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Model predictive control for adaptive cruise control with multi-objectives: comfort, fuel-economy, safety and car-following*

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Abstract: For automated vehicles, comfortable driving will improve passengers' satisfaction. Reducing fuel consumption brings economic profits for car owners, decreases the impact on the environment and increases energy sustainability. In addition to comfort and fuel-economy, automated vehicles also have the basic requirements of safety and car-following. For this purpose, an adaptive cruise control (ACC) algorithm with multi-objectives is proposed based on a model predictive control (MPC) framework. In the proposed ACC algorithm, safety is guaranteed by constraining the inter-distance within a safe range; the requirements of comfort and car-following are considered to be the performance criteria and some optimal reference trajectories are introduced to increase fuel-economy. The performances of the proposed ACC algorithm are simulated and analyzed in five representative traffic scenarios and multiple experiments. The results show that not only are safety and car-following objectives satisfied, but also driving comfort and fuel-economy are improved significantly.

Key words: Adaptive cruise control (ACC), Multi-objectives, Comfort, Fuel-economy, Model predictive control (MPC)
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1 Introduction

Vehicle automation is believed to be the trend in the automobile industry in future and will play an indispensable role in our lives. Adaptive cruise control (ACC) is an automated vehicle control system, which is widely adopted in modern vehicles (Vahidi and Eskandarian, 2003; Jiang and Wu, 2006). An ACC-equipped vehicle regulates the speed at a predefined value if there is no preceding vehicle. However, when a preceding vehicle is detected, an ACC-equipped vehicle automatically follows the preceding vehicle at a proper distance.

Safe car-following is a fundamental requirement when developing ACC algorithms. To maintain a safe car-following distance, proportional-integral-derivative (PID) controllers have been proposed to regulate the inter-distance and velocity (Ioannou *et al*., 1993; Zhang and Ioannou, 2006). Constant time headway policy, which specifies that the desired safe following distance is proportional to the speed of the vehicle, has received considerable attention (Ioannou *et al*., 1993; Liang and Peng, 1999; Naus *et al*., 2008). To avoid a collision during the car-following process, the actual inter-distance should be constrained within a safe range (Bageshwar *et al*., 2004; Martinez and Canudas-De-Wit, 2007). Moreover, fuzzy and neural controllers have been proposed to imitate human drivers' safe car-following behavior (Holzmann *et al*., 1997; Pasquier *et al*., 2001; Naranjo *et al*., 2007).

Besides safe car-following, driving comfort and fuel-economy are important as well. According to the survey report form National Highway Traffic Safety Administration (NHTSA) on ACC (James *et al*., 2008), driving comfort is often one of our most suggested areas for improvement, and some people refuse to use ACC due to its discomfort. So comfortable

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driving will improve passengers' satisfaction and possibly lead to increased usage of ACC in commercial vehicles. Reducing fuel consumption can save money for car owners, decrease the impact on the environment from the generation of carbon dioxide (CO_2), and increase energy sustainability. Therefore, it is very meaningful to consider comfort and fuel-economy in ACC design. In general, acceleration and jerk are important for evaluating the discomfort felt by drivers (Yi and Chung, 2001). In kinematics, acceleration represents the change of velocity over time (Crew, 2008), and jerk refers to the time derivative of acceleration (Sprott, 1997). Similar to velocity, acceleration and jerk also have two directions, longitudinal and lateral. In the longitudinal control of ACC, the longitudinal acceleration and jerk are considered as comfort metrics (Zhang and Ioannou, 2006; Martinez and Canudas-De-Wit, 2007). Martinez and Canudas-De-Wit (2007) improved comfort in ACC by restricting the values of acceleration and jerk, since the smaller the absolute values of acceleration and jerk are, the more comfortable passengers will feel (Yi and Chung, 2001; Zhang and Ioannou, 2006). Naus *et al.* (2008) proposed an ACC algorithm that considers driving safety and comfort. However, the comfort is satisfied by assuming that the actual acceleration equals the control command, ignoring the acceleration of the preceding vehicle. Regarding the requirement of fuel-economy, it is argued that the smooth responses of ACC will reduce fuel consumption (Bose and Ioannou, 2001; Vahidi and Eskandarian, 2003; Ioannou and Stefanovic, 2005), but the corresponding details about how to design an ACC algorithm are not mentioned clearly.

In this study, a new ACC algorithm is proposed to satisfy multi-objectives, not only including safety and car-following, but also driving comfort and fuel-economy. A model predictive control (MPC) framework (Mayne *et al.*, 2000) with multi-objectives is adopted for the design. In the proposed ACC algorithm, safety is regarded as constraints. Comfort and car-following requirements are synthesized into the performance criteria of MPC and some optimal reference trajectories are introduced to increase fuel-economy. By simulating various representative traffic scenarios, it is proved that the proposed ACC algorithm provides passengers improved comfort, significantly better fuel-economy and safe car-following driving.

2 ACC algorithm design

2.1 Modeling, control objectives and constraints

Generally, an ACC controller consists of an upper level controller and a lower level controller (Rajamani and Zhu, 2002). An upper level controller determines the required kinematics of the vehicle for fulfilling requirements and constraints; the lower level controller determines the throttle and brake to track the acceleration/deceleration command from the upper level controller. Since the emphasis of this research is on the upper level controller, we assume that the lower level controller is designed well. Its behavior is usually approximated by a first-order system (Bageshwar *et al.*, 2004; Zhou and Peng, 2005):

$$\tau \frac{\mathrm{d}\ddot{s}(t)}{\mathrm{d}t} + \ddot{s}(t) = u(t), \tag{1}$$

where τ refers to the time lag corresponding to the finite bandwidth of the lower level controller, u represents the acceleration command computed from the upper level controller. s, $\dot{s}$ and $\ddot{s}$ refer to the position, velocity and acceleration of the ACC-equipped vehicle, respectively.

The discrete-time expression of Eq. (1) is represented using difference approximation:

$$a(k+1) = \left(1 - \frac{T_\mathrm{s}}{\tau}\right) a(k) + \frac{T_\mathrm{s}}{\tau} u(k), \tag{2}$$

where T_s refers to the sampling period and $a(k)$ represents the acceleration of the ACC-equipped vehicle at the sampling time k.

