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Chimera state in a network of nonlocally coupled impact oscillators*

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Abstract: Chimera state is a peculiar spatiotemporal pattern, wherein the coherence and incoherence coexist in the network of coupled identical oscillators. In this paper, we study the chimera states in a network of impact oscillators with nonlocal coupling. We investigate the effects of the coupling strength and the coupling range on the network behavior. The results reveal the emergence of the chimera state for significantly small values of coupling strength, and higher coupling strength values lead to unbounded motions in the oscillators. We also study the network in the case of excitation failure. We observe that the coupling helps in the maintenance of an oscillatory motion with a lower amplitude in the failed oscillator.

Key words: Mechanical oscillators; Impact oscillator; Coupled network; Nonlocal coupling; Chimera state
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1 Introduction

Recently, the collective behaviors of the networks, such as synchronization (Arenas et al., 2008), spiral waves (Ma et al., 2016; Yao et al., 2017), and chimera states, have attracted the attention of scholars. The chimera state is a special state that is observed in the networks consisting of coupled identical oscillators (Majhi et al., 2016; Bera et al., 2017; Rakshit et al., 2017). The word "chimera" refers to any fantastic thing that is constructed by incongruous components (Abrams and Strogatz, 2004). This state is characterized when the oscillators split into coherent and incoherent groups (Abrams and Strogatz, 2004). Kuramoto and Battogtokh (2002) reported an interesting state with coexisting coherent and incoherent domains in a network of complex Ginzburg-Landau equations, for the first time. Abrams and Strogatz (2004) called this state "chimera." The chimera state is associated with several natural phenomena, including the uni-hemispheric sleep in birds and dolphins, bump states in neural systems, brain pathological states like seizures, or even in social systems (Omelchenko et al., 2015).

Since the discovery of the chimera state, it has been widely investigated in several studies with different oscillators ranging from phase oscillators (Suda and Okuda, 2015; Panaggio et al., 2016) to

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periodic and chaotic maps (Vadivasova et al., 2016), as well as neuronal networks (Wei et al., 2018b; Majhi et al., 2019; Wang et al., 2020). The studies have been done on several network topologies like 2D and 3D networks (Kundu et al., 2019; Liu et al., 2019), small-world networks (Tang et al., 2019), and hyper networks (Bera et al., 2019). There are also some experimental studies on the existence of chimera states. Tinsley et al. (2012) carried out their pioneer experimental study on the coupled chemical oscillators to study chimeras. Their simulations and results revealed different chimera behaviors from those observed in theoretical phase oscillators. Hart et al. (2016) revealed the existence of a chimera state in a minimum network of four chaotic optoelectronic oscillators with global coupling. Bera et al. (2016) found a specific pattern of coexistence of coherence and incoherence, named imperfect traveling chimera state, in a neuronal network with local synaptic gradient coupling. They also observed another type of chimera state with asynchronous spikes and coherent quiescent state (spike chimera) in a hyper network of neurons coupled by bidirectional electrical gap junctions and unidirectional chemical synapses (Bera et al., 2019).

Chaotic systems are of great interest in fields like mathematics (Pham et al., 2014; Wei et al., 2015a, 2015b, 2018a) and electronics (Wei et al., 2016). Many mechanical oscillators can exhibit chaotic behaviors (Wei et al., 2017a, 2017b). Mechanical oscillators have also been of interest from the perspective of chimera states. Martens et al. (2013) experimentally studied mechanical oscillators with hierarchical connectivity. They observed the chimera states as a result of a competition between two antagonistic synchronization patterns. Kapitaniak T et al. (2014) investigated a network of coupled pendula and found a new pattern, called imperfect chimera state, in which some of the pendula escape from the synchronized group of chimera state. Maistrenko et al. (2017) revealed the emergence of chimera states in a small network containing three identical pendulum-like oscillators. Dudkowski et al. (2019) investigated the traveling chimera state in connected self-excited pendulums that were hanged on a horizontally oscillating wheel.

The physical systems, especially the vibrating systems, can have intermittent contact by the collision of components (Blazejczyk-Okolewska and Kapitaniak, 1998; Lee, 2005; Ing et al., 2010). These impacts are relevant in many applications, such as in mechanical devices during the thickening and crushing process, working of minerals, the vibrations of helicopter rotor blades, and the fluctuation between the workpiece and cutting tool (Blazejczyk-Okolewska and Kapitaniak, 1998; Lee, 2005; Banerjee et al., 2009). Thus, the study of the impact dynamics is an important topic in the vibrating systems (Lee and Yan, 2006). In this study, we investigate the network of coupled impact oscillators. The employed impact oscillator is bilinear and has the coexistent attractors. We consider the network to be a 1D ring with nonlocal connections via the nearest neighbors. We study the behavior of the network by varying the coupling parameters: the coupling strength and coupling range. Further, we analyze the network when the excitation of one of the oscillators fails.

2 Model

The oscillator employed in our network is an impact oscillator. Fig. 1 represents its physical model. This model assumes that the discontinuity surface is static and time-independent. The motion of the mass is described by (Wiercigroch and Sin, 1998):

$$\begin{aligned} m\ddot{y} + c\dot{y} + k_1 y &= mA\Omega^2 \sin(\Omega t),\ y < g, \\ m\ddot{y} + c\dot{y} + k_1 y + k_2(y-g) &= mA\Omega^2 \sin(\Omega t),\ y \geq g, \end{aligned} \tag{1}$$

