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Quantitative measures for assessment of the hydraulic excavator digging efficiency*

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Abstract: In this paper, quantitative measures for the assessment of the hydraulic excavator digging efficiency are proposed and developed. The following factors are considered: (a) boundary digging forces allowed for by the stability of an excavator, (b) boundary digging forces enabled by the driving mechanisms of the excavator, (c) factors taking into consideration the digging position in the working range of an excavator, and (d) sign and direction of potential digging resistive force. A corrected digging force is defined and a mathematical model of kinematic chain and drive mechanisms of a five-member excavator configuration was developed comprising: an undercarriage, a rotational platform and an attachment with boom, stick, and bucket. On the basis of the mathematical model of the excavator, software was developed for computation and detailed analysis of the digging forces in the entire workspace of the excavator. By using the developed software, the analysis of boundary digging forces is conducted and the corrected digging force is determined for two models of hydraulic excavators of the same mass (around 17000 kg) with identical kinematic chain parameters but with different parameters of manipulator driving mechanisms. The results of the analysis show that the proposed set of quantitative measures can be used for assessment of the digging efficiency of existing excavator models and to serve as an optimization criterion in the synthesis of manipulator driving mechanisms of new excavator models.

Key words: Hydraulic excavators, Digging efficiency, Quantitative measures
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1 Introduction

The hydraulic excavators are popular multifunctional construction and mining machines. The main function of the hydraulic excavators of all types and sizes is the cyclic digging and transfer of soil. This function is achieved by use of an open kinematic chain consisting of undercarriage L_1, an upper structure L_2 and a front attachment with boom L_3, stick L_4 and work tool L_5 (Fig. 1). For digging operations below the ground level, the toward oneself technology (in relation to the excavator operator) is employed and a backhoe attachment is used (Fig. 1a). For digging operations above the ground level the away from oneself technology and a shovel attachment are used (Fig. 1b).

During digging operations, the occurring digging resistive force acting against the manipulator tool is overcome by digging forces. The digging forces of an excavator are exerted by the kinematic chain of the excavator, which is powered by the following driving mechanisms: (1) hydraulic motors for motion of swing and travelling bodies; (2) double acting hydraulic cylinders for powering of the manipulator links.

For an optimal design of the front manipulator and driving mechanisms, it is necessary to perform a detailed analysis of the digging forces and the digging resistive forces in the entire workspace of the excavator. The conducted analytical and experimental

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research that points out the importance of knowledge about the digging resistive forces for development and analysis of hydraulic excavators is related to: (1) analytical modeling and experimental determination of the value and the change in the digging resistive force during the excavation process (Maciejewski and Jarzebowski, 2002; Maciejewski *et al*., 2003; Yang *et al*., 2008); (2) development of mathematical models for kinematic and dynamic analysis of excavators (Budny *et al.*, 2003; Towarek, 2003; Hall and McAree, 2005; Gu *et al*., 2007); (3) development of control systems for automation of the digging process (Plonecki *et al*., 1998; Ha *et al*., 2000; Chang and Lee, 2002; Lee and Chang, 2002; Flores *et al*., 2007; Lin *et al*., 2008); and, (4) definition of quantitative measures for analysis and assessment of excavator digging efficiency in the workspace.

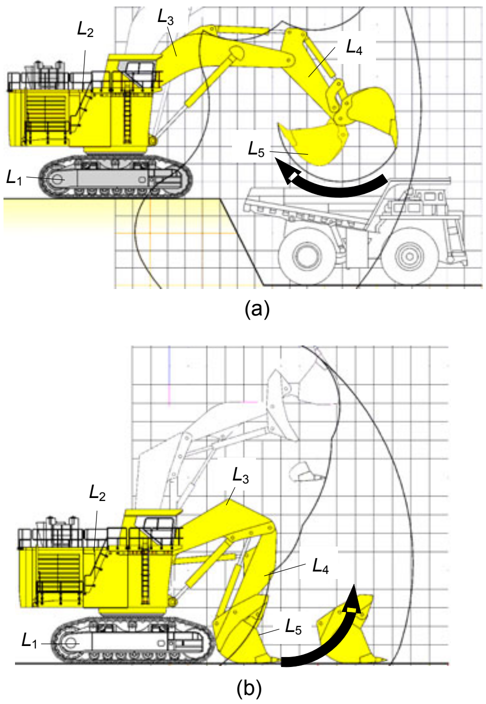

Fig. 1 Hydraulic excavators with backhoe (a) and shovel attachment (b)

As indicated by the manufacturer, stick and bucket digging forces are important parameters of the excavator. They are defined by appropriate standards (ISO 6015, 2006; SAE J1179, 2008) as one of the characteristics of the excavator digging function. For robotic manipulators, quantitative measures of workspace attributes are defined, which includes structural length index and manipulability measure (Craig, 2005). Some of these measures can be used for the assessment of the dynamic performance of shovels (Lipsett, 2009). In many cases, the above mentioned measures are not sufficient for assessment of digging possibilities and efficiency of the excavator in the workspace.

In this paper, a set of quantitative measures for assessment of the hydraulic excavator digging efficiency is proposed and developed.

For a comprehensive analysis of values and directions of digging forces in a specific position of the front manipulator, a hodograph of boundary digging forces at the top of the bucket tooth is defined. It has a form of a polygon whose sides are composed by vectors of boundary forces exerted by the manipulator driving mechanisms and vectors of boundary forces allowed by the stability of the excavator. The ratio of the computed digging force and the potential digging force is defined as a measure of the digging efficiency. The computed digging force is determined by a mathematical model of the excavator. Computations are performed for a given position and orientation of the manipulator and pressures in the hydraulic cylinders of driving mechanisms. The potential digging force represents the minimum value of boundary digging forces (Dudczak, 1977).

A hodograph of the effective digging forces is defined as a part of the hodograph of the boundary digging forces, for which the dot product of the digging velocity and digging force vector is positive. The ratio of the area and range of the effective digging forces hodograph is accepted as a criterion for excavator digging efficiency, whereas the area of the effective hodograph represents the mean value of the allowed digging forces. The range of the hodograph of the effective digging forces reflects the degree of compatibility of manipulator actuator parameters, excavator weight distribution, and digging force distribution (Budny, 1989).

A corrected digging force is set as a measure of digging efficiency in the entire workspace. The following considerations are taken into account: boundary digging forces allowed by the stability of an excavator; boundary forces exerted by the driving mechanisms of an excavator; and factors which relate to the digging position of the manipulator in the workspace as well as the sign and direction of the potential digging resistance action (Janosevic, 1997).

In this paper, mathematical models of the excavator kinematic chain, driving mechanisms and boundary digging forces are proposed and developed.

2 Mathematical modeling of excavator

A side view of the excavator is shown in Fig. 2. The mathematical model of the excavator consists of the mathematical model of the excavator kinematic chain and that of the manipulator driving mechanisms.

2.1 Model of the excavator kinematic chain

The excavator represents a five-link open kinematic chain which consists of the travelling body L_1, the swing body (rotational platform) L_2 and the three-link front manipulator consisting of a boom L_3, a stick L_4, and a bucket L_5 (Fig. 2).

The links of the front attachment kinematic chain are connected by kinematic pairs of the fifth class–rotational joints with a 1 DOF with different orientations in the space. The kinematic chain of the front manipulator which is a part of the excavator model is planar. The centers of the manipulator joints O_i (i=3, 4, 5) are penetration points of joint axis through the plane of symmetry of manipulator chain links.

Joint axes O_i (i=3, 4, 5) are parallel to each other, and the centers of joints lie in the same plane–the plane of the manipulator. The penetration of the cutting edge of the bucket through the plane of the manipulator represents the center of the cutting edge of the bucket O_{w}.

The adopted assumptions of the mathematical model of the excavator kinematic chain are: (1) The links of the excavator kinematic chain are modeled as rigid bodies; (2) The first joint between the travel body and the ground is a rotational joint O_{11} (or O_{12}), whose axes represent the lines along which the excavator could tip over; (3) During the working cycle, the excavator is stable and there is no motion in the first joint; (4) During the working cycle, the following forces act on the excavator: digging resistive force W, weight of kinematic chain and driving system links, weight of soil in the bucket; (5) During the digging operation, the kinematic chain of the excavator is considered as an open kinematic chain with a last link (bucket) that is subjected to the digging resistive force W (Janosevic, 1997).

To describe mathematically the kinematic model of the excavator, the following coordinate systems have been assigned:

(1) Global reference frame $OXYZ$ with unit vectors $\boldsymbol{i}$, $\boldsymbol{j}$, $\boldsymbol{k}$ along the coordinate axes OX, OY, and OZ, respectively. The support base lies in the horizontal plane OXZ and the vertical axis OY coincides with the axis of the rotational joint between the undercarriage and upperstructure O_2.

(2) Body fixed coordinate systems $O_ix_iy_iz_i$ (i=1, 2, 3, 4, 5), which are connected to each link L_i of the kinematic chain. The coordinate systems beginning O_i is situated in the center of joint by which the chain member L_i is connected to the previous member L_{i-1}. The bucket is connected to a coordinate system whose axis O_5x_5 passes through the center of the joint O_5 and the top of the cutting edge of the bucket O_{w}.

In the case of the stationary undercarriage, the coordinate system $O_1x_1y_1z_1$ coincides with the global coordinate system.

