Journal of Zhejiang University-SCIENCE A (Applied Physics & Engineering)
ISSN 1673-565X (Print); ISSN 1862-1775 (Online)
www.zju.edu.cn/jzus; www.springerlink.com
E-mail: jzus@zju.edu.cn

Parameter effects on the dynamic characteristics of a super-long-span triple-tower suspension bridge*

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Received Aug. 12, 2009; Revision accepted Nov. 13, 2009; Crosschecked Mar. 30, 2010

Abstract: A 3D finite element model for the Taizhou Yangtze River Bridge, the first triple-tower long-span suspension bridge in China, is established based on the nonlinear finite element software ABAQUS, and the dynamic characteristics of the bridge are analyzed using the LANCZOS eigenvalue solution method. The study focuses on the effects of the vertical, lateral and torsional stiffness of the steel box girder, the rigid central buckle and the elastic restraints connecting the towers and the steel box girder on the dynamic characteristics of the triple-tower suspension bridge. Our results show that, in general, the dynamic characteristics of the triple-tower suspension bridge are similar to those of two-tower suspension bridges. The vertical, lateral and torsional stiffness of the steel box girder have different effects on the dynamic characteristics of triple-tower suspension bridges. The elastic restraints have a more significant effect on the dynamic characteristics than the central buckle, and decreasing the stiffness of the elastic restraints results in the appearance of a longitudinal floating vibration mode of the bridge. Also, rigid central buckles have a greater influence on the dynamic characteristics of triple-tower suspension bridges than on those of two-tower suspension bridges. The results obtained could serve as a valuable numerical reference for analyzing and designing super-long-span triple-tower suspension bridges.

Key words: Suspension bridge, Taizhou Bridge, Triple-tower, Dynamic characteristics, Super-long-span
doi:10.1631/jzus.A0900496 **Document code:** A **CLC number:** U448.25

1 Introduction

With increasing global economic development and improved bridge construction techniques, suspension bridges, especially earth-anchored bridges, have proliferated all over the world because of their strong span ability. A suspension bridge is composed of such major parts as towers, anchors, main cables, suspenders, steel box girder, and saddle seats. Loads on the bridge deck are transferred from suspenders to main cables, and then from main cables to towers and anchors. The transfer path is short and clear. The main cables are the ‘lifeline’ of a suspension bridge, which is a geometrically changeable system with geometrical nonlinearity. The dominant tower, which is usually composed of tower columns and cross beams, undergoes mainly axial compression under the dead load condition and press bending under the live load condition.

At present, most research on static and dynamic analyses has focused on two-tower long-span suspension bridges (Abdel-Ghaffar, 2000; Zhang *et al.*, 2002; Maceri and Vairo, 2003; Almutairi *et al.*, 2006; Karoumi, 2007; Wang *et al.*, 2009; Xu *et al.*, 2009) and self-anchored suspension bridges (Ochsendorf and Billington, 1999; Zhang and Feng, 2004; Romeijn *et al.*, 2008). The triple-tower suspension bridge is a brand new type of structural form. Unlike a two-tower suspension bridge, a triple-tower suspension bridge has not only two towers at the end of the two main spans, but also a dominant tower between

* Project supported by the National Natural Science Foundation of China (NSFC) (No. 50978056), the NSFC for Young Scholars (No. 50908046), and the PhD Programs Foundation of MOE of China (No. 200802861012)

the two main spans which is included to alleviate the strain on the main cables and on the anchors at the two ends of the bridge. The middle tower is a vertical pivot for supporting the main cables through the saddle seats. Compared with a two-tower suspension bridge, the structural characteristics of a triple-tower suspension bridge are strikingly different owing to the addition of a middle tower and an additional main span. Deng *et al*. (2008) carried out a linear seismic responsive analysis on a triple-tower suspension bridge adopting a time-history analysis method and a linear response spectrum method based on the SAP2000 platform. Yoshida *et al*. (2004) studied the parameters influencing the deformation characteristics of a four-span suspension bridge which had two main 2000 m spans. In general, however, analyses of triple-tower bridges are few and insufficient, especially analyses of their dynamic characteristics.

Natural structural dynamic characteristics such as frequencies and vibration modes form the basis of earthquake, traffic and wind resistance analysis of structures (Kazama *et al*., 1995; Ding and Lee, 1999; Cheng *et al*., 2003; Maceri and Vairo, 2003, 2004; Chen and Kareem, 2006; Chen, 2006; Wang *et al*., 2006; Bartoli and Mannini, 2008; Hua and Chen, 2008; Xia *et al*., 2008). Therefore, it is important to obtain accurate and reliable data on the natural dynamic characteristics of structures. Vibration on long-span bridges has been one of the key research issues in the field of bridge engineering (Abdel-Ghaffar, 2000).

In this study, the Taizhou Yangtze River Highway Bridge (the Taizhou Bridge for short), the first triple-tower long-span suspension bridge in China, is taken as an example. A 3D dynamic finite element model was established for the Taizhou Bridge using the nonlinear finite element software ABAQUS. Based on the model, modal analysis was performed to obtain the dynamic characteristics (including the natural frequency and mode shapes of lateral, vertical, torsional and longitudinal vibrations) of this long-span triple-tower suspension bridge using the LANCZOS eigenvalue solution method. The effects of important design parameters on the dynamic characteristics of this type of bridge were also studied. Our results have theoretical applications for analyzing and designing triple-tower suspension bridges.