where y is the vertical displacement of the mass m, k_1 and k_2 are the primary and secondary stiffnesses, respectively, t is the time, A and Ω are the amplitude and frequency of the force, respectively, c is the damping, and g is the gap at the first moment. This equation can be nondimensional according to the amplitude of motion $x = y/y_0$ and the time $\tau = \omega_n t$, where $y_0 = 1$ mm is an arbitrary reference distance, and $\omega_n = \sqrt{k_1/m}$. By considering the velocity of the nondimensionalized displacement as $\dot{x} = dx/d\tau = v$, the dimensionless equations of motion of this oscillator are described by (Pavlovskaia et al., 2010)

$$\begin{aligned} \dot{x} &= v, \\ \dot{v} &= a\omega^2 \sin(\omega\tau) - 2\xi v - x - \beta(x-e)H(x-e), \end{aligned} \tag{2}$$

where $\beta = k_2/k_1$ is the stiffness ratio, $e = g/y_0$ is the nondimensional gap, $a = A/y_0$ and $\omega = \Omega/\omega_n$ are the nondimensional forcing amplitude and frequency, respectively, $\xi = c/(2m\omega_n)$ is the damping ratio, and $H(\cdot)$ is the Heaviside function, defined by

$$H(x-e) = \begin{cases} 0, & x < e, \\ 1, & x \geq e. \end{cases}$$

Depending on the values of the parameters, Eq. (2) can possess many coexisting states. We consider the values of the parameters to be $a = 0.7$, $\omega = 0.801$, $\xi = 0.01$, $\beta = 29$, and $e = 1.26$. Therefore, a single oscillator exhibits the coexistence of a period-5 oscillation (Fig. 2b) and a period-1 oscillation (Fig. 2d).

The equations describing the motion of coupled identical oscillators (Eq. (2)) can be given as

$$\begin{aligned} \dot{x}_i &= v_i + \frac{\sigma}{2P} \sum_{j=i-P}^{i+P} (x_j - x_i), \\ \dot{v}_i &= a\omega^2 \sin(\omega\tau) - 2\xi v_i - x_i - \beta (x_i - e) H(x_i - e) \\ &+ \frac{\sigma}{2P} \sum_{j=i-P}^{i+P} (v_j - v_i). \end{aligned} \quad (3)$$

The network consists of $N = 100$ oscillators, which are coupled in a ring, and i denotes the index of each oscillator in the network. Each oscillator is symmetrically coupled with its $2P$ nearest neighbors, and σ is the strength of coupling.

To find different behaviors of the network specific synchronization, many statistical measures such as the statistical factor of synchronization (R) (Wang and Ma, 2018) have been employed. One of the widely employed measures in chimera studies is the strength of incoherence (SI) (Majhi et al., 2019), which characterizes the transition from the asynchronization to the chimera state, and then to the synchronization. We employ this measure for finding the regions of chimera state. To calculate the strength of incoherence, firstly, the x variables are used to obtain new variables by setting $z_i = x_i - x_{i+1}$, then the network is divided into M small square bins of size n, and the standard deviation of each bin is calculated as:

$$\sigma_s(l) = \left\langle \sqrt{\frac{1}{n} \sum_{j=n(l-1)+1}^{ln} [z_j - \langle z \rangle]^2} \right\rangle_t, \quad (4)$$

where $\langle z \rangle = (1/N) \sum_{i=1}^{N} z_i$ and $\langle \dots \rangle_t$ denotes the time average ($l = 1, 2, \dots, M$). Then the strength of incoherence can be obtained by

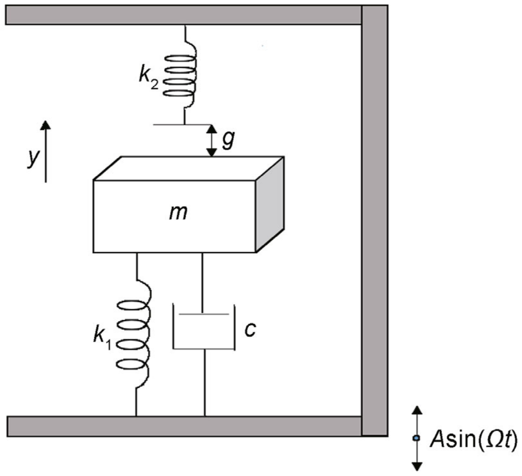

Fig. 1 Physical model of the oscillator (Pavlovskaia et al., 2010)

$$\begin{aligned} \mathrm{SI} &= 1 - \frac{\sum_{l=1}^{M} s_l}{M}, \\ s_l &= H[\delta_0 - \sigma_s(l)], \end{aligned} \quad (5)$$

where δ_0 is a predefined threshold. Finally, the synchronous, chimera, and asynchronous states can be determined by SI = 0, 0 < SI < 1, and SI = 1, respectively.

3 Results

We investigate the network of coupled oscillators for different values of coupling parameters. To solve the network, we have employed random initial conditions and the fourth-order Runge–Kutta method with a time step of 0.02 for $\tau = 5000$ time units.

Firstly, we considered that each oscillator is coupled with its $P = 5$ neighbors. When the coupling strength is very small, the motions of the oscillators are asynchronous. By increasing the coupling strength to $\sigma > 0.0001$, some oscillators tend to be synchronous, and therefore, a coherent group appears in the network. Fig. 3 (p.239) shows the obtained solutions of the network for different values of coupling strength and $P = 5$. The left panel of Fig. 3 shows the space-time plots, and the right panel shows the time snapshots of the positions of the oscillators after the transient time. We observe that when the coupling strength grows to $\sigma = 0.0002$, the chimera state emerges in the network. By further increasing σ, the number of oscillators in the synchronous group increases. Moreover, increasing the coupling