The geometry of the kinematic chain links L_i is defined in its local coordinate system $O_ix_iy_iz_i$ by a set of geometrical parameters:

$$L_i = \{e_i, s_i, t_i, m_i\}, \tag{1}$$

where $\widehat{\boldsymbol{e}}_i = \{e_{ix}, e_{iy}, e_{iz}\}$ is a unit vector (ort) of the axes of joint O_i used to connect links L_i and L_{i-1}; $\boldsymbol{s}_i$=$\{s_{ix}, s_{iy}, s_{is}\}$ is the vector of positions of the center of joint O_{i+1} used to connect links L_i and L_{i+1}, whereas the magnitude of vector $\boldsymbol{s}_i$ represents the kinematic length of member L_i; and $\boldsymbol{t}_i$=$\{t_{ix}, t_{iy}, t_{iz}\}$ is the vector of position of the center of mass m_i of link L_i.

To define the digging forces, the planar position of the kinematic chain configuration of an excavator is observed when potential (widthwise x-x or lengthwise z-z) lines along which an excavator can tip over are parallel to the axes of manipulator joints, where it is assumed that the centres of joints and masses of all members of the kinematic chain of an excavator lie in the vertical plane OXY of the absolute coordinate system of the model.

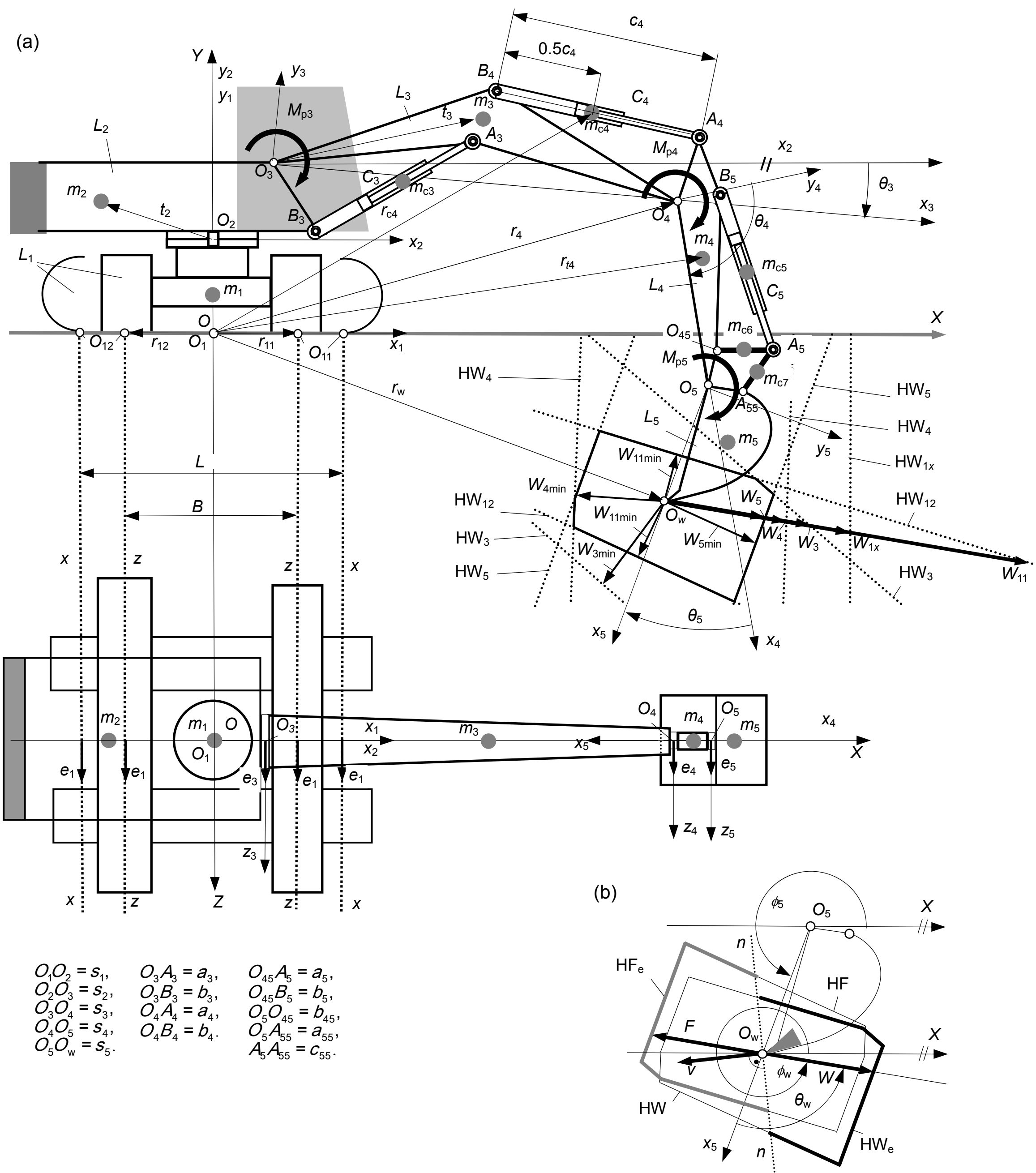

Fig. 2 Side view of a hydraulic excavator with a backhoe attachment
(a) Boundary digging resistive forces; (b) Hodographs of potential and effective digging resistive forces

2.2 Mathematical model of the manipulator driving mechanisms

The functions of the excavator are performed by the kinematic chain links powered by driving mechanisms. The front manipulator driving mechanisms are powered by double action hydraulic cylinders connected directly or through a lever system to the links L_{i-1} and L_i.

Hydraulic cylinders of the driving mechanisms of the boom C_3 and the stick C_4 are directly connected to the links. Cylinder of the driving mechanism of the bucket C_5 is connected to the bucket through a four-bar mechanism.

In the mathematical model of manipulator driving mechanisms, the following assumptions are made: (1) the position of the center of mass of cylinders is located in the middle of the current length of the cylinder; (2) the masses of the joint elements and transmission levers of driving mechanisms are added to the links of manipulator kinematic chain; (3) friction in the driving mechanisms joints, friction in the cylinders and the influence of pressure in return pipes of cylinders are neglected.

The driving mechanism C_i of the manipulator is determined by the set of parameters:

$$C_i = C_{ci} \cup C_{pi}, \quad \forall\ i = 3,4,5, \tag{2}$$

where C_{ci} is denoted as the set of parameters of the hydraulic cylinders, and C_{pi} is the set of transmission parameters of the driving mechanism.

The parameters of the cylinders are determined by the following set:

$$C_{ci} = \left\{ d_{i1}, d_{i2}, c_{ip}, c_{ik}, m_{ci}, n_{ci} \right\}, \quad \forall\ i = 3,4,5, \tag{3}$$

where d_{i1} and d_{i2} are the piston and rod diameters, respectively; c_{ip} is the minimum length of the cylinder with the connecting rod completely pulled in; c_{ik} is the maximum length of a hydro cylinder with the connecting rod completely pulled out; m_{ci} is the mass of cylinder; and n_{ci} is the number of cylinders in the driving mechanism.

The set of transmission parameters of the driving mechanisms of the boom C_3 and stick C_4 are determined by the set:

$$C_{pi} = \{\boldsymbol{a}_i, \boldsymbol{b}_i\}, \quad \forall i = 3,4, \tag{4}$$

where $\boldsymbol{a}_i = \{a_{ix}, a_{iy}\}$, $\boldsymbol{b}_i = \{b_{ix}, b_{iy}\}$ are the coordinates in the local coordinate systems of the position of the center of joints in which hydraulic cylinders are connected to the links of the driving mechanism (Fig. 2).

The subset of transmission parameters of the driving mechanism of the bucket C_5 is determined by the set:

$$C_{p5} = \{ a_5, \boldsymbol{b}_5, c_{55}, \boldsymbol{a}_{55}, \boldsymbol{b}_{45}, m_{c6}, m_{c7} \}, \tag{5}$$

where a_5 is the length of the lever of the bucket cylinder in the transmission part of the driving mechanism (Fig. 2); $\boldsymbol{b}_5 = \{b_{5x}, b_{5y}\}$ is the coordinates of the position of the center of joints in which a bucket hydro cylinder is connected to the manipulator stick; c_{55} is the length of the link in the transmission part of the driving mechanism; $\boldsymbol{a}_{55}=\{a_{55x}, a_{55y}\}$ and $\boldsymbol{b}_{45}=\{b_{45x}, b_{45y}\}$ are coordinates of the position of the center of joints in which transmission levers are connected to the links; m_{c6} is the mass of the lever of the bucket cylinder in the transmission part of the driving mechanism; and m_{c7} is the mass of the link in the transmission part of the driving mechanism.

2.3 Geometrical quantities of the model

The generalized coordinates of the excavator kinematic chain are represented by the relative angles θ_i (Fig. 2) between axes of two consecutively connected links L_i and L_{i-1}. The change of the length c_i of the hydraulic cylinders in the interval of its boundary values $c_i=[c_{ip}, c_{ik}]$ leads to the change of the generalized coordinates θ_i in the interval $\theta_i=[\theta_{ip}, \theta_{ik}]$, where θ_{ip} is the starting and θ_{ik} is the final angle of the relative position of link L_i in relation to the previous link L_{i-1}.

The relative angle θ_{io} between the links L_i and L_{i-1} is expressed as follows:

$$\theta_{io} = \theta_{ik} - \theta_{ip}. \tag{6}$$

The position of link L_i in relation to the horizontal OXZ plane of the absolute coordinate system is determined by the angle:

$$\phi_i = \sum_{i=3}^{i} \theta_i, \ \forall i = 3,4,5. \tag{7}$$

Unit vector $\boldsymbol{e}_i$ of the joint O_i axis, vector $\boldsymbol{r}_i$ of the center of joint O_i, vector $\boldsymbol{r}_w$ of the centre of the bucket cutting edge and vector $\boldsymbol{r}_{ii}$ of the center of mass of link L_i in the absolute coordinate system are determined by Eqs. (8)–(11), respectively:

$$\boldsymbol{e}_i = \boldsymbol{A}_{io}\hat{\boldsymbol{e}}_i, , \tag{8}$$

$$\boldsymbol{r}_i = \sum_{i=1}^{i-1} \boldsymbol{A}_{io}\boldsymbol{s}_i, \ \forall i = 2,3,4,5, \tag{9}$$

$$\boldsymbol{r}_w = \sum_{i=1}^{5} \boldsymbol{A}_{io}\boldsymbol{s}_i, \tag{10}$$

$$\boldsymbol{r}_{ti} = \boldsymbol{r}_i + \boldsymbol{A}_{io}\boldsymbol{t}_i, \tag{11}$$

where $\boldsymbol{A}_{io}$ is a transformation matrix, used to transfer the vectors from the local coordinate system $O_ix_iy_iz_i$ to the absolute coordinate system $OXYZ$. The elements of the transformation matrices $\boldsymbol{A}_{io}$ for the considered planar position of the excavator kinematic chain are given as follows.

