Journal of Zhejiang University-SCIENCE A (Applied Physics & Engineering) 2024 25(8):618-630
www.jzus.zju.edu.cn; www.springer.com/journal/11582
E-mail: jzus_a@zju.edu.cn

Research Article
https://doi.org/10.1631/jzus.A2300432

New formula for predicting the plastic buckling pressure of steel torispherical heads under internal pressure

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Abstract: Thin-walled torispherical heads under internal pressure can fail by plastic buckling because of compressive circumferential stresses in the head knuckle. However, existing formulas still have limitations, such as complicated expressions and low accuracy, in determining buckling pressure. In this paper, we propose a new formula for calculating the buckling pressure of torispherical heads based on elastic-plastic analysis and experimental results. First, a finite element (FE) method based on the arc-length method is established to calculate the plastic buckling pressure of torispherical heads, considering the effects of material strain hardening and geometrical nonlinearity. The buckling pressure results calculated by the FE method in this paper have good consistency with those of BOSOR5, which is a program for calculating the elastic-plastic bifurcation buckling pressure based on the finite difference energy method. Second, the effects of geometric parameters, material parameters, and restraint form of head edge on buckling pressure are investigated. Third, a new formula for calculating plastic buckling pressure is developed by fitting the curve of FE results and introducing a reduction factor determined from experimental data. Finally, based on the experimental results, we compare the predictions of the new formula with those of existing formulas. It is shown that the new formula has a higher accuracy than the existing ones.

Key words: Torispherical head; Plastic buckling; Elastic-plastic analysis; Prediction formula; Finite element method

1 Introduction

Torispherical heads are commonly used as end closures of pressure vessels in a variety of fields, including the petrochemical, aerospace, and food processing industries. Fig. 1 shows the geometry of a torispherical head made up of a spherical crown, a toroidal knuckle, and a short cylinder. These heads are subjected to internal pressure in many actual engineering situations. Due to compressive circumferential stresses in the head knuckle, local buckling can occur in internally pressurized torispherical heads, especially thin-walled ones. As a result, avoiding the failure of buckling has become an important issue in their design.

The buckling of torispherical heads under internal pressure is a classic problem. Galletly (1959) pointed out the existence of compressive circumferential stresses in their knuckles, which may lead to buckling failure. Fino and Schneider (1961) reported a case of bucking of a large torispherical head with a diameter of approximately 18 m. Bushnell (1976, 1977a) developed the BOSOR4 and BOSOR5 programs for bifurcation buckling analysis. BOSOR4 is used to analyze the buckling of elastic structures with large deformations. BOSOR5 can solve problems involving large deflections, elastic-plastic material behavior and creep, and it can be used for the analysis of plastic buckling and creep buckling. Bushnell (1977b), Bushnell and Galletly (1977), Aylward and Galletly (1979), Galletly and Radhamohan (1979), and Galletly (1981, 1986a, 1986b) performed a series of studies on the buckling of torispherical heads under internal pressure with the use of the programs BOSOR4 and BOSOR5. They investigated the buckling behavior of torispherical heads and developed formulas for predicting their buckling pressure. Recently, Li et al. (2017, 2019a, 2019b) and Zheng et al. (2018, 2020, 2021) conducted a series of

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Received Sept. 13, 2023; Revision accepted Nov. 28, 2023; Crosschecked July 11, 2024

failure experiments and finite element (FE) analysis on the failure of ellipsoidal and torispherical heads. Based on the experimental results and FE analysis, they developed new formulas for the prediction of the buckling pressure and collapse pressure of ellipsoidal heads under internal pressure. Recent research on the new theory and design of ellipsoidal heads is found in the book *New Theory and Design of Ellipsoidal Heads for Pressure Vessels* (Zheng and Li, 2021). Although ellipsoidal heads can be geometrically equivalent to torispherical heads in some pressure vessel codes, research has shown that ellipsoidal heads have higher buckling resistance compared to equivalent torispherical heads (Zheng et al., 2021). Błachut (2020) studied the impact of local and global shape imperfections on the buckling of externally pressurized domes. The results showed that the local dimple created by a concentrated force at the point of maximum deflection leads to a significant reduction of buckling pressure. Błachut (2023) investigated the effect of auxetic material on the buckling of externally pressurized torispherical heads and found that the inclusion of auxetic material can lead to an 89% increase in failure pressure compared with an auxetic-free configuration of the wall. Zhu et al. (2022) studied the buckling characteristics of spherical shells damaged by concentrated impact loads using experiments and numerical calculations. They found that the buckling load of spherical shells decreases with the increase of impact velocity and impact angle. Yang et al. (2021) deduced the thickness distribution of the constant strength of ellipsoidal heads under external pressure. The FE analysis showed that ellipsoidal heads with variable thicknesses had a higher buckling load than ellipsoidal heads with constant thickness. Zhang et al. (2022) studied the buckling of externally pressurized torispherical heads with uniform and stepwise thicknesses and found that thickening the whole knuckle and partial crown of the head was the best approach to strengthening them. Sowiński (2022, 2023a, 2023b) focused on stress distribution optimization of dished heads under internal pressure and developed a unique shape to minimize the von Mises stress.

