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CFD analysis of a transfer matrix of exhaust muffler with mean flow and prediction of exhaust noise

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Abstract: A multi-dimensional computational fluid dynamics (CFD) approach was proposed in this study aiming to calculate the transfer matrix of an engine exhaust muffler in the conditions with and without mean flow. The CFD model of the muffler with absorptive material defined as porous zone was calibrated with the measured noise reduction without mean flow, and was further employed to study the effect of the mean flow on the acoustic performance of the muffler. Furthermore, the exhaust acoustical source was derived from the calculated transfer matrices of six different additional acoustic loads obtained by the proposed CFD approach as well as the measured tail noise based on a multiload least squares method. Finally, the exhaust noise was predicted based on Thevenin's theorem. The proposed CFD approach was suggested to be able to predict the acoustic performance of a complex muffler considering mean flow (without and with mean flow) and heat transfer, and provide reasonable results of the exhaust noise.

Key words: Computational fluid dynamics (CFD), Transfer matrix, Mean flow, Acoustical source, Exhaust noise
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1 Introduction

Mufflers are widely used in intake and exhaust systems of internal combustion engines. The exhaust muffler comprising perforated pipes and porous media is employed to reduce the exhaust noise level, in both reactive and dissipative ways. Much work has been done in two fields: acoustic performance analysis of mufflers and exhaust noise prediction.

The acoustic performance of the exhaust muffler has been widely investigated in terms of the noise reduction, transmission loss, and insertion loss. Since the exhaust muffler is increasingly complex inside, the plane wave theory (Prasad and Crocker, 1984; Munjal and Prasad, 1986; Munjal, 1987) is not adequate for analyzing its acoustic performance. Numerical methods have been developed to meet the requirement of a higher accuracy. Yasuda *et al*. (2010) modeled a complex exhaust muffler by a 1D computational fluid dynamics (CFD) approach and predicted the tail pipe noise of an automotive muffler. The predicted noise showed a close agreement with the measured value at the 2nd order of the engine rotational frequency, while deviation existed at higher frequency, which is common for the 1D CFD approach. 3D approaches based on frequency domain, such as boundary element method (BEM) (Cheng *et al*., 1991; Ji and Selamet, 2000; Hao *et al*., 2005) and finite element method (FEM), have often been employed to investigate the acoustic characteristics of mufflers. Mehdizadeh and Paraschivoiu (2005) implemented an FEM to predict the transmission loss of mufflers. The perforated plates were modeled with the complex acoustic impedance, while the absorption linings were modeled as a bulk media with the complex speed of sound and mean density. In general, the propagation of sound in the absorptive material is determined by two complex quantities, the characteristic impedance and the propagation coefficient (Delany and Bazley, 1970). The impedance depends on the airflow resistance, which is an important factor

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for material with high porosity. The FEM seems to model the absorptive material quite well. Liu *et al*. (2010) adopted an FEM to model the filter element of an air-cleaner with five parameters: sound velocity, density, structural factor, resistivity, and porosity. However, the FEM is not suitable for modeling mufflers considering turbulent flow inside and heat exchange. Recently, a 3D CFD approach has been developed to tackle the mean flow and heat transfer problems in acoustic analysis (Middelberg *et al*., 2004). Broatch *et al*. (2005) adopted the time domain method to compute the transmission loss of some typical mufflers with mean flow and compared it with the experimental results.

There are mainly two ways to predict the exhaust noise of an internal combustion engine: CFD approach (Yasuda *et al*., 2010) and one-port source approach based on an electrical analogy. In the latter approach, the whole exhaust system is modeled by a one-port source model (Boden and Åbom, 1995; Davies and Holland, 1999) with acoustical source and impedance. The acoustical source can be obtained with multiload methods (Prasad, 1987; Desmons and Hardy, 1994; Jang and Ih, 2000). Jang and Ih (2000) implemented a refined multiload method to obtain the acoustical source of intake and exhaust systems and compared the refined method with the conventional four-load method and the least squares method. It was found that the refined method gains the most accurate source, followed by the least squares method and the four-load method. Normally the acoustic loads can be modeled by the transfer matrices (Davies, 1988; Craggs, 1989). Combined with the acoustic boundary at the exhaust tailpipe end (Davies *et al*., 1980; Davies, 1988) and the noise radiation characteristics (Munjal, 1987), the exhaust noise can be predicted with an acceptable accuracy.