1. Transformation matrix $\boldsymbol{A}_{1o}$ from the local system $O_1x_1y_1z_1$ between absolute system $OXYZ$ (Fig. 2):

$$\boldsymbol{A}_{1o} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}. \tag{12}$$

2. Transformation matrix $\boldsymbol{A}_{2o}$ from the local system $O_2x_2y_2z_2$ between the absolute system $OXYZ$:

$$\boldsymbol{A}_{2o} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}. \tag{13}$$

3. Transformation matrix $\boldsymbol{A}_{io}$ from the local system $O_ix_iy_iz_i$ (i=3,4,5) between the absolute system $OXYZ$:

$$\boldsymbol{A}_{io} = \begin{vmatrix} \cos\phi_i & -\sin\phi_i & 0 \\ \sin\phi_i & \cos\phi_i & 0 \\ 0 & 0 & 1 \end{vmatrix}, \ \forall\, i = 3,4,5. \tag{14}$$

2.4 Mechanical parameters of the model

The moment of gravitational forces of the links in the kinematic chain as well as the links of the excavator driving mechanisms for certain axes of joints O_i (i=3,4,5), when the bucket is empty, is determined by (Fig. 2)

$$M_{oi} = -g\sum_{k=i}^{k=5} m_k((\boldsymbol{r}_{tk} - \boldsymbol{r}_i)\times \boldsymbol{j})\cdot \boldsymbol{e}_i + M_{oci}, \quad \forall\, i = 3,4,5, \tag{15}$$

where $\boldsymbol{e}_i = \{0,0,1\}$ is the unit vector of axes of joints O_i (i=3,4,5), and M_{oci} is the moments of gravitational forces of the links of excavator driving mechanisms for certain axes of joints O_i (i=3,4,5).

Moments M_{oci} of gravitational forces of the links of excavator driving mechanisms for certain axes of joints O_i (i=3,4,5) are determined by the following equation (Fig. 2):

$$M_{oci} = \begin{cases} M_{oc3} = -g\dfrac{n_{c3}m_{c3}}{2}((\boldsymbol{r}_{A3} - \boldsymbol{r}_3)\times \boldsymbol{j})\cdot \boldsymbol{e}_3, \\ \quad -g\sum_{k=4}^{k=7} n_{ck}m_{ck}((\boldsymbol{r}_{ctk} - \boldsymbol{r}_3)\times \boldsymbol{j})\cdot \boldsymbol{e}_3 \\ \quad \forall i = 3, \\ M_{oc4} = -g\dfrac{n_{c4}m_{c4}}{2}((\boldsymbol{r}_{A4} - \boldsymbol{r}_4)\times \boldsymbol{j})\cdot \boldsymbol{e}_4 \\ \quad -g\sum_{k=5}^{k=7} n_{ck}m_{ck}((\boldsymbol{r}_{ctk} - \boldsymbol{r}_4)\times \boldsymbol{j})\cdot \boldsymbol{e}_4, \\ \quad \forall i = 4, \\ M_{oc5} = -g\dfrac{m_{c7}}{2}((\boldsymbol{r}_{A55} - \boldsymbol{r}_5)\times \boldsymbol{j})\cdot \boldsymbol{e}_5, \\ \quad \forall i = 5, \end{cases} \tag{16}$$

where $\boldsymbol{r}_{ctk}$ is the vector of the position of center of mass of hydro cylinders and the levers of the transmission part of driving mechanisms, where it is assumed that the centers of mass of hydro cylinders and the levers of the transmission part of driving mechanisms are at the middle of their kinematic lengths c_i, a_5 and c_{55} (Fig. 2a); $\boldsymbol{r}_{A3}, \boldsymbol{r}_{A4}$ are vectors of the position of centers of joints A_3 and A_4 where hydro cylinders are linked to the stick and boom of the attachment; $\boldsymbol{r}_{A55}$ is the vector of the position of center of joint A_{55} where the link of the transmission part of the mechanism is connected to the attachment bucket (Fig. 2a).

Gravitational moments for the first joint of the kinematic chain (i=1, with respect to x-x or z-z tip over line) are:

$$M_{o1} = \begin{cases} M_{o11} = -g\sum_{k=1}^{k=5} m_k((\boldsymbol{r}_{tk} - \boldsymbol{r}_{11})\times \boldsymbol{j})\cdot \boldsymbol{e}_1 \\ \quad -g\sum_{k=3}^{k=7} m_{ck}((\boldsymbol{r}_{ctk} - \boldsymbol{r}_{11})\times \boldsymbol{j})\cdot \boldsymbol{e}_1, \\ M_{o12} = -g\sum_{k=1}^{k=5} m_k(\boldsymbol{r}_{tk} - \boldsymbol{r}_{12})\times \boldsymbol{j})\cdot \boldsymbol{e}_1 \\ \quad -g\sum_{k=3}^{k=7} m_{ck}((\boldsymbol{r}_{ctk} - \boldsymbol{r}_{12})\times \boldsymbol{j})\cdot \boldsymbol{e}_1, \end{cases} \tag{17}$$

where $\boldsymbol{e}_1 = \{0,0,1\}$ is the unit vector of the first rotational joint (for *x-x* or *z-z* tip over line), and $\boldsymbol{r}_{11}, \boldsymbol{r}_{12}$ are vectors of the center of the appropriate first rotational joint.

The moments of gravitational forces of the links of the kinematic chain, driving mechanisms links and the weight of the soil in the bucket are related by

$$M_{zi} = M_{oi} - gm_z((\boldsymbol{r}_{t5} - \boldsymbol{r}_i) \times \boldsymbol{j}) \cdot \boldsymbol{e}_i, \ \forall \ i = 1,3,4,5, \tag{18}$$

where m_z is the mass of the soil in the bucket. It is assumed that the center of the mass of the soil coincides with that of the bucket.

Depending on the position of the bucket, the mass of the soil in the bucket is determined by the expression:

$$m_z = \begin{cases} \rho_z \cdot V \, |\cos\phi_5|, & \forall \ 270° \geq \phi_5 \geq 90°, \\ 0, & \forall \ 270° < \phi_5 < 90°, \end{cases} \tag{19}$$

where ρ_z is the density of the soil, V is the capacity of the bucket, and ϕ_5 is the angle of the position of the bucket in the absolute coordinate system (Fig. 2b).

Pressure p and flow Q of the hydrostatic excavator system are transformed according to the transmission function of driving mechanisms into angular speed $\dot{\theta}_i$ and driving moment M_{pi} of the link. The maximum driving moments $M_{pi\max}$ of the manipulator mechanisms in both directions of action are:

$$M_{pi\max} = \begin{cases} M_{pi1\max} = \text{sign}\ (\dot{\theta}_i) \cdot r_{ci} \cdot \ n_{ci} \dfrac{d_{i1}^2 \pi}{4} p_{\max}, \\ \forall i = 3,4,5, \ \dot{\theta}_3 > 0, \ \dot{\theta}_4 < 0, \ \dot{\theta}_5 < 0, \\ M_{pi2\max} = \text{sign}(\dot{\theta}_i) \cdot r_{ci} \cdot n_{ci} \dfrac{(d_{i1}^2 - d_{i2}^2)\ \pi}{4} p_{\max}, \\ \forall i = 3,4,5, \ \dot{\theta}_3 < 0, \ \dot{\theta}_4 > 0, \ \dot{\theta}_5 > 0, \end{cases} \tag{20}$$

where r_{ci} is the transmission function of the driving mechanisms which depends on the length of a hydraulic cylinder, the length of transmission levers and coordinates of the position of the joints of the hydraulic cylinders (Janosevic, 1997); and $p_{\max}$ is the maximum pressure of the hydrostatic excavator system.

3 Digging forces

The vector of digging force $\boldsymbol{F}$ lies in the plane of the manipulator and acts in the center of the bucket cutting edge. It has the same magnitude, direction and opposite sign to the vector of digging resistive force $\boldsymbol{W}$:

$$\boldsymbol{F} = - \ \boldsymbol{W}. \tag{21}$$

The direction of digging resistive force $\boldsymbol{W}$ with respect to the horizontal *OXZ* plane is determined by the angle:

$$\phi_w = \sum_{i=3}^{5} \theta_i + \theta_w, \tag{22}$$

where θ_w is the angle between the direction of the digging resistive force and O_5x_5 axis of the local coordinate system of the bucket L_5.

The direction and sign of digging resistive force $\boldsymbol{W}$ in the absolute coordinate system are determined by the unit vector ort$\boldsymbol{W}$:

$$\text{ort}\boldsymbol{W} = \boldsymbol{i} \cos\phi_w + \boldsymbol{j} \sin\phi_w. \tag{23}$$

3.1 Boundary digging resistive forces

The value of the digging resistive force is limited by the conditions of excavator tip over and maximum forces exerted by the driving mechanisms. The boundary digging resistive force limited by the excavator stability is determined by the equations of static equilibrium with respect to the first joint of the excavator kinematic chain. They are represented by the values of two boundary digging resistive forces.

