internal pressure (P_i). The winding angle of CFRC for each layer is α_i (i=2, 3, …, m) along the hoop direction of the cylinder. The thickness of each layer is t. The stress in longitude direction of fiber is σ_T.

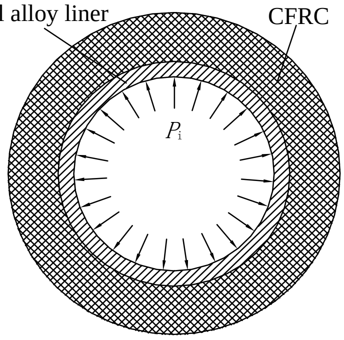

Fig.1 Cross section of CFRC wound Ti-Al alloy liner vessel. P_i: internal pressure

MODEL BUILDING

Mechanical analysis is based on the following assumptions (Zhou and Fan, 2001; Messager *et al.*, 2002; Wu *et al.*, 2003):

(1) Single CFRC layer is transversely isotropic. Macroscopically, the mechanical properties in the transverse direction are the same because the fibers are laid uniformly in the transverse section.

(2) Single CFRC layer is linear elastic. The resin is viscoelastic while the fiber is linear elastic and the module of resin is much smaller than that of fiber. So CFRC is assumed as linear elastic.

(3) There is no gliding between liner and CFRC wound layers or between CFRC layers.

The ratio of the hoop stress and the axial stress varies with the winding angle α_i (Fig.2), so it is difficult to have accurate stress distribution. If the model is simplified in some way, the hoop stress and the axial stress will be available. So the whole cylinder is simplified to two models. The first model is a cylinder loaded with the internal pressure in the hoop direction only, and the second one is a cylinder loaded with the internal pressure in the axial direction only.

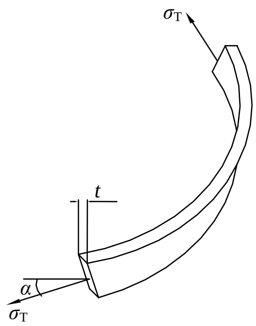

Fig.2 Carbon fiber wound layer

α: the winding angle of CFRC for each layer along the hoop direction of the cylinder; t: the thickness of each layer; σ_T: the stress in longitude direction of fiber

Model of the cylinder loaded with the internal pressure in the hoop direction only

Since the internal pressure in the hoop direction of a cylinder is loaded by the Ti-Al alloy liner and all CFRC layers together, we can build the models shown as Fig.3.

From the model, the following equation is obtained:

$$2\pi R_i p_i = 2\pi R_1 p_1 + 2\pi R_2 p_2 + \cdots + 2\pi R_n p_n + \cdots + 2\pi R_m p_m,$$

i.e.,

$$R_1p_1+R_2p_2+R_3p_3+\ldots+R_np_n+\ldots+R_mp_m=R_ip_i. \quad (1)$$

where R_n, R_m are the interior radii of the nth and mth layers, respectively; p_n, p_m are the interior pressures of the nth and mth layers, respectively.

Considering the boundary conditions between layers, the hoop strain at the same radius of r will be the same, which means that the interior wall's strain of the nth layer will be the same as the exterior wall's of the (n−1)th layer, and the exterior wall's strain of the nth layer will be the same as the interior wall's of the (n+1)th layer, expressed by the following equations:

$$\begin{cases} \varepsilon_{\theta 1\text{out}} = \varepsilon_{\theta 2\text{in}}, \\ \varepsilon_{\theta 2\text{out}} = \varepsilon_{\theta 3\text{in}}, \\ \quad\vdots \qquad\quad \vdots \\ \varepsilon_{\theta(m-2)\text{out}} = \varepsilon_{\theta(m-1)\text{in}}, \\ \varepsilon_{\theta(m-1)\text{out}} = \varepsilon_{\theta m\text{in}}, \end{cases} \quad (2)$$

where $\varepsilon_{\theta(m-1)\text{out}}$ is the exterior wall's strain of the (m−1)th layer, $\varepsilon_{\theta m\text{in}}$ is the interior wall's strain of the mth layer.