2 Bridge description

The Taizhou Bridge spans in the Yangtze River in Jiangsu Province, between the Jiangyin Yangtze River Bridge and the Runyang Yangtze River Bridge. The Taizhou Bridge connects Taizhou City on the north side with Zhenjiang City and Changzhou City on the south side. The overall layout of the Taizhou Bridge is shown in Fig. 1a. Each of the two main

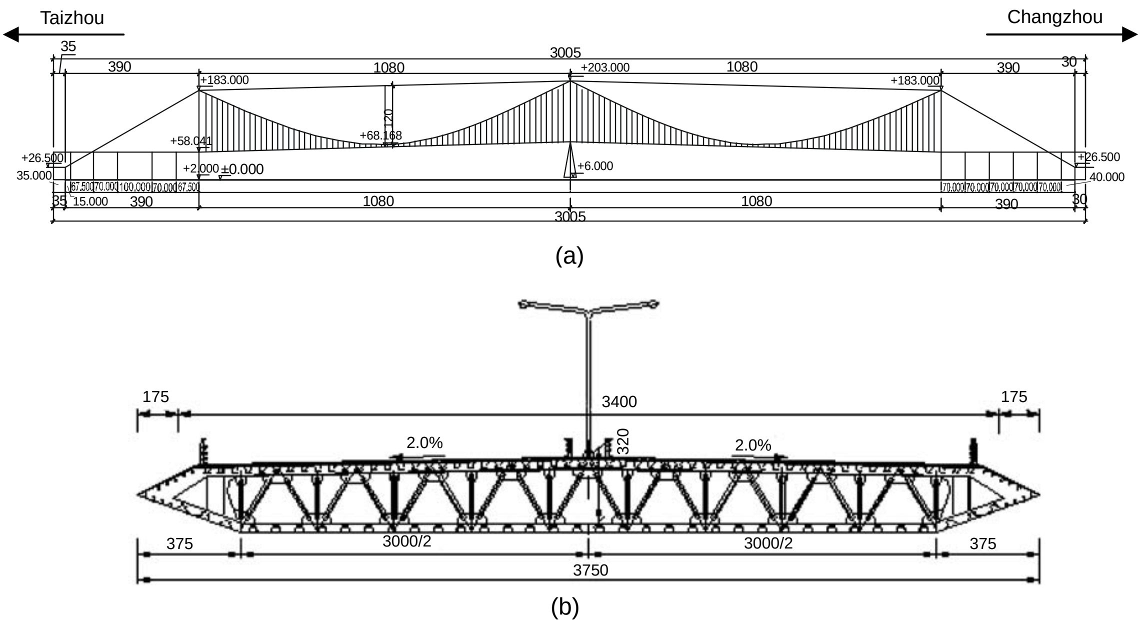

Fig. 1 The Taizhou Bridge. (a) Elevation (unit: m); (b) Cross-section of the steel box girder (unit: cm)

spans is 1080 m long and each of the two side spans is 390 m long. Therefore, the subspans of the bridge are 390 m+1080 m+1080 m+390 m. The sag-to-span length ratio is 1/9 and the distance between the two main cables is 34 m. The PPWS (pre-fabricated parallel wire strand) technique was used in the construction of the main cable. There are two suspenders at each suspension point and the distance between two adjacent suspension points is 16 m. A picture of the Taizhou Bridge is shown in Fig. 2.

Fig. 2 The Taizhou Triple-tower Suspension Bridge

Welded streamline flat steel box girders were used to form the main girder of the Taizhou Bridge (Fig. 1b). The length of a standard segment of the main girder is 16 m. The width of the main girder (including the wind mouth and road for inspection and repair) is 37.5 m and the height of the main girder in the middle is 3.2 m. Q345qD steel was mainly used in the main girder. A truss structure was adopted in the transverse clapboard for the purpose of decreasing the weight of dead loads and for ventilation inside the steel box girder. The distance between the transverse clapboards is 3.6 m.

The two end towers, the north and south towers, are reinforced concrete towers with an elevation of 180 m at the top, an elevation of 2 m at the base, and an overall height of 178 m. The middle tower is a non-uniform steel tower with an elevation of 200 m at the top, an elevation of 8 m at the base, and an overall height of 192 m. In the lateral direction it is a portal frame, and in the longitudinal direction is herringbone. Vertical bearings were not used on the lower cross beam of the middle tower, and lateral supports were used at the joints between the inner wall of the tower and the main girder to resist wind loads. Elastic restraints made of steel stranded wires were employed in the longitudinal direction. Vertical and lateral bearings were also included on the lower cross beams of the side towers.

This bridge uses a rigid central buckle fixed between the main cables and the steel box girder to replace the short suspenders at the midspan. The rigid central buckle is made up of a cable clamp connecting the main cable, one vertical component, two inclined components and the 18.4 m-length mid-span steel box girder (Fig. 3). The vertical and inclined components are welded into an ‘H’ shape using Q345D steel deck with 40 mm thickness, and are connected to the cable clamp by 396 10.9-grade M30-type high-strength bolts and to the mid-span steel box girder by the welds.

