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Effect of segments of soil-water characteristic curves on the estimated permeability function using statistical methods[*#]

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1 Introduction

A large amount of evidence indicates that rainwater infiltration is the most important and most common environmental factor affecting slope stability (Rahardjo et al., 2010). Water flows through an unsaturated soil are governed mainly by its coefficient of permeability, which is highly dependent on the soil suction. There are two approaches to obtain the coefficient of permeability of an unsaturated soil: direct measurement and an indirect method which uses estimation techniques from the soil-water characteristic curve (SWCC). Direct measurement provides the most accurate and reliable data, while the indirect method provides relatively accurate results with certain errors. However, the direct measurement of the coefficient of permeability of unsaturated soils is usually time consuming and expensive, especially in the case of low saturation (or a high suction range). Fredlund and Rahardjo (1993) and Fredlund et al. (2012) noted that the estimation procedures adopted in practical engineering had found increasing acceptance because the costs of direct measurement were prohibitively high for most projects, and the results from estimation techniques were sufficient for engineering design purposes. As a result, various models for estimating the permeability function of unsaturated soil have been proposed by different researchers (Childs and Collis-George, 1950; Gardner, 1958; Brooks and Corey, 1964; Kunze et al., 1968; van Genuchten, 1980; Mualem, 1986; Fredlund et al., 1994; Zhai and Rahardjo, 2015; Zhai et al., 2019). By comparing the estimated results and theoretical sophistication of different estimation models, Mualem (1986) categorized these models into three groups: empirical models, macroscopic models, and statistical models, and concluded that statistical models were the most rigorous.

In the statistical models, there are two approaches for dividing the SWCC into a certain number of segments: (i) even division within the domain of the volumetric water content, as recommended by Kunze et al. (1968), and (ii) even division within the domain of soil suction, as recommended by Zhai and Rahardjo (2015). However, the effects of the segments of a divided SWCC (i.e. the procedure of the division of SWCC segments and the number of segments) on the estimated permeability function have not been extensively discussed.

In this study, both approaches as recommended by Kunze et al. (1968) and Zhai and Rahardjo (2015) were adopted to investigate the effects of SWCC segments on the estimated permeability function of soil. Based on the analyzed results, a minimum number of 40 SWCC segments is recommended for the estimation of the permeability function using these two equations.

The theories of the statistical model and the criterion of the evaluation are provided in Data S1,

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including the equations of both Kunze et al. (1968) and Zhai and Rahardjo (2015).

2 Effects of SWCC segments on the estimated permeability function

Two sets of soil data, including those from soils with unimodal and bimodal SWCCs, were selected for investigating the effects of SWCC segments on the estimated permeability function. The selected unimodal SWCCs of volcanic sand and glass beads from Brooks and Corey (1964) and fine sand from Gonzalez and Adams (1980) are illustrated in Fig. 1a. The bimodal SWCCs of K50S50 (50% sand mixed with 50% kaolin) and K50S50 with 9% mica (50% sand mixed with 50% kaolin and an additional 9% mica) from Zhai et al. (2017) are illustrated in Fig. 1b. No index properties were reported by Brooks and Corey (1964) nor Gonzalez and Adams (1980), therefore only the index properties of K50S50 and K50S50 with 9% mica are shown in Table 1. The experimental data of the relative coefficient of permeability for these soils with either unimodal or bimodal SWCCs are illustrated in Figs. 2a and 2b, respectively. The experimental data for the unimodal SWCCs were best fitted with Fredlund and Xing (1994)'s equation following procedures as recommended by Zhai and Rahardjo (2012), and the best fitted parameters are shown in Table 2. The experimental data for the