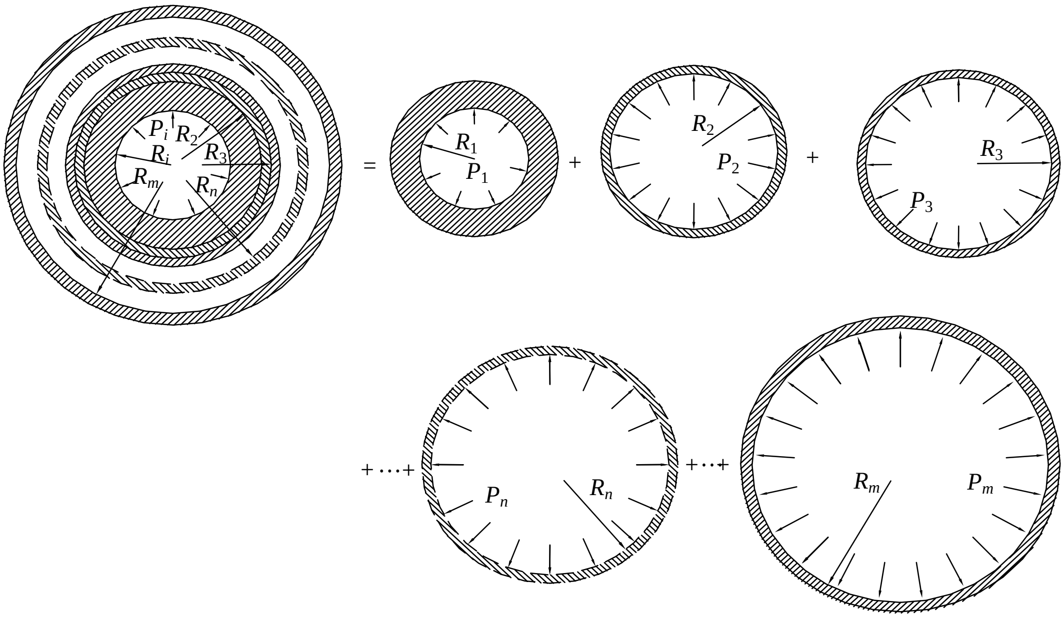

Fig.3 Model of the cylinder loaded with the internal pressure in the hoop direction only. R_n, R_m are the interior radii of the *n*th and *m*th layers, respectively; p_n, p_m are the interior pressures of the *n*th and *m*th layers, respectively

Stress formulae of single CFRC layer with winding angle α

The axial elastic module and the transverse elastic module of a single direction CFRC are assumed as E_{L} and E_{T}. The Poisson's ratio of axial direction to transverse direction of a simple direction CFRC is assumed as ν_{TL}. We define θ, r and z as the hoop, radial and axial directions of the cylinder, respectively. The elastic modules in θ direction, r direction and z direction of a single CFRC layer are E_θ, E_r and E_z, respectively. The Poisson's ratios of θ direction to r direction, θ direction to z direction, and r direction to z direction are defined as $\nu_{\theta r}$, $\nu_{\theta z}$ and ν_{rz}, respectively. According to the formulas of offset-axis stiffness, we obtain the following formulas deduced from Chen *et al.*(1997), Wild and Vickers (1997) and Zhou (2001):

$$E_\theta=\frac{1}{\dfrac{\cos^4\alpha}{E_{\mathrm{L}}}+\left(\dfrac{1}{G_{\mathrm{LT}}}-2\dfrac{\nu_{\mathrm{TL}}}{E_{\mathrm{L}}}\right)\sin^2\alpha\cos^2\alpha+\dfrac{\sin^4\alpha}{E_{\mathrm{T}}}},$$

$$E_z=\frac{1}{\dfrac{\cos^4\alpha}{E_{\mathrm{L}}}+\left(\dfrac{1}{G_{\mathrm{LT}}}-2\dfrac{\nu_{\mathrm{LT}}}{E_{\mathrm{L}}}\right)\sin^2\alpha\cos^2\alpha+\dfrac{\sin^4\alpha}{E_{\mathrm{T}}}},$$

$$\nu_{\theta z}=\left[(\cos^4\alpha+\sin^4\alpha)\frac{\nu_{\mathrm{TL}}}{E_{\mathrm{T}}}-\cos^2\alpha\sin^2\alpha\left(\frac{1}{E_{\mathrm{T}}}+\frac{1}{E_{\mathrm{L}}}\right)\right]E_{\mathrm{T}}.$$

