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A semi-analytical method for the analysis of pile-supported embankments*

Wan-huan ZHOU[†1], Ren-peng CHEN[†‡2], Lin-shuang ZHAO[1], Zheng-zhong XU[2], Yun-min CHEN[2]

([1]*Department of Civil and Environmental Engineering, University of Macau, Macau, China*)

([2]*MOE Key Laboratory of Soft Soils and Geoenvironmental Engineering, Department of Civil Engineering, Zhejiang University, Hangzhou 310058, China*)

[†]E-mail: hannahzhou@umac.mo; chenrp@zju.edu.cn

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Abstract: In this paper, a semi-analytical method for the analysis of pile-supported embankments is proposed. The mathematic model describes the cooperative behavior of pile, pile cap, foundation soil, and embankment fills. Based on Terzaghi's 1D consolidation theory of saturated soil, the consolidation of foundation soil is calculated. The embankments with two different types of piles: floating piles and end-bearing piles are investigated and discussed. The results of axial force and skin friction distributions along the pile and the settlements of pile-supported embankments are presented. It is found that it takes a longer time for soil consolidation in the embankment with floating piles, compared with the case using end-bearing piles. The differential settlement between the pile and surrounding soil at the pile top is larger for the embankment with end-bearing piles, compared with the case of floating piles.

Keywords: Pile-supported embankment, Soil arching, Load transfer, Equal settlement plane, Soft soils
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1 Introduction

Settlement develops after the embankment is built on the ground and it is an essential concern for civil engineers, especially when the embankment is seated on soft soils. Recently, more attention has been paid to the foundations of high-speed railways as the high-speed trains will induce vibration forces to the ground (Bian *et al.*, 2011). Geotechnical engineers have developed various methods to improve soft soils such as preloading, vertical drains, and grouting injection (Magnan, 1994; Shen *et al.*, 2005). The pile-supported embankment, consisting of piles, pile caps, foundation soil, and embankment fills, has been increasingly used in soft soils due to its advantage of rapid construction and small settlement. However, the mechanics of load transfer is very complex in the pile-supported embankment. Differential settlement exists between the pile and foundation soil owing to the distinct compression modulus of them. As a result, a relative displacement would occur on the interface of the slippage in the embankment fill. The slippage would be resisted by shear stresses, and thus the overburden stress on the foundation soil is reduced while the pressure on the pile cap is increased. This phenomenon is called 'arching effect' (Terzaghi, 1943; Terzaghi and Peck, 1967).

Many researchers have investigated the effect of soil arching in the embankment construction. Russell and Pierpoint (1997) presented 3D numerical analyses for soil arching to analyze the behavior of pile-supported embankments. The British Standard BS8006 (1995) introduced Marston's formula to estimate the vertical stress on the top of the pile cap. Han and Gabr (2002) conducted numerical analysis on the embankments and the analysis indicated that the soil arching ratio decreases with an increase in the height of embankment fill. Low *et al*. (1993) investigated soil arching in the pile-supported embankments and proposed the theoretical solution by comparing with the model-test results.

[‡] Corresponding author

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The mechanics of load transfer owing to the interaction between the pile and soil is another essential problem for pile-supported embankment design. Randolph and Worth (1978) presented an approximate closed-form solution to the problem of a vertically loaded pile in a linear elastic soil. The solution gives the load-settlement ratio of the pile in terms of the pile geometry and stiffness, and soil stiffness. Zhou *et al*. (2006) and Chen *et al*. (2007; 2009) investigated the development of negative skin friction of piles in soft soils. Zhou and Yin (2008) proposed a simple mathematical model for the interaction analysis between soil inclusion and soils. Chen R.P. *et al.* (2008) studied the interactions among the rigid pile cap, the pile, the soil, and the embankment fill and proposed a closed-form solution based on a simplified unit-cell concept. Chen Y.M. *et al.* (2008) conducted a series of 2D model testes to investigate soil arching in pile-supported embankments with or without geosynthetic reinforcements. Model tests showed that settlements were influenced significantly by embankment height, cap beam width and net spacing. Chen *et al.* (2005a) studied the mechanism and settlement of pile-supported embankments with floating piles and end-bearing piles.