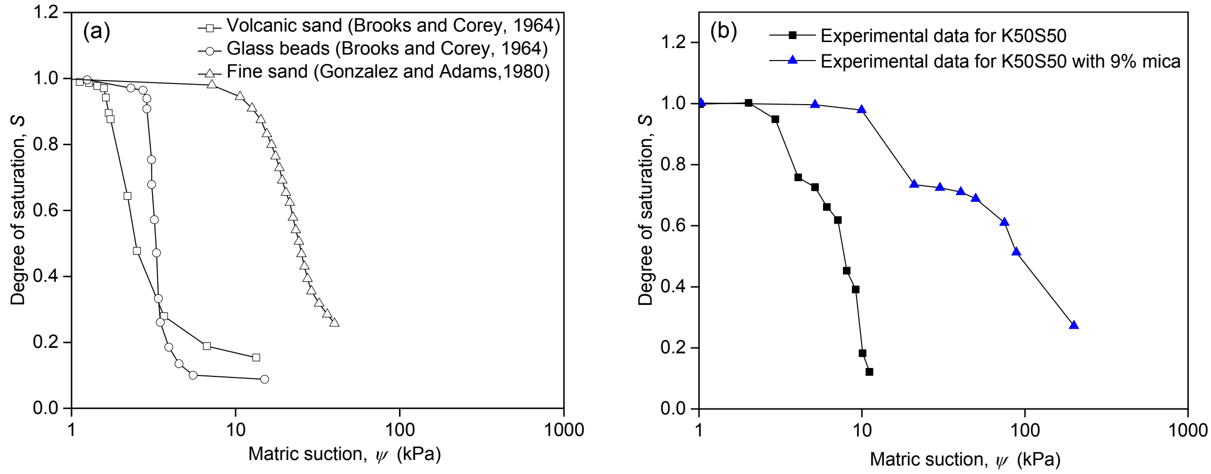

Fig. 1 SWCCs of selected soils
(a) Unimodal SWCCs for volcanic sand, glass beads, and fine sand; (b) Bimodal SWCCs for mixtures of sand and kaolin

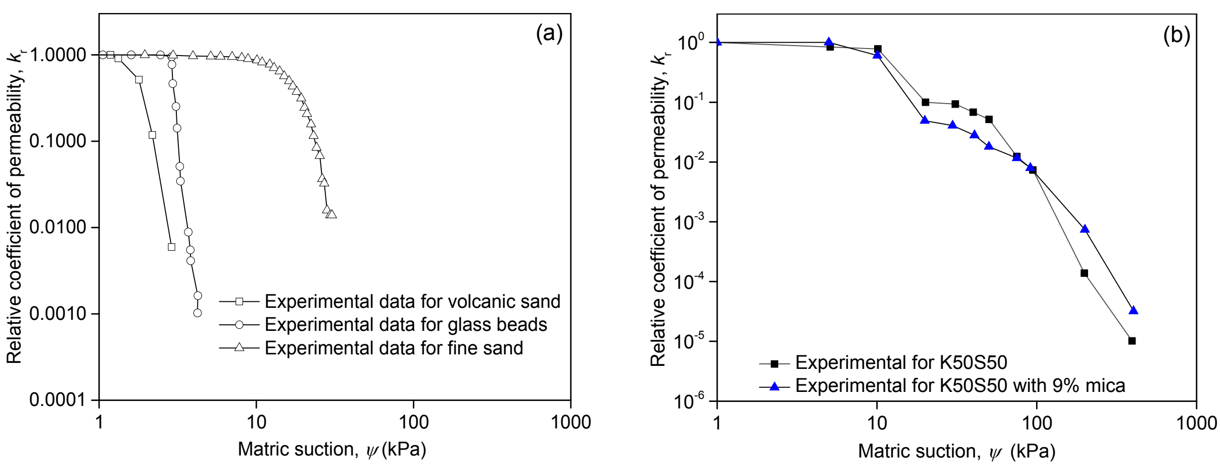

Fig. 2 Measured relative coefficients of permeability of selected soils
(a) Volcanic sand, glass beads, and fine sand; (b) Mixtures of sand and kaolin

bimodal SWCCs were also best fitted with Fredlund and Xing (1994)'s equation following procedures as recommended by Zhai et al. (2017), and the best fitted parameters are illustrated in Table 3.

The fitting parameters as illustrated in Tables 2 and 3 were subsequently used for estimating the relative coefficients of permeability using the equations of both Kunze et al. (1968) and Zhai and Rahardjo (2015). The estimated results from adopting different numbers of SWCC segments (i.e. 10, 20, 30, 50, and 80) using these two equations are illustrated in Figs. 3–7. The performance of estimation models with different numbers of SWCC segments was evaluated using the coefficient of determination, R^2. The results are shown in Fig. 8 (p.631).