The objective of the present study was to analyze the acoustic performance of an exhaust muffler considering mean flow and heat exchange by a multi-dimensional time-domain CFD approach. Since the transfer matrices were computed, the approach can be called the CFD matrix approach. In the CFD approach, the absorptive material, E-glass roving filled in one of the chambers, was modeled as a porous zone.

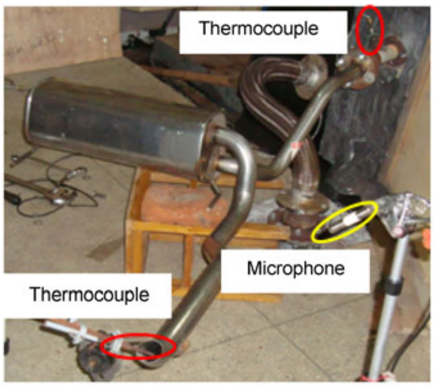
In this study, the exhaust noise of an internal combustion engine was measured when a complex exhaust muffler and six different acoustic loads were installed. Next, the acoustic properties of the loads were analyzed by the CFD matrix approach, and the exhaust acoustical source was obtained by the multi-load least squares method. Finally, the exhaust noise with the exhaust muffler installed was calculated and compared with the measured one.

2 Measurement

2.1 Exhaust noise

A four-stroke, 1.6-L gasoline engine was mounted on an engine test bench in a semi-anechoic room, and the exhaust muffler and six different loads (Fig. 1b) were connected to the outlet of the catalyst. The exhaust noise was measured as shown in Fig. 1a. Two thermocouples were adopted to measure the temperature of the exhaust gas at the inlet and outlet of the muffler, and the mass flow rate was monitored. The microphone was placed 50 cm from the center of the exhaust port, and its axis was 45° deviated from the normal axis of the port. The exhaust noise and the temperature at the inlet and outlet were measured under the conditions of 10 stable speeds from 1000 to 5500 r/min, in an interval of 500 r/min.

(a)



(b)

Fig. 1 (a) Exhaust noise measuring field with the exhaust muffler installed; (b) Six different acoustic loads

2.2 Noise reduction (NR)

The noise reduction (NR) of the muffler without mean flow was measured to validate the calculated transfer matrix. The NR is defined as

$$\mathrm{NR}=L_{\mathrm{Pi}}-L_{\mathrm{Po}}, \tag{1}$$

where L_{Pi} and L_{Po} are sound pressure levels upstream and downstream of the muffler, respectively. The NR measuring field is shown in Fig. 2. The muffler was placed in the semi-anechoic room, while a standard sound source of B&K HP1001 was placed outside of the room. The sound was introduced to the semi-anechoic room by a plastic pipe through a hole in the wall, and the exhaust muffler was connected to the pipe end. The inlet microphone was flushed with the pipe wall at the inlet section, while the outlet microphone was placed at the center of the outlet section. NR was obtained by subtracting the inlet pressure level from the outlet level.

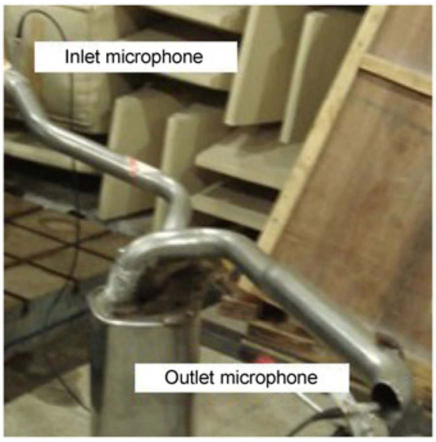


Fig. 2 Noise reduction measuring field of the exhaust muffler without mean flow

3 CFD analysis of exhaust muffler

The transfer matrix of a muffler can be expressed as

$$\begin{pmatrix} p_1 \\ v_1 \end{pmatrix}=\begin{bmatrix} A\,B \\ C\,D \end{bmatrix}\begin{pmatrix} p_2 \\ v_2 \end{pmatrix}, \tag{2}$$

where p stands for the acoustic pressure and v is the mass velocity. When p_2=0, B and D equal to p_1/v_2 and v_1/v_2, respectively, and when v_2=0, A and D equal to p_1/p_2 and v_1/p_2, respectively.

First, the transfer matrix of the muffler without mean flow was calculated. The NR computed from the transfer matrix and the acoustic boundary at the exhaust end was compared with the measured result, in order to validate the CFD matrix approach. Further, the transfer matrix of the muffler with mean flow corresponding to the engine operating condition was calculated for predicting exhaust noise.