The first boundary digging resistive force W_{1x} (Fig. 2) is limited by the friction force between the undercarriage and the ground as follows:

$$W_{1x} = \frac{mg \cdot \mu_p}{|\cos\phi_w|}, \tag{24}$$

where m is the total mass of the excavator; μ_p is the dry friction coefficient between the undercarriage and the ground. The minimum value of this boundary digging resistive force $W_{1x\min}$ is

$$W_{1x\,\min} = mg \cdot \mu_{\mathrm{p}}. \quad (25)$$

The second boundary digging resistive force $W_{1\mathrm{o}}$ is determined from the excavator tip over conditions. Depending on the position of the excavator kinematic chain and ort$\boldsymbol{W}$, the boundary digging resistance $W_{1\mathrm{o}}$ limited by the static excavator stability is determined from the equations of equilibrium with respect to one of the rotational joints O_{11} and O_{12}, whose axes represent the potential tip over lines of the excavator as follows (Fig. 2):

$$W_{1\mathrm{o}} = \begin{cases} W_{11} = \dfrac{-M_{\mathrm{o}11}}{\left((\boldsymbol{r}_{\mathrm{w}} - \boldsymbol{r}_{11}) \times \mathrm{ort}\boldsymbol{W}\right) \cdot \boldsymbol{e}_1}, \\ \forall\, y_{\mathrm{w}} > 0,\ \ \phi_2 > \phi_{\mathrm{w}} > \phi_1 + 180^{\mathrm{o}}, \\ \forall\, y_{\mathrm{w}} < 0,\ \ \phi_1 > \phi_{\mathrm{w}} > \phi_2 - 180^{\mathrm{o}}, \\ W_{12} = \dfrac{-M_{\mathrm{o}12}}{\left((\boldsymbol{r}_{\mathrm{w}} - \boldsymbol{r}_{12}) \times \mathrm{ort}\boldsymbol{W}\right) \cdot \boldsymbol{e}_1}, \\ \forall\, y_{\mathrm{w}} > 0,\ \ \phi_2 + 180^{\mathrm{o}} > \phi_{\mathrm{w}} > \phi_1, \\ \forall\, y_{\mathrm{w}} < 0,\ \ \phi_1 - 180^{\mathrm{o}} > \phi_{\mathrm{w}} > \phi_2, \end{cases} \quad (26)$$

where y_{w} is coordinate of the bucket top, and ϕ_1, ϕ_2 are determined by

$$\phi_1 = \arccos\left(\frac{(\boldsymbol{r}_{\mathrm{w}} - \boldsymbol{r}_{11}) \cdot \boldsymbol{i}}{|\boldsymbol{r}_{\mathrm{w}} - \boldsymbol{r}_{11}|}\right), \quad \phi_2 = \arccos\left(\frac{(\boldsymbol{r}_{\mathrm{w}} - \boldsymbol{r}_{12}) \cdot \boldsymbol{i}}{|\boldsymbol{r}_{\mathrm{w}} - \boldsymbol{r}_{12}|}\right), \quad (27)$$

where $(\boldsymbol{r}_{\mathrm{w}} - \boldsymbol{r}_{11})$ and $(\boldsymbol{r}_{\mathrm{w}} - \boldsymbol{r}_{12})$ are vectors built with OX axes.

The minimum values of the boundary digging resistive forces $W_{1\min}$ limited by the static excavator stability are obtained from

$$W_{1\mathrm{o}\,\min} = \begin{cases} W_{11\min} = \dfrac{-M_{\mathrm{o}11}}{|\boldsymbol{r}_{\mathrm{w}} - \boldsymbol{r}_{11}|}, & \forall (\boldsymbol{r}_{\mathrm{w}} - \boldsymbol{r}_{11}) \perp \mathrm{ort}\boldsymbol{W}, \\ W_{12\min} = \dfrac{-M_{\mathrm{o}12}}{|\boldsymbol{r}_{\mathrm{w}} - \boldsymbol{r}_{12}|}, & \forall (\boldsymbol{r}_{\mathrm{w}} - \boldsymbol{r}_{12}) \perp \mathrm{ort}\boldsymbol{W}. \end{cases} \quad (28)$$

Boundary digging resistive forces W_i (i=3, 4, 5) which can be overcome by the driving mechanisms of the manipulator for a known ort$\boldsymbol{W}$, and the position of the excavator kinematic chain along with the maximum driving moments $M_{\mathrm{p}i\max}$, are determined from the equations of the static equilibrium with respect to the axes of manipulator joints O_i (i=3,4,5). These are given by (Fig. 2)

$$W_i = \frac{-M_{\mathrm{p}i\max} - M_{\mathrm{o}i}}{((\boldsymbol{r}_{\mathrm{w}} - \boldsymbol{r}_i) \times \mathrm{ort}\boldsymbol{W}) \cdot \boldsymbol{e}_i}, \ \forall\, i\text{=3,4,5}. \quad (29)$$

The minimum values of boundary digging resistive forces $W_{i\min}$ which can be overcome by the driving mechanisms are obtained:

$$W_{i\,\min} = \frac{-M_{\mathrm{p}i\max} - M_{\mathrm{o}i}}{|\boldsymbol{r}_{\mathrm{w}} - \boldsymbol{r}_i|}, \quad \forall\, (\boldsymbol{r}_{\mathrm{w}} - \boldsymbol{r}_i) \perp \mathrm{ort}\boldsymbol{W},\ i = 3,4,5. \quad (30)$$

3.2 Hodographs of digging resistive forces

For a specified position of the kinematic chain links, the hodograph of the boundary resistive force HW$_i$ (it is equivalent to the hodograph of the digging force HF$_i$) (Fig. 2a) is a line, perpendicular to the vector of the boundary resistive force $W_{i\min}$ passing through the point O_{w}. The value of the force $W_{i\min}$ is the minimum value, determined from the operation of the driving mechanism in both directions (Dudczak, 1977; Janosevic, 1997).

Boundary resistive force W_{g} for the known ort$\boldsymbol{W}$ and the position of the kinematic chain consists of a set determined by certain boundary digging resistive forces (or digging forces):

$$|-\boldsymbol{F}_{\mathrm{g}}| = W_{\mathrm{g}} = \{W_{1\mathrm{o}}, W_{1x}, W_3, W_4, W_5\}. \quad (31)$$

Potential resistive force W_{m} (or digging force F_{m}) is the value of the minimum resistive force from the set of boundary digging resistive forces:

$$|-\boldsymbol{F}_{\mathrm{m}}| = W_{\mathrm{m}} = \min\{W_{1\mathrm{o}}, W_{1x}, W_3, W_4, W_5\}. \quad (32)$$

The hodograph of potential digging resistive force HW (Fig. 2b) is a polygon bounded by the hodographs of boundary digging resistive forces limited by the excavator stability and the hodographs of boundary digging resistive forces limited by the manipulator driving mechanisms.

The line *n*-*n* (Fig. 2b) perpendicular to the current digging velocity vector $\boldsymbol{v}$ divides the hodograph HW into two parts. The first part is the zone of directions of action, which corresponds to the digging by toward oneself technology and it is called hodograph of effective digging resistances $\mathrm{HW_e}$. In this zone, the dot product of the vector of the potential digging resistive forces and the digging velocity vector is negative:

$$\boldsymbol{W}_{\mathrm{e}} = \boldsymbol{v} \cdot \boldsymbol{W}_{\mathrm{m}} < 0, \tag{33}$$

and the dot product of the potential digging forces and the digging velocity vector is positive:

$$\boldsymbol{F}_{\mathrm{e}} = \boldsymbol{v} \cdot \boldsymbol{W}_{\mathrm{m}} > 0. \tag{34}$$

3.3 Rated digging forces

Digging forces rated by the manufacturer are determined according to standards (ISO 6015, 2006; SAE J1179, 2008) and they include bucket digging force $F_{5\mathrm{d}}$ and stick digging force $F_{4\mathrm{d}}$ (Fig. 3).

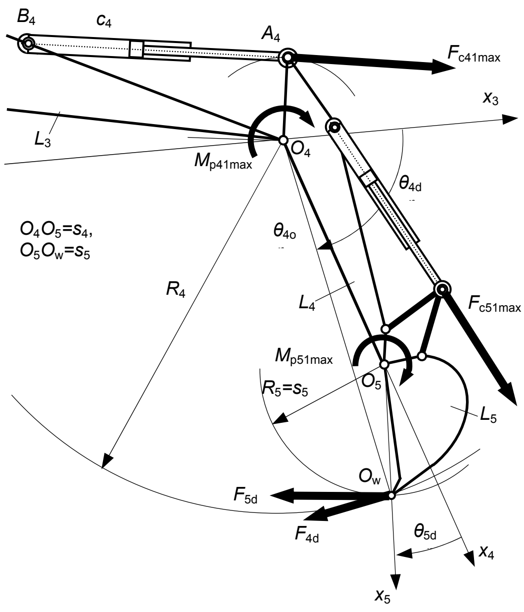

Fig. 3 Rated digging forces of a hydraulic excavator backhoe attachment

The bucket digging force $F_{5\mathrm{d}}$ represents the maximum force which the bucket driving mechanism exerts at maximum work pressure of the hydrostatic system for the direction of digging force perpendicular to the bucket cutting radius.

The relative position of the bucket with respect to the stick for rated bucket digging force is determined by angle $\theta_{5\mathrm{d}}$.

In the same fashion, the stick digging force $F_{4\mathrm{d}}$ represents the maximum boundary force which the stick driving mechanism exerts at maximum work pressure of the hydrostatic system. This force is for the direction of the digging force perpendicular to the bucket cut cutting radius in position $\theta_{5\mathrm{d}}$ when the rated bucket digging force is achieved. The relative position of the stick with respect to the boom upon rated stick digging force is determined by angle $\theta_{4\mathrm{d}}$.