Figs. 3–7 indicate that Kunze et al. (1968)'s model performs as well as Zhai and Rahardjo (2015)'s model in the high suction range. However, Kunze et al. (1968)'s model gives only a rough estimation in the low suction range. As explained, the first segment from using Kunze et al. (1968)'s method covers a much wider suction range than other segments. As a result, the representation of the big pores (which correspond to low suction) using that model becomes rough, which makes the estimation of the permeability function in the low suction range also rough. The accuracy of estimation results can be improved by increasing the numbers of SWCC segments.

Fig. 8 shows that R^2 decreases with a decrease in the number of SWCC segments both for soils with unimodal and bimodal SWCCs. Fig. 8a indicates, for a unimodal SWCC, that R^2 for the estimation with 30 pieces of segments is very close to the R^2 for the estimation with 100 pieces of segments. This indicates that 30 pieces of segments are enough for the estimation for a unimodal SWCC using both the methods of Kunze et al. (1968) and Zhai and Rahardjo (2015). Similarly, Fig. 8b indicates that 40 pieces of segments are sufficient for the estimation for a bimodal SWCC using both methods. Consequently, a minimum of 40 pieces of segments are recommended for the estimation of the permeability function using the statistical methods.

Table 1 Index properties of K50S50 and K50S50 with 9% mica

Index property	Description	
	K50S50	K50S50 with 9% mica
Dry density, ρ_d ($\times10^3$ kg/m^3)	1.75	1.77
Water content, w (%)	12.1	16.0
Void ratio, e	0.48	0.47
Liquid limit, LL (%)	46.7	49.6
Plastic limit, PL (%)	27.4	31.9
Plasticity index, PI (%)	19.3	18.7
Specific gravity, G_s	2.59	2.61
GSD–sand (%)	50	59.8
GSD–silt (%)	37.5	29.1
GSD–clay (%)	12.5	11.1
Unified soil classification system (USCS)	SM-ML	SM

GSD represents grain-size distribution data; SM-ML represents silty sand with low plasticity

Table 2 Fitting parameters of unimodal SWCCs

Soil	a_f (kPa)	n_f	m_f	C_r (kPa)
Volcanic sand	1.88	9.44	0.67	1500
Glass beads	3.03	28.97	0.82	1500
Fine sand	18.97	4.23	1.26	1500

a_f, n_f, m_f are fitting parameters in Fredlund and Xing (1994)'s equation; C_r is the input parameter, a rough estimation of the residual suction

3 Conclusions

The principles of the estimation of the permeability function from SWCCs using statistical methods

Table 3 Fitting parameters of bimodal SWCCs in Fredlund and Xing (1994)'s equation

Soil	S_{s1}	a_1 (kPa)	n_1	m_1	S_{s2}	a_2 (kPa)	n_2	m_2
K50S50	1	10.59	14.96	0.14	0.71	71.40	5.45	0.59
K50S50 with 9% mica	1	12.31	3.97	0.35	0.75	51.44	5.05	0.72

S_{s1} and S_{s2} are the initial degree of saturation for the first and second segments of bimodal SWCC, respectively; a_1, n_1, m_1, a_2, n_2, and m_2 are fitting parameters in Fredlund and Xing (1994)'s equation for the bimodal SWCC