The acoustic boundary at the exhaust end can be modeled as the radiation impedance. The radiation impedance without mean flow (M=0, where M is the Mach number) and with mean flow (M>0) can be expressed as (Munjal, 1987)

$$Z_{\mathrm{r}}/Y_0=(0.25k^2r^2-M)+\mathrm{j}k(0.6r), \tag{3}$$

where Z_{r} is the radiation impedance, Y_0 is the characteristic impedance corresponding to the mass velocity, k is the wave number, and r is the tail pipe radius.

3.1 Model description

A CFD commercial code was used for calculations. A 3D segregated implicit solver with the 2nd order implicit time stepping method was used. A time step of 10 μs was chosen, which is accurate enough for the frequency range limited below 1.5 kHz. The fluid was air based on the ideal gas model, with default setup of the code. Laminar model and standard k-ε model were chosen to simulate viscous behavior of the gas in the CFD model without and with mean flow, respectively.

The muffler consisted of three chambers connected by perforated pipes and perforated baffle, with E-glass roving filled in the first chamber (Fig. 3a). The CFD model contained 750 000 grids of tetrahedrons and prisms, with the maximum size of 10 mm and the minimum size of 2 mm. At the muffler inlet, an extended pipe of 0.5 m was built, and the extended pipe end was set up as the non-reflexive model inlet (Broatch *et al.*, 2005). Since the non-reflexive boundary in the CFD code still reflects minor pressure wave, the extended pipe was adopted to dissipate the reflected wave and reduce pollution at the muffler inlet (Fig. 3b).

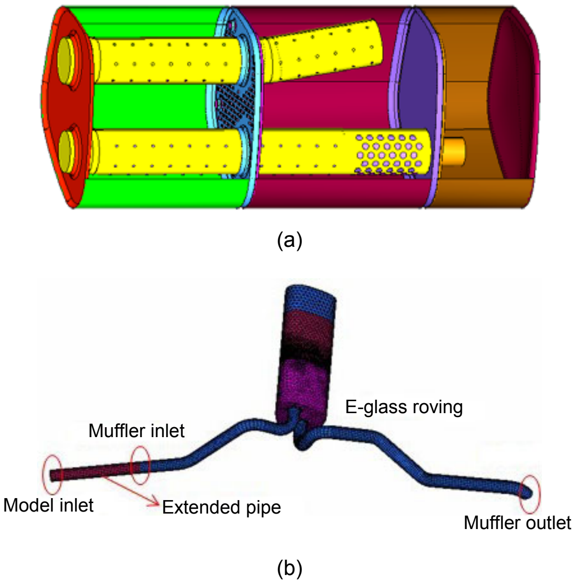


Fig. 3 (a) CAD model of the exhaust muffler chambers; (b) CFD model of the whole exhaust muffler

3.2 Boundary setup

The boundaries of the CFD model without mean flow were set up as follows:

1. The model inlet was set up as the non-reflective boundary. The pressure pulse was generated by fluctuated Mach number of a half sinusoidal period. The pulse sustained 16 μs with a magnitude of 0.003. The temperature was set as 300 K.

2. The muffler outlet was set up either as pressure outlet or as mass flow boundary. When it was pressure outlet with 0 Pa, parameters B and D could be solved by signal processing technique. When it was mass flow boundary of 0 kg/s, parameters A and C could be solved.

3. The first chamber filled with the E-glass roving was modeled as homogeneous porous zone, with given porosity of 0.95 and resistivity of 2500 Pa·s/m^2.

4. The wall was set up as adiabatic and non-slipping.

To simulate the pulse propagating in mean flow, a steady solution of the flow field must be first obtained. There were 10 cases of steady flow field corresponding to different engine speeds. Table 1 shows the boundary of mean flow at the speed of 2500 r/min, where the wall temperature was estimated.

The unsteady boundaries with mean flow were set up as follows:

Table 1 Measured and simulated boundaries of mean flow at 2500 r/min

Parameter	Value	
	Measured	Simulated
Speed (r/min)	2500	2500
Inlet temperature (K)	780	780
Outlet temperature (K)	520	545
Wall temperature (K)	–	400
Mach number	0.13	0.13
Inlet pressure (kPa)	–	4.2
Outlet pressure (kPa)	0	0

1. The same fluctuated Mach number as the value in the condition without mean flow was added at the model inlet after the steady flow had been established.

