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Theoretical relationships between first flush of roof runoff and influencing factors*

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Abstract: Considering the short length of building roofs, a theoretical analysis of the first flush of roof runoff was conducted based on the kinematic wave and pollutant erosion equations. This mathematical derivation with analytical solutions predicts pollutant mass first flush (MFF), mean concentration of initial runoff (MCIF), mean concentration of roof runoff (MCRR) with diversion of initial portion and residual mass available on the bed surface (RS) after the entire runoff under the condition of constant excess rainfall. And the effects of the associated influencing factors (roof length, roof gradient, roof surface roughness, rainfall intensity, rainfall duration, and erosion coefficients) on them were discussed while the values of parameters referred to the previous studies. The results showed that for roofs whose length is shorter than 20 m, both the increase in roof length and roof gradient and the decrease in roof surface roughness result in larger MFF and MCIF and smaller MCRR and RS, which is beneficial to water reuse and pollution reduction. The theoretical relationship between the first flush and the influencing factors may aid the planning and design of roof in terms of rainwater utilization or diffuse pollution control.

Key words: Roof runoff, Kinematic wave equation, Pollutant erosion equation, First flush
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INTRODUCTION

Roof runoff is one of the important non-point pollutant sources in cities (van Metre and Mahler, 2003; Chang *et al*., 2004). However, it is much cleaner than surface runoff (such as highway runoff) because of less anthropological pollution, thus able to become potential urban water resources.

Although the presence of first flush of surface runoff depends on meteorological and geometrical characteristics of watershed, it is generally concluded that small size watersheds with high imperviousness tend to show strong first flush (Ma *et al.*, 2002). As roof is normally impervious with a small area, the first flush phenomenon is frequently observed in roof runoff (Förster, 1999; Zobrist *et al*., 2000; Simmons *et al*., 2001). First flush is defined as a greater fraction of constituent mass or higher concentration in the early part of the runoff volume (Sansalone and Cristina, 2004). If the first portion of the runoff contains a large portion of pollutant mass, diversion or treatment of the first portion may be economically advantageous in roof rainwater utilization and pollutant removal, so characterizing the first flush phenomena is of practical significance.

So far, there have been many studies on the relationship between the first flush phenomenon and associated parameters, such as watershed area, imperious area, rainfall intensity, rainfall duration, and antecedent dry weather period based on the analysis of a large number of runoff events (Gupta and Saul, 1996; Charbeneau and Barrett, 1998; Lee *et al*., 2002; Cristina and Sansalone, 2003). However, no consensus was found, which may be attributable to the complexity of rainfall events and the variability of study sites. Then a deterministic model was

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developed (Kang *et al.*, 2006), which successfully predicted the variation course of pollutant concentrations of storm runoff. Then the researchers discussed the application of the model to the first flush design. In this model, the kinematic wave equation, pollutant erosion equation, and advection-dispersion equation (ADE) were used for the calculation of flow, pollutant erosion rate, and pollutant transport rate, respectively. Among them, ADE has no analytical solution even in idealized situations (such as constant rainfall), so the complex numerical method must be applied.

It is surmised that the deterministic model is also applicable to building roofs, which can be considered as a special kind of ground without infiltration. Moreover, roof has a shorter slope and larger slope gradient, suggesting that the washed pollutants will reach the runoff outlet immediately. Then the pollutant transport plays a minor role in the variation of pollutant concentration at the outlet. In accordance with the above characteristics of a construction roof, by omitting the pollutant transport, a theoretical analysis of the first flush of roof runoff based on constant excess rainfall was done to predict the pollutant mass of the first flush (MFF), the mean concentration of initial runoff (MCIF), the mean concentration of roof runoff (MCRR) with a diversion of initial portion and the residual mass available on the bed surface (RS) after the entire runoff in this study. Finally, the effects of the associated parameters such as slope length, slope gradient, roof roughness coefficients on MFF, MCIF, MCRR, and RS were discussed.

METHODOLOGY

The basic principles of the theoretical analysis consist of first flush quantification, the rainfall-runoff model and the pollutant erosion model, which are explained in detail as follows.

First flush quantification

To quantify the first flush phenomenon, Ma *et al.*(2002) advanced the concept of MFF ratio, which can be defined as follows:

$$MFF_r=\frac{F_r/F}{V_r/V}=\frac{F_r/F}{r/100}, \tag{1}$$

where r is the volume percentage of the initial portion to that of the total runoff, F_r the pollutant load of the first flush, F the total pollutant load of the entire runoff, V_r the volume of the initial portion of runoff, and V the total runoff volume.

Rainfall-runoff model

The kinematic wave equation approximation to the Saint-Venant hydrodynamic equations is frequently applied to the overland flow modeling because the calculation is simple and accurate where the backwater effects and the flow acceleration are not important (Akan *et al.*, 2000; Cristina and Sansalone, 2000; Singh, 2002a; 2002b). In actual rainfall situations, the kinematic wave equation can be solved with the method of characteristics and the lax-wendroff method. However, it offers an exact analytical solution for idealized situations, such as constant excess rainfall. Accordingly, a 1D kinematic wave model considering constant excess rainfall was suggested as follows:

$$\frac{\partial h}{\partial t}+\left(\frac{\sqrt{S_0}}{n}\right)\frac{\partial h^m}{\partial x}=R, \tag{2}$$

where h is the flow depth (m), t the time elapsed since the beginning of the overland flow (s), m the constant (=5/3), x the distance along the flow direction (m), S_0 the slope gradient, n Manning's roughness coefficient of the overland surface, and R the excess rainfall (mm/h).

Eq.(2) can be solved with the boundary and initial conditions:

$$h(0,t)=0,\ t\geq 0, \tag{3}$$
$$h(x,0)=0,\ 0\leq x\leq L, \tag{4}$$

where L is the slope length (m).