In this paper, a semi-analytical method is proposed for pile-supported embankment analysis. The arching effect is taken into account in the interaction between embankment fill, pile, pile cap, and foundation soil. The consolidation of the foundation soil is calculated by Terzaghi's theory. The settlements of some key points such as the pile top, pile toe, and the foundation soil surface at the elevation of the pile top are analyzed. By comparing two kinds of embankment cases, the distributions of axial force and skin friction of pile are also presented.

2 Mathematical model for pile-supported embankment

2.1 Modeling of arching effect in the embankment

The modeling of arching effect in the embankment is described by the plan of pile arrangement (Fig. 1a) and the profile of a pile-supported embankment (Fig. 2). In the model, the pile (with the pile cap), the foundation soil and the embankment fill are taken into consideration and it is assumed that the compression deformations are only in the vertical direction. The piles are arranged in a square pattern and the pile spacing is S_a. The geometry of the pile includes the cross-sectional area A_p, the diameter D_p, and the length L. The area of the rigid pile cap is A_i and the thickness is δ. In the analytical model, the embankment fill above the pile cap is simplified to an inner column with a diameter of D_i and the cross-sectional area of A_i, which is equal to the area of the pile cap. A hollow outer column (Fig. 1b) of embankment fill is defined within the area $S_a \times S_a$, with an inner diameter of D_i and outer diameter of D_o and the cross-sectional area of the outer column is A_o.

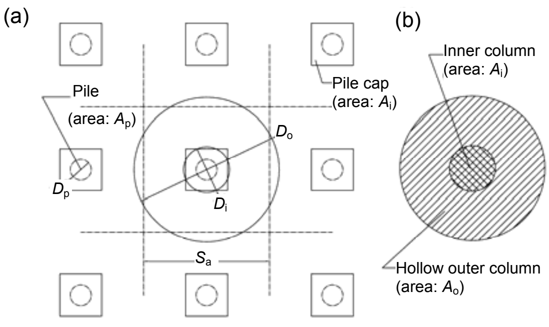

Fig. 1 Plan of pile arrangement (a) and the inner and hollow outer column soil model (b)

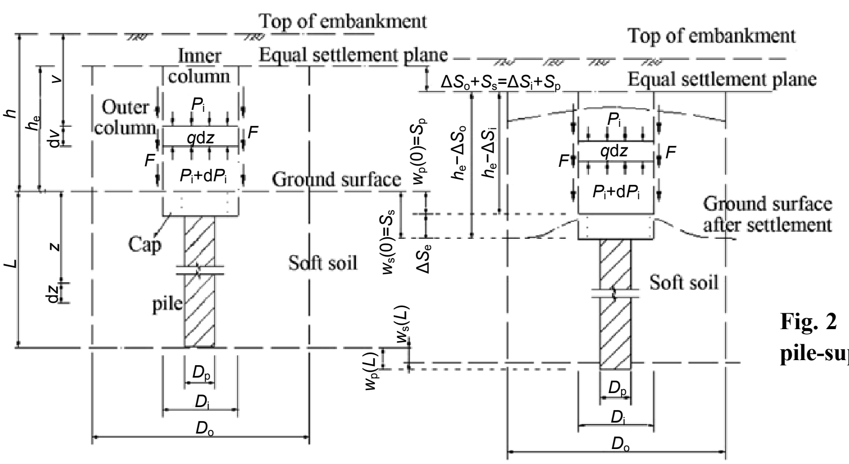

Fig. 2 Analytical model for the pile-supported embankment

The settlement of the pile is less than that of the foundation soil at the top surface of the pile cap. Therefore, the relative movement would occur at the interface between the inner and outer columns (Fig. 2). Shear stresses act downwards on the surface of the inner column but upwards to the outer column. Taking the surface of the embankment as datum level and the downward direction as positive, the equilibrium of the unit element with the thickness of dv can be obtained in the vertical direction by

$$A_\mathrm{i}\mathrm{d}P_\mathrm{i} = (\gamma A_\mathrm{i} + \pi D_\mathrm{i} F)\mathrm{d}v, \quad F=fP_\mathrm{i}K_0, \tag{1}$$

where K_0 is the coefficient of at-rest earth pressure, $K_0=1-\sin\varphi$, f is the coefficient of friction on the interface between the inner and outer columns and $f=\tan\varphi$, φ is the friction angle of the embankment fill, $P_\mathrm{i}(v)$ is the pressure of the inner column at the elevation v, and dP_i is the increment.