The constant time headway (CTH) policy is used:

$$\Delta s_\mathrm{des}(k) = d_0 + t_\mathrm{h} \cdot v(k), \tag{3}$$

where $\Delta s_\mathrm{des}(k)$ and $v(k)$ refer to the desired following distance and actual velocity at sampling time k, respectively, d_0 represents a fixed safety distance when the vehicle is at low or zero speeds, and t_h refers to the desired time headway.

The spacing error, δ, and the relative velocity, v_e, are defined as

$$\delta(k) = \Delta s(k) - \Delta s_\mathrm{des}(k), \tag{4}$$

$$v_\mathrm{e}(k) = v_\mathrm{l}(k) - v(k), \tag{5}$$

where $\Delta s(k)$ and $v_l(k)$ represent the actual inter-distance and the velocity of the preceding vehicle at the sampling time k, respectively.

The jerk caused by the variation of current acceleration is presented:

$$j(k)=\frac{a(k)-a(k-1)}{T_s}. \tag{6}$$

Then the state vector is defined:

$$\boldsymbol{x}(k)=[\Delta s(k), v(k), v_e(k), a(k), j(k)]^{\mathrm{T}}. \tag{7}$$

In this study, the acceleration of the preceding vehicle is modeled as the disturbance, $w(k)=a_l(k)$. Then the state-space equations which represent the dynamics of an ACC-equipped vehicle and its preceding vehicle are presented:

$$\boldsymbol{x}(k+1)=\boldsymbol{A}\boldsymbol{x}(k)+\boldsymbol{B}u(k)+\boldsymbol{G}w(k), \tag{8a}$$

$$\boldsymbol{y}(k)=\boldsymbol{C}\boldsymbol{x}(k)-\boldsymbol{Z}, \tag{8b}$$

where spacing error, relative velocity, acceleration and jerk constitute a performance vector:

$$\boldsymbol{y}(k)=[\delta(k), v_e(k), a(k), j(k)]^{\mathrm{T}}.$$

The system matrices are represented as follows:

$$\boldsymbol{A}=\begin{pmatrix} 1 & 0 & T_s & -\frac{1}{2}T_s^2 & 0 \\ 0 & 1 & 0 & T_s & 0 \\ 0 & 0 & 1 & -T_s & 0 \\ 0 & 0 & 0 & 1-\frac{T_s}{\tau} & 0 \\ 0 & 0 & 0 & -\frac{1}{\tau} & 0 \end{pmatrix},\ \boldsymbol{B}=\begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{T_s}{\tau} \\ \frac{1}{\tau} \end{pmatrix},$$

$$\boldsymbol{G}=\begin{pmatrix} \frac{1}{2}T_s^2 \\ 0 \\ T_s \\ 0 \\ 0 \end{pmatrix},\ \boldsymbol{C}=\begin{bmatrix} 1 & -t_h & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix},\ \boldsymbol{Z}=\begin{bmatrix} d_0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

The upper level controller is designed to follow the preceding vehicle safely, comfortably and fuel-economically.

The objective of car-following behavior is that the ACC system regulates its speed to that of the preceding vehicle, v_l, and keeps the inter-distance to the desired value, Δs_{des}:

$$\text{Objectives: } \delta(k)\to 0,\ v_e(k)\to 0,\ \text{as}\ k\to\infty. \tag{9}$$

To provide comfort to the passengers, the absolute values of acceleration and jerk should be as small as possible during the car-following process:

$$\text{Objectives: } \begin{cases} \min|a(k)|, \\ \min|j(k)|. \end{cases} \tag{10}$$

For the purpose of reducing fuel consumption, smooth responses are preferred. So some smooth curves which gradually converge to the desired values are introduced. They can be called the reference trajectories. We make the performance variables move along the references trajectories, rather than minimize them to zero directly. In this study, the reference trajectories are chosen to be the exponential attenuation functions. For instance, the reference trajectory of acceleration a_r is defined as

$$a_r(k+i)=a(k)+[0-a(k)]\left[1-\mathrm{e}^{\frac{-iT_0}{\alpha}}\right]=\rho^i a(k), \tag{11}$$

where

$$\rho=\mathrm{e}^{\frac{-T_0}{\alpha}}\ (0<\rho<1),$$

where α and T_0 refer to the time constant and sampling period of reference trajectory, respectively, and ρ is the parameter to be designed. The larger the value of ρ, the slower the response, and the more robust ACC system behaves. The references trajectories of other performance variables are designed in a similar form as Eq. (11).

To avoid a collision with the preceding vehicle, the distance between the ACC-equipped and preceding vehicles should be larger than a critical minimum safe distance d_c:

$$\text{Hard constraint: } \Delta s(k)\geq d_c. \tag{12}$$

This constraint should be considered as hard, because a collision cannot be tolerated.

Besides, the minimum, maximum values of velocity, acceleration, jerk and control command are constrained:

$$\text{Hard constraint:}\quad v_{\min} \le v(k) \le v_{\max}, \tag{13a}$$

$$\text{Hard constraint:}\quad a_{\min} \le a(k) \le a_{\max}, \tag{13b}$$

$$\text{Hard constraint:}\quad u_{\min} \le u(k) \le u_{\max}, \tag{13c}$$

$$\text{Hard constraint:}\quad j_{\min} \le j(k) \le j_{\max}. \tag{13d}$$

Since the vehicle capabilities associated with velocity, accelerating, braking, jerk (the variation of acceleration) and control command are standardized in ACC systems (Hiraoka *et al.*, 2005), these constraints are considered as hard in this study.

Above all, the ACC system and its multi-objectives, including driving comfort and fuel-economy, in addition to safety and car-following, are modeled respectively. Then the control design is transformed to be a constrained multi-objectives optimal control problem.