Assuming that the duration of constant excess rainfall is T_R (h), Parlange *et al.*(1981) obtained the exact analytical solution of Eq.(2). For $t\leq T_R$, which is the assumed condition in this study, the solution can be expressed as

$$h=Rt,\ \text{for}\ x\geq KR^{m-1}t^m,\ \ t\leq T_R, \tag{5}$$
$$h=(Rx/K)^{1/m},\ \text{for}\ x\leq KR^{m-1}t^m,\ \ t\leq T_R, \tag{6}$$

where $K=\sqrt{S_0}/n$.

Pollutant erosion model

The pollutant erosion rate was generally assumed as a first-order reaction that acts as a function of mass available on the bed surface (Singh, 1996; Tomanovic and Maksimovic, 1996). However, it was found that the above assumption cannot model both the initial high pollutant concentrations observed in the early runoff and the low residual concentrations at the latter part of the storm. Then Kang *et al.*(2006) made modifications by suggesting the concepts of short-term pollutant source and long-term pollutant source. The modified equation is presented as follows:

$$\frac{\mathrm{d}m_{x,t}}{\mathrm{d}t}=-\varepsilon_{\mathrm{s}}u^2m_{x,t}-\varepsilon_{\mathrm{l}}u^2, \tag{7}$$

where $m_{x,t}$ is the mass available on the bed surface (g/m^2), u is the flow velocity (m/s), ε_{s} (s/m^2) and ε_{l} ($g\cdot s/m^4$) are erosion coefficients of the pollutant mass from the short-term and long-term sources, respectively.

MATHEMATICAL DERIVATION

Duration of the first part of runoff (T_r)

Assuming that the volume of the first portion of runoff on the unit width basis is V_r, T_r can be deduced as follows.

According to Eq.(5), we have

$$V_r=\int_0^{T_r}q\mathrm{d}t=\int_0^{T_r}KR^mt^m\mathrm{d}t=\frac{KR^mT_r^{\,m+1}}{m+1},\quad \text{for } L\geq KR^{m-1}T_r^{\,m}, \tag{8}$$

where q is the flow rate per unit width and $q=Kt$.

According to Eq.(5) and Eq.(6), we get

$$\begin{aligned}V_r&=\int_0^{T_r}q\mathrm{d}t\\&=\int_0^{(L/(KR^{m-1}))^{1/m}}KR^mt^m\mathrm{d}t+\int_{(L/(KR^{m-1}))^{1/m}}^{T_r}K(RL/K)\mathrm{d}t\\&=\frac{L^{(m+1)/m}R^{1/m}}{(m+1)K^{1/m}}+RL\left(T_r-\left(L/(KR^{m-1})\right)^{1/m}\right),\end{aligned} \tag{9}$$

for $L\leq KR^{m-1}T_r^{\,m}$.

The volume of a total runoff on the unit width basis V can be written as

$$V=\frac{LRT_R}{\sqrt{1+S_0^{\,2}}}. \tag{10}$$

When $T_r=T_R=T_{\mathrm{c}}$ (T_{c}, time of concentration, is $(L/(KR^{m-1}))^{1/m}$) in Eq.(9) and Eq.(10), $V-V_r$ will be equilibrium detention storage for the roof, which is almost the same as the previous research (Wong and Li, 2000), although the very limited influence of S_0 on V was considered in this study.

Then the following equation can be obtained:

$$\frac{V_r}{V}\times 100=r. \tag{11}$$

Integrating Eq.(8) and Eq.(11) yields:

$$T_r=\left(\frac{(r/100)LT_R(m+1)}{\sqrt{1+S_0^{\,2}}KR^{m-1}}\right)^{1/(m+1)}. \tag{12}$$

Then whether the above T_r satisfies the inequality "$L\geq KR^{m-1}T_r^{\,m}$" shall be checked. If not, integrating Eq.(9) and Eq.(11) yields:

$$T_r=\frac{rT_R}{100}-\frac{R^{(1-m)/m}L^{1/m}}{(m+1)(1+S_0^{\,2})^{1/2m}K^{1/m}}+\left(\frac{L}{\sqrt{1+S_0^{\,2}}KR^{m-1}}\right)^{1/m}. \tag{13}$$

Flushed pollutant mass at a position up to time T_r (F_{x,T_r})

Assuming the initial pollutant mass available on the roof is constant and its value is m_0 (g/m^2), Eq.(7) can be rewritten as follows:

$$\frac{\mathrm{d}m_{x,t}}{m_{x,t}-\varepsilon_2/\varepsilon_1}=-\varepsilon_1u^2\mathrm{d}t. \tag{14}$$

If $t=0$, $m_{x,t}=m_0$. Integrating Eq.(14) from 0 to T_r, we obtain

$$\frac{m_{x,T_r}-\varepsilon_2/\varepsilon_1}{m_0-\varepsilon_2/\varepsilon_1}=\mathrm{e}^{-\int_0^{T_r}\varepsilon_1u^2\mathrm{d}t},$$

and we can further obtain from the above equation:

$$\frac{m_{x,T_r}}{m_0}=\left(1-\left(\frac{\varepsilon_2}{\varepsilon_1}\right)\Big/m_0\right)\mathrm{e}^{-\int_0^{T_r}\varepsilon_1u^2\mathrm{d}t}+\left(\frac{\varepsilon_2}{\varepsilon_1}\right)\Big/m_0. \quad (15)$$

Then the ratio of F_{x,T_r} to m_0 is:

$$\frac{F_{x,T_r}}{m_0}=1-\frac{m_{x,T_r}}{m_0}=\left(1-\left(\frac{\varepsilon_2}{\varepsilon_1}\right)\Big/m_0\right)\left(1-\mathrm{e}^{-\int_0^{T_r}\varepsilon_1u^2\mathrm{d}t}\right)=f(x). \quad (16)$$

In practice the duration of diversion of the first flush must be less than the rainfall duration, namely $T_r\leq T_R$.