This bridge has a triple-tower suspended-cable structure, the first of its kind in China. To increase the longitudinal stiffness of the middle tower, it was

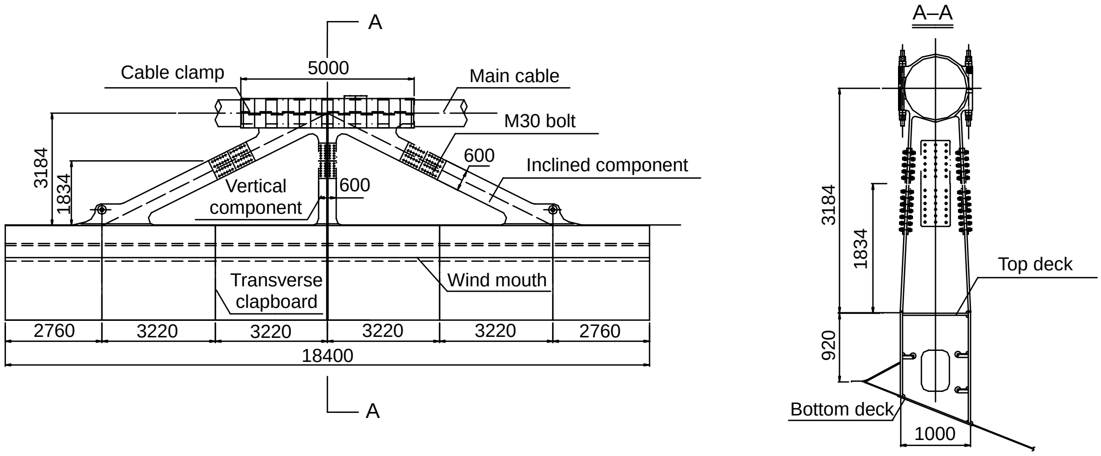

Fig. 3 Diagram of central buckle of the Taizhou Bridge (unit: mm)

designed to be herringbone in the longitudinal direction. Based on the design of the Taizhou Bridge, the dynamic characteristics of a triple-tower suspension bridge and the effects of various parameters were studied.

3 Finite element model of the bridge

A backbone model is generally used to simulate the bridge floor system when conducting dynamic analysis on long-span suspension bridges (Hua and Chen, 2008). As an initial theoretical model of the Taizhou Bridge, our model used the backbone model to simulate the steel box girder. The model was established based on the nonlinear finite element software ABAQUS (Fig. 4).

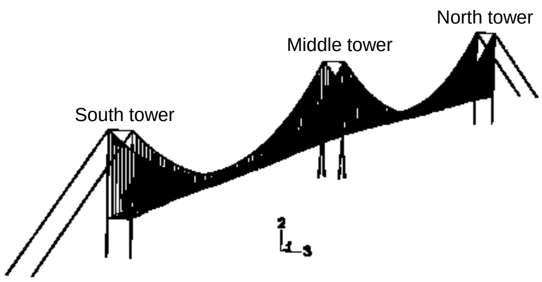

Fig. 4 Full finite element model

The model represents the bridge with the geometry and structural properties estimated from the design drawings. Four types of finite elements were used to model different structural members:

1. The steel girders and three towers are modeled by spatial beam elements (B31). B31 is a 3D 2-node linear Timoshenko beam element with six degrees of freedom (DOFs) at each node, and allows transverse shear deformations, biaxial bending and axial strain. A total of 884 beam elements were used. The stiffness and mass of the steel box girder were concentrated on the middle nodes. For the steel box girder the element stiffness was taken as its actual stiffness, while the element density was taken as the transformed density of the dead loads including the first and second phases of the bridge deck. A segment of steel box girder between two adjacent suspenders was divided into 4 elements and the general sectional characteristics were used to describe sectional properties of the girders. According to the length of the structures, the higher and lower tower columns in the model were divided into 46 elements in total, and the higher and lower cross beams of the middle tower and end towers were all divided into 12 elements.

2. The cross beams connecting the suspenders and the steel box girder, as well as the central buckles, were also modeled by B31. The difference is that the elastic modulus of these B31 elements was set so high that they are actually rigid arms.

3. T3D2, which is a 3D 2-node truss element with linear approximation of displacements, was adopted to simulate the suspenders and main cables, and the main cables were meshed at the suspending points. The T3D2 elements were assigned by the property of only tension and no compression based on the real condition.

4. The spring element SPRING2, which links a global DOF at one node with a global DOF at another node and can be nonlinear, was used to simulate the longitudinal elastic restraints between the lower cross beam of the middle tower and the steel box girder, the stiffness of which is 640 kN/mm.

For the purpose of taking into account the geometric stiffness of the main cables and suspenders under dead loads, initial stress was exerted on the main cables and the suspenders of the finite element model. The cables' nonlinearity was accounted for by an equivalent tangent formulation (Dischinger, 1949; Ernst, 1965), and the Ernst equivalent elastic modulus was adopted. The concept of equivalent elastic modulus was first proposed by Ernst (1965), and the formula is

$$E_{\mathrm{eq}}=\frac{E}{1+(w_0 l)^2 AE/(12T^2)}, \tag{1}$$

where E is the elastic modulus of the cables, A is the area of their cross-sections, w_0 is the cables' weight per unit length, l is the horizontal distance between the two ends of the cable, and T is the tension force on the cable.