On the plane of equal settlement (Fig. 2) $v=h-h_\mathrm{e}$, the pressure is described as $P_\mathrm{i}=\gamma(h-h_\mathrm{e})$. Integrating Eq. (1) from $h-h_\mathrm{e}$ to v yields:

$$P_\mathrm{i}(v) = \frac{\gamma D_\mathrm{i}}{4fK_0}\left[\exp\left(4fK_0\frac{v-h+h_\mathrm{e}}{D_\mathrm{i}}\right)-1\right] + \gamma(h-h_\mathrm{e})\exp\left(4fK_0\frac{v-h+h_\mathrm{e}}{D_\mathrm{i}}\right). \tag{2}$$

According to the force equilibrium in the vertical direction:

$$mP_\mathrm{i}(v) + (1-m)P_\mathrm{o}(v) = \gamma v, \tag{3}$$

where $m=A_\mathrm{i}/(A_\mathrm{o}+A_\mathrm{i})$ and $P_\mathrm{o}(v)$ is the stress of the outer column at the elevation v. Let $P_\mathrm{i}(h)$ denote the pressure of inner column at $z=h$ (i.e., the stress at the top of the pile cap). The pressure P_p transfers from the pile cap to the pile top can be expressed as $P_\mathrm{p}A_\mathrm{p}=P_\mathrm{i}(h)$ when the weight of the pile cap is ignored. Hence, the proportion of the load acting on the pile is defined as

$$n = \frac{P_\mathrm{p}A_\mathrm{p}}{\gamma h(A_\mathrm{o}+A_\mathrm{i})} = \frac{P_\mathrm{i}(h)A_\mathrm{i}}{\gamma h(A_\mathrm{o}+A_\mathrm{i})} = \frac{mP_\mathrm{i}(h)}{\gamma h}. \tag{4}$$

According to the assumption of the plane of equal settlement (Terzaghi, 1943), there is no differential settlement at the plane of the equal settlement between the inner and outer columns. Hence,

$$\Delta S_\mathrm{i} + S_\mathrm{p} = \Delta S_\mathrm{o} + S_\mathrm{s}, \tag{5}$$

where $\Delta S_\mathrm{i} = \int_{h-h_\mathrm{e}}^{h}\frac{P_\mathrm{i}(v)}{E_\mathrm{c}}\mathrm{d}v$ and $\Delta S_\mathrm{o} = \int_{h-h_\mathrm{e}}^{h}\frac{P_\mathrm{o}(v)}{E_\mathrm{c}}\mathrm{d}v$ are the compressions of the inner and outer columns, respectively; E_c is the modulus of the embankment fill; S_p and S_s denote the settlements of the pile cap and foundation soil, respectively. Therefore, the differential settlement S_e can be obtained:

$$\begin{aligned} S_\mathrm{e} &= \Delta S_\mathrm{i} - \Delta S_\mathrm{o} = S_\mathrm{s} - S_\mathrm{p} = \int_{h-h_\mathrm{e}}^{h}\frac{P_\mathrm{i}(v)-P_\mathrm{o}(v)}{E_\mathrm{c}}\mathrm{d}v \\ &= \left(1+\frac{A_\mathrm{i}}{A_\mathrm{o}}\right)\left\{\frac{\gamma D_\mathrm{c}}{4fK_0E_\mathrm{c}}\left[\left(\frac{D_\mathrm{c}}{4fK_0}+h-h_\mathrm{e}\right)\right.\right. \\ &\quad \left.\left.\cdot\left(\exp\left(\frac{4fK_0h_\mathrm{e}}{D_\mathrm{c}}\right)-1\right)-h_\mathrm{e}\right] - \frac{\gamma h_\mathrm{e}(2h-h_\mathrm{e})}{2E_\mathrm{c}}\right\}. \end{aligned} \tag{6}$$

2.2 Consolidation of foundation soil

The consolidation of the foundation soil is simplified as 1D consolidation under uniform surcharge, as shown in Fig. 3. The surface of the pile cap is taken as datum level and the downward direction as positive.