2.2 Model predictive control

In this subsection, ACC with multi-objectives is designed in a model predictive control (MPC) framework. The basic principle of MPC is that the current control action is obtained by solving an on-line optimization, and the first value of the solved control sequence is applied. Then the horizon moves forward a step and the process is repeated, which can be called the receding horizon optimization (Mayne *et al.*, 2000). The future system states are predicted, based on the model and the current state $\boldsymbol{x}(k)$. The feedback scheme is used to compensate the predictive error and modeling inaccuracy. Therefore, the predictive state and performance vectors of ACC are calculated as

$$\hat{\boldsymbol{X}}_p(k+p\,|\,k) = \bar{\boldsymbol{A}}\boldsymbol{x}(k) + \bar{\boldsymbol{B}}\boldsymbol{U}(k+m) + \bar{\boldsymbol{G}}\boldsymbol{W}(k+p) + \bar{\boldsymbol{H}}\boldsymbol{e}_x(k), \tag{14a}$$

$$\hat{\boldsymbol{Y}}_p(k+p\,|\,k) = \bar{\boldsymbol{C}}\boldsymbol{x}(k) + \bar{\boldsymbol{D}}\boldsymbol{U}(k+m) + \bar{\boldsymbol{E}}\boldsymbol{W}(k+p) - \bar{\boldsymbol{Z}} + \bar{\boldsymbol{F}}\boldsymbol{e}_y(k), \tag{14b}$$

where

$$\hat{\boldsymbol{X}}_p(k+p\,|\,k) = \begin{bmatrix} \hat{\boldsymbol{x}}_p(k+1\,|\,k) \\ \hat{\boldsymbol{x}}_p(k+2\,|\,k) \\ \vdots \\ \hat{\boldsymbol{x}}_p(k+p\,|\,k) \end{bmatrix},$$

$$\hat{\boldsymbol{Y}}_p(k+p\,|\,k) = \begin{bmatrix} \hat{\boldsymbol{y}}_p(k+1\,|\,k) \\ \hat{\boldsymbol{y}}_p(k+2\,|\,k) \\ \vdots \\ \hat{\boldsymbol{y}}_p(k+p\,|\,k) \end{bmatrix},$$

$$\boldsymbol{U}(k+m) = \begin{bmatrix} u(k) \\ u(k+1) \\ \vdots \\ u(k+m-1) \end{bmatrix}, \boldsymbol{W}(k+p) = \begin{bmatrix} w(k) \\ w(k+1) \\ \vdots \\ w(k+p-1) \end{bmatrix},$$

$$\bar{\boldsymbol{A}} = \begin{bmatrix} \boldsymbol{A} \\ \boldsymbol{A}^2 \\ \vdots \\ \boldsymbol{A}^{p-1} \end{bmatrix},\ \bar{\boldsymbol{B}} = \begin{bmatrix} \boldsymbol{B} & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ \boldsymbol{AB} & \boldsymbol{B} & & \vdots \\ \vdots & \vdots & & \boldsymbol{0} \\ \boldsymbol{A}^{p-1}\boldsymbol{B} & \boldsymbol{A}^{p-2}\boldsymbol{B} & \cdots & \sum_{l=0}^{p-m}\boldsymbol{A}^l\boldsymbol{B} \end{bmatrix},$$

$$\bar{\boldsymbol{G}} = \begin{bmatrix} \boldsymbol{G} & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ \boldsymbol{AG} & \boldsymbol{G} & & \vdots \\ \vdots & \vdots & & \boldsymbol{0} \\ \boldsymbol{A}^{p-1}\boldsymbol{G} & \boldsymbol{A}^{p-2}\boldsymbol{G} & \cdots & \boldsymbol{G} \end{bmatrix},\ \bar{\boldsymbol{C}} = \begin{bmatrix} \boldsymbol{CA} \\ \boldsymbol{CA}^2 \\ \vdots \\ \boldsymbol{CA}^{p-1} \end{bmatrix},$$

$$\bar{\boldsymbol{D}} = \begin{bmatrix} \boldsymbol{CB} & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ \boldsymbol{CAB} & \boldsymbol{CB} & & \vdots \\ \vdots & \vdots & & \boldsymbol{0} \\ \boldsymbol{CA}^{p-1}\boldsymbol{B} & \boldsymbol{CA}^{p-2}\boldsymbol{B} & \cdots & \sum_{l=0}^{p-m}\boldsymbol{CA}^l\boldsymbol{B} \end{bmatrix},$$

$$\bar{\boldsymbol{E}} = \begin{bmatrix} \boldsymbol{CG} & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ \boldsymbol{CAG} & \boldsymbol{CG} & & \vdots \\ \vdots & \vdots & & \boldsymbol{0} \\ \boldsymbol{CA}^{p-1}\boldsymbol{G} & \boldsymbol{CA}^{p-2}\boldsymbol{G} & \cdots & \boldsymbol{CG} \end{bmatrix},$$

$$\bar{\boldsymbol{H}} = \begin{bmatrix} \boldsymbol{H}_1 \\ \boldsymbol{H}_2 \\ \vdots \\ \boldsymbol{H}_p \end{bmatrix},\ \bar{\boldsymbol{Z}} = \begin{bmatrix} \boldsymbol{Z} \\ \boldsymbol{Z} \\ \vdots \\ \boldsymbol{Z} \end{bmatrix},\ \bar{\boldsymbol{F}} = \begin{bmatrix} \boldsymbol{F}_1 \\ \boldsymbol{F}_2 \\ \vdots \\ \boldsymbol{F}_p \end{bmatrix},$$

$$\boldsymbol{e}_x(k) = \boldsymbol{x}(k) - \hat{\boldsymbol{x}}_m(k\,|\,k-1),\ \boldsymbol{e}_y(k) = \boldsymbol{y}(k) - \hat{\boldsymbol{y}}_m(k\,|\,k-1),$$

where p refers to the predicted horizon, m refers to the control horizon, the control sequence $u(k)$, $u(k+1)$, $u(k+2)$, …, $u(k+m-1)$ are to be solved, $\boldsymbol{x}(k)$ represents the current measured state, $\boldsymbol{y}(k)$ refers to the current measured performance vector, $\hat{\boldsymbol{x}}_m(k|k-1)$ and $\hat{\boldsymbol{y}}_m(k|k-1)$ refer to the open loop predictive state and performance vectors at the sampling time k, respectively, based on the information available at the previous sampling time k−1. $\boldsymbol{e}_x$ and $\boldsymbol{e}_y$ represent the predictive errors regarding state and performance vectors, respectively, $\boldsymbol{H}_i$ and $\boldsymbol{F}_i$ refer to the corresponding error feedback matrices.

In real use, the disturbance vector of the preceding vehicle $\boldsymbol{W}(k+p)$ can be modeled in the following way.

At the sampling time k, the current disturbance (the acceleration of the preceding vehicle) $w(k)$ is unknown, but it can be approximated by the value of the previous sampling time k−1, which can be estimated according to the measured relative velocity and acceleration of the ACC-equipped vehicle:

$$w(k)=\hat{w}(k-1|k), \tag{15}$$

$$\hat{w}(k-1|k)=\frac{v_{\mathrm{e}}(k)-v_{\mathrm{e}}(k-1)}{T_{\mathrm{s}}}+a(k-1), \tag{16}$$

where $\hat{w}(k-1|k)$ refers to the disturbance of sampling time k−1 estimated at the sampling time k.