The geometrically nonlinear control option should be turned on when conducting static analysis under dead loads, and the modal analysis (linear perturbation) was carried out using the nonlinear analysis results. Fig. 5 shows a part of the finite element model corresponding to the steel box girder and the middle tower of the Taizhou Bridge.

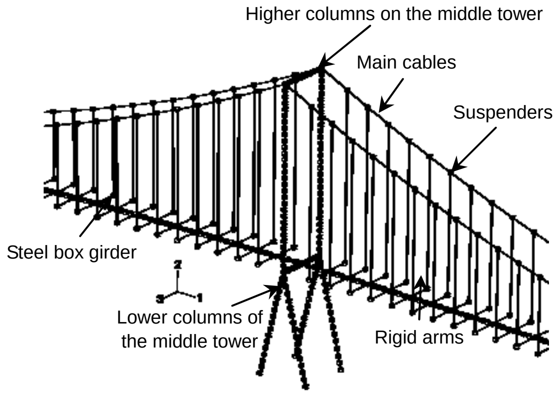

Fig. 5 Part of the finite element model

The DOF of the lateral displacement, vertical displacement and the rotation around the longitudinal axis between the side towers and the steel box girder were coupled (kinematic coupling). As for the middle tower, only the lateral DOF was coupled with the steel box girder. The bottom of the middle tower, side towers and the piers of the approach bridge were entirely fixed on the earth, ignoring the influence of soil-pile-structure interaction. The ends of the two side main cables were also fixed and all of the DOF of the side cables' ends in the model were restrained.

4 Parameter study on the dynamic characteristics of the Taizhou Bridge

4.1 Dynamic characteristics of the bridge

Comparative analysis showed that whether the eigenvalue solver SUBSPACE or LANCZOS is used, the calculation results of the dynamic characteristics of the bridge are barely affected. Therefore, the LANCZOS eigenvalue solver was adopted for dynamic characteristic analysis with the aim of increasing the calculation speed.

The first 50 natural frequencies and vibration modes were calculated during the analysis, and the first 30 vibration modes were classified according to the characteristics of the vibration modes (Tables 1 and 2). Figs. 6 and 7 show the first four vertical and lateral vibration modes of the bridge, respectively. The longitudinal floating vibration mode of the bridge is not obvious due to the great stiffness of the elastic restraints employed between the steel box girder and lower cross beams at the middle tower.

The first natural frequency of the bridge is 0.07755 Hz, and the natural period is about 12.90 s (Tables 1 and 2) corresponding to the first antisymmetric lateral bending mode of the main cables and the steel box girder. The second frequency of the bridge is 0.08267 Hz and the period is about 12.10 s corresponding to the first antisymmetric vertical bending mode of the main cables and the steel box girder. The first vibration mode of two-tower suspension bridges is usually symmetric lateral bending of the main girder. The first antisymmetric lateral bending of this triple-tower suspension bridge within each of its main spans resembles the first symmetric lateral bending of a two-tower suspension bridge, which shows the similarity between the characteristics of the two types of suspension bridge. The first torsional mode of the bridge is the torsional vibration of the steel box girder with a frequency of 0.22969 Hz and a natural period of about 4.35 s, which is accompanied by the lateral vibration of the main cables.

The modal coupling phenomenon between the deck, the main cable and the tower of the triple-tower suspension bridges can be easily found in Tables 1 and 2, and is the same as that of two-tower suspension bridges (Abdel-Ghaffar, 2000). Considering the significance of torsional modes on the wind-resistance performance of long-span suspension bridges, the lateral and torsional mode coupling phenomenon in triple-tower suspension bridges deserves special attention.

We made a simplified theoretical prediction about the first four frequencies of the bridge based on some widely used simplified frequency estimation equations (Li, 1996). We list the prediction results in Table 3. Our calculation results agree quite well with the prediction results, indicating that we have not made any serious mistakes during the calculation. We also compare the first 10 frequencies with corresponding data from the Construction Authority of the Yangtze River Highway Bridges of Jiangsu Province (Deng *et al.*, 2008) and provide the relative errors (Table 3). This comparison also shows good agreement.

4.2 Effect of the stiffness of the steel box girder

The effects of key design parameter values on the dynamic characteristics of the triple-tower suspension bridge were studied. When the vertical stiffness multiple of the steel box girder changes from 1.0 to 2.0, the frequencies of the first lateral and

Table 1 Characteristics of lateral modes (some coupling with the torsional modes)