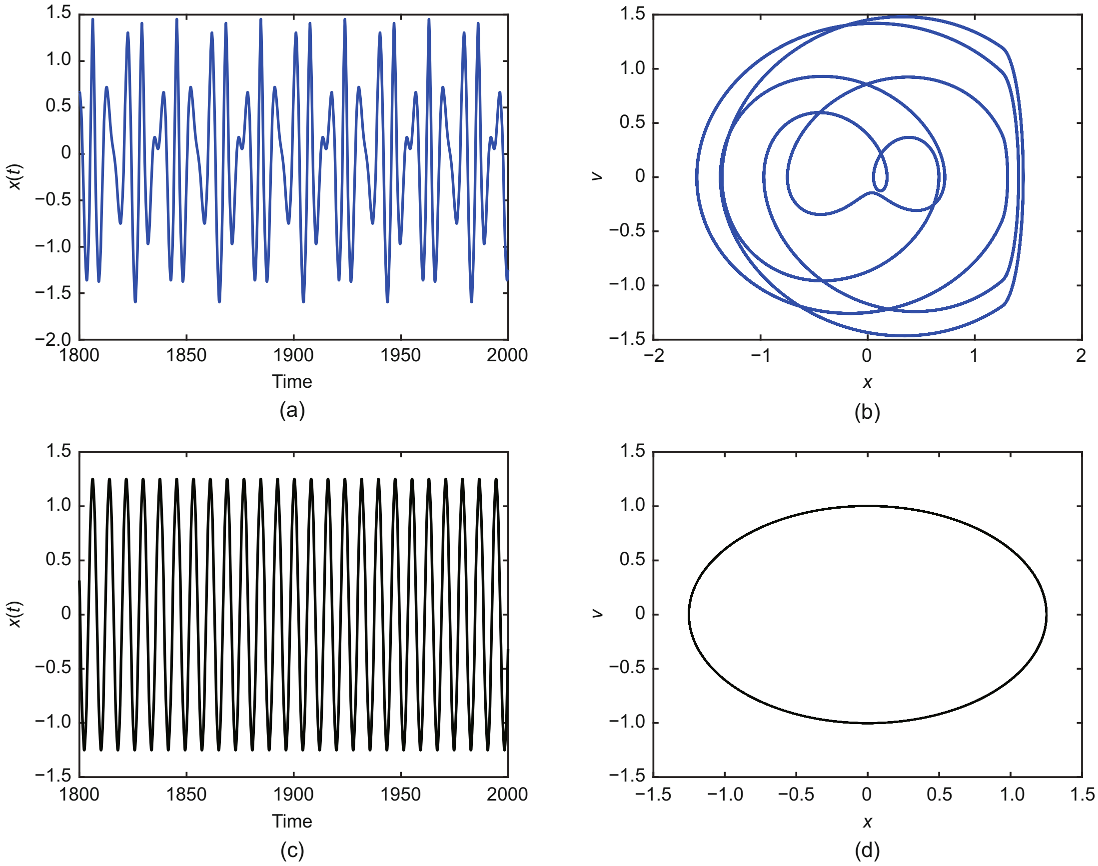

Fig. 2 Coexisting solutions of system (Eq. (2)) for $a = 0.7$, $\omega = 0.801$, $e = 1.26$, $\xi = 0.01$, and $\beta = 29$: (a) time series of the position variable of the oscillator and (b) the corresponding attractor by setting the initial conditions at $(-2, 2)$, showing the period-5 oscillation, (c) time series of the position variable of the oscillator and (d) the corresponding attractor by setting the initial conditions at $(0, 0)$, showing the period-1 oscillation

strength to some values leads to the increment of the width of the coherent groups (Figs. 3b–3d). Further increase of coupling strength causes a change in the observed chimera state, such that the spatial positions of the coherent and incoherent groups are not static in time. This state that is called nonstationary chimera, is observed in Fig. 3e for $\sigma = 0.0009$. If the coupling strength further increases, the oscillators will be unstable, and the positions of the oscillators will increase unboundedly. Fig. 4a (p.240) shows the response of the network by setting σ at 0.001. Fig. 4b shows the position variable of the first oscillator of the network, which is diverging.

As shown in Fig. 2, the parameters of the oscillators were set at values in which a single oscillator shows the coexistence of the period-1 and period-5 attractors. However, when the oscillators are coupled in the network, the coupling effects on their behavior and the attractors change. Fig. 5 (p.240) shows some of the observed attractors of the coupled oscillators. We observe that not only the period-5 attractor with different topologies appears but also the chaotic attractors emerge.

The results of numerical simulations of the network for different values of σ and P are presented in Fig. 6 (p.241) as (P, σ) parameter plane. Fig. 6a demonstrates the observed states of the network for $P = 1, 2, \ldots, 49$. The black, gray, and white colors show the asynchronous oscillations, the chimera state, and the state of divergence of the oscillators, respectively. Clearly, when each oscillator is coupled to the nearest neighboring of $P < 5$, the chimera state is observed in a wide range of coupling strength. When $P > 5$, the chimera state is only observed for minimal values of the coupling strength, and the network becomes unstable quickly. To indicate the range of coupling strength at $5 < P < 49$, where the chimera state occurs, we presented Fig. 6b. Fig. 6