Taking into account the gravity and the shear stress, the equilibrium equation of force about a single CFRC layer on the cylinder is

$$\sigma_\theta-\sigma_r=r\frac{\mathrm{d}\sigma_r}{\mathrm{d}r}. \tag{3}$$

The geometrical equations are

$$\varepsilon_r=\frac{\mathrm{d}w}{\mathrm{d}r},\ \varepsilon_\theta=\frac{w}{r}, \tag{4}$$

where w is the radial displacement of random radius.

From Eq.(4), we obtain

$$\frac{\mathrm{d}\varepsilon_\theta}{\mathrm{d}r}=\frac{1}{r}(\varepsilon_r-\varepsilon_\theta). \tag{5}$$

The constitutive equations are

$$\begin{cases}\varepsilon_\theta=\dfrac{1}{E_\theta}\sigma_\theta-\dfrac{\nu_{r\theta}}{E_r}\sigma_r-\dfrac{\nu_{z\theta}}{E_z}\sigma_z,\\ \varepsilon_r=-\dfrac{\nu_{\theta r}}{E_\theta}\sigma_\theta+\dfrac{1}{E_r}\sigma_r-\dfrac{\nu_{zr}}{E_z}\sigma_z,\\ \varepsilon_z=-\dfrac{\nu_{\theta z}}{E_\theta}\sigma_\theta-\dfrac{\nu_{rz}}{E_r}\sigma_r+\dfrac{1}{E_z}\sigma_z,\end{cases} \tag{6}$$

where σ_θ, σ_r, σ_z are the hoop stress, radial stress, and axial stress, respectively. ε_θ, ε_r, ε_z are the hoop strain, radial strain, and axial strain, respectively. As shown in Fig.2, we can obtain:

$$\sigma_\theta=\sigma_T\cdot\cos^2\alpha,\ \ \sigma_z=\sigma_T\cdot\sin^2\alpha,$$

where σ_T is the stress in the longitude direction of the fiber. So,

$$\sigma_z=\sigma_\theta\cdot\tan^2\alpha, \tag{7}$$

then

$$\varepsilon_\theta=\left(\frac{1}{E_\theta}-\frac{\nu_{z\theta}}{E_z}\tan^2\alpha\right)\sigma_\theta-\frac{\nu_{r\theta}}{E_r}\sigma_r, \tag{8}$$

therefore

$$\frac{\mathrm{d}\varepsilon_\theta}{\mathrm{d}r}=\left(\frac{1}{E_\theta}-\frac{\nu_{z\theta}}{E_z}\tan^2\alpha\right)\frac{\mathrm{d}\sigma_\theta}{\mathrm{d}r}-\frac{\nu_{r\theta}}{E_r}\frac{\mathrm{d}\sigma_r}{\mathrm{d}r}. \tag{9}$$

According to constitutive Eq.(6),

$$\begin{aligned}\varepsilon_r-\varepsilon_\theta=&\left(\frac{1}{E_r}+\frac{\nu_{r\theta}}{E_r}\right)\sigma_r\\&-\left(\frac{\nu_{\theta r}}{E_\theta}+\frac{\nu_{zr}}{E_z}\tan^2\alpha+\frac{1}{E_\theta}-\frac{\nu_{z\theta}}{E_z}\tan^2\alpha\right)\sigma_\theta;\end{aligned} \tag{10}$$