Order number	Frequency (Hz)	Characteristics of modes
1	0.07755	Antisymmetric lateral vibration of the main cables and the steel box girder (a half wave)
2	0.10170	Symmetric lateral vibration of the main cables and the steel box girder (a half wave)
3	0.22969	Lateral vibration of the main cables accompanied by torsion of the steel box girder
4	0.23157	Symmetric lateral vibration of the main cables accompanied by lateral bending of the middle tower
5	0.25524	Antisymmetric lateral vibration of the main cables
6	0.28501	Symmetric lateral vibration of the main cables between the central buckle and the middle tower accompanied by torsion of the steel box girder
7	0.29455	Symmetric lateral vibration of the main cables between the central buckle and the middle tower
8	0.29457	Symmetric lateral vibration of the main cables between the central buckle and the middle tower
9	0.29600	Antisymmetric lateral vibration of the main cables accompanied by lateral vibration of the side towers
10	0.30052	Symmetric longitudinal vibration of the main cables between the north tower and the central buckle
11	0.30052	Symmetric longitudinal vibration of the main cables between the south tower and the central buckle
12	0.30430	Symmetric lateral vibration of the main cables accompanied by torsion of the steel box girder
13	0.31585	Antisymmetric lateral vibration of the main cables between the central buckle and the middle tower
14	0.32915	Symmetric lateral vibration of the main cables between the central buckle and the middle tower
15	0.33612	Antisymmetric lateral vibration of the main cables between the central buckle and the side towers
16	0.33665	Symmetric lateral vibration of the main cables between the central buckle and the side towers
17	0.36936	Symmetric lateral vibration of the main cables accompanied by torsion of the steel box girder
18	0.37313	Symmetric lateral vibration of the main cables between the central buckle and the middle tower accompanied by lateral vibration of the middle tower

Table 2 Characteristics of vertical modes

Order number	Frequency (Hz)	Characteristics of modes
1	0.08267	Antisymmetric vertical vibration of the main cables and the steel box girder (a half wave)
2	0.14607	Symmetric vertical vibration of the main cables and the steel box girder (a wave)
3	0.15015	Symmetric vertical vibration of the main cables and the steel box girder between the side towers and central buckle (a half wave)
4	0.16771	Antisymmetric vertical vibration of the main cables and the steel box girder (a half wave)
5	0.17050	Symmetric vertical vibration of the main cables and the steel box girder (a half wave)
6	0.22491	Symmetric vertical vibration of the main cables and the steel box girder (a half wave)
7	0.22834	Antisymmetric vertical vibration of the main cables and steel box girder within two spans (two waves)
8	0.24840	Symmetric vertical vibration of the main cables and the steel box girder (a half wave)
9	0.29297	Antisymmetric vertical vibration of the main cables and the steel box girder (two waves)
10	0.30201	Symmetric vertical vibration of the main cables and the steel box girder (two half waves)
11	0.35736	Symmetric vertical vibration of the main cables and the steel box girder (two half waves)
12	0.36045	Antisymmetric vertical vibration of the main cables and the steel box girder (two half waves)

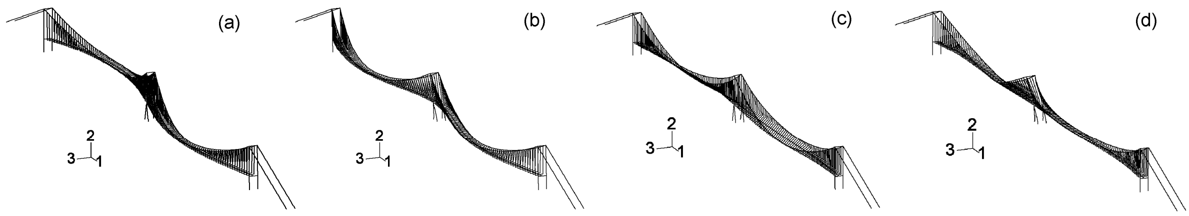

Fig. 6 The first 4 lateral modes. (a) First; (b) Second; (c) Third; (d) Fourth

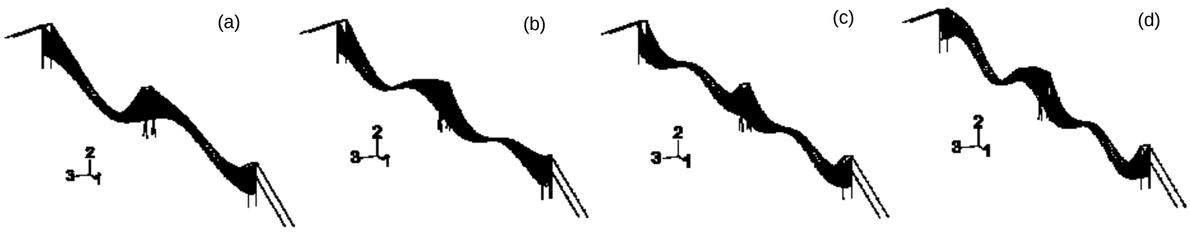

Fig. 7 The first 4 vertical modes. (a) First; (b) Second; (c) Third; (d) Fourth

Table 3 Comparison of the first 10 frequencies with official data*

Order number	Theoretical prediction frequencies (Hz)	Frequencies (Hz)	Official frequencies (Hz)	Relative error compared to official frequencies (%)
1	0.07176	0.07755	0.07742	0.17
2	0.08679	0.08267	0.08055	2.63
3	0.11324	0.10170	0.09467	7.43
4	0.15183	0.14607	0.11776	24.04
5	–	0.15015	0.12032	24.79
6	–	0.16771	0.15401	8.90
7	–	0.17050	0.16898	0.90
8	–	0.22491	0.19516	15.24
9	–	0.22834	0.19619	16.39
10	–	0.22969	0.22168	3.61

* Official data comes from the Construction Authority of the Yangtze River Highway Bridges of Jiangsu Province (Deng *et al*., 2008)

first vertical bending also change (Fig. 8).