According to Eq.(5) and Eq.(6), the flow velocity u can be obtained as

$$u=Kh^{m-1}=KR^{m-1}t^{m-1},\ \text{for}\ t\leq(x/(KR^{m-1}))^{1/m}, \quad (17)$$
$$u=Kh^{m-1}=K(Rx/K)^{(m-1)/m},\ \text{for}\ t\geq(x/(KR^{m-1}))^{1/m}, \quad (18)$$

then we have

$$\int_0^{T_r}u^2\mathrm{d}t=\int_0^{T_r}K^2R^{2(m-1)}t^{2(m-1)}\mathrm{d}t=\frac{1}{2m-1}K^2R^{2(m-1)}T_r^{2m-1},$$
$$\text{for}\ T_r\leq(x/(KR^{m-1}))^{1/m}, \quad (19)$$

and

$$\begin{aligned}\int_0^{T_r}u^2\mathrm{d}t=&\int_0^{(x/(KR^{m-1}))^{1/m}}K^2R^{2(m-1)}t^{2(m-1)}\mathrm{d}t\\&+\int_{(x/(KR^{m-1}))^{1/m}}^{T_r}K^{2/m}R^{2(m-1)/m}x^{2(m-1)/m}\mathrm{d}t\\=&\frac{1}{2m-1}K^{1/m}R^{(m-1)/m}x^{(2m-1)/m}\\&+K^{2/m}R^{2(m-1)/m}T_rx^{2(m-1)/m}-K^{1/m}R^{(m-1)/m}x^{(2m-1)/m},\end{aligned}$$
$$\text{for}\ T_r\geq(x/(KR^{m-1}))^{1/m}. \quad (20)$$

Let $K^{1/m}R^{(m-1)/m}=P$, Eq.(20) can be rewritten as

$$\int_0^{T_r}u^2\mathrm{d}t=\frac{1}{2m-1}Px^{(2m-1)/m}+P^2T_rx^{(2m-1)/m}-Px^{(2m-1)/m}. \quad (21)$$

Flushed pollutant mass of the entire roof up to time T_r (F_{T_r})

During a rainfall, the roof pollutants are flushed off the surface and transported to the downstream by the runoff. The pollutants leaving the roof surface can be considered to reach the outlet without delay in this analysis as the length of the roof is normally a few meters. Based on the above assumption, the difference between the assumed F_{T_r} and its real value is the pollutant load (S_{T_r}) of runoff available on the roof at time T_r. Then it is necessary to evaluate the error by omitting S_{T_r}, which is equal to its concentration (C_{T_r}) multiplied by its volume (V_{T_r}). On one hand, C_{T_r} is much less than MCIF (C_r) when r is large enough (such as $r\geq20$), as the pollutant concentration of roof runoff decreases dramatically with time; on the other hand, the volume (V_{T_r}) of runoff available on roof at time T_r is also much less than that (V_r) of the initial part of runoff. Therefore it is easy to infer that F_{T_r} ($F_{T_r}=V_rC_r$) is much larger than S_{T_r} ($S_{T_r}=V_{T_r}C_{T_r}$), which can be neglected with an extremely minor error. In engineering practices, the condition of $r\geq20$ can be generally satisfied and the simplification is probably meaningful in practice.

For the entire roof, the ratio (M_{T_r}) of F_{T_r} to m_0 can be derived from Eq.(16) as follows:

$$M_{T_r}=\frac{F_{T_r}}{m_0}=\frac{\int_0^Lf(x)\mathrm{d}x}{L}. \quad (22)$$

Eq.(22) can be further expressed as

$$M_{T_r}=\left((m_0-\varepsilon_l/\varepsilon_s)/(m_0L)\right)\times\int_0^L1-\mathrm{e}^{-\varepsilon_s\left(\frac{1}{2m-1}Px^{(2m-1)/m}+P^2T_rx^{(2m-1)/m}-Px^{(2m-1)/m}\right)}\mathrm{d}x,$$
$$\text{for}\ L\leq KR^{m-1}T_r^m, \quad (23)$$

and

$$M_{T_r}=\frac{\int_0^{KR^{m-1}T_r^m}f(x)\mathrm{d}x+\int_{KR^{m-1}T_r^m}^Lf(x)\mathrm{d}x}{L}$$
$$=\left((m_0-\varepsilon_1/\varepsilon_s)/(m_0L)\right)$$

$$\times\left\{\int_0^{KR^{m-1}T_r^{\,m}} 1-\mathrm{e}^{-\varepsilon_s\left(\frac{1}{2m-1}Px^{(2m-1)/m}+P^2T_rx^{(2m-1)/m}-Px^{(2m-1)/m}\right)}\mathrm{d}x\right.$$

$$\left.+(L-KR^{m-1}T_r^{\,m})\left(1-\mathrm{e}^{-\varepsilon_s\left(\frac{1}{2m-1}K^2R^{2(m-1)}T_r^{\,2m-1}\right)}\right)\right\},$$

$$\text{for } L\geq KR^{m-1}T_r^m. \tag{24}$$

Flushed pollutant mass of the entire roof by the designed rainfall (F_{T_R})

When the rainfall stops ($t\geq T_R$), the roof runoff will last for a very short time and the pollutant load of runoff after the time T_R, like S_{T_R}, will be too low to be taken into account. Then the flushed pollutant mass of the entire roof by the designed rainfall (F_{T_R}) can be approximated by that flushed up to time T_R.

Similarly with respect to Eqs.(23) and (24), the ratio (M_{T_R}) of F_{T_R} to m_0 can be expressed as follows:

$$M_{T_R}=\left((m_0-\varepsilon_1/\varepsilon_s)/(m_0L)\right)$$

$$\times\int_0^L 1-\mathrm{e}^{-\varepsilon_s\left(\frac{1}{2m-1}Px^{(2m-1)/m}+P^2T_Rx^{(2m-1)/m}-Px^{(2m-1)/m}\right)}\mathrm{d}x,$$

$$\text{for } L\leq KR^{m-1}T_R^m, \tag{25}$$

and

$$M_{T_r}=\frac{\int_0^{KR^{m-1}T_R^{\,m}}f(x)\mathrm{d}x+\int_{KR^{m-1}T_R^{\,m}}^{L}f(x)\mathrm{d}x}{L}$$