according to Eqs.(5) and (10),

$$\begin{aligned}&\left(\frac{1}{E_\theta}-\frac{\nu_{z\theta}}{E_z}\tan^2\alpha\right)\frac{\mathrm{d}\sigma_\theta}{\mathrm{d}r}-\frac{\nu_{r\theta}}{E_r}\frac{\mathrm{d}\sigma_r}{\mathrm{d}r}=\frac{1}{r}\left[\left(\frac{1}{E_r}+\frac{\nu_{\theta r}}{E_r}\right)\sigma_r\right.\\&\left.-\left(\frac{\nu_{\theta r}}{E_\theta}+\frac{\nu_{zr}}{E_z}\tan^2\alpha+\frac{1}{E_\theta}-\frac{\nu_{z\theta}}{E_z}\tan^2\alpha\right)\sigma_\theta\right];\end{aligned} \tag{11}$$

according to Eqs.(3) and (11),

$$\begin{aligned}&\left(\frac{1}{E_\theta}-\frac{\nu_{z\theta}}{E_z}\tan^2\alpha\right)\frac{\mathrm{d}\left(\sigma_r+r\frac{\mathrm{d}\sigma_r}{\mathrm{d}r}\right)}{\mathrm{d}r}-\frac{\nu_{r\theta}}{E_r}\frac{\mathrm{d}\sigma_r}{\mathrm{d}r}\\&=\frac{1}{r}\left[\left(\frac{1}{E_r}+\frac{\nu_{r\theta}}{E_r}\right)\sigma_r-\left(\frac{\nu_{\theta r}}{E_\theta}+\frac{\nu_{zr}}{E_z}\tan^2\alpha+\frac{1}{E_\theta}\right.\right.\\&\left.\left.-\frac{\nu_{z\theta}}{E_z}\tan^2\alpha\right)\left(\sigma_r+r\frac{\mathrm{d}\sigma_r}{\mathrm{d}r}\right)\right].\end{aligned} \tag{12}$$

Because of the symmetry of compliance matrix $\frac{\nu_{r\theta}}{E_r}=\frac{\nu_{\theta r}}{E_\theta}$, simplifing Eq.(12), we can obtain

$$\begin{aligned}&r^2\left(\frac{1}{E_\theta}-\frac{\nu_{z\theta}}{E_z}\cdot\tan^2\alpha\right)\sigma_r''+\left[\frac{\nu_{zr}}{E_z}\tan^2\alpha\right.\\&\left.+3\left(\frac{1}{E_\theta}-\frac{\nu_{z\theta}}{E_z}\tan^2\alpha\right)\right]r\sigma_r'+\left(\frac{1}{E_r}+\frac{\nu_{zr}}{E_z}\tan^2\alpha\right.\\&\left.+\frac{1}{E_\theta}-\frac{\nu_{z\theta}}{E_z}\tan^2\alpha\right)\sigma_r=0.\end{aligned} \tag{13}$$

Let
$$A=\frac{1}{E_\theta}-\frac{\nu_{z\theta}}{E_z}\tan^2\alpha,$$
$$B=\frac{\nu_{zr}}{E_z}\tan^2\alpha+3\left(\frac{1}{E_\theta}-\frac{\nu_{z\theta}}{E_z}\tan^2\alpha\right),$$
$$C=\frac{1}{E_2}+\frac{\nu_{zr}}{E_3}\tan^2\alpha+\frac{1}{E_1}-\frac{\nu_{z\theta}}{E_3}\tan^2\alpha,$$

then Eq.(13) is simplified as

$$r^2\sigma_r''+\frac{B}{A}r\sigma_r'+\frac{C}{A}\sigma_r=0. \tag{14}$$

Eq.(14) is an Euler equation, and the solution is

$$\sigma_r=C_1r^{x_1}+C_2r^{x_2}, \tag{15}$$

where

$$x_1=-\frac{B}{2A}+\frac{1}{2}\sqrt{(B/A)^2-4C/A},$$
$$x_2=-\frac{B}{2A}-\frac{1}{2}\sqrt{(B/A)^2-4C/A},$$

and

$$\sigma_\theta=\sigma_r+r\frac{\mathrm{d}\sigma_r}{\mathrm{d}r}=C_1(1+x_1)r^{x_1}+C_2(1+x_2)r^{x_2}. \tag{16}$$

The boundary conditions on the internal (r=b) and external (r=c) surfaces are given by

$$\begin{cases}P_0=C_1b^{x_1}+C_2b^{x_2},\\0=C_1c^{x_1}+C_2c^{x_2}.\end{cases} \tag{17}$$