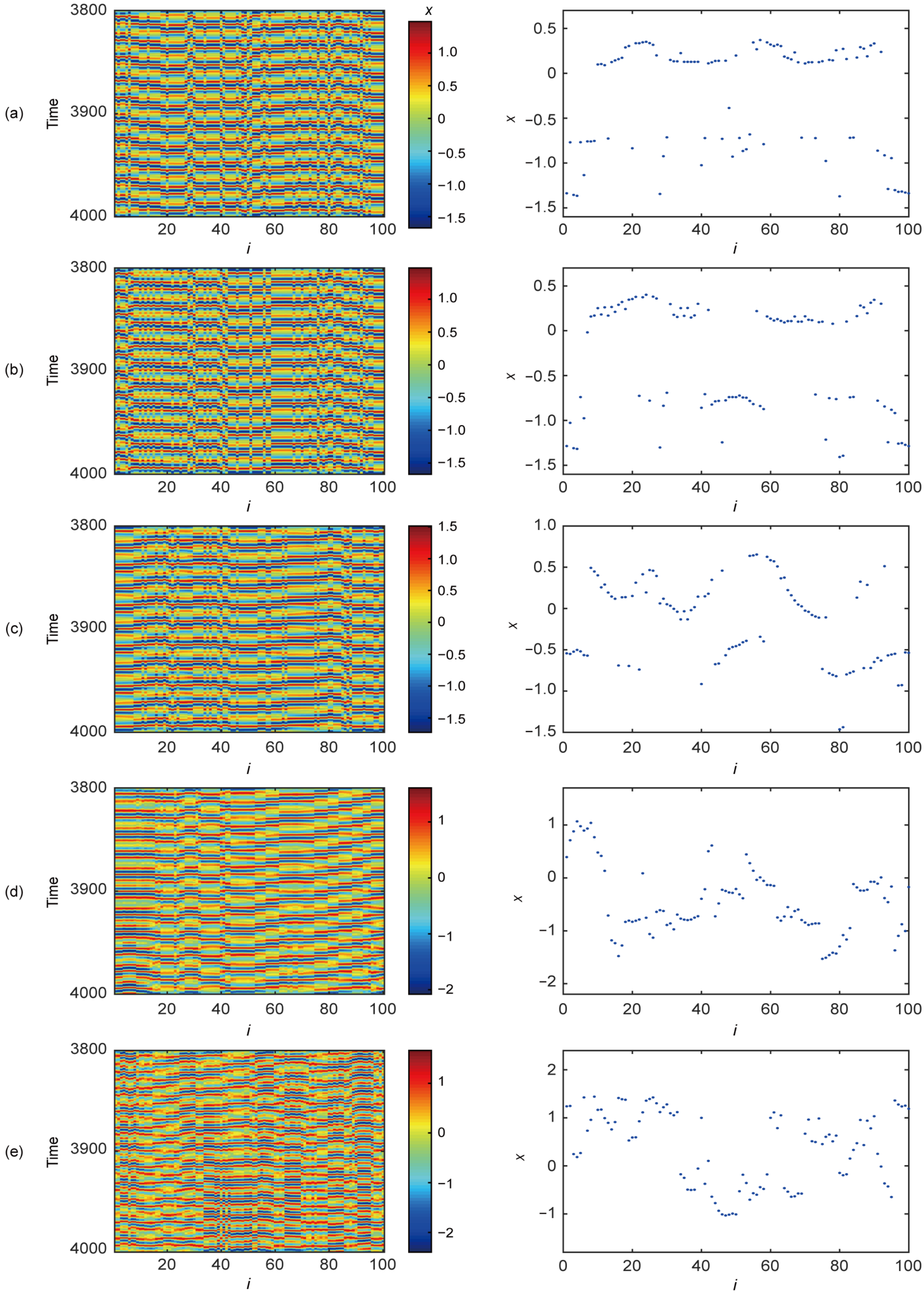

Fig. 3 Space-time plots of the network (left panel) and time snapshots of the positions of the coupled oscillators (right panel) in different coupling strengths for $P = 5$: (a) $\sigma = 0.0002$, (b) $\sigma = 0.0003$, (c) $\sigma = 0.0005$, (d) $\sigma = 0.0006$, and (e) $\sigma = 0.0009$. This figure shows the formation of chimera states for specific values of coupling strength

shows that at most of the values of the parameters, the oscillators become divergent.

In the mechanical oscillators, there is a possibility that the external excitation fails due to some unpredicted side effects like manufacturing defects (Dudkowski et al., 2016). Thus, we discuss the result of the failure of one of the oscillators. In an uncoupled oscillator, if the excitation fails, the

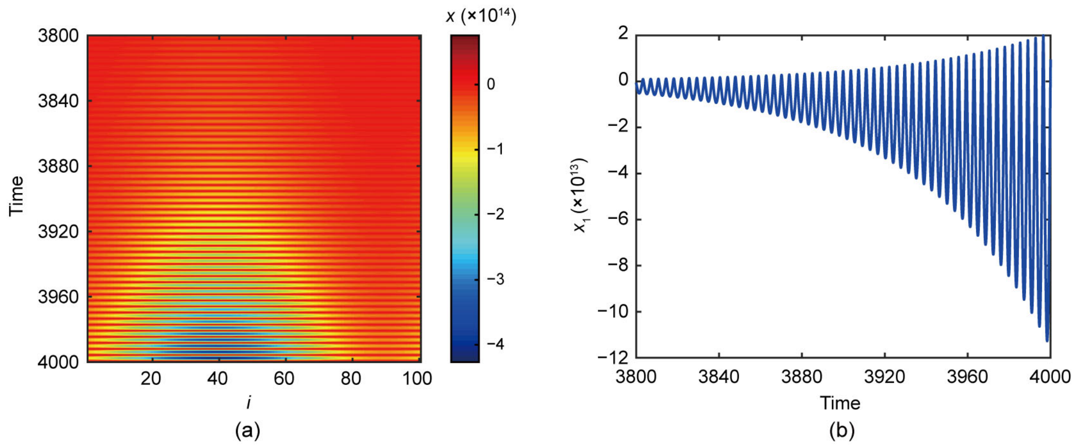

Fig. 4 Space-time plot of the network for P = 5 and σ = 0.001 at which the oscillators become unstable (a) and the diverging position of the first oscillator of the network (b)