2. The muffler outlet was set up either as the pressure outlet with 0 Pa or as the mass flow boundary with a constant mass flow rate corresponding to the mean flow Mach number at the outlet.

3. Different wall temperatures, corresponding to different engine operating conditions, were given to the outside wall of the chambers and the outside pipes. Heat transfer exists between the gas and the wall.

4. The porous zone was set to be the same as it was in the condition without mean flow.

3.3 Results

NR without mean flow calculated from the transfer matrix and the impedance was compared with the measured one (Fig. 4). The calculated NR shows a good agreement with the measured one in general, though there are some obvious deviations at some frequency ranges, e.g., 250–400 Hz and 1220–1340 Hz. The deviations might be mainly attributed to the assumption of homogeneous porous zone, since the actual absorptive zone is not totally homogeneous. Other reasons may be mesh modeling error and signal processing error. Based on the result, the CFD approach has been validated.

Two mean flow cases of the muffler were analyzed in this study, corresponding to the speeds of 2500 and 5500 r/min. The NR spectra are shown in Fig. 5. Compared with the case without mean flow, the spectra of the mean flow cases are smoother, which may be attributed to the effects of the porous zone. With the engine speed running up from 2500 to 5500 r/min, the Mach number and temperature at the inlet of the muffler changed from 0.13 and 780 K to

0.3 and 950 K, respectively. Also, NR spectra have a trend of shifting toward the higher frequency and lower magnitude.

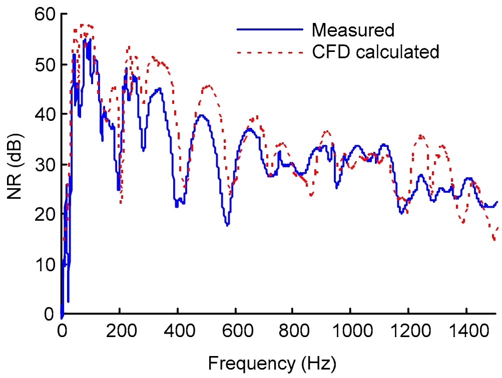


Fig. 4 Noise reduction (NR) of exhaust muffler

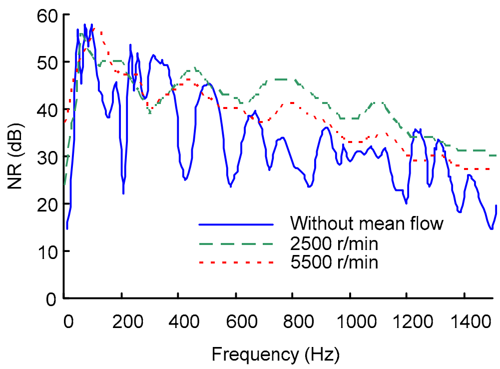


Fig. 5 Calculated noise reduction (NR) of exhaust muffler with mean flow

4 Acoustical source and exhaust noise

The duct system from the catalyst outlet to the exhaust end can be modeled as the electrical circuit based on Thevenin's theorem (Desmons *et al.*, 1995) (Fig. 6). The source P_s and impedance Z_s are such that

$$P=\frac{Z}{Z+Z_s}\cdot P_s, \tag{4}$$

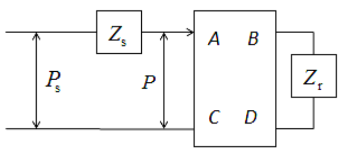


Fig. 6 An electrical circuit representation of acoustic ducts

where Z is the impedance of the muffler and P is the pressure at the muffler inlet. The impedance Z and pressure P can be calculated from the transfer matrix and the impedance at the exhaust end Z_r.

4.1 CFD analysis of acoustic loads

Six acoustic loads of four straight pipes and two expansion chambers (Fig. 1b) were simulated as 2D axisymmetrical CFD models. The CFD model setup and boundary setup were similar as the exhaust muffler setup with mean flow. All 10 mean flow cases of the loads were calculated.

With the calculated transfer matrices of the loads and measured exhaust noise, a multiload least squares method was employed to obtain the acoustical source of the exhaust system (Prasad, 1987; Desmons and Hardy, 1994). Then the exhaust noise with one of the pipes and the exhaust muffler installed was predicted based on the source.

4.2 Results

Figs. 7 and 8 show the calculated sound pressure level (SPL) of exhaust noise when a pipe was installed. The calculated noise spectra and order spectra are in good accordance with the measured results. The errors might be mainly attributed to the regenerated flow-noise and wall temperature setup. It proves that the acoustical source and the CFD-calculated transfer matrices are accurate enough for further application.