The displacement of the surrounding foundation soil of the pile is

$$w_\mathrm{s}(z,t) = \int\frac{\sigma'_\mathrm{s}(z,t)}{E_\mathrm{s}}\mathrm{d}z, \tag{7}$$

where $\sigma'_\mathrm{s}(z, t)$ denotes the effective stress of soil in the vertical direction at elevation z and time t.

It is known that the load of the embankment is supported by the pile, the skeleton of the soil, and the pore water pressure. Hence, the soil effective stress at time t can be expressed as

$$\sigma'_\mathrm{s}(z,t) = \left(\gamma HS_\mathrm{a}^2 - \sigma_\mathrm{p}(z,t)A_\mathrm{p} - u(z,t)A_\mathrm{s}\right)/A_\mathrm{s}, \tag{8}$$

where $A_\mathrm{s}=S_\mathrm{a}\times S_\mathrm{a}-A_\mathrm{p}$ is the area of the soil within the area $S_\mathrm{a}\times S_\mathrm{a}$.

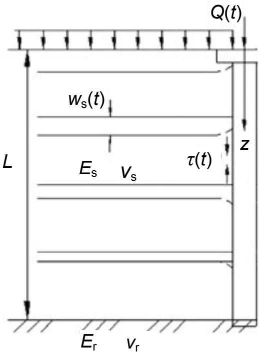

Fig. 3 Model for consolidation of foundation soil and pile-soil interatiocn

The consolidation of the foundation soil among the piles can be calculated based on the classic Terzaghi 1D consolidation theory (Terzaghi, 1943):

$$\left(\frac{\partial u}{\partial t}\right)=C_v\left(\frac{\partial^2 u}{\partial z^2}\right). \tag{9}$$

Chen *et al*. (2005b) have derived the differential quadrature method (DQM) solution for 1D nonlinear consolidation in multi-layered soils.

2.3 Load transfer model

A unit element with the thickness of dz in the pile is taken into consideration and the equilibrium of the force in the vertical direction can be written as

$$\frac{\mathrm{d}^2 w_\mathrm{p}(z,t)}{\mathrm{d}z^2}=-\frac{2\pi r_0}{E_\mathrm{p}A_\mathrm{p}}\tau(z,t), \tag{10}$$

where $w_\mathrm{p}(z, t)$ is the displacement of the pile at depth z and time t, E_p is the modulus of pile, r_0 is the radius of the pile, and $\tau(z, t)$ is the shear stress at depth z and time t.

The relationship between the shear stress $\tau(z, t)$ and the differential displacement of pile relative to the foundation soil $w_\mathrm{p}(z, t)-w_\mathrm{s}(z, t)$ can be written as

$$\tau(z,t)=\frac{w_\mathrm{p}(z,t)-w_\mathrm{s}(z,t)}{\dfrac{1}{k_\mathrm{s}}+R_\mathrm{fs}\dfrac{\left|w_\mathrm{p}(z,t)-w_\mathrm{s}(z,t)\right|}{\tau_\mathrm{f}}}, \tag{11}$$

where $w_\mathrm{s}(z,t)=\int\frac{\sigma'_\mathrm{s}(z,t)}{E_\mathrm{s}}\mathrm{d}z$ denotes the displacement of the surrounding foundation soil; R_fs is ratio of the critical shear stress τ_ult to shear strength τ_f at the interface between the pile and soil; $R_\mathrm{fs}=1$ (i.e., $\tau_\mathrm{ult}=\tau_\mathrm{f}$) is typically chosen; k_s is the initial stiffness at the pile-soil interface.

The differential settlement makes contribution to the toe resistance of the pile; however, the total settlement of the foundation soil below the pile toe has no influence on it. It is assumed herein that the end-bearing soil has elastic-plastic behavior (Fig. 4). The relationship between the pile toe resistance and the differential settlement can be expressed as

$$q_\mathrm{p}=\begin{cases}k_\mathrm{p}\left(w_\mathrm{p}(L)-w_\mathrm{s}(L)\right), & w_\mathrm{p}(L)-w_\mathrm{s}(L)\le\delta_\mathrm{u},\\ q_\mathrm{u}, & w_\mathrm{p}(L)-w_\mathrm{s}(L)>\delta_\mathrm{u},\end{cases} \tag{12}$$

where k_p is the stiffness of the soil around the pile toe, q_u is the critical toe resistance, $w_\mathrm{p}(L)-w_\mathrm{s}(L)$ is the total differential settlement at the pile toe, and $\delta_\mathrm{u}=q_\mathrm{u}/k_\mathrm{p}$ is the critical toe displacement.