Because the proportion of gravity stiffness in the vertical stiffness of the full bridge is greater than 80% (Fig. 8), the increase in vertical stiffness has little effect on the first symmetric and antisymmetric lateral bending frequencies, while the first symmetric and antisymmetric vertical bending frequencies increase a little. The first antisymmetric vertical bending frequency increases from 0.08267 Hz to 0.08372 Hz and the first symmetric vertical bending frequency increases from 0.14607 Hz to 0.14810 Hz, which is only about 1.27% and 1.39%, respectively.

Fig. 9 shows the changes in the first lateral and first vertical bending frequencies when the lateral stiffness multiple of the steel box girder changes from 1.0 to 2.0.

The increase in the lateral stiffness barely affects the first vertical bending frequency, while the first lateral bending frequency increases significantly, by about 26.7%. The first antisymmetric and symmetric lateral bending frequencies increase from 0.07755 Hz to 0.09734 Hz and from 0.10170 Hz to 0.12892 Hz, respectively.

Fig. 10 shows the changes in the first lateral, vertical and torsional bending frequencies when the torsional stiffness multiple of the steel box girder changes from 1.0 to 2.0.

The increase in the torsional stiffness affects neither the first symmetric lateral and vertical bending frequencies, nor the antisymmetric lateral and vertical frequencies. However, its influence on the first torsional frequency is notable, increasing by about 9.7% from 0.22969 Hz to 0.25197 Hz. Obviously, the influence of the torsional stiffness of the steel box girder on the dynamic characteristics is significant. Also, to determine how stiff the restraint

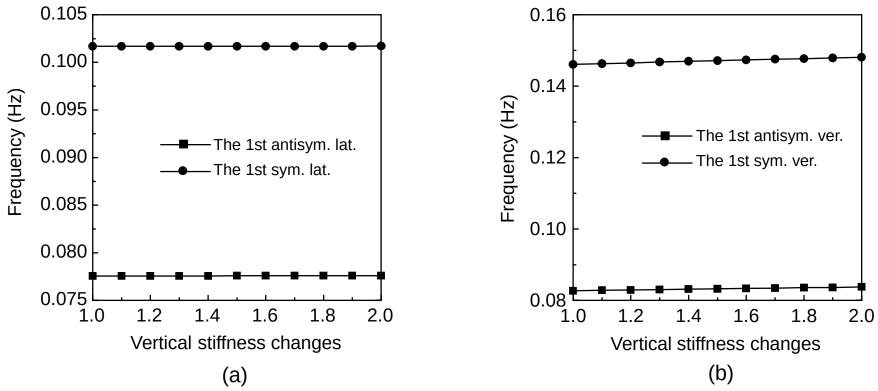

Fig. 8 The influence of vertical stiffness of the steel box girder on dynamic characteristics. (a) Lateral frequencies; (b) Vertical frequencies

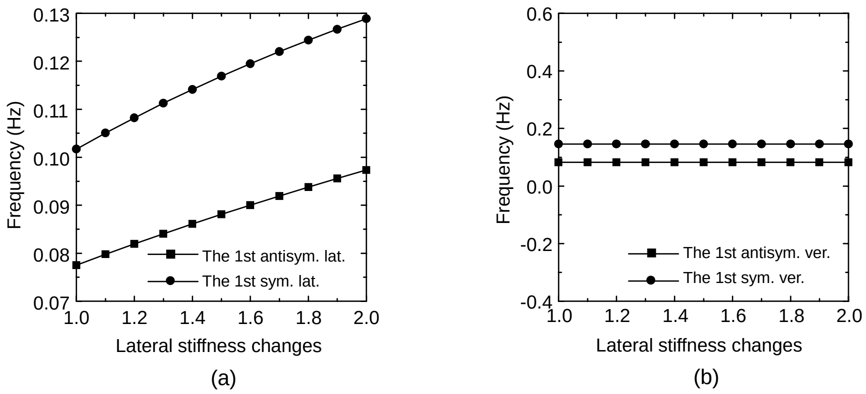

Fig. 9 The influence of lateral stiffness of the steel box girder on dynamic characteristics. (a) Lateral frequencies; (b) Vertical frequencies

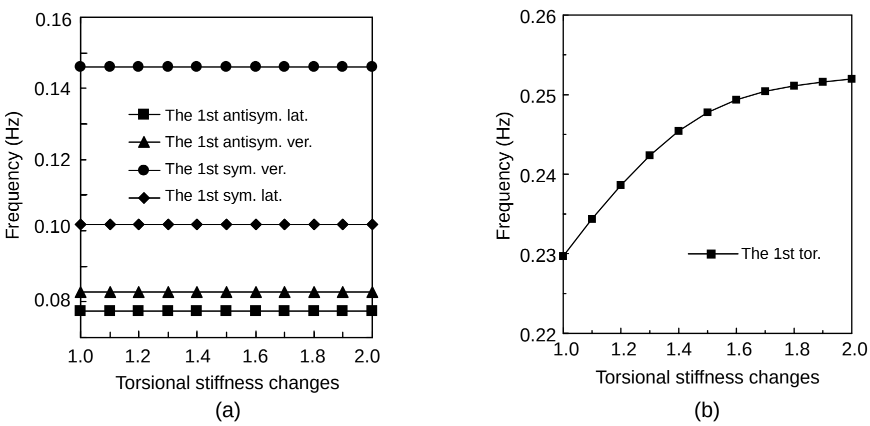

Fig. 10 The influence of torsional stiffness of the steel box girder on dynamic characteristics. (a) Lateral and vertical frequencies; (b) Torsional frequencies