Assume that the disturbance remains constant in the predictive horizon, which is a common way of modeling the disturbance vector:

$$w(k+i)=w(k),\ i=1,2,...,p-1. \tag{17}$$

Then the disturbance vector $\boldsymbol{W}(k+p)$ is obtained:

$$\boldsymbol{W}(k+p)=\begin{bmatrix}\hat{w}(k-1|k)\\ \hat{w}(k-1|k)\\ \vdots \\ \hat{w}(k-1|k)\end{bmatrix}. \tag{18}$$

The inaccuracy in modeling the disturbance vector can be compensated by the feedback scheme and receding horizon optimization, which is also the advantage of MPC (Mayne *et al.*, 2000; Bemporad *et al.*, 2002).

Then the objectives of comfort, fuel-economy and car-following are synthesized into a performance criterion of MPC by weighted sum:

$$J=\sum_{i=1}^{p}[\hat{\boldsymbol{y}}_p(k+i|k)-\boldsymbol{y}_r(k+i)]^{\mathrm{T}}\boldsymbol{Q}[\hat{\boldsymbol{y}}_p(k+i|k)-\boldsymbol{y}_{\mathrm{r}}(k+i)] +\sum_{i=0}^{m-1}u(k+i)^{\mathrm{T}}\boldsymbol{R}u(k+i), \tag{19}$$

where $\boldsymbol{y}_r(k+i)$ represents the reference trajectory of the performance vector, $\boldsymbol{Q}$ and $\boldsymbol{R}$ refer to the weighting matrices of objectives and control, respectively. $\boldsymbol{Q}$=diag$\{q_\delta, q_{\mathrm{v}}, q_{\mathrm{a}}, q_{\mathrm{j}}\}$.

The objectives of driving safety together with the ability limit of vehicles are regarded as constraints of MPC:

$$\text{s.t.}\ \begin{cases}\Delta s(k)\ge d_{\mathrm{c}},\\ v_{\min}\le v(k)\le v_{\max},\\ a_{\min}\le a(k)\le a_{\max},\\ j_{\min}\le j(k)\le j_{\max},\\ u_{\min}\le u(k)\le u_{\max}.\end{cases} \tag{20}$$

Then the multi-objectives are synthesized in an MPC framework. This allows us to extend the range of design requirements for future research, such as mechanical stress and tailpipe emissions, by modeling them respectively.

Substituting Eq. (14) into Eq. (19), the matrix form of performance criteria is obtained:

$$\boldsymbol{J}=\boldsymbol{V}^{\mathrm{T}}\bar{\boldsymbol{Q}}\boldsymbol{V}+\boldsymbol{U}(k+m)^{\mathrm{T}}\bar{\boldsymbol{R}}\boldsymbol{U}(k+m), \tag{21}$$

where

$$\boldsymbol{V}=\bar{\boldsymbol{C}}\boldsymbol{x}(k)+\bar{\boldsymbol{D}}\boldsymbol{U}(k+m)+\bar{\boldsymbol{E}}\boldsymbol{W}(k+p)-\bar{\boldsymbol{Z}}+\bar{\boldsymbol{F}}\boldsymbol{e}_y(k) -\bar{\boldsymbol{\Phi}}\boldsymbol{C}\boldsymbol{x}(k)+\bar{\boldsymbol{\Phi}}\boldsymbol{Z},$$

$$\bar{\boldsymbol{Q}}=\begin{pmatrix}\boldsymbol{Q} & \cdots & \boldsymbol{0}\\ \vdots & & \vdots\\ \boldsymbol{0} & \cdots & \boldsymbol{Q}\end{pmatrix},\ \bar{\boldsymbol{R}}=\begin{pmatrix}\boldsymbol{R} & \cdots & \boldsymbol{0}\\ \vdots & & \vdots\\ \boldsymbol{0} & \cdots & \boldsymbol{R}\end{pmatrix},$$

$$\bar{\boldsymbol{\Phi}}=\begin{bmatrix}\boldsymbol{\psi}_1\\ \boldsymbol{\psi}_2\\ \vdots\\ \boldsymbol{\psi}_p\end{bmatrix},\ \boldsymbol{\psi}_i=\begin{bmatrix}\rho_\delta^i & & & \\ & \rho_{\mathrm{v}}^i & & \\ & & \rho_{\mathrm{a}}^i & \\ & & & \rho_{\mathrm{j}}^i\end{bmatrix},$$

where ρ_δ, ρ_{v}, ρ_{a} and ρ_{j} represent the parameters of reference trajectories corresponding to spacing error, relative velocity, acceleration and jerk, respectively.

Expanding Eq. (21) and omitting the terms which are independent of the control output, we can obtain:

$$\begin{aligned} \boldsymbol{J} &= \boldsymbol{U}(k+m)^{\mathrm{T}}\left(\bar{\boldsymbol{R}}+\bar{\boldsymbol{D}}^{\mathrm{T}}\bar{\boldsymbol{Q}}\bar{\boldsymbol{D}}\right)\boldsymbol{U}(k+m) \\ &\quad +2\{\boldsymbol{x}(k)^{\mathrm{T}}\left[\bar{\boldsymbol{C}}^{\mathrm{T}}-\boldsymbol{C}^{\mathrm{T}}\bar{\boldsymbol{\Phi}}^{\mathrm{T}}\right]\bar{\boldsymbol{Q}}\bar{\boldsymbol{D}}+\boldsymbol{W}(k+p)^{\mathrm{T}}\bar{\boldsymbol{E}}^{\mathrm{T}}\bar{\boldsymbol{Q}}\bar{\boldsymbol{D}} \\ &\quad -(\bar{\boldsymbol{Z}}^{\mathrm{T}}-\boldsymbol{Z}^{\mathrm{T}}\bar{\boldsymbol{\Phi}}^{\mathrm{T}})\bar{\boldsymbol{Q}}\bar{\boldsymbol{D}}+\boldsymbol{e}_y(k)^{\mathrm{T}}\bar{\boldsymbol{F}}^{\mathrm{T}}\bar{\boldsymbol{Q}}\bar{\boldsymbol{D}}\}\boldsymbol{U}(k+m). \quad (22)\end{aligned}$$