3.4 Corrected digging forces

The corrected digging force F_{u} is defined for the entire workspace of the excavator using the following equation:

$$F_{\mathrm{u}} = k_{\theta} \frac{\sum_{s=1}^{N_3}\sum_{r=1}^{N_4}\sum_{k=1}^{N_5}\sum_{w=1}^{N_w} k_{xy} \cdot k_{\mathrm{w}} \cdot F_{\mathrm{e}\ srkw}}{N_3 \cdot N_4 \cdot N_5 \cdot N_{\mathrm{w}}}, \tag{35}$$

where $F_{\mathrm{e}\ srkw}$ is the magnitude of the effective digging force, with subscripts which show that the force relates to a specific position of the boom (*s*), stick (*r*) and bucket (*k*) of the manipulator and a specific direction of digging force (*w*); k_{θ} is the factor of the digging area in the workspace of the excavator; k_{w} is the factor of digging force direction, k_{xy} is the factor of position; N_3 is the number of positions of the manipulator boom; N_4 is the number of stick positions; N_5 is the number of bucket positions; and N_{w} is the number of directions of digging resistive force.

Defined by Eq. (35), the corrected digging force represents the mean value of the effective digging force for the entire workspace of the excavator. It is corrected by the coefficients which takes into account the influence of the following parameters: (1) the size of the excavator workspace; (2) the direction of the digging forces action; and (3) the position of the cutting edge top in the excavator workspace.

The meanings of the correctional factors in Eq. (35) are as follows.

1. Digging area factor k_{θ}

When determining the corrected digging force, one should take into account the relative size of the entire workspace of the excavator by using the digging area factor k_θ, which is defined by the ratio:

$$k_\theta = \frac{\theta_{3o} + \theta_{4o} + \theta_{5o}}{\theta_{3or} + \theta_{4or} + \theta_{5or}}, \tag{36}$$

where θ_{3o}, θ_{4o}, and θ_{5o} are angles of ranges of relative motion of manipulator boom, stick, and bucket, θ_{3or}, θ_{4or}, and θ_{5or} are referential constant angles of ranges of relative movement of manipulator boom, stick, and bucket selected so that they have slightly larger values than the potential appropriate angles θ_{3o}, θ_{4o}, and θ_{5o}.

2. Factor of direction of the digging force k_w

It is known that in the hydraulic excavators which work according to toward oneself technology, the direction of the digging resistive force is variable and has random values, which is usually dispersed around the direction of the vector of the digging velocity (Dudczak, 1977). This points out that digging forces in all possible directions covered by the hodograph of digging forces do not have the same significance for the toward oneself technology. Therefore, when defining the corrected digging force, it is necessary to introduce the factor of direction of the digging force k_w with a value that is defined within the limits $0 \le k_w \le 1$, depending on the digging technology and angle θ_w, where the digging velocity is taken to be perpendicular to the bucket cutting radius (Fig. 4).

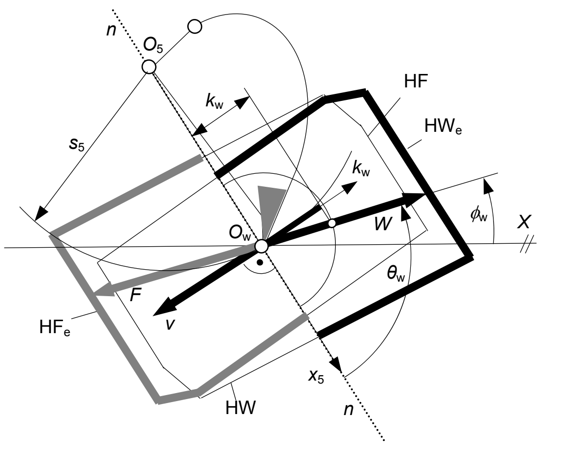

Fig. 4 Defining factor of direction of the digging force k_w

For the digging toward oneself technology, the factor of direction of the digging force k_w is defined by

$$k_w = \sin\theta_w, \ \forall\theta_w = [\theta_{wp}, \theta_{wk}], \tag{37}$$

where θ_{wp}=0° and θ_{wk}=180° are the minimum and the maximum angles of direction of the digging resistive force, respectively.

3. Digging position factor k_{xy}

Excavators of all sizes do not work in a uniform fashion in every part of their workspace for their lifecycle. There are zones where, they usually perform the toward oneself technology. According to this technology, to determine the corrected digging force, it is necessary to introduce the digging position factor k_{xy}, whose value is defined as a function of the coordinates x_w, y_w of the bucket tooth top within the boundaries: $0 \le k_{xy} \le 1$. For the digging toward oneself technology, the digging position factor k_{xy} is determined in relation to the radius R and angle ψ (Fig. 5) of the polar coordinate system with a center in point O_3.

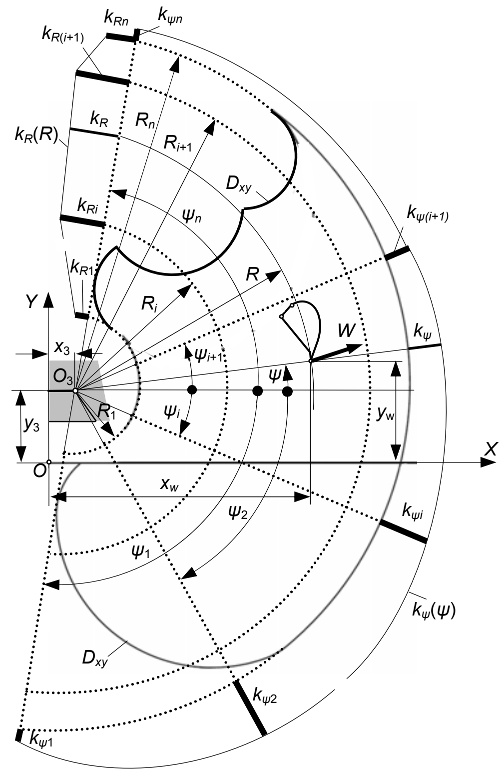

Fig. 5 Definition of the factor of excavator digging position k_{xy}

The value of the digging position factor k_{xy} is determined by

$$k_{xy} = k_R \cdot k_\psi, \tag{38}$$

where k_R is the digging position factor that depends on the digging position radius R; and k_ψ is the digging position factor that depends on the digging position angle ψ.

The connection between the digging coordinates (R, ψ) in the polar and the coordinates in the absolute coordinate system (x_w, y_w) of an excavator is determined by (Fig. 5)

$$R=[(x_w-x_3)^2+(y_w-y_3)^2]^{0.5}, \tag{39}$$

$$\psi=\mathrm{arctg}\frac{y_w-y_3}{x_w-x_3}, \tag{40}$$

where x_3 and y_3 are the coordinates of the joint O_3.

By selecting the desired number n_R of digging radii R_i in the closed interval between the minimum and maximum values (R_i=[R_1, R_2, …, R_n]), and by selecting the desired number n_ψ of the digging position angles ψ_i in the interval of its minimum and maximum values (ψ_i=[ψ_1, ψ_2, …, ψ_n]), a grid of digging position factor values is formed. These values cover and divide the entire workspace of the excavator into a desired number of working zones.

For the selected digging position radii R_i and angles ψ_i, the values of the factor are set within the boundaries: $0\le k_{Ri}\le 1$, $0\le k_{\psi i}\le 1$, so that a digging position factor k_{xy} is determined for every workspace part of the excavator.

Within the digging zone bounded by adjacent digging position radii R_i, R_{i+1} and adjacent digging position angles ψ_i, ψ_{i+1}, the digging position factors have the values of (Fig. 5):

$$k_R=k_{R_i}+(k_{R_{i+1}}-k_{R_i})\frac{R-R_i}{R_{i+1}-R_i},\quad \forall R_{i+1}\ge R\ge R_i,\ i=1,2,..,n_R-1, \tag{41}$$

$$k_\psi=k_{\psi_i}+(k_{\psi_{i+1}}-k_{\psi i})\frac{\psi-\psi_i}{\psi_{i+1}-\psi_i},\quad \forall \psi_{i+1}\ge\psi\ge\psi_i,\ i=1,2,..,n_\psi-1. \tag{42}$$

The boundary digging position radii R_1 and R_n and the boundary digging position angles ψ_1 and ψ_n are selected so that the grid of digging position factors covers the entire workspace of the excavator.