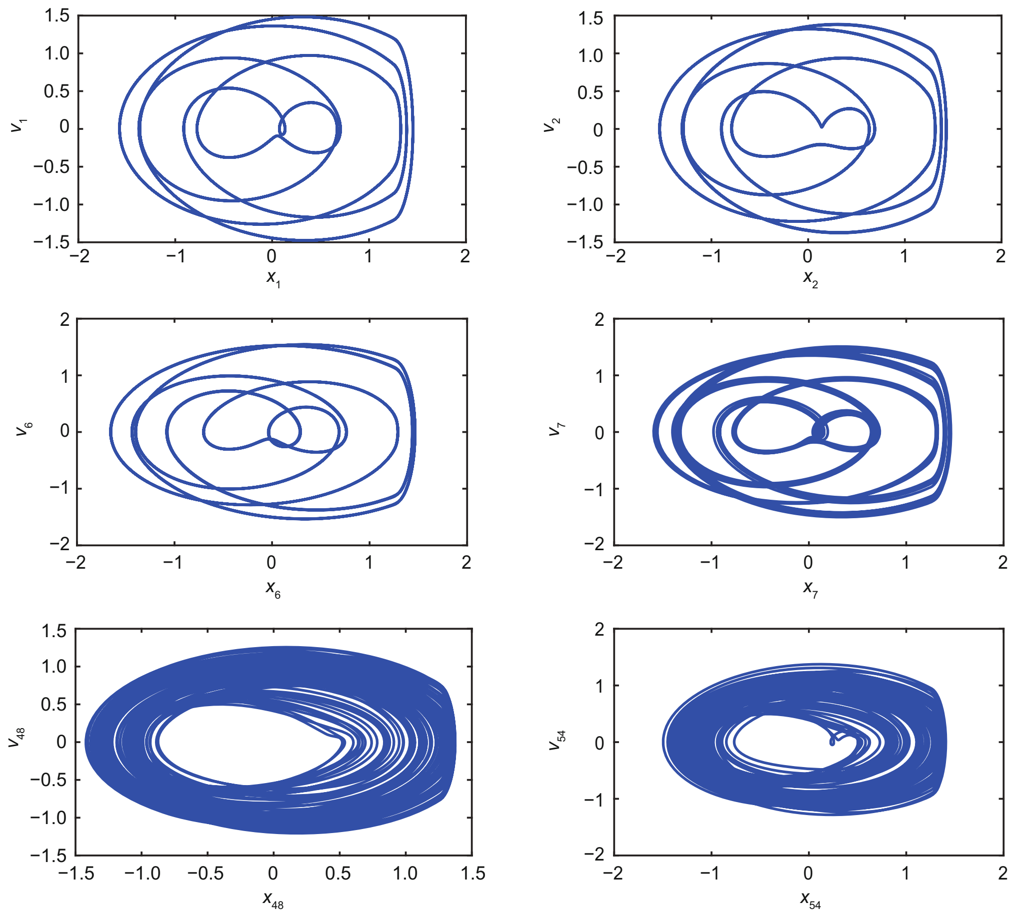

Fig. 5 Some of the observed attractors of the oscillators of the network for P = 5 and σ = 0.0002. The interaction of the oscillators leads to the emergence of different attractors

position and velocity of the oscillators decay to zero. We assume that all of the oscillators are oscillating correctly until $\tau = 2000$ times unit, and at $\tau = 2000$, the excitation of one of the oscillators fails. Fig. 7 shows the result. In Fig. 7a, we use a black square to depict the initiation of the failure. Fig. 7b shows the time series of the failed oscillator and Fig. 7c shows the corresponding attractor when oscillating correctly. After the failure of the excitation, the amplitude of variations of the oscillator's position sub-

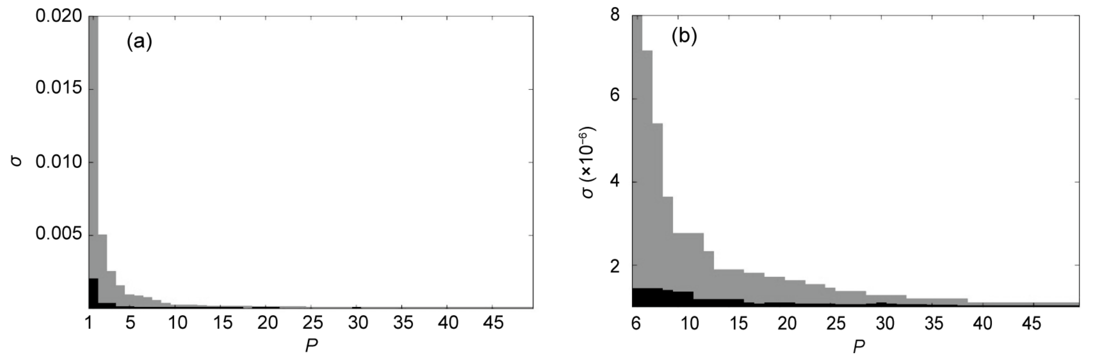

Fig. 6 Regions of different observed states of the network in the (P, σ) parameter plane. The black, gray, and white colors show the asynchronization, the chimera state, and the divergence, respectively. Fig. 6b is the enlargement of Fig. 6a for $P > 5$