The solutions are

$$C_1=\frac{-P_0c^{x_2}}{c^{x_1}b^{x_2}-c^{x_2}b^{x_1}},\ C_2=\frac{P_0c^{x_1}}{c^{x_1}b^{x_2}-c^{x_2}b^{x_1}},\quad (18)$$

then

$$\sigma_r=\frac{P_0(c^{x_1}r^{x_2}-c^{x_2}r^{x_1})}{c^{x_1}b^{x_2}-c^{x_2}b^{x_1}},\quad (19)$$

$$\begin{aligned}\sigma_\theta&=\frac{P_0(c^{x_1}r^{x_2}-c^{x_2}r^{x_1})+P_0(x_2c^{x_1}r^{x_2}-x_1c^{x_2}r^{x_1})}{c^{x_1}b^{x_2}-c^{x_2}b^{x_1}}\\&=\frac{P_0\left[(1+x_2)c^{x_1}r^{x_2}-(1+x_1)c^{x_2}r^{x_1}\right]}{c^{x_1}b^{x_2}-c^{x_2}b^{x_1}}.\end{aligned}\quad (20)$$

Model of the cylinder loaded with the internal pressure in the axial direction only

In this model, the internal pressure in the axial direction of the cylinder is also loaded by the Ti-Al alloy liner and all CFRC layers together. So, this model is built in the same way as the first one, and expressed as follows:

$$\pi R_i^2p_i=\pi R_1^2q_1+\pi R_2^2q_2+\cdots+\pi R_n^2q_n+\cdots+\pi R_m^2q_m,$$

i.e.,

$$R_1^2q_1+R_2^2q_2+\cdots+R_n^2q_n+\cdots+R_m^2q_m=R_\mathrm{i}^2p_\mathrm{i},\quad (21)$$

where q_1, q_2, …, q_m are the axial internal pressures loaded by each layer.

Assuming that the axial strain of the whole cylinder as well as the axial stress of every CFRC is evenly distributed, we can get

$$\begin{cases}\varepsilon_{z1}=\varepsilon_{z2},\\ \varepsilon_{z1}=\varepsilon_{z3},\\ \quad\vdots\qquad\vdots\\ \varepsilon_{z1}=\varepsilon_{zn},\\ \quad\vdots\qquad\vdots\\ \varepsilon_{z1}=\varepsilon_{zm},\end{cases}\quad (22)$$

where ε_{zn} is the axial strain of the nth CFRC wound layer. Then

$$\sigma_{zn}=q_nR_n/(2t).\quad (23)$$

STRAIN COMPUTATION AND EQUATIONS SOLUTION

Hoop strain computation of CFRC

According to Eqs.(6) and (20), we can obtain (Braun, 1992; Smerdov, 2000; Parnas and Nuran, 2002; Zheng, 2006a):

$$\begin{aligned}\varepsilon_{\theta n\mathrm{in}}&=\frac{1}{E_{\theta n}}p_nS_{n\mathrm{in}}-\frac{\nu_{r\theta n}}{E_{rn}}p_n-\frac{\nu_{z\theta n}}{E_{zn}}\frac{q_nR_n}{2t}\\&=K_{np}p_n+K_{nq}q_n,\ n\geq 2,\end{aligned}\quad (24)$$

$$\begin{aligned}\varepsilon_{\theta n\mathrm{out}}&=\frac{1}{E_{\theta n}}p_nS_{n\mathrm{out}}-\frac{\nu_{r\theta n}}{E_{rn}}p_n-\frac{\nu_{z\theta n}}{E_{zn}}\frac{q_nR_n}{2t}\\&=K'_{np}p_n+K_{nq}q_n,\ n\geq 2,\end{aligned}\quad (25)$$

where

$$S_{n\mathrm{in}}=\frac{\left[(1+x_2)c_n^{x_1}b_n^{x_2}-(1+x_1)c_n^{x_2}b_n^{x_1}\right]}{c_n^{x_1}b_n^{x_2}-c_n^{x_2}b_n^{x_1}},$$