$$=\left((m_0-\varepsilon_1/\varepsilon_s)/(m_0L)\right)$$

$$\times\left\{\int_0^{KR^{m-1}T_R^{\,m}} 1-\mathrm{e}^{-\varepsilon_s\left(\frac{1}{2m-1}Px^{(2m-1)/m}+P^2T_Rx^{(2m-1)/m}-Px^{(2m-1)/m}\right)}\mathrm{d}x\right.$$

$$\left.+(L-KR^{m-1}T_R^{\,m})\left(1-\mathrm{e}^{-\varepsilon_s\left(\frac{1}{2m-1}K^2R^{2(m-1)}T_R^{\,2m-1}\right)}\right)\right\},$$

$$\text{for } L\geq KR^{m-1}T_R^m. \tag{26}$$

Determination of MFF_r, C_r, D_r, and m_R

MFF_r can be derived from Eq.(1), which is expressed as

$$MFF_r=\frac{M_{T_r}/M_{T_R}}{r/100}. \tag{27}$$

And MCIF (C_r), MCRR (D_r) and RS (m_R) can also be easily obtained as follows:

$$C_r=\frac{F_{T_r}}{rV/100}=\frac{m_0M_{T_r}}{rV/100}, \tag{28}$$

$$D_r=\frac{(M_{T_R}-M_{T_r})m_0}{(1-r/100)V}, \tag{29}$$

$$m_R=m_0-F_{T_R}=(1-M_{T_R})m_0. \tag{30}$$

The above contents present the derivation process of MFF_r, C_r, D_r and m_R and their analytical expressions. Eqs.(23)~(26) require a simple numerical integration (such as Cotes formula). And it is noted that MFF_r depends on S_0, L, R, T_R, r, ε_s and n except for ε_1 and m_0; all above factors except r decide the value of m_R; both C_r and D_r rely on all factors. Among these factors, S_0, L, R and T_R can be obtained by a direct measurement or local rainfall characteristics; normally n has a reference value; however, ε_s and ε_1 must be calibrated with measurements.

RESULTS AND DISCUSSION

It is worth mentioning that the theoretical relationship is difficult to verify by empirical data, which will consume much money and energy. The similar difficulties were also encountered by Wong (2008). However, to make the analysis as reliable as possible, this study tried to compare some of the current results with those of the previous study. Consequently, the values or ranges of parameters referred to the published results in the following parts.

Relationship between MFF_{20} and the influencing factors (S_0, n, R, T_R, ε_s, and L)

As shown in Figs.1a, 1c, 1d and 1e, the increase of S_0, R, T_R and ε_s leads to the increase of MFF_{20} and its value asymptotically approaches a maximum. The longer the L, the larger the MFF_{20} yields for the same S_0, R, T_R, and ε_s. Fig.1f shows MFF_{20} with respect to L ($L\leq16$ m). As is shown, a longer slope causes a larger MFF_{20}. The above results agree well with those of Kang *et al.*(2006)'s work in the range of watershed length less than 20 m. Furthermore, they discussed the effect of ε_l and found that MFF_{20} is not sensitive to ε_l in the shorter watersheds, especially for the watersheds whose length is less than 20 m; i.e., MFF_{20} is nearly constant, which corresponds to the conclusion of this study, indicating that ε_l has no effect on MFF_{20}.

Fig.1b displays the effect of the Manning's roughness coefficient on MFF_{20}. As n increases, MFF_{20} becomes smaller in an approximately linear way. Moreover, a longer slope (L=16 m) results in a larger MFF_{20} and a slower descending speed of MFF_{20}.

Relationship between C_{20} and the influencing factors (S_0, n, R, T_R, ε_s, and L)

As shown in Figs.2a and 2e, C_{20} increases as S_0 and ε_s mount up. For a shorter slope (L=4 m), C_{20} is