should be in relation to the dynamic behavior of the bridge, we examined the changes in the natural frequencies of the bridge as the torsional stiffness changed from 0 to 1.0. Fig. 11 shows that when the multiple of the torsional stiffness changes from 0.0 to 1.0, the larger the torsional stiffness, the larger the vertical and lateral frequencies become, which is especially apparent for the 1st symmetrical vertical frequency and the 1st antisymmetrical lateral frequency. But when the multiple approaches 1.0, the increase in the frequency slows down, and the curve becomes very flat. Therefore, the design torsional stiffness is appropriate in terms of the efficiency of increasing the vertical and lateral frequencies.

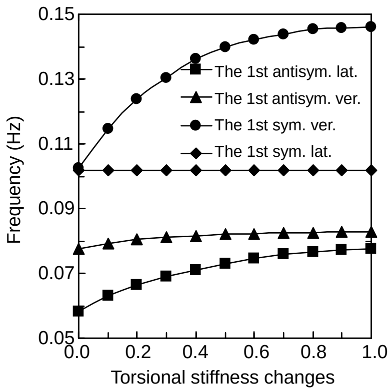

Fig. 11 The changes dynamic characteristics when torsional stiffness changes from 0 to 1

All the vibration modes addressed in Fig. 8a, Fig. 9b and Fig. 10a are practically uncoupled with any vibration modes. Note that all these vibration modes are the basic frequencies in different directions, which is not surprising because basic frequencies of two-tower suspension bridges seldom couple with any other vibration modes, and coupling of higher vibration modes occurs more easily (Lin *et al.*, 2007; Sun and Liu, 2007).

The influence of changes in stiffness of the steel box girder on the dynamic characteristics of the long-span triple-tower suspension bridge can be summarized as follows:

1. Because gravity stiffness forms a large part of the vertical stiffness of the full bridge, the vertical stiffness of the steel box girder has little influence on the dynamic characteristics of triple-tower suspension bridges.

2. The increase in the lateral stiffness of the steel box girder has little influence on the vertical and torsional bending frequencies, but has a significant effect on the lateral bending frequencies.

3. The increase in torsional stiffness has little influence on the lateral and vertical bending frequencies, but has a significant effect on the first torsional frequency.

4.3 Effect of the longitudinal elastic restraints of the steel box girder

When the stiffness multiple of longitudinal elastic restraints of the steel box girder changes from 1.0 to 2.0, our calculations show that the frequencies of the triple-tower suspension bridge barely change. Therefore, the stiffness of the longitudinal elastic restraints originally designed for the bridge is sufficient, and a further increase in their stiffness would have little effect.

The first 10 modes of the triple-tower suspension bridge with or without the longitudinal elastic restraints are listed in Table 4. The elastic restraints between the lower cross beam of the middle tower and the steel box girder have a significant effect on the vertical modes of the bridge. The fundamental frequency without the elastic restraints reduces by about 24.7%. The first vibration mode without the elastic restraints corresponds to an antisymmetric vertical vibration mode of the main cables and the steel box girder. The 1st longitudinal floating vibration mode is shown in Fig. 12. The frequencies of the bridge increase with the elastic restraints, which suggests that the adoption of the elastic restraints enhances the stiffness of the bridge.

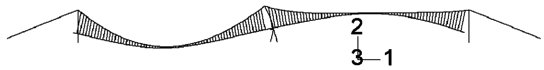

Fig. 12 Longitudinal floating mode

4.4 Effect of the central buckle and elastic restraints

To solve the problem of bending and breaking of the short suspenders at midspan and to repress displacement responses of the main girder, rigid central buckles were adopted in the Taizhou Bridge. These make a rigid connection between the main cables and the main girder and substitute for the short suspenders at midspan. Since this is the first time that a central buckle has been used in a triple-tower suspension bridge, it is important to study the effect of the central buckle on the dynamic characteristics of the bridge.

Our results show that employing the rigid central buckle and adopting elastic restraints between the steel box girder and the lower cross beam of the middle tower in the longitudinal direction of the bridge can effectively control the longitudinal floating of the steel box girder. Table 5 shows the effect of the central buckle and the elastic restraints on the first 4 bending vibration modes of the bridge.