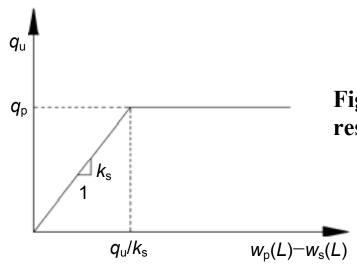

Fig. 4 Pile toe resistance model

2.4 Boundary and compatibility conditions

The settlement of the pile at the top surface of the pile cap is

$$S_\mathrm{p}=w_\mathrm{p}(0,t). \tag{13}$$

The settlement of the foundation soil at the top surface of the pile cap is

$$S_\mathrm{s}=w_\mathrm{s}(0,t). \tag{14}$$

The differential settlement between the pile and the foundation soil at the top surface of pile cap is

$$w_\mathrm{s}(0,t)-w_\mathrm{p}(0,t)=S_\mathrm{e}. \tag{15}$$

The boundary condition at the pile toe can be expressed as

$$E_{\mathrm{p}}\frac{\mathrm{d}w_{\mathrm{p}}(z)}{\mathrm{d}z}\bigg|_{z=L}=\begin{cases}-k_{\mathrm{p}}\left(w_{\mathrm{p}}(L)-w_{\mathrm{s}}(L)\right), \\ \qquad w_{\mathrm{p}}(L)-w_{\mathrm{s}}(L)\le q_{\mathrm{u}}/k_{\mathrm{p}}, \\ -q_{\mathrm{u}},\ \ w_{\mathrm{p}}(L)-w_{\mathrm{s}}(L)> q_{\mathrm{u}}/k_{\mathrm{p}}.\end{cases} \tag{16}$$

3 Semi-analysis solution

The solution can be obtained by combining the governing Eqs. (2) and (6)–(12) with the boundary and compatibility conditions Eqs. (13)–(16). The procedure for the solution is briefly described below.

At the beginning when the embankment load is applied on the foundation soil, the initial conditions are that the stress of the pile at depth i is $\sigma_{\mathrm{p}}(i, t_0)$ and the pressure on the foundation soil is supported by pore water pressure $u(i, t_0)$. Then at time t_1, with the time increment Δt, the variables such as the differential settlement $\Delta w(t_1)$, the effective stress of the foundation soil $\sigma'_{\mathrm{p}}(i, t_1)$, the settlement of foundation soil $w_{\mathrm{s}}(i, t_1)$ at depth i, the skin friction $\tau(i, t_1)$, and the settlement of pile $w_{\mathrm{p}}(i, t_1)$ at depth i can be obtained by the iterative method. Hence, all variables can be solved in the whole time domain.

4 Case study and discussion

In the case study, the soft soil located in the southeast of China was chosen and the parameters of the embankment fill, foundation soil, pile and pile-soil interface are listed in Tables 1 and 2. Two different types of piles are investigated: end-bearing and floating piles. The thickness of soft soil is 20 m for the embankment with end-bearing piles embedded in the firm soil. The thickness of soft soil is 25 m for the case of embankment with floating piles.

4.1 Axial force and skin friction

The distributions of axial force of the pile versus time for the cases of end-bearing piles and floating piles are shown in Figs. 5a and 5b, respectively. In Fig. 5a, the maximum axial force appears at z=8.5 m in the later stage of consolidation (t=1852 d) for the embankment with end-bearing piles. However, in the case of floating piles (Fig. 5b), the maximum axial force occurs at 3 m in depth after 5556 d of consolidation.