The constraints Eq. (20) can be written as the following matrix form:

$$\text{s.t.} \begin{cases} \bar{\boldsymbol{M}} \le \bar{\boldsymbol{L}}\hat{\boldsymbol{X}}_p(k+p) \le \bar{\boldsymbol{N}}, \\ \boldsymbol{U}(k+m) \le \boldsymbol{U}_{\max}, \\ -\boldsymbol{U}(k+m) \le -\boldsymbol{U}_{\min}, \end{cases} \quad (23)$$

where

$$\boldsymbol{M}=\begin{bmatrix} d_{\mathrm{c}} \\ v_{\min} \\ a_{\min} \\ j_{\min} \end{bmatrix},\ \boldsymbol{N}=\begin{bmatrix} Inf \\ v_{\max} \\ a_{\max} \\ j_{\max} \end{bmatrix},\ \boldsymbol{L}=\begin{bmatrix} 1&0&0&0&0 \\ 0&1&0&0&0 \\ 0&0&0&1&0 \\ 0&0&0&0&1 \end{bmatrix},$$

$$\bar{\boldsymbol{L}}=\begin{bmatrix} \boldsymbol{L} &&& \\ & \boldsymbol{L} && \\ && \ddots & \\ &&& \boldsymbol{L} \end{bmatrix},\ \bar{\boldsymbol{M}}=\begin{bmatrix} \boldsymbol{M} \\ \boldsymbol{M} \\ \vdots \\ \boldsymbol{M} \end{bmatrix},\ \bar{\boldsymbol{N}}=\begin{bmatrix} \boldsymbol{N} \\ \boldsymbol{N} \\ \vdots \\ \boldsymbol{N} \end{bmatrix},$$

$$\boldsymbol{U}_{\max}=\begin{bmatrix} u_{\max} \\ \vdots \\ u_{\max} \end{bmatrix},\ \boldsymbol{U}_{\min}=\begin{bmatrix} u_{\min} \\ \vdots \\ u_{\min} \end{bmatrix}.$$

where *Inf* represents a real positive infinite value, indicating that the inter-distance has no upper bound.

Substituting Eq. (14) into Eq. (23), the constraints are rewritten as

$$\boldsymbol{\Omega}\boldsymbol{U}(k+m) \le \boldsymbol{T}, \quad (24)$$

where

$$\boldsymbol{\Omega}=\left[\bar{\boldsymbol{L}}\bar{\boldsymbol{B}} \quad -\bar{\boldsymbol{L}}\bar{\boldsymbol{B}} \quad \boldsymbol{I} \quad -\boldsymbol{I}\right]^{\mathrm{T}},$$

$$\boldsymbol{T}=\begin{bmatrix} \bar{\boldsymbol{N}}-\bar{\boldsymbol{L}}\bar{\boldsymbol{G}}\boldsymbol{W}(k+p)-\bar{\boldsymbol{L}}\bar{\boldsymbol{A}}\boldsymbol{x}(k)-\bar{\boldsymbol{L}}\bar{\boldsymbol{H}}\boldsymbol{e}_x(k) \\ -\bar{\boldsymbol{M}}+\bar{\boldsymbol{L}}\bar{\boldsymbol{G}}\boldsymbol{W}(k+p)+\bar{\boldsymbol{L}}\bar{\boldsymbol{A}}\boldsymbol{x}(k)+\bar{\boldsymbol{L}}\bar{\boldsymbol{H}}\boldsymbol{e}_x(k) \\ \boldsymbol{U}_{\max} \\ -\boldsymbol{U}_{\min} \end{bmatrix}.$$

In summary, the MPC optimization problem of ACC can be described as

$$\min\left\{\boldsymbol{U}(k+m)^{\mathrm{T}}\boldsymbol{K}\boldsymbol{U}(k+m)+2\boldsymbol{S}\boldsymbol{U}(k+m)\right\},$$
$$\text{s.t. } \boldsymbol{\Omega}\boldsymbol{U}(k+m) \le \boldsymbol{T}. \quad (25)$$

Then the problem of solving the control law is transformed to an on-line quadratic programming (QP) problem, which can be solved using a mathematical programming solver (Boyd and Vandenberghe, 2004).

At each sampling time, the ACC system measures the current inter-spacing, relative velocity, own velocity, acceleration and jerk. Using these measurements, the control sequence is computed by solving a QP problem and only the first command is applied. Then the process is repeated at the next sampling time.

3 Simulation and discussion

3.1 Methods

The objective of this subsection is to evaluate the performance of the proposed ACC algorithm. Two algorithms are simulated and compared, which are the proposed ACC algorithm that considers comfort, fuel-economy, safety and car-following (ACC_CFSC) and the algorithm that considers only safety and car-following (ACC_SC). Whether a collision can be avoided effectively is considered as the criterion of safety. The objective of car-following is assessed by the performance of regulating the velocity and inter-distance to the desired values. Driving comfort is estimated by the magnitudes of acceleration and jerk (Naus *et al*., 2008). The comprehensive modal emissions model (CMEM) developed at the University of California (UC), Riverside, USA, is used to calculate the fuel consumption (Barth *et al*., 2004). The inputs for CMEM are vehicle speed and acceleration. In a nominal case, the road grade is taken as 0 and no wind is considered (Scora and Barth, 2006).

The following traffic scenarios are simulated: (1) Following a preceding vehicle with varying speed; (2) Cut in; (3) Cut out; (4) Approaching a stationary vehicle; (5) Hard stop.

These five scenarios are basic and representative in the traffic world. In real implementation of ACC, most complicated traffic cases lasting a longer time are often combinations of these scenarios. This indicates that the results of these five basic scenarios can be generalized. For each kind of scenario, 40 simulation experiments are carried out, which vary in the initial inter-distance, the initial relative velocity and the acceleration disturbance of the preceding vehicle, with the purpose of covering more traffic cases. Take scenario 1 (following a preceding vehicle with varying speed) for example. In the corresponding 40 different experiments, the initial inter-distance is chosen to be 30, 50, 70 and 90 m, respectively (from near to far); the relative velocity is chosen to be −10, −5, 0, 5 and 10 m/s, respectively (from negative large to positive large); and the acceleration amplitude of the preceding vehicle is chosen to be 0.8 and 2 m/s^2, respectively (from low to high maneuvers). The methods of experiment design for other scenarios are similar and need no further explanation. Moreover, as we are concerned with the performance of the proposed ACC algorithm, the initial conditions in each simulation experiment are guaranteed feasible (Bageshwar *et al*., 2004).