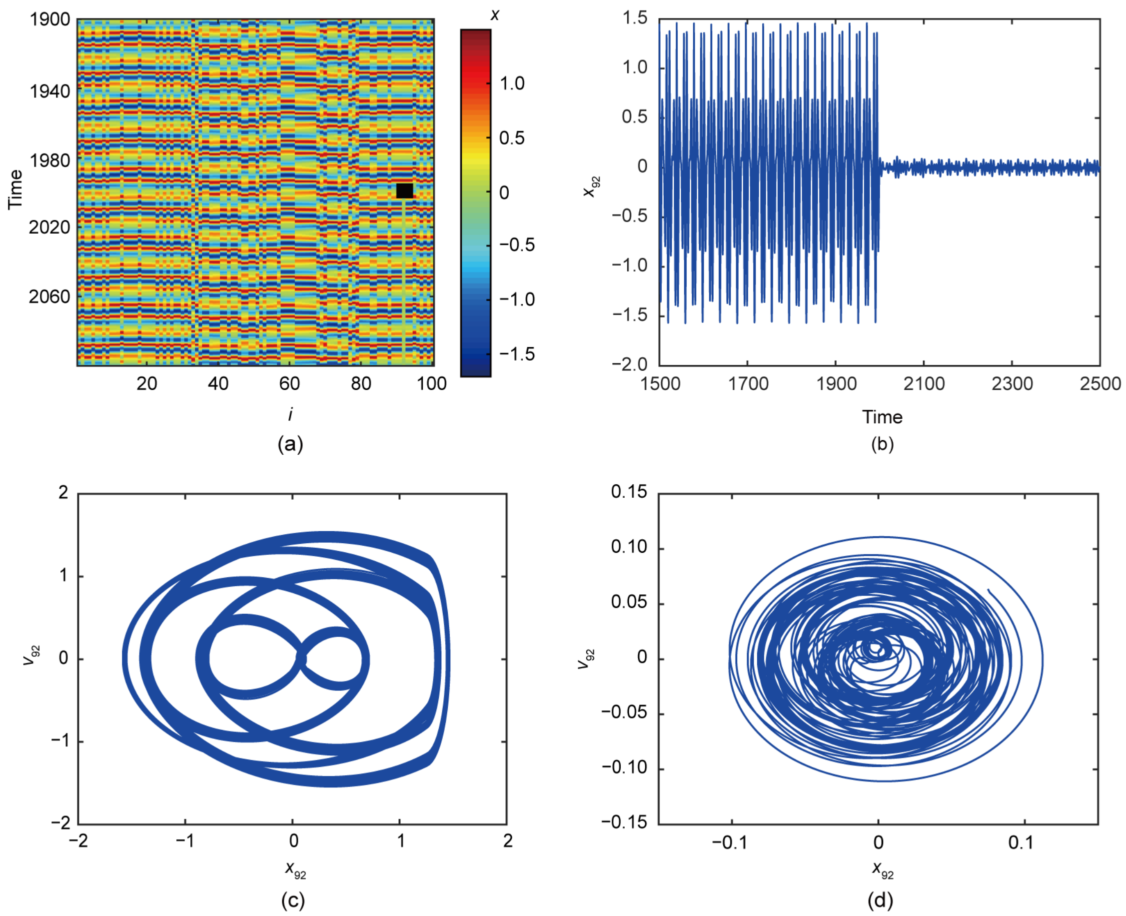

Fig. 7 Results of the network in chimera state, in the case of one vibrator failure for $P = 5$ and $\sigma = 0.0002$: (a) space-time plot; (b) time series of the failed oscillator; (c) attractor of the oscillator before failure; (d) attractor of the oscillator after failure

stantially decreases but does not become zero, which happens when the oscillators are uncoupled. Fig. 7d shows the attractor of the oscillator after the failure.

We also investigate the effect of the failure by adjusting higher coupling strength where the oscillators are unstable. Fig. 8 shows the result of this case by setting $\sigma = 0.001$. It indicates that in this case, the failed oscillator continues its diverging oscillation. Thus, the excitation failure does not qualitatively change the behavior of the oscillator in comparison to the lack of failure.

4 Conclusions

In this study, we studied a network of nonlocally coupled impact oscillators. We investigated the behaviors of the network by varying the coupling parameters. To our best knowledge, the network of this oscillator has not been studied earlier. The numerical simulations indicated that the coupled oscillators are asynchronous for significantly small coupling strengths. By increasing the coupling strength, the chimera state emerges in the network, and a further increase of the coupling strength leads to the divergence of the oscillators. Therefore, we could not obtain a complete synchronous state. Plotting the parameter plane of the network showed that as the number of neighbors in the coupling increases, the region of the chimera state reduces considerably. The results also revealed that the coupling has significant effects on the attractors of the coupled oscillators and can even give rise to the emergence of chaotic attractors. Finally, we studied the network with the assumption of the failure of one of the oscillators' excitation. In the case of the lack of coupling, the excitation failure causes the oscillator motion to decay to zero. However, the coupling can influence the motion of the failed oscillator. We observed that in the network, the amplitude of the failed oscillator's motion is decreased, but not attracted by the zero equilibrium point. This observation confirms that the coupling can preserve the desired behavior of the network in the case of the failure of some nodes (Kapitaniak M et al., 2014). Furthermore, when the network is in the divergence state, the failure of one oscillator has no significant effect, and the failed oscillator continues diverging.

Contributors

Jerzy WOJEWODA and Karthikeyan RAJAGOPAL: conceptualization, methodology, validation, writing-original draft. Viet-Thanh PHAM and Fatemeh PARASTESH: formal analysis, conceptualization, investigation, writing-original draft. Tomasz KAPITANIAK and Sajad JAFARI: supervision, validation, visualization, writing-review and editing.

Conflict of interest

Jerzy WOJEWODA, Karthikeyan RAJAGOPAL, Viet-Thanh PHAM, Fatemeh PARASTESH, Tomasz KAPITANIAK, and Sajad JAFARI declare that they have no conflict of interest.