The elastic restraints have a greater effect on the dynamic characteristics of the triple-tower suspension bridge than does the central buckle. This is mainly because, unlike elastic restraints with proper stiffness

Table 4 The influence of elastic restraints on dynamic characteristics of the bridge

Frequency (Hz)	Characteristics of the modes (with elastic restraints)	Frequency (Hz)	Characteristics of the modes (without elastic restraints)
0.07755	Antisymmetric lateral vibration of the main cables and the steel box girder (half wave)	0.05836	Antisymmetric vertical vibration of the main cables and the steel box girder (a half wave)+longitudinal floating
0.08267	Antisymmetric vertical vibration of the main cables and the steel box girder (half wave)	0.07755	Antisymmetric lateral vibration of the main cables and the steel box girder (a half wave)
0.10170	Symmetric lateral vibration of the main cables and the steel box girder (half wave)	0.10170	Symmetric lateral vibration of the main cables and the steel box girder (a half wave)
0.14607	Antisymmetric vertical vibration of the main cables and the steel box girder (a wave)	0.10235	Antisymmetric vertical vibration of the main cables and the steel box girder (a wave) +longitudinal floating
0.15015	Symmetric vertical vibration of the main cables and the steel box girder (a half wave)	0.15015	Symmetric vertical vibration of the main cables and the steel box girder (a half wave)
0.16771	Antisymmetric vertical vibration of the main cables and the steel box girder (a half wave)	0.16433	Antisymmetric vertical vibration of the main cables and the steel box girder (a half wave)
0.17050	Symmetric vertical vibration of the main cables and the steel box girder between the side towers and central buckle (half wave)	0.17050	Symmetric vertical vibration of the main cables and the steel box girder between the side towers and central buckle (a half wave)
0.22491	Symmetric vertical vibration of the main cables and the steel box girder (a half wave)	0.22482	Antisymmetric vertical vibration of the main cables and the steel box girder (two waves)
0.22834	Antisymmetric vertical vibration of the main cables and the steel box girder within two spans (two waves)	0.22491	Symmetric lateral vibration of the main cables and the steel box girder (a half wave)
0.22969	Lateral vibration of the main cables accompanied by torsion of the steel box girder	0.22969	Lateral vibration of the main cables accompanied by torsion of the steel box girder

Table 5 The influence of the central buckle and elastic restraints on the dynamic characteristics

Order number	Frequency (Hz)			Characteristics of modes
	Central buckle+ elastic connections	Central buckle	Elastic restraints	
1	0.07755	0.05836	0.06945	The 1st antisymmetric lateral vibration
2	0.08267	0.07755	0.07706	The 1st antisymmetric vertical vibration
3	0.10170	0.10170	0.10089	The 1st symmetric lateral vibration
4	0.14607	0.10235	0.11449	The 1st symmetric vertical vibration

values, flexible main cables cannot effectively control the longitudinal floating of the steel box girder through the central buckle. But the central buckle can effectively control the vibration of the main cables and plays an essential role in preventing the relative longitudinal displacement between the steel box girder and the main cables becoming too large. Also, compared with traditional two-tower suspension bridges (Lin *et al*., 2007; Sun and Liu, 2007; Hua and Chen, 2008), the effect of the rigid central buckle on the dynamic characteristics of triple-tower suspension bridges is greater owing to the effect of the middle tower. Thus, much attention should be given to the middle tower in the design of a triple-tower suspension bridge.

5 Conclusion

The following conclusions can be drawn from this parameter study on the dynamic characteristics of a triple-tower suspension bridge:

1. The first natural frequency of the Taizhou Bridge is 0.07755 Hz which corresponds to the first antisymmetric lateral bending mode of the main cables and the steel box girder. The second frequency of the bridge is 0.08267 Hz, corresponding to the first antisymmetric vertical bending mode of the main cables and the steel box girder. These frequencies are very similar to the dynamic characteristics of two-tower suspension bridges.

2. The modal coupling phenomenon between the

deck and the main cable of the triple-tower suspension bridge can be found easily, as is the case with two-tower suspension bridges. The coupling vibration phenomenon of the lateral and the torsional vibration modes deserves special attention during wind-resistance analyses of long-span triple-tower suspension bridges.

3. An increase in the vertical stiffness of the steel box girder has little effect on the first lateral and vertical bending frequencies. An increase in the lateral stiffness of the steel box girder has little effect on the first vertical bending frequency, but has a significant effect on the lateral bending frequencies. An increase in torsional stiffness has little effect on the first lateral and vertical bending frequencies, symmetric or anti-symmetric, but has a significant effect on the first torsional frequency.

4. Employing the elastic restraints between the lower cross beam of the middle tower and the steel box girder has a significant effect on the vertical modes of the triple-tower suspension bridge: the fundamental frequency reduces by about 24.7% without the elastic restraints. The first vibration mode is an antisymmetric vertical bending of the main cables and the steel box girder, and longitudinal floating vibration modes appear when there are no elastic restraints. Our analysis shows that the designed stiffness of the elastic restraint of the Taizhou Bridge is sufficient, and increasing the stiffness has no effect on the vibration frequency of the bridge.

5. The effect of the elastic restraints over the dynamic characteristics of the triple-tower suspension bridge is more significant than that of the central buckle. However, the central buckle not only effectively controls the vibration of the main cables, but also prevents relative longitudinal displacement between the steel box girder and the main cables from becoming too large. Moreover, the effect of the rigid central buckle on the dynamic characteristics of triple-tower suspension bridges is larger than on those of traditional two-tower suspension bridges, and therefore deserves special attention in the design of triple-tower suspension bridges.

As this was an initial finite element model of the Taizhou Suspension Bridge, many complicating factors such as the temperature and the soil-pile-structure interaction were not considered. Therefore, in the future, our finite element model needs to be updated and validated by field measurements.

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