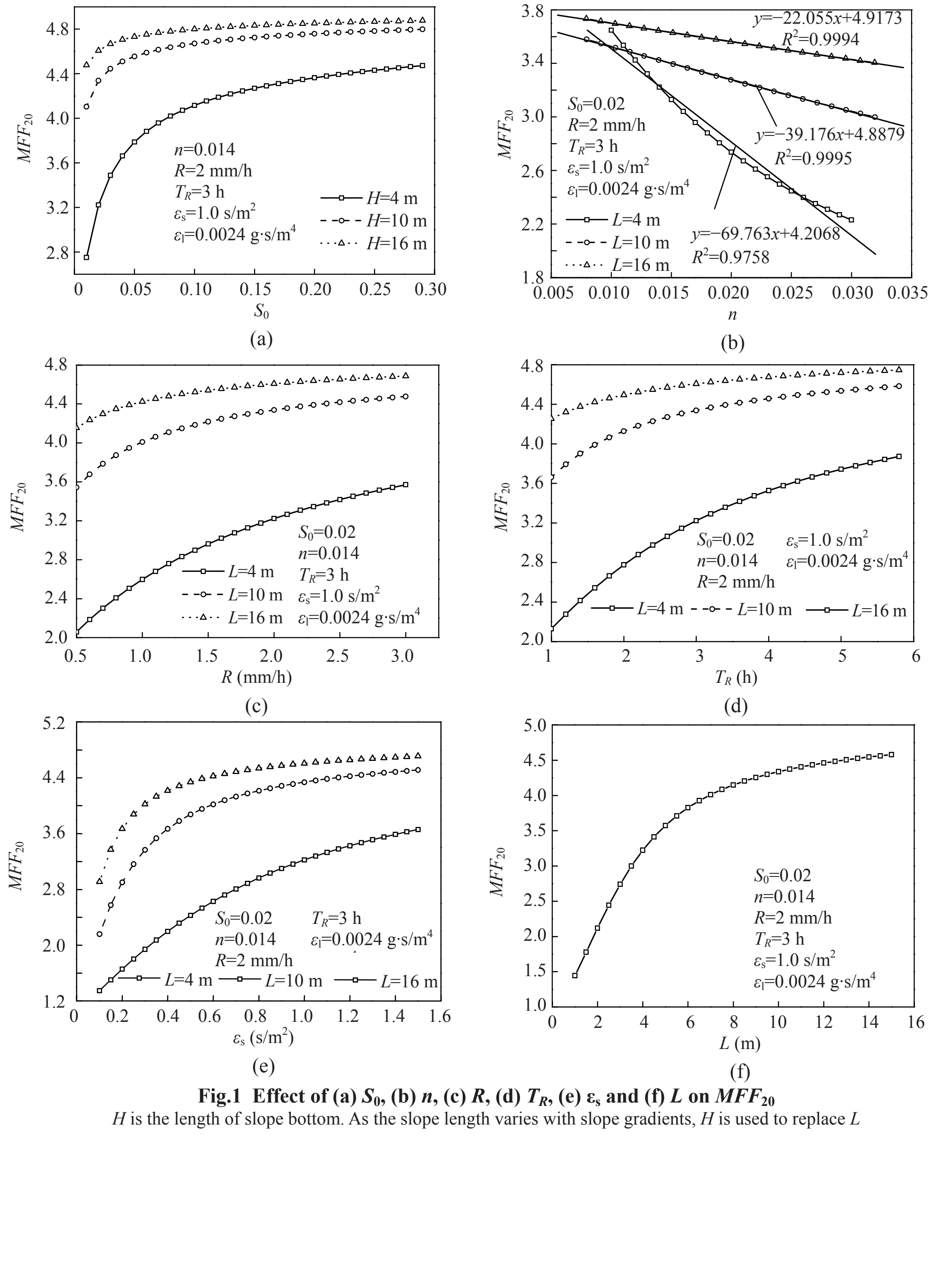

Fig.1 Effect of (a) S_0, (b) n, (c) R, (d) T_R, (e) ε_s and (f) L on MFF_{20}

H is the length of slope bottom. As the slope length varies with slope gradients, H is used to replace L

larger and more sensitive to S_0 and ε_s. Fig.2b displays C_{20} with respect to n and L. C_{20} proportionally decreases as n increases and a longer slope (L=16 m) yields a smaller C_{20}. There has been little data to verify the effect of L on C_{20}; however, Pitt (1987) examined the relationship between surface roughness and the total washoff load and found that smooth surfaces produced a larger washoff load than did rough surfaces under the same condition of rainfall, which proves the reasonability of the result in this study. According to Figs.2c and 2d, larger R and T_R produce smaller C_{20}. Fig.2f examines the relationship between C_{20} and L. It is observed that a maximum C_{20} exists, which may be due to the fact that a shorter slope (L≤2 m) has no enough flow energy to flush roof pollutants and that a longer slope (L≥2 m) provides too much volume of the first portion of runoff.

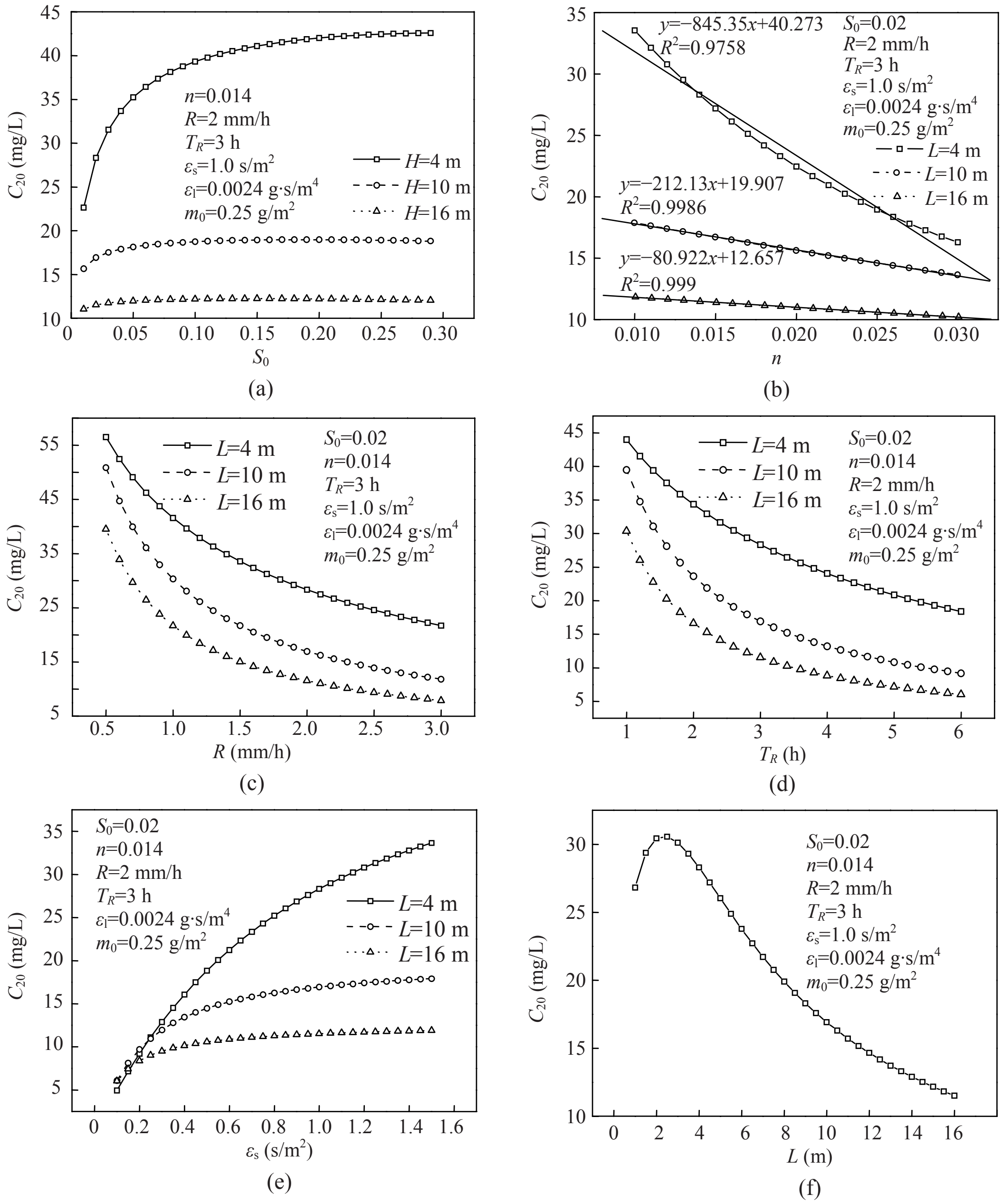

Fig.2 Effect of (a) S_0, (b) n, (c) R, (d) T_R, (e) ε_s and (f) L on C_{20}

H is the length of slope bottom. As the slope length varies with slope gradients, H is used to replace L