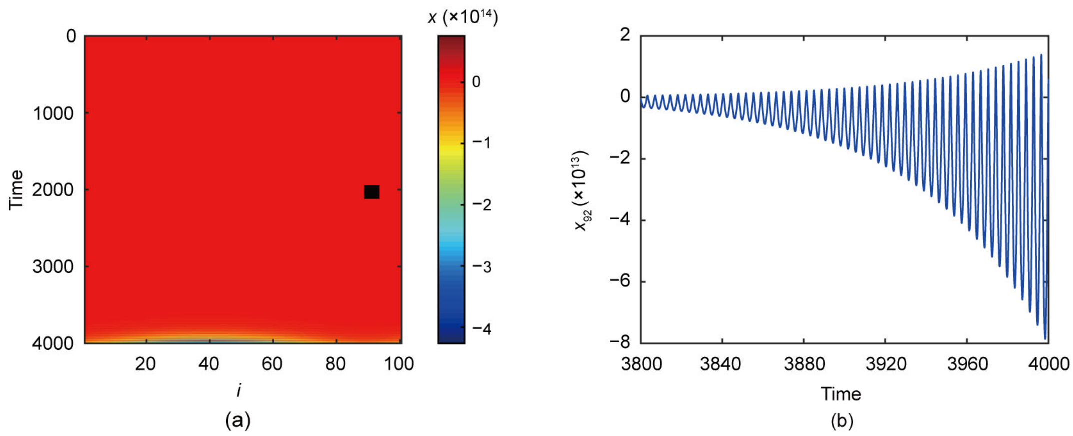

Fig. 8 Results of the network in diverging state, in the case of one vibrator failure for $P = 5$ and $\sigma = 0.001$: (a) space-time plot; (b) time series of the failed oscillator. This figure shows that after the failure, the oscillator continues diverging

References

Abrams DM, Strogatz SH, 2004. Chimera states for coupled oscillators. *Physical Review Letters*, 93(17):174102. https://doi.org/10.1103/PhysRevLett.93.174102

Arenas A, Díaz-Guilera A, Kurths J, et al., 2008. Synchronization in complex networks. *Physics Reports*, 469(3):93-153. https://doi.org/10.1016/j.physrep.2008.09.002

Banerjee S, Ing J, Pavlovskaia E, et al., 2009. Invisible grazings and dangerous bifurcations in impacting systems: the problem of narrow-band chaos. *Physical Review E*, 79(3):037201. https://doi.org/10.1103/PhysRevE.79.037201

Bera BK, Ghosh D, Banerjee T, 2016. Imperfect traveling chimera states induced by local synaptic gradient coupling. *Physical Review E*, 94(1):012215. https://doi.org/10.1103/PhysRevE.94.012215

Bera BK, Majhi S, Ghosh D, et al., 2017. Chimera states: effects of different coupling topologies. *EPL (Europhysics Letters)*, 118(1):10001. https://doi.org/10.1209/0295-5075/118/10001

Bera BK, Rakshit S, Ghosh D, et al., 2019. Spike chimera states and firing regularities in neuronal hypernetworks. *Chaos*, 29(5):053115. https://doi.org/10.1063/1.5088833

Blazejczyk-Okolewska B, Kapitaniak T, 1998. Co-existing attractors of impact oscillator. *Chaos, Solitons & Fractals*, 9(8):1439-1443. https://doi.org/10.1016/S0960-0779(98)00164-7

Dudkowski D, Maistrenko Y, Kapitaniak T, 2016. Occurrence and stability of chimera states in coupled externally excited oscillators. *Chaos*, 26(11):116306. https://doi.org/10.1063/1.4967386

Dudkowski D, Czolczynski K, Kapitaniak T, 2019. Traveling chimera states for coupled pendula. *Nonlinear Dynamics*, 95(3):1859-1866. https://doi.org/10.1007/s11071-018-4664-5

Hart JD, Bansal K, Murphy TE, et al., 2016. Experimental observation of chimera and cluster states in a minimal globally coupled network. *Chaos*, 26(9):094801. https://doi.org/10.1063/1.4953662

Ing J, Pavlovskaia E, Wiercigroch M, et al., 2010. Bifurcation analysis of an impact oscillator with a one-sided elastic constraint near grazing. *Physica D: Nonlinear Phenomena*, 239(6):312-321. https://doi.org/10.1016/j.physd.2009.11.009

Kapitaniak M, Lazarek M, Nielaczny M, et al., 2014. Synchronization extends the life time of the desired behavior of globally coupled systems. *Scientific Reports*, 4(1):4391. https://doi.org/10.1038/srep04391

Kapitaniak T, Kuzma P, Wojewoda J, et al., 2014. Imperfect chimera states for coupled pendula. *Scientific Reports*, 4(1):6379. https://doi.org/10.1038/srep06379

Kundu S, Bera BK, Ghosh D, et al., 2019. Chimera patterns in three-dimensional locally coupled systems. *Physical Review E*, 99(2):022204. https://doi.org/10.1103/PhysRevE.99.022204

Kuramoto Y, Battogtokh D, 2002. Coexistence of coherence and incoherence in nonlocally coupled phase oscillators. *Nonlinear Phenomena in Complex Systems*, 5(4):380-385.

Lee JY, 2005. Motion behavior of impact oscillator. *Journal of Marine Science and Technology*, 13(2):89-96.