To analyze the performances regarding comfort and fuel-economy, three metrics are considered, which are the magnitudes of acceleration and jerk (Naus *et al*., 2008) and the fuel consumption predicted by CMEM. For every simulation experiment in each traffic scenario, these three metrics of ACC_CFSC and ACC_SC are collected respectively. The data of ACC_SC are used as a basis for comparison and the relative results of ACC_CFSC are obtained. Then the average values of the three metrics for each 40 experiments are calculated respectively, to reflect the corresponding traffic scenario, as shown in Table 1. Therefore, all the data in Table 1 are the average results from 40 experiments and a positive number means improvement while a negative number means deterioration. For illustration and analysis, one experiment (among 40) is selected randomly for each scenario, and the corresponding responses of ACC_CFSC and ACC_SC are represented in Figs. 1–5.

The parameters for the ACC system are chosen as follows according to (Yi and Chung, 2001; Rajamani and Zhu, 2002; Bageshwar *et al*., 2004; Zhou and Peng, 2005; Martinez and Canudas-De-Wit, 2007): T_s=0.2 s, t_h=1.5 s, τ=0.5 s, d_0=7 m, d_c=5 m, v_{min}=0 m/s, v_{max}=36 m/s, a_{min}=−5.5 m/s^2, a_{max}=2.5 m/s^2, u_{min}=−5.5 m/s^2, u_{max}=2.5 m/s^2, j_{min}=−2 m/s^3, j_{max}=2 m/s^3, ρ=0.94, $\boldsymbol{Q}$=diag{1,10,1,1}, R=1.

3.2 Discussion

By analyzing the corresponding responses and the data regarding comfort and fuel-economy from multiple experiments, we have the following conclusions:

(1) Both ACC_CFSC and ACC_SC can guarantee safety. The simulation results in five scenarios indicate that safety is satisfied in these two algorithms, as the inter-distances are non-negative and collision is avoided (Figs. 1–5).

(2) Car-following behavior is satisfied in ACC_CFSC and ACC_SC. According to Figs. 1–5, it is shown that the tracking ability of velocity behaves well in the two algorithms, and the inter-distances are finally adjusted to the desired values. Take the responses in Fig. 1 (following a preceding vehicle with varying speed) for example. Both the two algorithms take acceleration at the beginning, to catch up with the preceding vehicle, and then regulate the speed to adapt the velocity variation of the preceding vehicle, and meanwhile keep the desired following distance calculated by CTH policy. These are in accord with human drivers' car-following behavior.

(3) Regarding the comfort objective, it can be concluded that passengers will feel more comfortable in vehicles equipped with ACC_CFSC, since ACC_CFSC reduces the magnitudes of acceleration and jerk compared to ACC_SC, as shown in Table 1. In the scenario of approaching a stationary vehicle, the average reduction in acceleration and jerk magnitudes are 41.39% and 74.18%, respectively, which means driving comfort is improved drastically. While in the scenario of hard stop, the improvement in acceleration magnitude is slight (4.13% reduction on average), because when the preceding vehicle brakes fiercely, an ACC-equipped vehicle should perform a

Table 1 Benefits regarding comfort and fuel consumption by comparing ACC_CFSC to ACC_SC

Benefit	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5
Acceleration magnitude benefits	18.28%	37.61%	24.14%	41.39%	4.13%
Jerk magnitude benefits	63.92%	72.52%	68.55%	74.18%	69.91%
Fuel consumption benefits	12.86%	12.23%	17.03%	19.69%	7.59%

Note: each value is the average from the corresponding 40 experiments

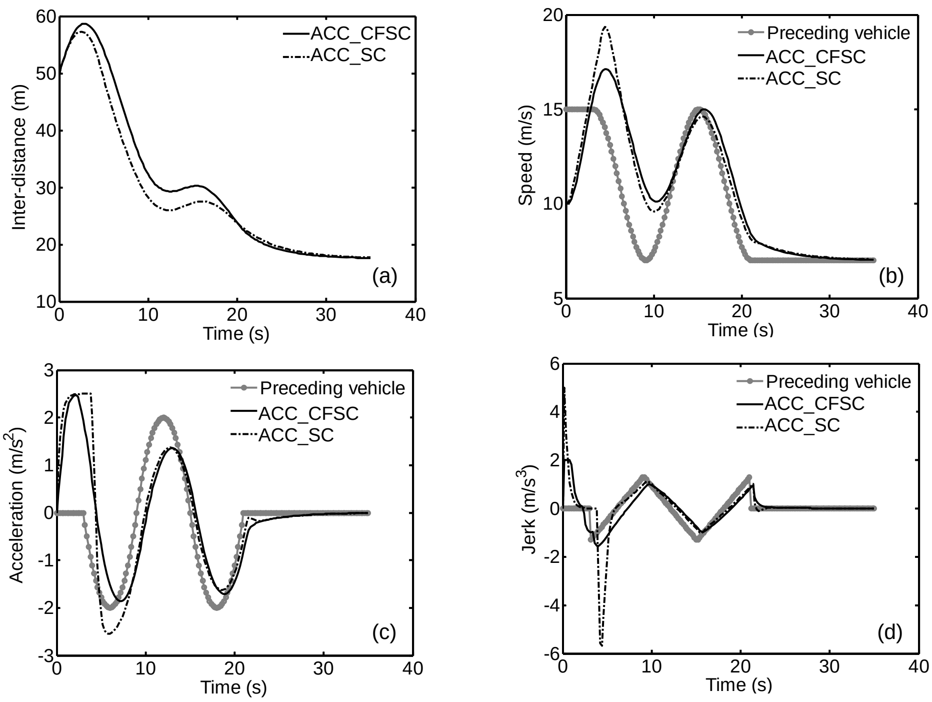

Fig. 1 Responses in the scenario of following a preceding vehicle with varying speed. (a) Inter-distance responses; (b) Speed responses; (c) Acceleration responses; (d) Jerk responses

In this simulation experiment, the ACC-equipped vehicle follows a preceding vehicle with varying speed. The initial inter-distance is 50 m, the initial velocities of preceding vehicle and ACC-equipped vehicle are 15 m/s and 10 m/s, respectively (5 m/s for relative velocity). The acceleration amplitude of preceding vehicle is 2 m/s^2