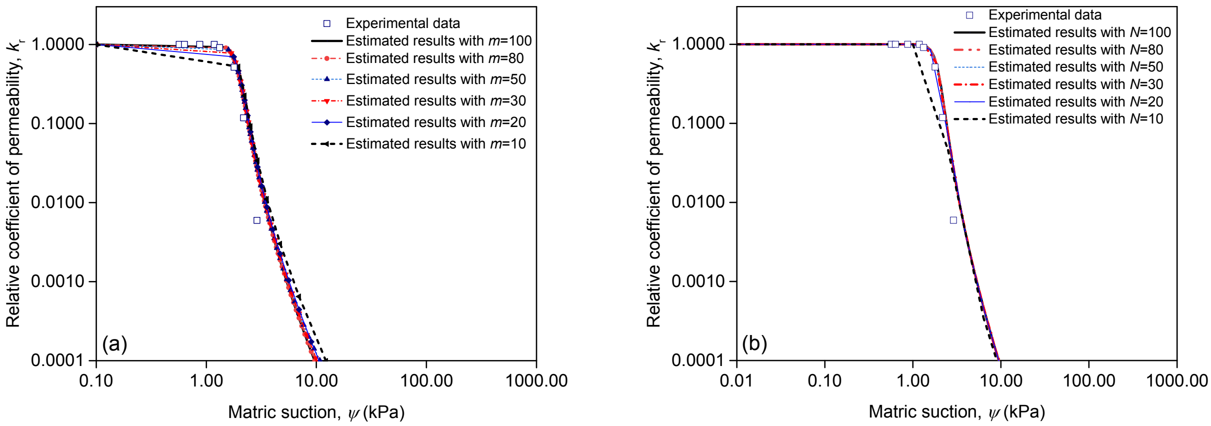

Fig. 3 Comparison of estimated and measured relative coefficients of permeability for volcanic sand with different numbers of segments (*m* or *N* is the number of SWCC segments)
(a) Volcanic sand using Kunze et al. (1968)'s method; (b) Volcanic sand using Zhai and Rahardjo (2015)'s method

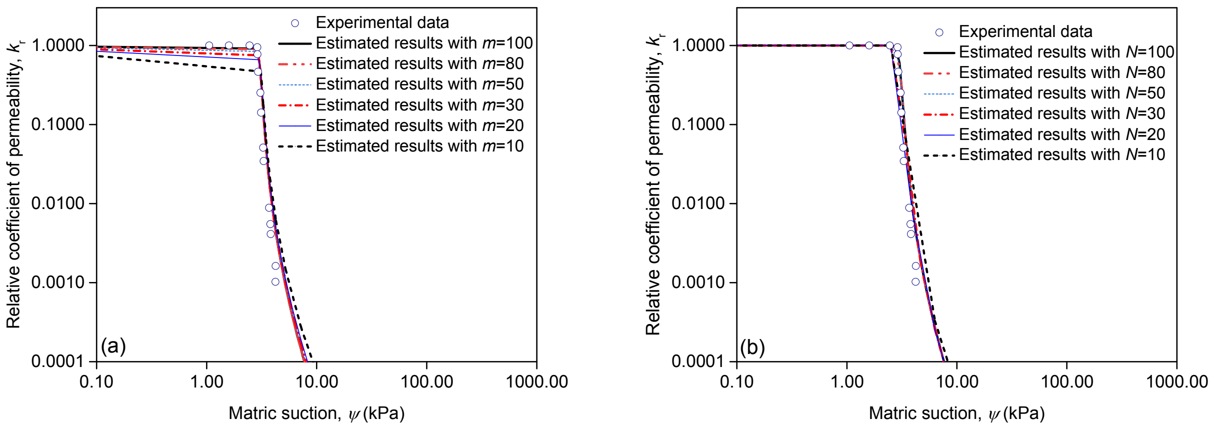

Fig. 4 Comparison of estimated and measured relative coefficients of permeability for glass beads with different numbers of segments
(a) Glass beads using Kunze et al. (1968)'s method; (b) Glass beads using Zhai and Rahardjo (2015)'s method