Lee JY, Yan JJ, 2006. Control of impact oscillator. *Chaos, Solitons & Fractals*, 28(1):136-142. https://doi.org/10.1016/j.chaos.2005.05.018

Liu YJ, Khalaf AJM, Jafari S, et al., 2019. Chimera state in a two-dimensional network of coupled genetic oscillators. *EPL (Europhysics Letters)*, 127(4):40001. https://doi.org/10.1209/0295-5075/127/40001

Ma J, Xu Y, Ren GD, et al., 2016. Prediction for breakup of spiral wave in a regular neuronal network. *Nonlinear Dynamics*, 84(2):497-509. https://doi.org/10.1007/s11071-015-2502-6

Maistrenko Y, Brezetsky S, Jaros P, et al., 2017. Smallest chimera states. *Physical Review E*, 95(1):010203. https://doi.org/10.1103/PhysRevE.95.010203

Majhi S, Perc M, Ghosh D, 2016. Chimera states in uncoupled neurons induced by a multilayer structure. *Scientific Reports*, 6(1):39033. https://doi.org/10.1038/srep39033

Majhi S, Bera BK, Ghosh D, et al., 2019. Chimera states in neuronal networks: a review. *Physics of Life Reviews*, 28:100-121. https://doi.org/10.1016/j.plrev.2018.09.003

Martens EA, Thutupalli S, Fourriere A, et al., 2013. Chimera states in mechanical oscillator networks. *Proceedings of the National Academy of Sciences of the United States of America*, 110(26):10563-10567. https://doi.org/10.1073/pnas.1302880110

Omelchenko I, Provata A, Hizanidis J, et al., 2015. Robustness of chimera states for coupled FitzHugh-Nagumo oscillators. *Physical Review E*, 91(2):022917. https://doi.org/10.1103/PhysRevE.91.022917

Panaggio MJ, Abrams DM, Ashwin P, et al., 2016. Chimera states in networks of phase oscillators: the case of two small populations. *Physical Review E*, 93(1):012218. https://doi.org/10.1103/PhysRevE.93.012218

Pavlovskaia E, Ing J, Wiercigroch M, et al., 2010. Complex dynamics of bilinear oscillator close to grazing. *International Journal of Bifurcation and Chaos*, 20(11):3801-3817. https://doi.org/10.1142/S0218127410028069

Pham VT, Volos C, Jafari S, et al., 2014. Constructing a novel no-equilibrium chaotic system. *International Journal of Bifurcation and Chaos*, 24(5):1450073. https://doi.org/10.1142/S0218127414500734

Rakshit S, Bera BK, Perc M, et al., 2017. Basin stability for chimera states. *Scientific Reports*, 7(1):2412. https://doi.org/10.1038/s41598-017-02409-5

Suda Y, Okuda K, 2015. Persistent chimera states in nonlocally coupled phase oscillators. *Physical Review E*, 92(6):060901. https://doi.org/10.1103/PhysRevE.92.060901

Tang J, Zhang J, Ma J, et al., 2019. Noise and delay sustained chimera state in small world neuronal network. *Science China Technological Sciences*, 62(7):1134-1140. https://doi.org/10.1007/s11431-017-9282-x

Tinsley MR, Nkomo S, Showalter K, 2012. Chimera and phase-cluster states in populations of coupled chemical oscillators. *Nature Physics*, 8(9):662-665. https://doi.org/10.1038/nphys2371

Vadivasova TE, Strelkova GI, Bogomolov SA, et al., 2016. Correlation analysis of the coherence-incoherence transition in a ring of nonlocally coupled logistic maps. *Chaos*, 26(9):093108.
https://doi.org/10.1063/1.4962647

Wang CN, Ma J, 2018. A review and guidance for pattern selection in spatiotemporal system. *International Journal of Modern Physics B*, 32(06):1830003.
https://doi.org/10.1142/S0217979218300037

Wang Z, Baruni S, Parastesh F, et al., 2020. Chimeras in an adaptive neuronal network with burst-timing-dependent plasticity. *Neurocomputing*, 406:117-126.
https://doi.org/10.1016/j.neucom.2020.03.083

Wei ZC, Sprott JC, Chen H, 2015a. Elementary quadratic chaotic flows with a single non-hyperbolic equilibrium. *Physics Letters A*, 379(37):2184-2187.
https://doi.org/10.1016/j.physleta.2015.06.040

Wei ZC, Zhang W, Yao MH, 2015b. On the periodic orbit bifurcating from one single non-hyperbolic equilibrium in a chaotic jerk system. *Nonlinear Dynamics*, 82(3):1251-1258.
https://doi.org/10.1007/s11071-015-2230-y

Wei ZC, Pham VT, Kapitaniak T, et al., 2016. Bifurcation analysis and circuit realization for multiple-delayed Wang-Chen system with hidden chaotic attractors. *Nonlinear Dynamics*, 85(3):1635-1650.
https://doi.org/10.1007/s11071-016-2783-4

Wei ZC, Moroz I, Sprott JC, et al., 2017a. Detecting hidden chaotic regions and complex dynamics in the self-exciting homopolar disc dynamo. *International Journal of Bifurcation and Chaos*, 27(2):1730008.
https://doi.org/10.1142/S0218127417300087

Wei ZC, Moroz I, Sprott JC, et al., 2017b. Hidden hyperchaos and electronic circuit application in a 5D self-exciting homopolar disc dynamo. *Chaos*, 27(3):033101.
https://doi.org/10.1063/1.4977417

Wei ZC, Pham VT, Khalaf AJM, et al., 2018a. A modified multistable chaotic oscillator. *International Journal of Bifurcation and Chaos*, 28(7):1850085.
https://doi.org/10.1142/S0218127418500852

Wei ZC, Parastesh F, Azarnoush H, et al., 2018b. Nonstationary chimeras in a neuronal network. *EPL (Europhysics Letters)*, 123(4):48003.
https://doi.org/10.1209/0295-5075/123/48003

Wiercigroch M, Sin VWT, 1998. Experimental study of a symmetrical piecewise base-excited oscillator. *Journal of Applied Mechanics*, 65(3):657-663.
https://doi.org/10.1115/1.2789108

Yao YG, Deng HY, Yi M, et al., 2017. Impact of bounded noise on the formation and instability of spiral wave in a 2D lattice of neurons. *Scientific Reports*, 7(1):43151.
https://doi.org/10.1038/srep43151