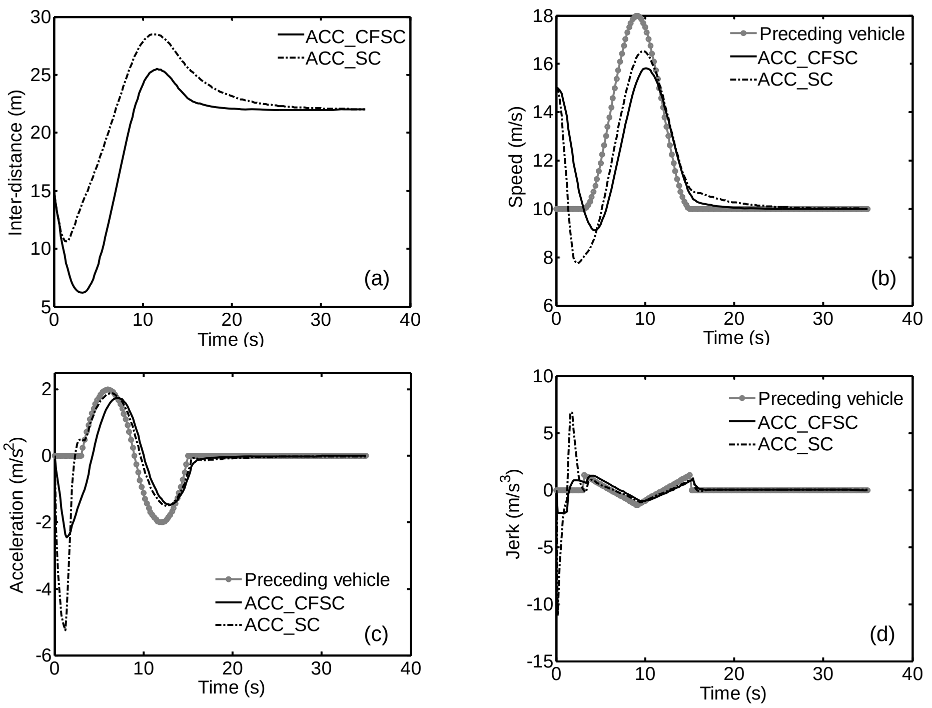

Fig. 2 Responses in the scenario of cut in. (a) Inter-distance responses; (b) Speed responses; (c) Acceleration responses; (d) Jerk responses

In this simulation experiment, the initial velocity of ACC-equipped vehicle is 15 m/s, a vehicle in the other lane performs cut in with an inter-distance of 15 m and a velocity of 10 m/s (−5 m/s for relative velocity). The preceding vehicle varies its speed with the acceleration amplitude of 2 m/s^2

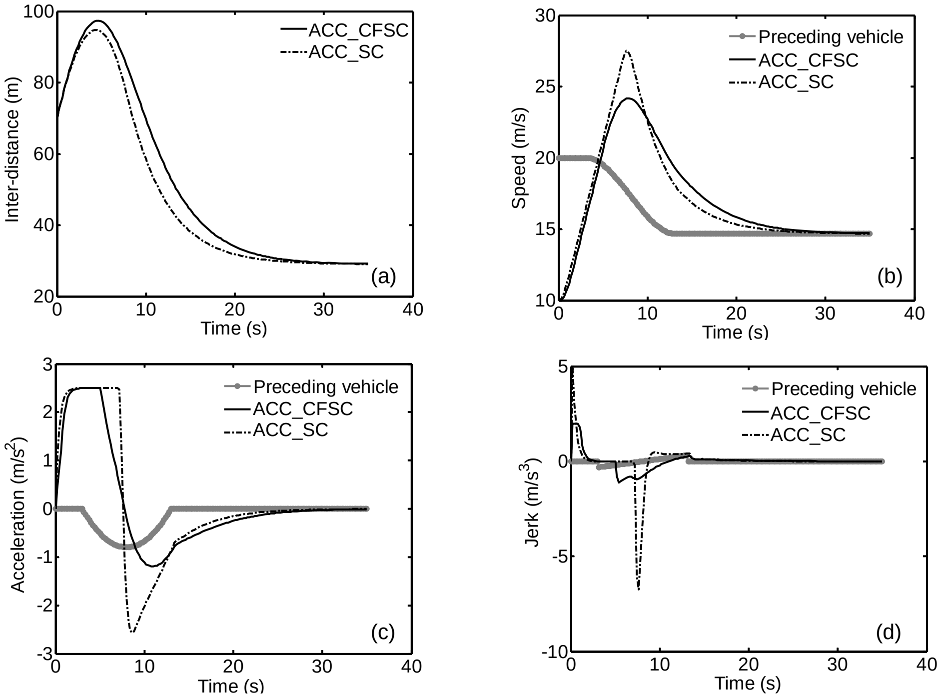

Fig. 3 Responses in the scenario of cut out. (a) Inter-distance responses; (b) Speed responses; (c) Acceleration responses; (d) Jerk responses

In this simulation experiment, the preceding vehicle performs cut out, which causes the ACC-equipped vehicle follows a new preceding vehicle in the same lane. The inter-distance is suddenly increased to 70 m due to cut out. The velocities of ACC-equipped and new preceding vehicles are 10 m/s and 20 m/s, respectively (10 m/s for relative velocity). The acceleration amplitude of new preceding vehicle is 0.8 m/s^2