$$S_{n\mathrm{out}}=\frac{\left[(1+x_2)c_n^{x_1}c_n^{x_2}-(1+x_1)c_n^{x_2}c_n^{x_1}\right]}{c_n^{x_1}b_n^{x_2}-c_n^{x_2}b_n^{x_1}},$$

$$K_{np}=\frac{1}{E_{\theta n}}S_{n\mathrm{in}}-\frac{\nu_{r\theta n}}{E_{rn}},$$

$$K'_{np}=\frac{1}{E_{\theta n}}S_{n\mathrm{out}}-\frac{\nu_{r\theta n}}{E_{rn}},$$

$$K_{nq}=-\frac{\nu_{z\theta n}}{E_{zn}}\frac{R_n}{2t}.$$

Axial strain of CFRC

Assuming that the axial strain of the whole cylinder is evenly distributed, the distribution of axial strain of the cylinder can be obtained:

$$\varepsilon_{zn}=\frac{q_nR_n}{2tE_{zn}}-\nu_{\theta z}\frac{p_nR_n}{tE_{\theta n}}.\quad (26)$$

Equations solution

According to Eqs.(1), (2), (21) and (22), we can obtain the simultaneous equations:

$$\begin{aligned}&R_1p_1+R_2p_2+R_3p_3+\cdots+R_np_n+\cdots+R_mp_m=R_\mathrm{i}p_\mathrm{i},\\&K_{1p}p_1+K_{1q}q_1-K'_{2p}p_2-K'_{2p}q_2=0,\\&K_{2p}p_2+K_{2q}q_2-K'_{3p}p_2-K_{3p}q_3=0,\\&\quad\vdots\\&K_{(m-1)p}p_{m-1}+K_{(m-1)q}q_{m-1}-K'_{mp}p_m-K'_{mp}q_m=0,\\&R_1^2q_1+R_2^2q_2+R_3^2q_3+\cdots+R_n^2q_n+\cdots+R_m^2q_m=R_\mathrm{i}^2p_\mathrm{i},\\&\frac{q_1R_1}{2tE_{z_1}}+\nu_{\theta z_2}\frac{p_2R_2}{tE_{\theta_2}}-\nu_{\theta z_1}\frac{p_1R_1}{tE_{\theta_1}}-\frac{q_2R_2}{2tE_{z_2}}=0,\end{aligned}$$

$$\frac{q_1R_1}{2tE_{z_1}}+\nu_{\theta z_3}\frac{p_3R_3}{tE_{\theta_3}}-\nu_{\theta z_1}\frac{p_1R_1}{tE_{\theta_1}}-\frac{q_3R_3}{2tE_{z_3}}=0,$$
$$\vdots$$
$$\frac{q_1R_1}{2tE_{z_1}}+\nu_{\theta z_m}\frac{p_mR_m}{tE_{\theta_m}}-\nu_{\theta z_1}\frac{p_1R_1}{tE_{\theta_1}}-\frac{q_mR_m}{2tE_{z_m}}=0. \qquad (27)$$

Express them in matrix (Agarwal and Broutman, 1990; Verijenko *et al.*, 2001; Xia *et al.*, 2001):

$$\begin{bmatrix}\boldsymbol{A} & \boldsymbol{B}\\ \boldsymbol{C} & \boldsymbol{D}\end{bmatrix}[\boldsymbol{P}\quad \boldsymbol{Q}]^{\mathrm{T}}=[\boldsymbol{V}\quad \boldsymbol{W}]^{\mathrm{T}}, \qquad (28)$$

where $\boldsymbol{P}=[p_1, p_2, \ldots, p_m]$,
$\boldsymbol{Q}=[q_1, q_2, \ldots, q_m]$,
$\boldsymbol{V}=\begin{bmatrix}R_{\mathrm{i}}p_{\mathrm{i}} & 0 & 0 & \cdots & 0\end{bmatrix}^{\mathrm{T}}$,
$\boldsymbol{W}=\begin{bmatrix}R_{\mathrm{i}}^2p_{\mathrm{i}} & 0 & 0 & \cdots & 0\end{bmatrix}^{\mathrm{T}}$,