Table 1 Parameters of embankment fill and foundation soil

Parameter	Embankment fill	Soft soil	Firm soil
Height (m)	6.5	20	5
Angle of friction (°)	32	–	30
Cohesion (kPa)	0	–	0
Unit weight (kN/m^3)	21	–	–
Young's modulus (GPa)	15	–	40
Poisson's ratio	0.30	0.35	0.27
Coefficient of permeability (cm/s)	–	1.5×10^{-7}	–
Coefficient of consolidation (cm/s)	–	1.05×10^{-2}	–
Critical toe resistance (MPa)	–	400	5000

Table 2 Parameters of pile and pile-soil interface

Parameter	Value
Pile and cap	
Diameter, d (cm)	40
Thickness, δ (cm)	5
Length, L (m)	20
Density, ρ (kg/m^3)	2500
Young's modulus, E (GPa)	25
Pile spacing, S_{a} (m)	2.5
Cross-sectional area of pile cap, A_{i} (m^2)	1.21
Poisson's ratio, v	0.15
Pile-soil interface	
Critical relative displacement, w_{u} (mm)	2
Angle of friction between pile and soil, ϕ_{m} (°)	10

Fig. 6 shows the distributions of skin friction along the pile at different time. It is found that negative skin friction occurs in the upper part of pile and the neutral point (i.e., zero skin friction point in the pile) moves downward with time in both cases. Fig. 6a shows that the neutral point occurs at z=0.9 m after 31 d of consolidation, and then it moves downward to 8.5 m in depth after 1852 d of consolidation for the case of end-bearing piles. However, in the case of floating piles (Fig. 6b), the location of the neutral point varies in a small range.

4.2 Settlement of the pile-supported embankment

Trends of settlement with time at some key points for the case of end-bearing piles and floating piles are presented in Figs. 7a and 7b, respectively. The settlements of pile top, pile toe, and foundation soil at the elevation of pile top and pile toe are plotted. In the case of end-bearing piles (Fig. 7a), it is found that the settlements increase with time. At the end of consolidation, the settlements at pile top and pile toe are 5.5 and 4.8 cm, respectively. That is, the pile compression is 0.7 cm. In addition, the foundation soil

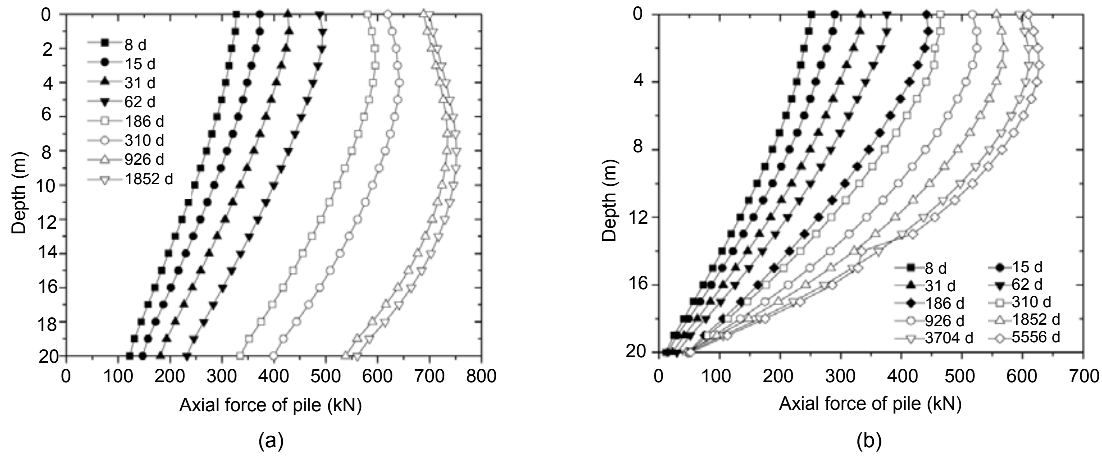

Fig. 5 Variation of axial force along pile for the embankment with end-bearing piles (a) and floating piles (b)