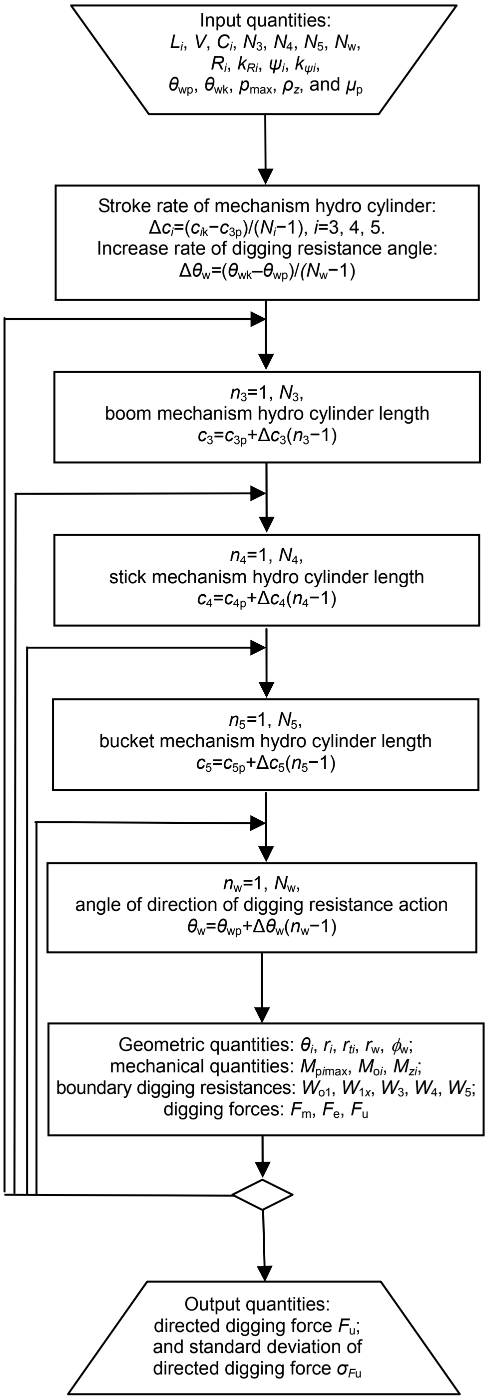

Fig. 6 Computer program algorithm for analysis of hydraulic excavator digging forces

4 Analysis

To show the possibilities of a corrected digging force for assessment of the excavator digging efficiency, an analysis for a hydraulic excavator of mass m=16945 kg and V=0.6 m^3 bucket capacity is conducted and presented. Two variants of the excavator with the same parameters of kinematic chain links L_i (Table 1) and different parameters of manipulator driving mechanisms C_{3j}, C_{4j}, and C_{5j} (Tables 2–4) were comparatively analysed.

The analysis covered the following variants of excavators: variant A={L, C_{31}, C_{41}, C_{51}} and variant B={L, C_{32}, C_{42}, C_{52}}. The variants of the manipulator driving mechanisms had the identical transformation parameters–hydraulic cylinder piston and connecting rod diameters but different transmission parameters–position coordinates of joints by which hydraulic cylinders and transmission levers were connected to the links.

4.1 Simulation software

The analysis was performed by a computer program, with an algorithm (Fig. 6) that is based on the defined mathematical models of the kinematic chain, driving mechanisms, and digging forces.

On the basis of input quantities, and by using the cyclic change of numbers N_w, N_5, N_4 and N_3, the following quantities are determined: (1) geometrical parameters (θ_i, r_i, r_{ti}, r_w) which define the position of joint centers and mass centers of the kinematic chain of an excavator; (2) loading moments (M_{oi}, M_{zi}) and moments ($M_{pi\max}$) of the driving mechanisms; (3) statistical set of boundary (F_g), potential (F_m) and effective (F_e) digging forces for the entire workspace. The number of values of the statistical set parameters was equal to the product: $N_w \times N_5 \times N_4 \times N_3$; (4) corrected digging force (F_u) and the standard deviation of the corrected digging force (σ_{Fu}) for the entire workspace.

The standard deviation of the corrected digging force σ_{Fu} is defined with the following equation:

$$\sigma_{Fu} = \sqrt{\frac{\sum_{s=1}^{N_3}\sum_{r=1}^{N_4}\sum_{k=1}^{N_5}\sum_{w=1}^{N_w}(k_{xy}\cdot k_w \cdot F_{e\ srkw} - F_u)^2}{N_3 \cdot N_4 \cdot N_5 \cdot N_w}}. \tag{43}$$

The appropriate input files and quantities used in the analysis are given in Table 5.

Table 1 Parameters of kinematic chain members of computational hydraulic excavator model (Fig. 2)

L_i	e_i			s_i			t_i			m_i (kg)	B (mm)	L (mm)
	e_{ix}	e_{iy}	e_{iz}	s_{ix} (mm)	s_{iy} (mm)	s_{iz} (mm)	t_{ix} (mm)	t_{iy} (mm)	t_{iz} (mm)			
L_1	0	0	1	0	1020	0	0	450	0	7500	2000	3200
L_2	0	1	0	410	665	0	−1360	435	0	6500		
L_3	0	0	1	5000	0	0	2260	565	0	1250		
L_4	0	0	1	2200	0	0	535	135	0	530		
L_5	0	0	1	1300	0	0	580	335	0	455		

Note: B is the track gauge, and L is the length between centers of rollers

Table 2 Parameters of variant solutions C_{3j} of boom drive mechanism of computational excavator model (Fig. 2)

C_{3j}	d_{31} (mm)	d_{32} (mm)	c_{3p} (mm)	c_{3k} (mm)	a_{3x} (mm)	a_{3y} (mm)	b_{3x} (mm)	b_{3y} (mm)	m_{c3} (kg)	n_{c3}
C_{31}	115	80	1555	2605	430	−520	1870	955	205	2
C_{32}	115	80	1525	2560	545	−410	1915	990	205	2

Table 3 Parameters of variant solutions C_{4j} of stick drive mechanism of computational excavator model (Fig. 2)

C_{4j}	d_{41} (mm)	d_{42} (mm)	c_{4p} (mm)	c_{4k} (mm)	a_{4x} (mm)	a_{4y} (mm)	b_{4x} (mm)	b_{4y} (mm)	m_{c4} (kg)	n_{c4}
C_{41}	140	90	1710	2885	−545	350	1985	1085	280	1
C_{42}	140	90	1723	2910	−620	187	2076	1104	280	1

Table 4 Parameters of variant solutions C_{5j} of bucket drive mechanism of computational excavator model (Fig. 2)

C_{5j}	d_{51} (mm)	d_{52} (mm)	c_{5p} (mm)	c_{5k} (mm)	a_5 (mm)	b_{5x} (mm)	b_{5y} (mm)	b_{45x} (mm)	b_{45y} (mm)	c_{55} (mm)	a_{55x} (mm)	a_{55y} (mm)	m_{c5} (kg)	n_{c5}
C_{51}	115	80	1449	2381	575	1850	470	230	15	515	0	300	190	1
C_{52}	115	80	1386	2270	510	1770	570	290	30	475	−62	351	190	1

Table 5 Input quantities of computer program for analysis of digging forces of variant A and B excavators

Parameter	Value	Parameter	Value
V (m^3)	0.6	θ_{3or} (°)	130°
N_3	50	θ_{4or} (°)	170°
N_4	30	θ_{5or} (°)	200°
N_5	20	p_{max} (MPa)	33
N_w	20	ρ_z (kg/m^3)	1800
θ_{wp} (°)	0°	μ_p	0.9
θ_{wk} (°)	180		

Parameters of excavator kinematic chain links file are given in Table 1; parameters of variant solutions of manipulator drive mechanisms file are given in Tables 2–4; and radii R_i and angles ψ_i of digging position and their corresponding values of factors k_{Ri} and $k_{\psi i}$ are given in Table 6

4.2 Analysis of the results

The obtained simulation results were analyzed. The analysis shows that the excavator A and excavator B have different transmission functions of the driving mechanisms (r_{c3}, r_{c4}, r_{c5}) and different changes in the driving moments of the boom (M_{p3max}) (Fig. 7a), the stick (M_{p4max}) (Fig. 7b) and the bucket (M_{p5max}) (Fig. 7c); whereas they have almost identical ranges of motion of the manipulator links (θ_{3o}, θ_{4o}, θ_{5o}). The absolute values of maximum driving moments of the boom, stick and bucket mechanisms are greater than the absolute values of the maximum loading moments (M_{z3}, M_{z4}, M_{z5}).

The analysis of the digging forces exerted by the motion of the stick (F_4) (Fig. 8a) and the bucket (F_5) (Fig. 8b), determined under the conditions prescribed by the standard (ISO 6015, 2006) show that the excavator A and the excavator B have the same values of rated stick (F_{4d}) and bucket (F_{5d}) digging forces. They are exerted under different angles of the relative position of the stick in relation to the boom (θ_{4d}) and the bucket in relation to the stick (θ_{5d}).

For the excavator variant A the rated stick (F_{4d}) (Fig. 9a) and bucket (F_{5d}) digging forces could not be exerted in the range (θ_{3o}) of the manipulator boom since they are greater than the potential digging force $F_{31}=\min\{F_3,F_{1o},F_{1x}\}$. The force F_{31} represents the minimum value of the boundary digging force that could be exerted by the driving mechanism of the boom (F_3) and the boundary forces allowed by the excavator stability (F_{1o}, F_{1x}). On the other hand, for the excavator variant B, the rated stick (F_{4d}) and bucket (F_{5d}) digging forces could be exerted only in the partial range ($\Delta\theta_{34}$, $\Delta\theta_{35}$) of motion of the manipulator boom. For the relative position of the stick and bucket, determined by angles θ_{4d}, θ_{5d}, the potential digging force F_{31} was greater in variant B,