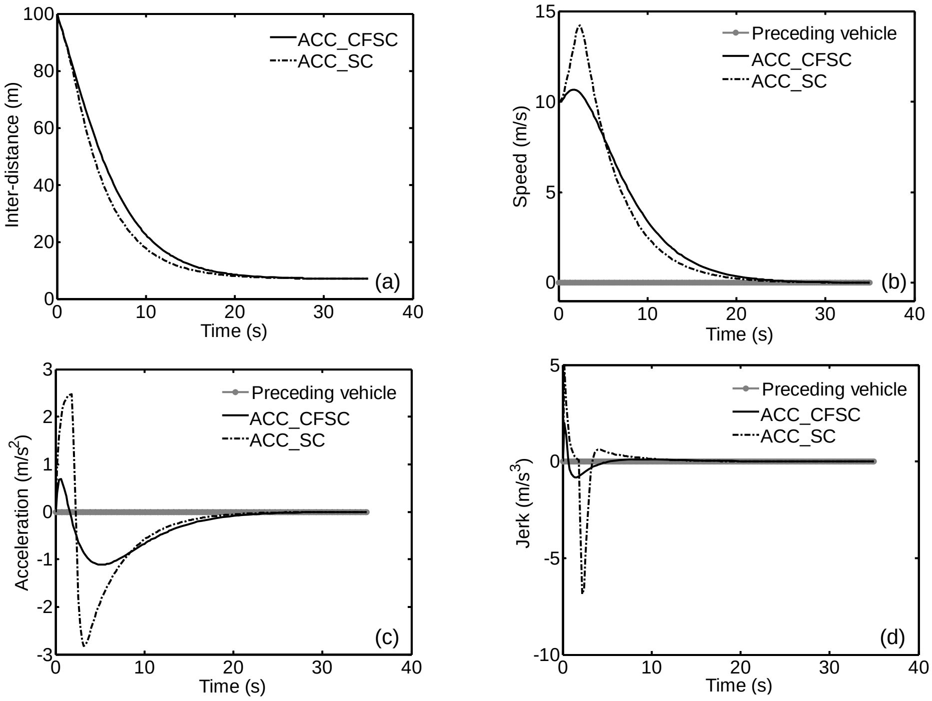

Fig. 4 Responses in the scenario of approaching a stationary vehicle. (a) Inter-distance responses; (b) Speed responses; (c) Acceleration responses; (d) Jerk responses

In this simulation experiment, the ACC-equipped vehicle approaches a stationary vehicle. The initial inter-distance and velocity are 100 m and 10 m/s, respectively

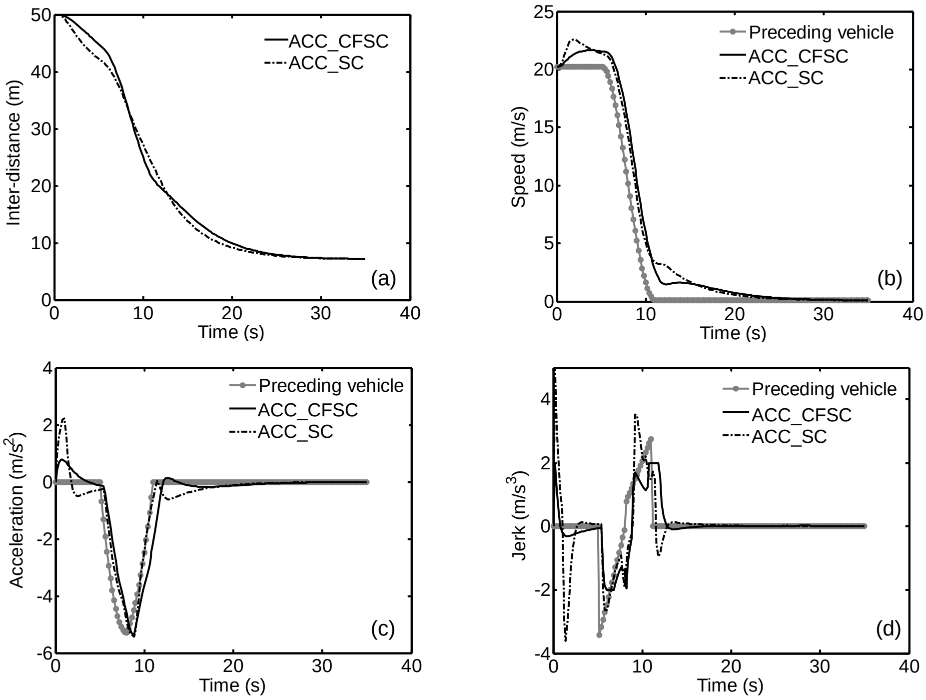

Fig. 5 Responses in the scenario of hard stop. (a) Inter-distance responses; (b) Speed responses; (c) Acceleration responses; (d) Jerk responses

In this simulation experiment, ACC-equipped vehicle is following the preceding vehicle at a velocity of 20 m/s initially. The initial inter-distance is 50 m. Then the preceding vehicle performs a hard stop at *t*=5 s

hard brake to avoid a collision, and the corresponding acceleration magnitudes are similar in the two algorithms. Moreover, according to the response figures (Figs. 1–5), it can be seen that jerk magnitudes are less than 2 m/s^3 in ACC_CFSC, while exceeding 2 m/s^3, even achieving 11 m/s^3 (Fig. 2) in ACC_SC. This clearly demonstrates that driving comfort is improved in the proposed ACC algorithm, as jerk does not exceed the threshold value of 2 m/s^3 (Martinez and Canudas-De-Wit, 2007).

(4) Based on CMEM, the aforementioned method of predicting fuel consumption, it can be concluded that ACC_CFSC should increase fuel-economy, as Table 1 shows. This is because the responses of ACC_CFSC are smoother than those of ACC_SC. Take the responses in Fig. 3 (cut out) for example, it can be seen that ACC_CFSC maintains a similar inter-distance to ACC_SC, while executing smoother (smaller fluctuations) speed and acceleration responses. This indicates that ACC_CFSC performs with similar safety as ACC_SC, but with less fuel consumption. According to Table 1, the fuel-economy of the proposed ACC algorithm is improved by different degrees in different scenarios by multiple simulation experiments. Especially in the scenario of approaching a stationary vehicle, the predicted fuel consumption is reduced by 19.69% on average. However, there is a relatively small improvement (7.59% on average) in the hard stop scenario, since both the two algorithms take a hard brake to guarantee safety, and the corresponding velocity and acceleration responses are similar.

Above all, the proposed ACC algorithm achieves good performance in these basic and representative traffic scenarios. Driving comfort and fuel-economy are improved and safe car-following is guaranteed.

4 Conclusion

In this paper, a new ACC algorithm based on an MPC framework is proposed to satisfy multi-objectives, which are comfort, fuel-economy, safety and car-following. The performances of the proposed ACC algorithm are simulated and analyzed in five representative traffic scenarios and multiple experiments. It is shown that the proposed ACC algorithm met safety and car-following requirements and outperformed traditional algorithms by improving driving comfort and increasing fuel efficiency. It can be expected that the proposed ACC algorithm will possibly lead to the increased usage of ACC in commercial vehicles by providing more comfortable driving for passengers and bringing economic benefits to car owners. Future research includes looking at the lower level control and its integration with the upper level control in ACC.

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