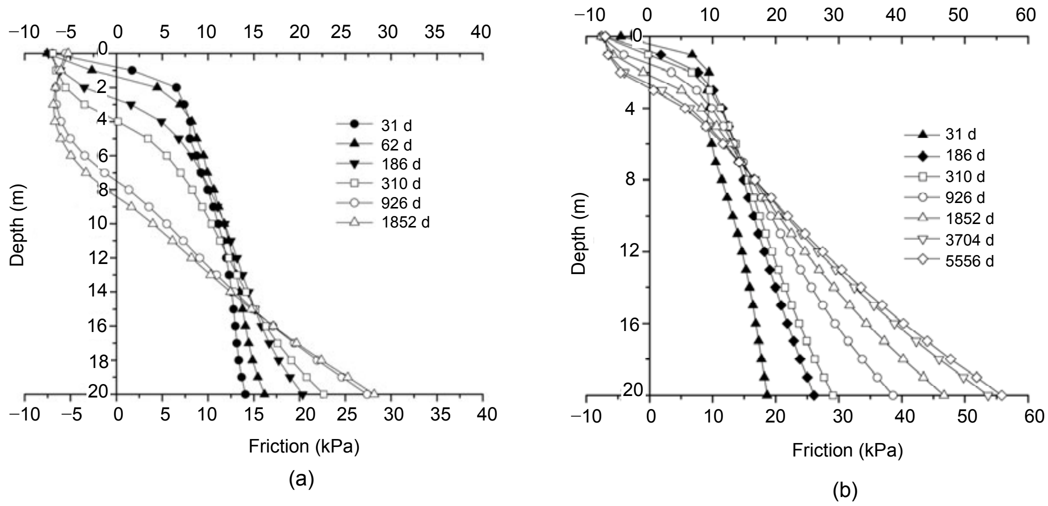

Fig. 6 Variation of skin friction along pile for the embankment with end-bearing piles (a) and floating piles (b)

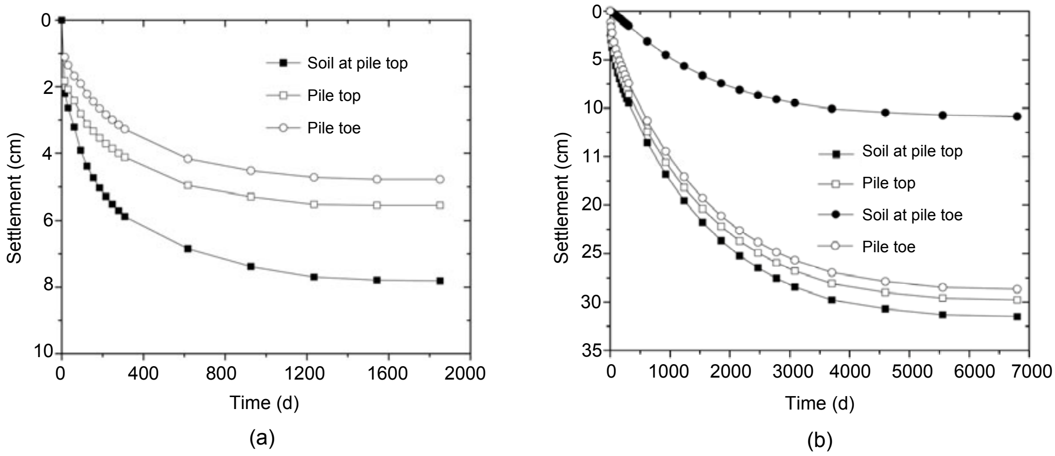

Fig. 7 Settlements vs. time for the embankment with end-bearing piles (a) and floating piles (b)

settles 7.8 cm at the end of consolidation. The differential settlement between pile and soil at the pile top is 2.3 cm.

It takes a much longer time for soil consolidation in the case of floating piles, compared with the case of end-bearing piles. Fig. 7b shows that at the end of consolidation, the settlements at pile top and pile toe are 29.8 and 28.7 cm, respectively. That is, the pile compression is 1.1 cm. The soil settlements at pile top and pile toe are 31.5 and 10.9 cm, respectively. The differential settlement between pile and soil at pile top is 1.7 cm, which is smaller than that in the case of end-bearing piles.

5 Conclusions

A semi-analytical method is proposed for the analysis of pile-supported embankment. Both the effect of soil arching and the interaction between the embankment fill, pile and foundation soil are considered in the mathematical model. The consolidation of foundation soil is calculated by Terzaghi's 1D consolidation theory. A case study is presented for the embankment with two different types of piles: floating and end-bearing piles. The distributions of axial force and skin friction at different consolidation time are plotted. The settlements at pile top, pile toe and foundation soil are presented. It is found that the neutral point moves down to 8.5 m in depth after 1852 d in the case of end-bearing piles, while the location of neutral point varies in a small range in the case of floating piles during consolidation. It is also found that it takes a longer time for soil consolidation in the embankment with floating piles, compared with the case using end-bearing piles. The differential settlement between the pile and surrounding soil at the pile top is larger for the embankment with end-bearing piles, compared with the case of floating piles.

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