The exhaust noise with muffler installed is shown in Fig. 9 (p.715). The predicted noise spectra at 2500 r/min fits to the measured values at frequencies below 600 Hz, especially at frequencies corresponding to the 2nd, 4th, 8th, and 10th orders of the engine rotational frequency. The deviation at the 6th order frequency of 250 Hz can be attributed to the error of the predicted NR (Fig. 4). The NR error also influences the accuracy of the predicted noise from 250 to 400 Hz at 5500 r/min. Compared to the case of 2500 r/min, the predicted noise at 5500 r/min has a larger error, which might be due to the flow-generated noise in the high-speed flow field. Meanwhile, the CFD modeling of the porous zone with mean flow has not been studied thoroughly, and it may be the main reason for the large deviation at frequencies from 800 to 1500 Hz in both cases. As in the case of acoustic loads, the heat transfer setup at the wall results in some error.

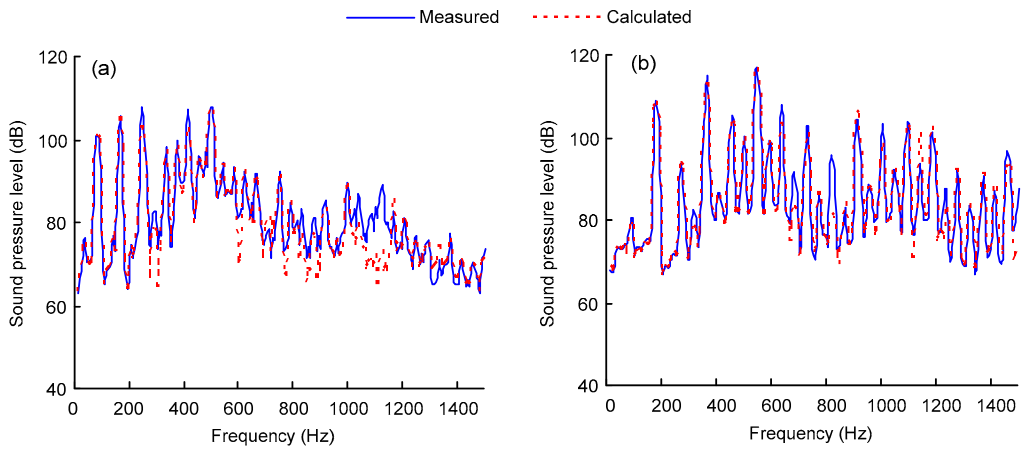


Fig. 7 Measured and calculated noise spectra with one of the pipes installed at 2500 r/min (a) and 5500 r/min (b)