$$\boldsymbol{A}=\begin{bmatrix}R_1 & R_2 & \cdots & R_n & \cdots & R_m\\ K'_{1p} & -K_{2p} & 0 & 0 & \cdots & 0\\ 0 & K'_{2p} & -K_{3p} & 0 & \cdots & 0\\ & & & & \vdots & \\ 0 & \cdots & K'_{np} & -K_{(n+1)p} & \cdots & 0\\ & & & & \cdots & \\ 0 & 0 & \cdots & 0 & K'_{(m-1)p} & -K_{mp}\end{bmatrix},$$

$$\boldsymbol{B}=\begin{bmatrix}K_q & -K_{2q} & & & & \\ & K_{2q} & -K_{3q} & & & \\ & & \cdots & & & \\ & & K_{(n-1)q} & -K_{nq} & & \\ & & \cdots & & & \\ & & & & K_{(m-1)q} & -K_{mq}\end{bmatrix},$$

$$\boldsymbol{C}=\begin{bmatrix}0 & & & & 0\\ -\frac{\nu_{\theta z_1}R_1}{tE_{\theta_1}} & \frac{\nu_{\theta z_2}R_2}{tE_{\theta_2}} & & & \\ -\frac{\nu_{\theta z_1}R_1}{tE_{\theta_1}} & & \frac{\nu_{\theta z_3}R_3}{tE_{\theta_3}} & & \\ -\frac{\nu_{\theta z_1}R_1}{tE_{\theta_1}} & & & \frac{\nu_{\theta z_4}R_4}{tE_{\theta_4}} & \\ \vdots & & & \cdots & \\ -\frac{\nu_{\theta z_1}R_1}{tE_{\theta_1}} & & & & \frac{\nu_{\theta z_m}R_m}{tE_{\theta_m}}\end{bmatrix},$$

$$\boldsymbol{D}=\begin{bmatrix}R_1^2 & R_2^2 & \cdots & R_n^2 & \cdots & R_{m-1}^2 & R_m^2\\ \frac{R_1}{2tE_{z_1}} & -\frac{R_2}{2tE_{z_2}} & & & & & \\ \frac{R_1}{2tE_{z_1}} & & -\frac{R_3}{2tE_{z_3}} & & & & \\ \frac{R_1}{2tE_{z_1}} & & & -\frac{R_4}{2tE_{z_4}} & & & \\ \vdots & & & & & & \\ \frac{R_1}{2tE_{z_1}} & & & & & & -\frac{R_m}{2tE_{z_m}}\end{bmatrix}.$$

The solutions are

$$[\boldsymbol{P}\quad \boldsymbol{Q}]^{\mathrm{T}}=\begin{bmatrix}\boldsymbol{A} & \boldsymbol{B}\\ \boldsymbol{C} & \boldsymbol{D}\end{bmatrix}^{-1}[\boldsymbol{V}\quad \boldsymbol{W}]^{\mathrm{T}}. \qquad (30)$$

Taking the $p_1, p_2, \ldots, p_m$ back into Eq.(20), the hoop stresses of the cylinder will be obtained. Usually, the reasonable expected distribution of hoop stresses of each layer is even in the whole cylinder (Zheng, 2006a):

$$\sigma_{\theta n}=\frac{p_nR_n}{t_n}. \qquad (31)$$

The axial stresses of the cylinder can be obtained according to Eq.(23), i.e.,

$$\sigma_{zn}=\frac{q_nR_n}{2t_n}. \qquad (32)$$

Example and validation

In order to examine the theory, a Ti-Al alloy liner wound with carbon fiber resin composite pressure vessel was designed. The uniform hoop stress and axial stress were required in this vessel. The parameters for this vessel are as follows: the elastic module of liner is 109 GPa, the tensile strength of CFRC layer is σ_b=2878.8 MPa, and E_{L}=181 GPa, E_{T}=10.3 GPa, G_{TL}=7.17 GPa, ν_{TL}=0.28, t=0.42 mm, R_{i}=100 mm, R_2=104 mm, R_m=114.5 mm, l=701 mm. The cylinder is made of Ti-Al alloy liner and 13 hoop winding layers with the same angle 0° combined 12 axial layers with the same angle 76°, whose arrangement is that the hoop layers and the axial layers are alternately wound in turn. p_{i}=70 MPa.