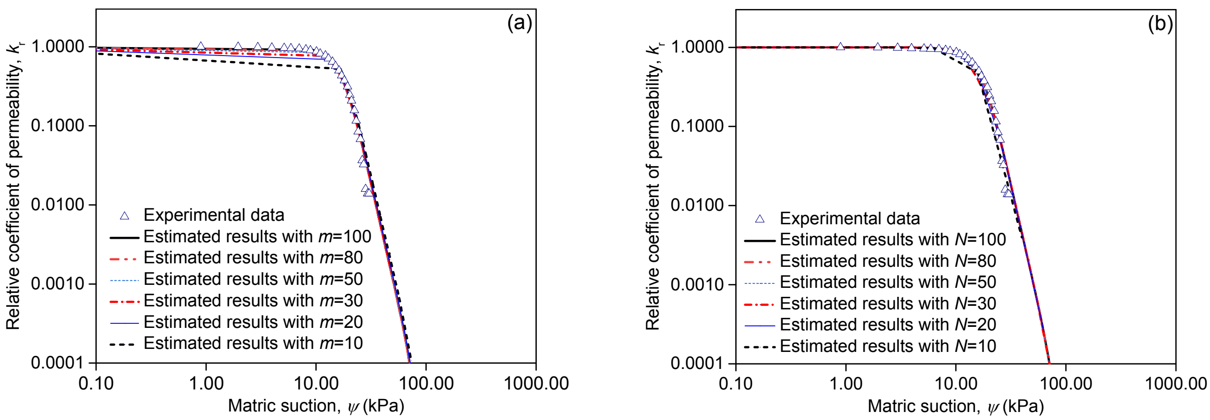

Fig. 5 Comparison of estimated and measured relative coefficients of permeability for fine sand with different numbers of segments
(a) Fine sand using Kunze et al. (1968)'s method; (b) Fine sand using Zhai and Rahardjo (2015)'s method

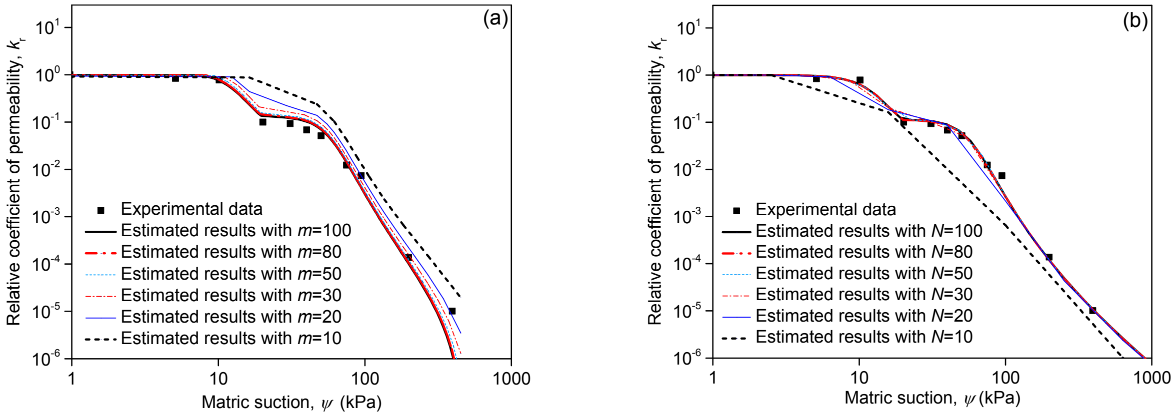

Fig. 6 Comparison of estimated and measured relative coefficients of permeability for K50S50 with different numbers of segments

(a) K50S50 using Kunze et al. (1968)'s method; (b) K50S50 using Zhai and Rahardjo (2015)'s method

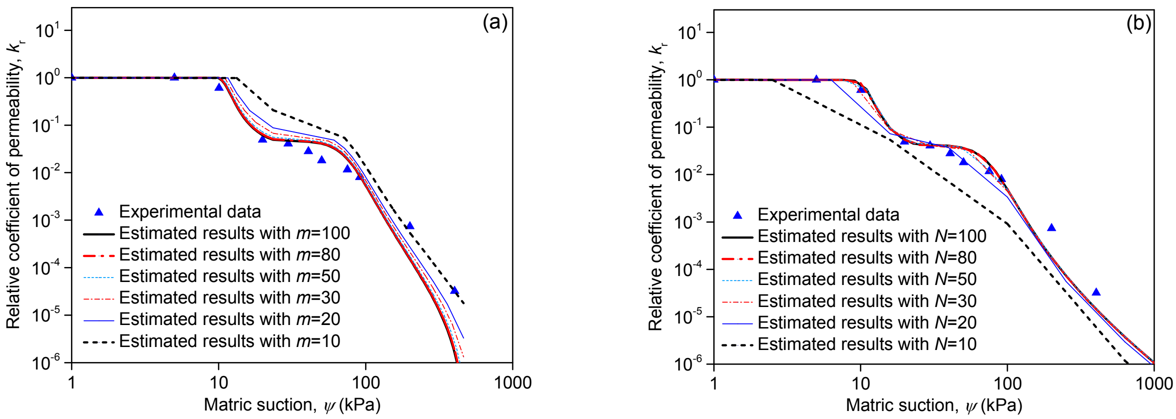

Fig. 7 Comparison of estimated and measured relative coefficients of permeability for K50S50 with 9% mica with different numbers of segments