Relationship between D_{20} and the influencing factors (S_0, n, R, T_R, ε_s, and L)

D_{20} is an important index of water quality of roof runoff after the first part of runoff being diverged, so it is meaningful to investigate the affecting factors of D_{20}. Figs.3a, 3c and 3d describe the relationship between three factors (S_0, R and T_R) and D_{20} for three different slope lengths (4, 10 and 16 m). D_{20} diminishes rapidly as S_0, R and T_R increases; and a longer slope produces a smaller D_{20}, which means that rainwater from a long slope (L=16 m) after a diversion of the same portion is cleaner than that from a short slope (L=4 m). As shown in Fig.3b, as n grows, D_{20} also increases approximately linearly. Furthermore, the longer the slope length, the smaller the increase rates. The effect of ε_s on D_{20} is a little complex as

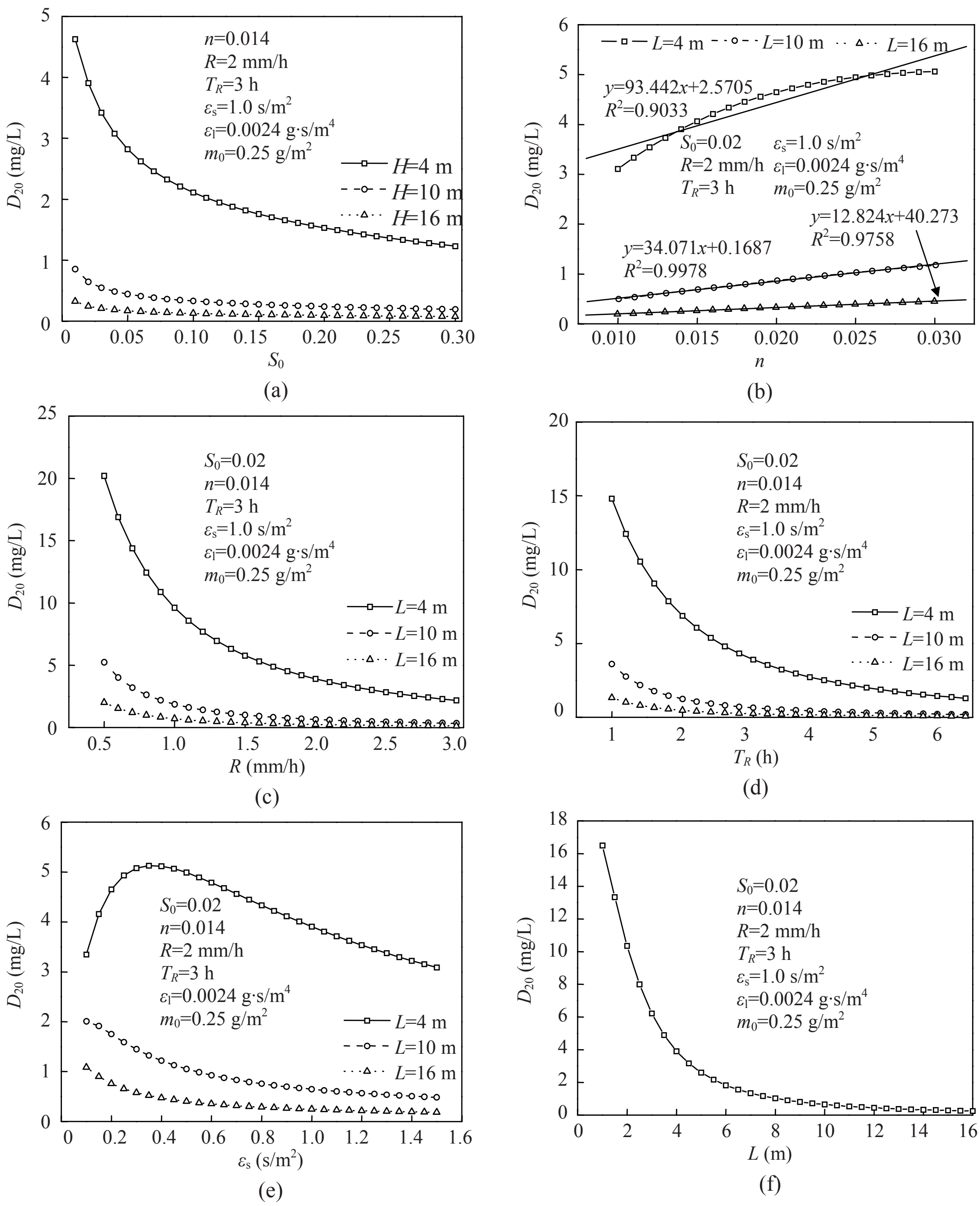

Fig.3 Effect of (a) S_0, (b) n, (c) R, (d) T_R, (e) ε_s and (f) L on D_{20}

H is the length of slope bottom. As the slope length varies with slope gradients, *H* is used to replace *L* represented in Fig.3e. For L=4 m, there exists a peak of D_{20}; for L=10 or 16 m, D_{20} decreases as ε_s increases. And it can be found that a shorter slope always produces a larger D_{20}. Fig.3f specially examines the influence of L on D_{20}, which indicates that D_{20} becomes smaller as the slope length increases and especially, it diminishes rapidly when the slope length is in a shorter range (L≤6 m).