To simplify the computation, the axial stress of the hoop winding layers with the same angle 0° is ignored. From Eq.(27), the solutions of p_n (n=1, 2, …, 26) are [14.599, 3.698, 0.413, 3.641, 0.401, 3.629, 0.402, 3.633, 0.392, 3.529, 0.391, 3.547, 0.391, 3.561, 0.389, 3.527, 0.382, 3.528, 0.390, 3.499, 0.378, 3.498, 0.381, 3.469, 0.371, 3.402] MPa. The solutions of q_n (n=1, 2, …, 26) are [23.191, 0, 3.420, 0, 3.401, 0, 3.387, 0, 3.351, 0, 3.327, 0, 3.301, 0, 3.282, 0, 3.245, 0, 3.223, 0, 3.201, 0, 3.175, 0, 3.148, 0] MPa.

The maximum hoop stress of CFRC wound layer is 920.385 MPa, and the uniform axial stress of CFRC is 419.639 MPa. The uniform hoop stress in the liner is 376.607 MPa, and the uniform axial stress in the liner is 302.382 MPa. Usually, low stresses are required in a thin Ti-Al alloy liner, which can be done by controlling pretension stresses of the carbon fibers during the winding of CFRC layers. By computing and optimizing (Zheng, 2006a), the reasonable winding pretension stresses of each layer is [251, 298, 246, 295, 241, 289, 236, 286, 231, 279, 226, 274, 216, 269, 212, 264, 208, 259, 206, 254, 203, 249, 201, 245, 199] MPa.

Finally, under an interior pressure of 70 MPa, the hoop stresses of carbon fiber layers are between 1015 and 1039 MPa, the axial stresses of carbon fiber layers are between 622 and 655 MPa. The evenly distributed hoop stress of the liner is 157.3 MPa, and the evenly distributed axial stress of the liner is 163.9 MPa. The stress distribution is reasonable for a safe operation. A vessel designed above pretension stresses is shown in Fig.4.

Fig.4 Reasonable designed composite pressure vessel

To validate the theoretical result, we measured the dimensions of the Ti-Al alloy liner and the finished vessel. Usually, the residual stresses in CFRC vessel are obtained by measuring the axial and circumferential shrinkages after manufacture. Before the fibers are wound, the interior perimeter and length of the Ti-Al alloy liner are measured to be 602.51 and 628.61 mm, respectively. After all fibers are wound, its axial length and interior perimeter become 602.18 and 627.92 mm, respectively. This means that the axial and the circumferential shrinkages are 0.33 and 0.69 mm, respectively. The axial residual stress and the hoop residual stress in the inner shell can then be calculated to be approximately 65.2 and 130.8 MPa, respectively. When the vessel is tested under a hydraulic pressure of 70 MPa, the length and interior perimeter of the liner are 603.39 and 629.60 mm, respectively. That is 159.3 (axial stress) and 172.3 MPa (hoop stress) in the liner, respectively. This proves that the axial stress under the operating pressure in the Ti-Al alloy liner is low. Though we cannot measure the stress in CFRC layers, it matches the theoretical value very well because the wall is thin and the pretension force is almost evenly distributed relative to the traditional steel vessels. In conclusion, the experimental results are close to the theoretical ones.

CONCLUSION

Based on the principle that the internal pressure is loaded by the Ti-Al alloy liner and all CFRC layers together, we built two cylinder models loaded with the stress either in the hoop or axial direction only under the internal pressure. At the same time, combined with the conditions of displacement continuous boundary between layers, a group of equations have been built. The internal pressures in the hoop direction and axial direction loaded by every layer have been obtained from the solution of these equations. The stress distribution on the cylinder has been gained from the internal pressure loaded by each layer. After the stresses are obtained, an optimal design can be done by adjusting the pretension stresses in the fibers. According to this method, a reasonable design of a CFRC wound light weight high pressure hydrogen storage vessel has been made.

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