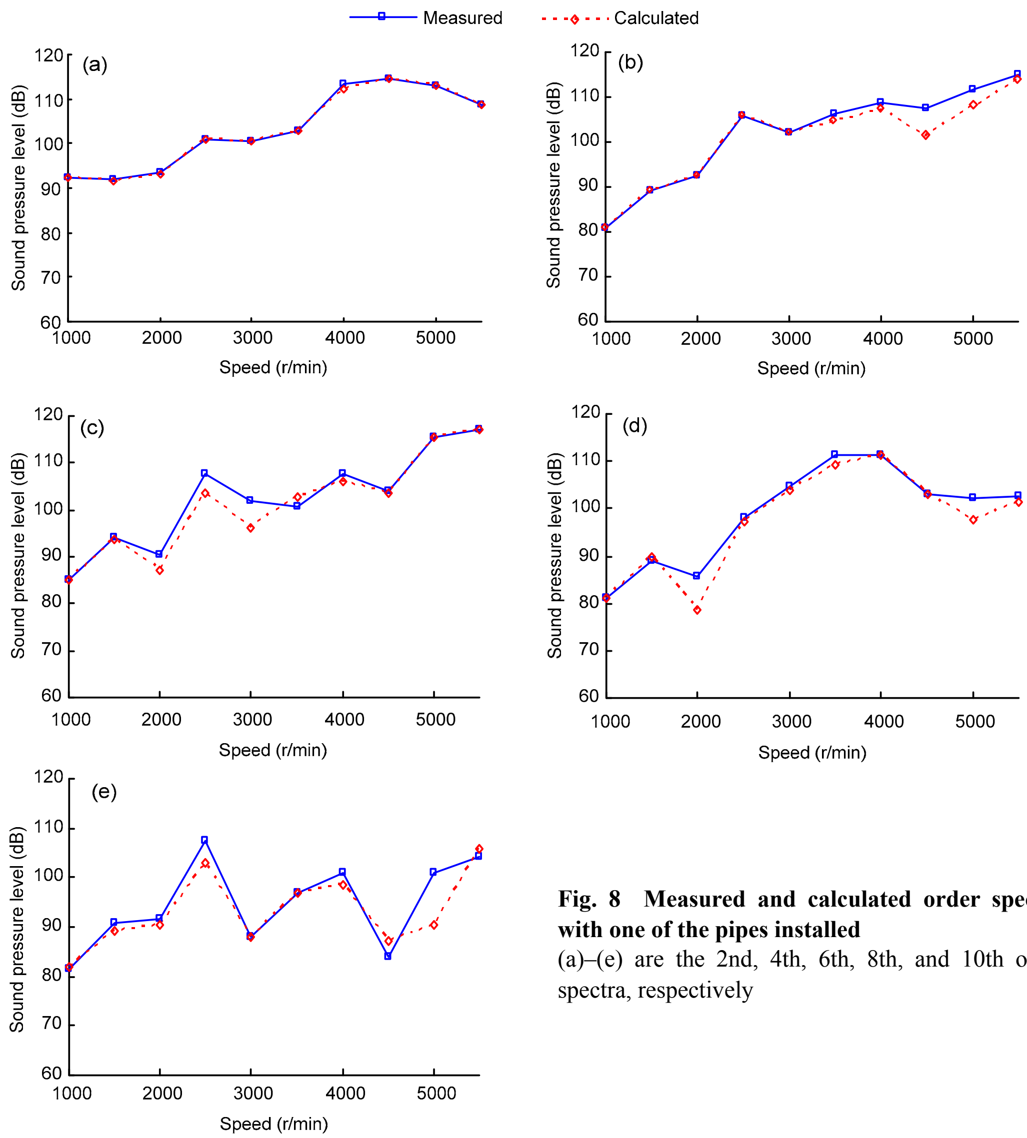


Fig. 8 Measured and calculated order spectra with one of the pipes installed

(a)–(e) are the 2nd, 4th, 6th, 8th, and 10th order spectra, respectively

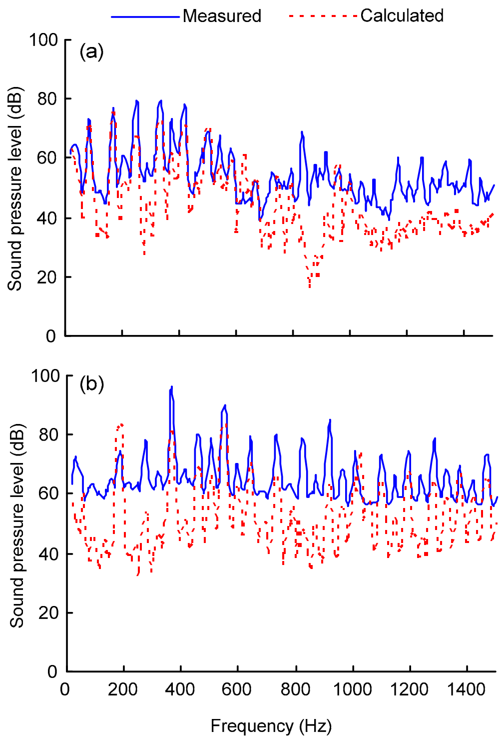


Fig. 9 Measured and calculated noise spectra with exhaust muffler installed at 2500 r/min (a) and 5500 r/min (b)

5 Conclusions

This paper presents some pioneer work on the CFD analysis of the acoustic characteristics of the exhaust muffler considering the absorptive material, heat transfer, and mean flow effects. A multi-dimensional CFD approach was proposed to calculate the transfer matrices of six acoustic loads and an exhaust muffler. Furthermore, the acoustical source of the exhaust system was obtained by a multiload least squares method and the exhaust noise with one of the loads and the exhaust muffler installed was predicted based on Thevenin's theorem.

The predicted noise with the load installed has shown a close agreement with the measured results since the simulated model is simple and the flow-generated noise is low, while the prediction with the exhaust muffler has not been as accurate as the former case. For higher accuracy, more work should be done in the areas of porous zone modeling, heat transfer modeling at walls, and flow-generated noise of turbulence.

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