Relationship between m_R and the influencing factors (S_0, n, R, T_R, ε_s and L)

As roof pollutants cannot be completely washed away, roof is still covered with a certain concentration of contaminants after rainfall, which will be a pollution source for next runoff event. Then examining the influencing factors of m_R is beneficial to pollution reduction. Strictly speaking, m_R—having no relation to the first flush—reflects the behavior of the total runoff.

Figs.4a, 4c, 4d and 4e show that m_R diminishes as S_0, R, T_R and ε_s increase; a longer slope leads to a smaller m_R. Fig.4b displays m_R with respect to n. m_R is positively proportional to n for a longer slope (L=16 m); both m_R and its variation are smaller than those

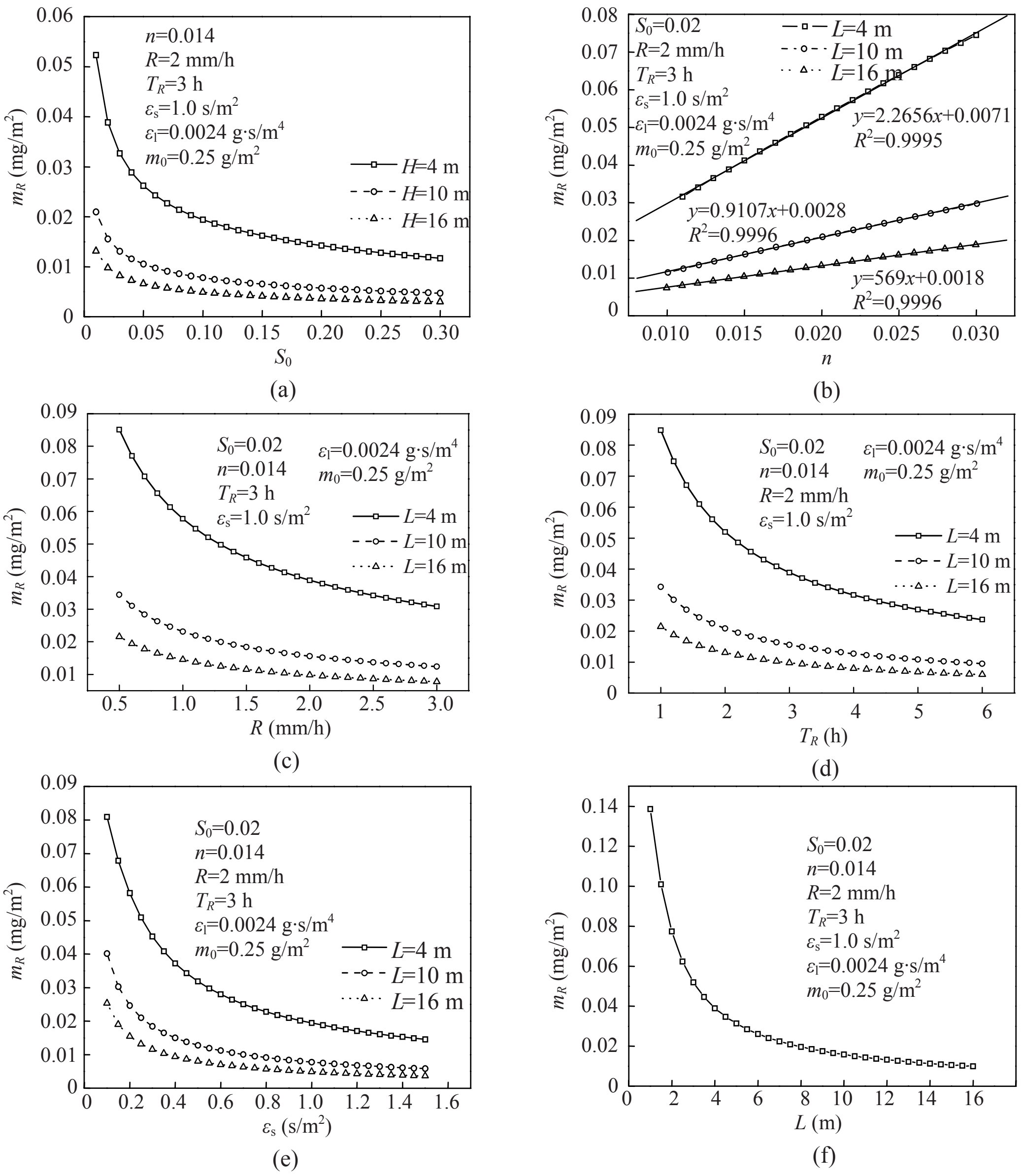

Fig.4 Effect of (a) S_0, (b) n, (c) R, (d) T_R, (e) ε_s and (f) L on m_R

H is the length of slope bottom. As the slope length varies with slope gradients, H is used to replace L

for a shorter slope (L=4 m). Different values of L ranged from 1 to 16 m are simulated in Fig.4f. A longer slope produces a smaller m_R while L strongly affects m_R when L is less than 6 m.

Relationship between ε_l and the parameters (C_{20}, D_{20} and m_R)

As mentioned above, ε_l has nothing to do with MFF_{20}; however, it affects C_{20}, D_{20} and m_R. In view of this, different values of ε_l (0.0001 g·s/m^4≤ε_l≤0.00015 g·s/m^4) are simulated in Fig.5. The results showed that C_{20} and D_{20} are inversely proportional to ε_l, while m_R is just the reverse. However, the variation of C_{20}, D_{20} and m_R is much small in the simulated range.