(a) K50S50 with 9% mica using Kunze et al. (1968)'s method; (b) K50S50 with 9% mica using Zhai and Rahardjo (2015)'s method

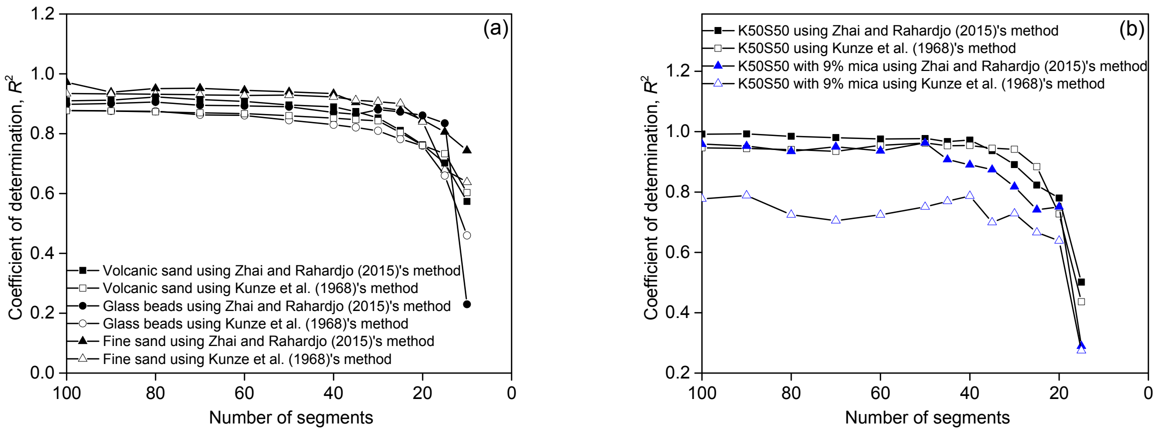

Fig. 8 Illustration of the degree of curve matching between estimated and measured relative coefficients of permeability for soils with unimodal (a) and bimodal (b) SWCCs

are explained. Analysis of the estimation results for five types of soil, including both unimodal and bimodal SWCCs, showed that the effects of the SWCC segments on the estimated results were significant if the number of segments was less than 40. As a result, a minimum number of 40 is recommended for the estimation. We also observed that the methodology of the division of the SWCC in the domain of volumetric water content gives only a rough estimation of the relative coefficient of permeability in the low suction range. Therefore, it is recommended that the SWCC should be divided within the domain of the matric suction rather than the volumetric water content for estimating the permeability function using statistical methods.

Contributors

Qian ZHAI and Chen-feng ZHANG wrote the first draft of the manuscript. Qian ZHAI, Chen-feng ZHANG, Guo-liang DAI, and Xue-liang ZHAO revised and edited the final version of the manuscript.

Conflict of interest

Qian ZHAI, Chen-feng ZHANG, Guo-liang DAI, and Xue-liang ZHAO declare that they have no conflict of interest.

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List of electronic supplementary materials

Data S1 Statistical models and evaluation criterion.

中文概要

题　目： 水-土特征曲线的分段对估测非饱和渗透系数的影响

目　的： 在采用统计法估测非饱和渗透系数的过程中，探讨水-土特征曲线分段的不同方法和不同数量对

估测结果的影响。
创新点： 比较在体积含水量区间和土吸力区间分割水-土特征曲线对估测非饱和渗透系数的影响；最终建议在土吸力区间分割水-土特征曲线。
方　法： 采用现有模型和数学公式，对比不同类型土质（单峰和双峰水-土特征曲线）估测和实验测量的非饱和渗透系数。
结　论： 1. 传统的在体积含水量区间分割水-土特征曲线会造成对低吸力区域的非饱和渗透系数的估测不够精确。2. 综合比较单峰和双峰的水-土特征曲线及 2 种不同分割水-土特征曲线的方法，发现在分段大于 40 片时，估测结果非常接近实验结果。因此，在采用统计法估测土体的非饱和渗透系数时，建议在土吸力区间对水-土特征曲线进行分割，且分割数量应该大于 40。

关键词： 渗透方程；统计法；间接法；水-土特征曲线