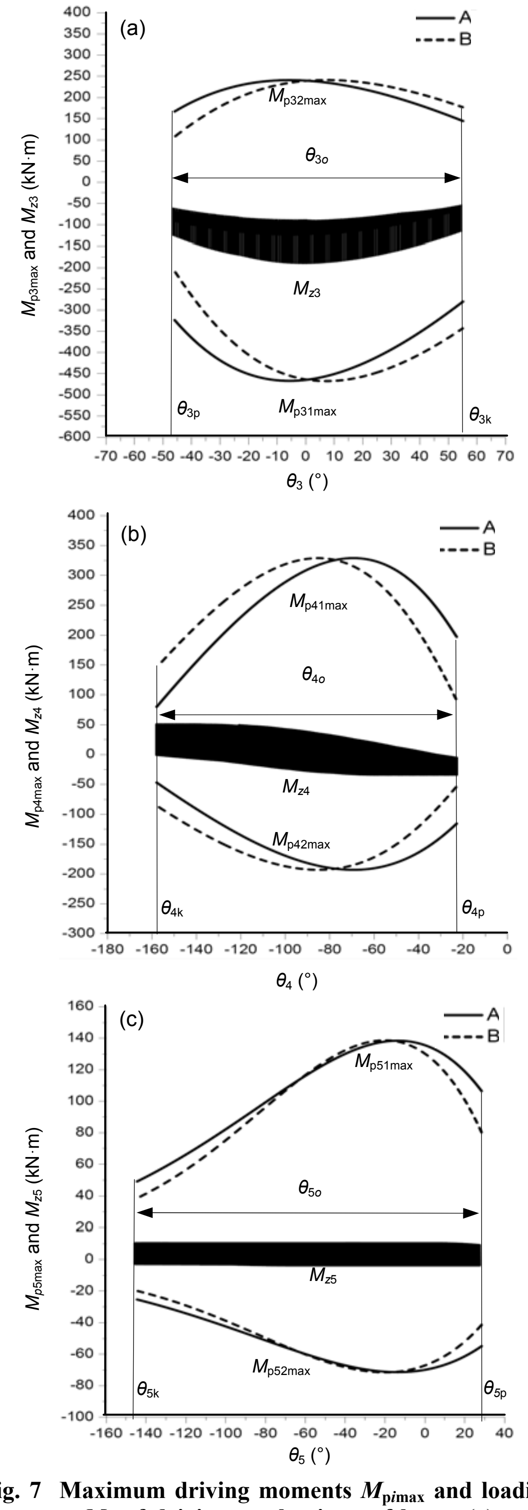

Fig. 7 Maximum driving moments M_{pimax} and loading moments M_{zi} of driving mechanisms of boom (a), stick (b) and bucket (c) of variant A and B excavators

because digging reaches x_w, y_w (Fig. 9b) within the range of motion of boom θ_{3o}, and less for variant B in comparison to variant A.

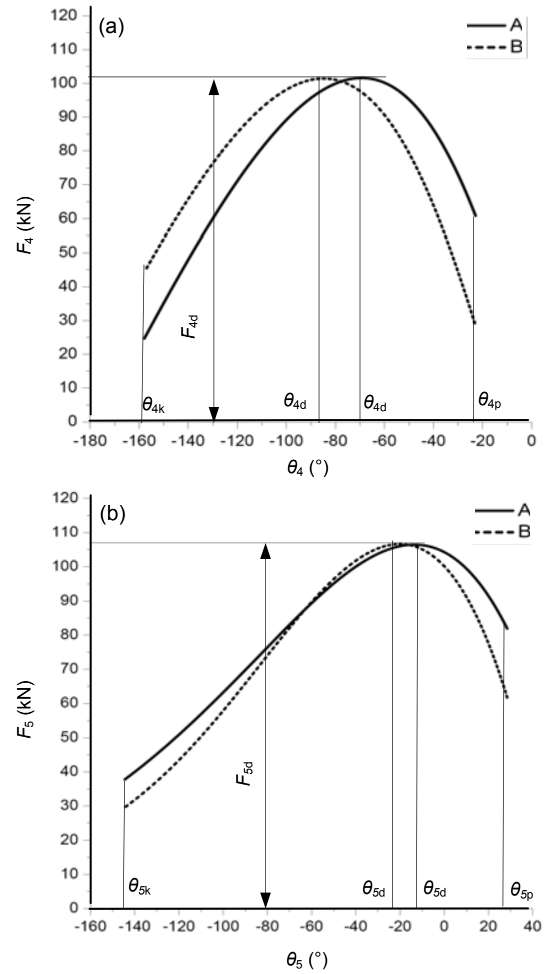

Fig. 8 Rated digging forces of stick F_{4d} (a) and bucket F_{5d} (b) of variant A and B excavators, determined according to standard

The analysis of the hodograph of digging forces for a specified number of the kinematic chain positions in the entire workspace of the excavator was performed for both excavators. For two isolated different manipulator positions defined by generalized coordinates (θ_3, θ_4, θ_5), the analysis showed that the hodographs of effective digging forces HF_e (Fig. 10) were significantly different for variant A and B excavators.

The comparative analysis of the spectra of values of effective digging forces F_e in the entire working range of variant A (Fig. 11a) and B (Fig. 11b) excavators, provides only visual but not quantitative assessment of the digging efficiency of the excavators.

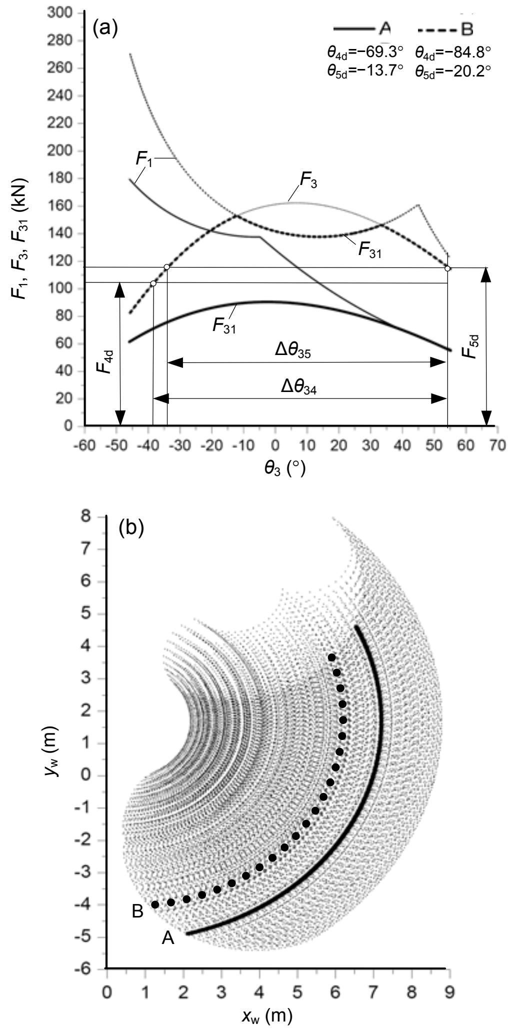

Fig. 9 Rated digging forces

(a) Ratio of rated to potential digging forces in the range of motion of boom; (b) Trajectory of central bucket cutting edge upon potential action of rated forces in working range of variant A and B excavators

A specific color of the spectrum corresponds to each of the positions of the bucket tooth tip. Each spectrum color represents a certain interval of the effective digging force value. The spectrum shows that in the same position (same coordinates x_w, y_w) of the bucket tooth tip, different values of the effective digging force can be achieved, since the same position of the bucket tooth tip can be achieved by various relative positions of boom, stick and bucket.

Spectra of effective digging forces are obtained using the Origin Pro 8 program, based on the files of effective digging forces obtained using a developed program whose algorithm is shown in Fig. 6.

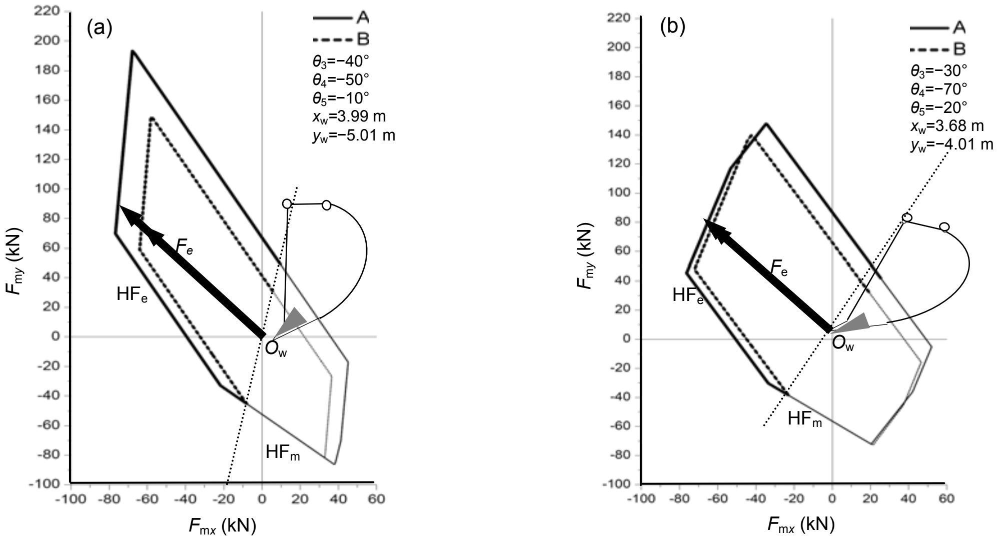

Fig. 10 Hodographs of potential (HF_m) and effective (HF_e) digging forces of variant A and B excavators for two digging positions in workspace

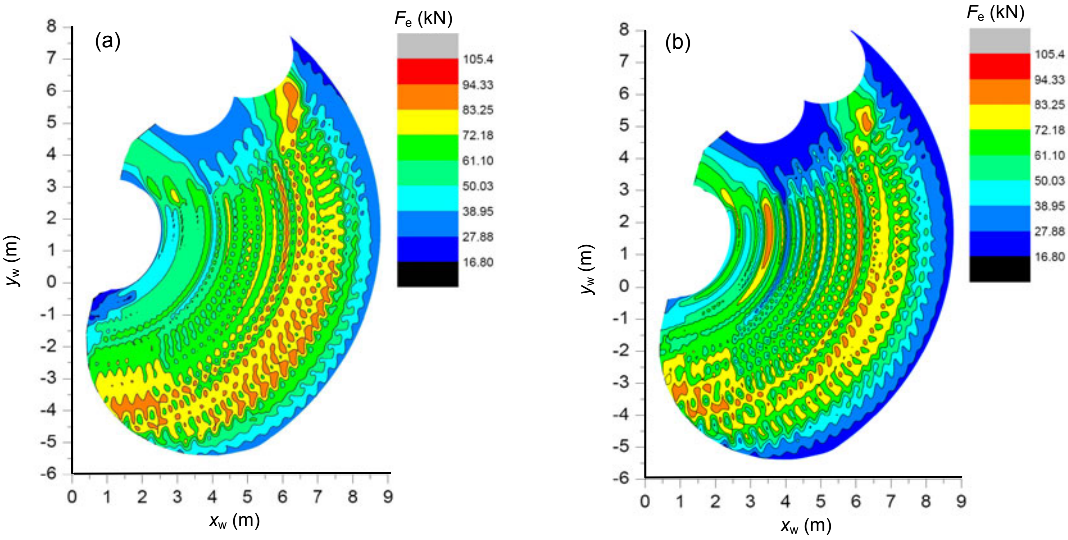

Fig. 11 Spectrum of values of effective digging forces F_e in the entire workspace of excavator
(a) Variant A; (b) Variant B

For the given input conditions and quantities (Table 5), the corrected digging forces F_u and the standard deviation of the corrected digging force σ_{Fu} were determined for variant A and B excavators for constant and variable values of the digging area factor (k_θ), the digging position factor (k_{xy}) and the factor of direction of the digging force (k_w) (Table 6).