Relationship between r and the parameters (MFF_r, C_r and D_r)

In engineering practices, it is necessary to determine an optimum r in terms of technology and economy. Among technical considerations, pollution reduction is an important factor, and MFF_r, C_r and D_r are three important parameters for runoff pollution control. Fig.6 examines the effect of r on them. It shows that MFF_r, C_r and D_r, diminishing as r

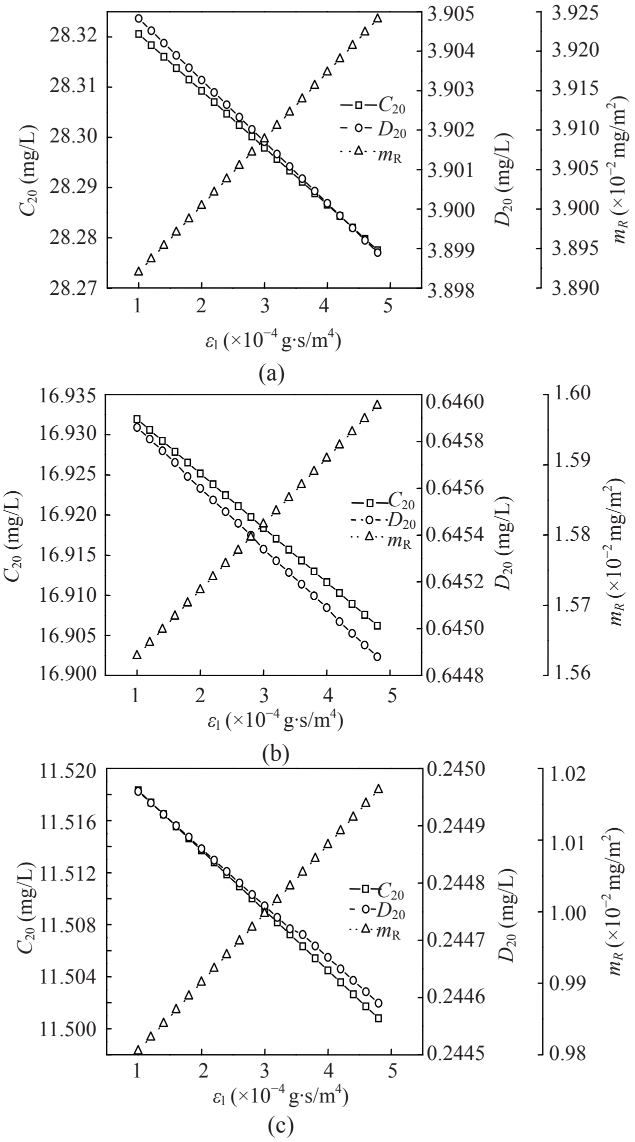

Fig.5 Effect of ε_l on C_{20}, D_{20} and m_R. (a) L=4 m; (b) L=10 m; (c) L=16 m. S_0=0.02, n=0.014, R=2 mm/h, T_R=3 h, ε_s=1.0 s/m^2, m_0=0.25 g/m^2

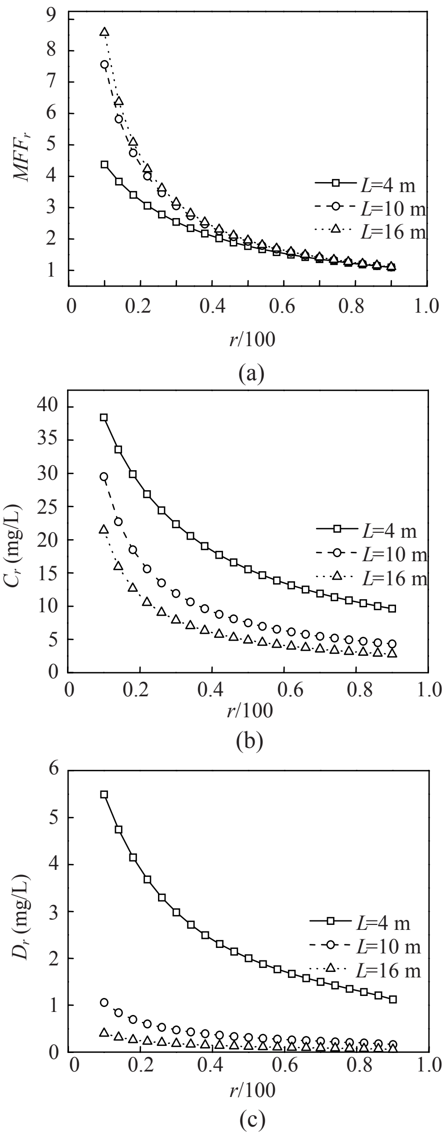

Fig.6 Effect of r on (a) MFF_r, (b) C_r and (c) D_r. S_0=0.02, n=0.014, R=2 mm/h, T_R=3 h, ε_s=1.0 s/m^2, ε_l=0.0024 g·s/m^4, m_0=0.25 g/m^2

increases, are more sensitive to r in a smaller range ($r \leq 40$). Furthermore, it is interesting to note that the influence of r on MFF_r, C_r and D_r is greater for a short roof (L=4 m) than for a long roof (L=16 m).

CONCLUSION

A theoretical analysis was carried out to establish the relationship between the first flush of roof runoff and the associated factors. The theoretical derivation used the kinematic wave model and pollutant erosion equation and disregarded the pollutant transport based on a short building roof (≤20 m). The theoretical analysis predicted pollutant MFF, MCIF, MCRR with a diversion of the initial portion and RS after the entire runoff. Some of the results are accorded with those of other studies, so it may provide reference for further studies on roof runoff in respect of rainwater utilization or runoff pollution control.

(1) Analytical relationship between first flush parameters (MFF_r, C_r, D_r and m_R) and roof characteristics such as slope length, slope, roof roughness, rainfall intensity, duration, initial mass, and erosion coefficients were obtained.

(2) For common roofs whose slope length is less than 20 m, the increase of L, S_0 and decrease of n generally leads to the increase of MFF_{20} and C_{20} and the decrease of D_{20} and m_R, which indicates that a longer roof, steeper roof gradient and smoother roof surface will be beneficial to rainwater reuse or pollution control.

(3) Long-term pollution sources have a small effect on C_{20}, D_{20} and m_R, even though a larger ε_1 produces smaller C_{20} and D_{20} and a larger m_R.

(4) MFF_r, C_r and D_r diminish as r increases, while their decreasing is more sensitive to r in a smaller range ($r \leq 40$).

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