In the first case, unit values of correctional factors ($k_\theta=k_{xy}=k_w=1$) are selected (Table 6), hence, they have no impact on the value of the corrected digging force. For this case, the working range of the excavator, digging position and the direction of force are neglected, so the corrected digging force, in fact represents the mean value of the effective digging force for the entire workspace of the excavator. For the selected unit values of the correctional factors, the results of the analysis show that the corrected digging force of the excavator variant B is greater, while its standard deviation is smaller than the deviation of the excavator variant A.

The variables of the following values are selected in the second case: digging area factor ($k_\theta \le 1$)

(Table 6), digging position factor (k_{xy}≤1) and the factor of direction of the digging force (k_w≤1), where greater values are given to the digging zones below the level of ground and to the directions of digging force action perpendicular to the bucket cutting radius. The analysis shows that the corrected digging force of the excavator variant A was greater, while its standard deviation was smaller than that of the excavator variant B.

Results of the analysis show that when designing an excavator, the defined corrected digging force can be used in the analysis and assessment of possible variant solutions for excavator driving mechanisms.

Factors of the corrected digging force k_θ, k_{xy} and k_w do not always have the same values but are chosen according to the set demands of the excavator digging function with the aim of a more efficient, coordinated and directed action of all driving mechanisms in the desired (priority) zone of the working range of an excavator. When a possible set of variant excavator solutions is analyzed and assessed comparatively, the selected values of factors k_θ, k_{xy} and k_w do not change, where the optimal solution corresponds to the maximum value of the corrected digging force.

5 Conclusions

In this paper, quantitative measures for the assessment of the hydraulic excavator digging efficiency have been introduced.

Rated digging forces of hydraulic excavators, defined according to standards, do not present the full possibilities of the digging function in the entire workspace of an excavator truthfully. They represent the maximum potential digging forces which can be achieved in certain positions of the kinematic chain within the narrow zone of the workspace.

The analysis of the digging forces of hydraulic excavators is somewhat difficult due to the big number of possible positions of the manipulator, different digging technologies and various exploitation conditions in the spatial workspace.

Boundary digging forces limited by the excavator stability depend on the distribution of the total mass of the excavator and the support contours of the excavator base. The support contours are outlined by the tip over lines of the excavator. Boundary digging forces that can be exerted by the driving mechanisms of the excavator manipulator depend on the transmission functions of the driving mechanisms.

Hodographs of boundary digging forces and hodographs of effective digging forces are suitable for the analysis of digging forces in a specific position of the excavator kinematic chain. However, hodographs of digging forces cannot be considered as the resulting indicators of digging efficiency for the entire workspace of the excavator.

The corrected digging force defined in this paper, represents a contribution to the development of quantitative measures of digging efficiency in the entire workspace of an excavator. In the comparative analysis of the different variants of the identical excavators, the defined corrected digging force show the following: (a) the way that actions of manipulator mechanisms are compatible mutually and in relation to the boundaries conditioned by the excavator stability; (b) how many maximum possibilities of active operations of manipulator mechanisms are corrected toward the desired zones of the working range and expected directions of digging resistive forces.

The results of the analysis obtained by the developed software show that the proposed quantitative measures can be used for assessment of the digging efficiency of the excavators and to serve as an optimization criterion for the synthesis of driving mechanisms. The defined corrected digging force can be employed in the assessment of the digging efficiency of existing excavator models, and as a criterion during the synthesis of manipulator driving mechanisms.

Table 6 Corrected digging forces of variant A and B excavators for constant and variable values of factors k_θ, k_w, k_{Ri}, and $k_{\psi i}$

k_θ	k_w	k_{Ri}						$k_{\psi i}$						F_u (kN)		σ_{Fu}	
		R_i=1.5 m	1.8 m	3.4 m	5.1 m	6.8 m	8.5 m	Ψ_i=−90°	−70°	−20°	0°	20°	90°	Variant A	Variant B	Variant A	Variant B
1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	69442	70689	29146	29105
$\frac{\theta_{3o}+\theta_{4o}+\theta_{5o}}{\theta_{3or}+\theta_{4or}+\theta_{5or}}$	$\sin\theta_w$	0.1	0.2	0.3	1.0	1.0	0.8	0.1	1.0	1.0	0.5	0.2	0.1	23942	22001	23315	24051

References

Budny, E., 1989. Probleme des optimalen entwurfs von arbeits-ausrustungen der hydraulikbagger. *Maschinenbautehnik*, **38**(7):12-14 (in German).

Budny, E., Chlosta, M., Gutkowski, W., 2003. Load-independent control of a hydraulic excavator. *Automation in Construction*, **12**(3):245-254. [doi:10.1016/S0926-5805(02)00088-2]

Chang, P.H., Lee, S.J., 2002. A straight-line motion tracking control of hydraulic excavator system. *Mechatronics*, **12**(1):119-123. [doi:10.1016/S0957-4158(01)00014-9]

Craig, J., 2005. Introduction to Robotics, Mechanics and Control. Pearson Educational International, USA.

Dudczak, A., 1977. Kryteria Doboru Parametrow Mechanizm ow Napedowych Osprzetu Koparki Hidraulicznej. Instytut Mechanizacji Budownictwa, Warszawa (in Polish).

Flores, G.F., Kecskemethy, A., Pottker, A., 2007. Workspace Analysis and Maximum Force Calculation of a Face-Shovel Excavator using Kinematical Transformers. 12th IF to MM World Congress, Besancon.

Gu, J., Taylor, J., Seward, D., 2007. Modelling of a hydraulic excavator using simplified refined instrumental variable (SRIV) algorithm. *Journal of Control Theory and Applications*, **5**(4):391-396. [doi:10.1007/s11768-006-6180-2]

Ha, Q.P., Nguyen, Q.H., Rye, D.C., Durrant-Whyte, H.F., 2000. Impedance control of a hydraulically actuated robotic excavator. *Automation in Construction*, **9**(5-6):421-435. [doi:10.1016/S0926-5805(00)00056-X]

Hall, A.S., McAree, P.R., 2005. Robust bucket position tracking for a large hydraulic excavator. *Mechanism and Machine Theory*, **40**(1):1-16. [doi:10.1016/j.mechmachtheory.2004.05.005]

ISO 6015, 2006. Earth-Moving Machinery–Hydraulic Excavators and Manipulator Loaders–Methods of Determining Tool Forces. International Organization for Standardization.

Janosevic, D., 1997. Optimal Synthesis of Drive Mechanisms in Hydraulic Excavators. PhD Thesis, Faculty of Mechanical Engineering, University of Nis, Serbia.

Lee, S.U., Chang, P.H., 2002. Control of a heavy-duty robotic excavator using time delay control with integral sliding surface. *Control Engineering Practice*, **10**(7):697-711. [doi:10.1016/S0967-0661(02)00027-8]

Lin, X., Pan, S.X., Wang, D.Y., 2008. Dynamic simulation and optimal control strategy for a parallel hybrid hydraulic excavator. *Journal of Zhejiang University SCIENCE-A*, **9**(5):624-632. [doi:10.1631/jzus.A071552]

Lipsett, M.G., 2009. Methods for Assessing Dynamic Performance of Shovels. Proc. 18th International Symposium Mine Planning and Equipment Selection, Banff, p.9.

Maciejewski, J., Jarzebowski, A., 2002. Laboratory optimization of the soil digging process. *Journal of Terramechanics*, **39**(3):161-179. [doi:10.1016/S0022-4898(02)00022-8]

Maciejewski, J., Jarzebowski, A., Trampczynski, W., 2003. Study on the efficiency of the digging process using the model of excavator bucket. *Journal of Terramechanics*, **40**(4):221-233. [doi:10.1016/j.jterra.2003.12.003]

Plonecki, L., Trampczynski, W., Cendrowicz, J., 1998. A concept of digital control system to assist the operator of hydraulic excavators. *Automation in Construction*, **7**(5):401-411. [doi:10.1016/S0926-5805(98)00045-4]

SAE J1179, 2008. Hydraulic Excavator and Manipulator Digging Forces. SAE International.

Towarek, Z., 2003. Dynamics of a single-bucket excavator on a deformable soil foundation during the digging of ground. *International Journal of Mechanical Sciences*, **45**(6-7):1053-1076. [doi:10.1016/j.ijmecsci.2003.09.004]

Yang, J.X., Zhang, C.Y., Huang, H.Y., Zhang, X.F., 2008. Reducing-resistance mechanism of vibratory excavation of hydraulic excavator. *Journal of Central South University of Technology*, **15**:535-539.