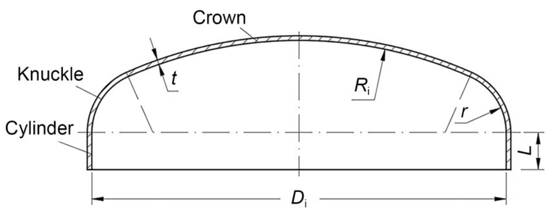

Fig. 1 Geometry of a torispherical head (D_i is the inside diameter, L is the cylinder length, r is the knuckle radius, R_i is the crown radius, and t is the head thickness)

The current formulas for predicting the buckling pressure of torispherical heads under internal pressure are summarized here. Using the program BOSOR4, Aylward and Galletly (1979) developed a formula for the elastic buckling pressure of torispherical heads, as given by

$$P_b = 167E\alpha_1\alpha_2\alpha_3\left(\frac{t}{D_i}\right)^{\beta}, \tag{1}$$

where E is the elastic modulus, P_b is the buckling pressure, $\alpha_1=1.1(R_i/D_i)^2-1.5R_i/D_i+1.0$, $\alpha_2=48.0(r/D_i)^2-6.0r/D_i+1.0$, $\alpha_3=[-35.0(r/D_i)^2+8.0r/D_i-0.32](R_i/D_i)^2+1.0$, and $\beta=2.05+0.4R_i/D_i$.

The ranges of applicability of Eq. (1) are $500\leqslant D_i/t\leqslant 2000$, $0.75\leqslant R_i/D_i\leqslant 1.50$, and $0.06\leqslant r/D_i\leqslant 0.15$. For torispherical heads with $R_i/D_i=1.0$, Eq. (1) can be reduced to:

$$P_b = 100E\left(\frac{3.7r}{D_i}+0.68\right)\left(\frac{t}{D_i}\right)^{2.45}. \tag{2}$$

Galletly and Radhamohan (1979) developed a formula for the plastic buckling pressure of torispherical heads using an elastic perfectly plastic material model, as given by

$$P_b = \frac{285S_y\left(1-\dfrac{125S_y}{E}\right)\left(\dfrac{r}{D_i}\right)^{0.84}}{\left(\dfrac{D_i}{t}\right)^{1.53}\left(\dfrac{R_i}{D_i}\right)^{1.1}}, \tag{3}$$

where S_y is the yield strength. The ranges of applicability of Eq. (3) are $500\leqslant D_i/t\leqslant 1500$, $0.75\leqslant R_i/D_i\leqslant 1.5$, $0.06\leqslant r/D_i\leqslant 0.18$, and 138 MPa$<S_y<$517 MPa. According to the mechanical properties of steels used for most torispherical heads, the average values of yield strength S_y and Young's modulus E are 310 MPa and 200 GPa, respectively. Eq. (3) then becomes:

$$P_b = \frac{230S_y\left(\dfrac{r}{D_i}\right)^{0.84}}{\left(\dfrac{D_i}{t}\right)^{1.53}\left(\dfrac{R_i}{D_i}\right)^{1.1}}. \tag{4}$$

Galletly and Błachut (1985) expanded the parametric analysis ranges of head geometric parameters and material yield strength. In particular, the diameter-to-thickness ratio (D_i/t) was decreased to 300. The prediction formulas were obtained by fitting the buckling pressure results calculated by the BOSOR5 computer program. Two formulas for predicting the buckling pressure were developed by Eqs. (5) and (6). Eq. (6) has a smaller fitting error than Eq. (5) but is more complex in form.

$$P_b = \frac{120S_y\left(\frac{r}{D_i}\right)^{0.81}}{\left(\frac{D_i}{t}\right)^{1.46}\left(\frac{R_i}{D_i}\right)^{1.18}}, \tag{5}$$

$$P_b = \frac{200S_y\left(\frac{r}{D_i}\right)^{1.5}}{\left(\frac{D_i}{t}\right)^{1.42}\left(\frac{R_i}{D_i}\right)^{1.17}}\left[1+0.05\left(\frac{r}{D_i}\right)^{-1.315}\right]. \tag{6}$$

To predict the buckling pressure of torispherical heads with high strength (517 MPa<S_y<724 MPa), Galletly (1981) developed a formula for the buckling pressure of torispherical heads with R_i/D_i=1.0, as given by

$$\ln\left(\frac{P_b}{100}\right) = -0.60+0.89(A+B)-1.49C+ 0.42B^2+0.33A^2B+0.305AC+0.78ABC, \tag{7}$$

where $A=\ln(S_y/60000)$, $B=\ln(8r/D_i)$, and $C=\ln[D_i/(1000t)]$. The ranges of applicability of Eq. (7) are $500\leqslant D_i/t\leqslant 1250$ and $0.06\leqslant r/D_i\leqslant 0.2$, and P_b and S_y are in lbf/in^2 (1 lbf=4.45 N, and 1 in=2.54 cm).

Miller (1999, 2001) developed formulas for predicting the failure pressure of torispherical heads under internal pressure. The formulas corresponding to the yielding pressure and elastic buckling pressure are given by thin shell theory and elastic buckling theory. The formulas for the buckling pressure are derived by applying reduction factors to the formulas corresponding to the yielding pressure and elastic buckling pressure. The reduction factors obtained from test data account for the effects of geometric imperfections, residual stresses, and nonlinear material properties. Miller (1999, 2001)'s formulas are presented below.

(a) Elastic buckling stress S_e

$$S_e = C_1E\left(\frac{t}{r}\right), \tag{8}$$

where

$$C_1 = \begin{cases} 9.31\left(\frac{r}{D_i}\right)-0.086, & \frac{r}{D_i}\leqslant 0.08, \\ 0.692\left(\frac{r}{D_i}\right)+0.605, & \frac{r}{D_i}>0.08. \end{cases}$$

(b) Maximum membrane stress in the knuckle S_k (negative value is compression)

$$S_k = C_2\left(0.5\frac{R_e}{r}-1\right)p\frac{R_e}{t}, \tag{9}$$

where p is the internal pressure, and

$$C_2 = \begin{cases} 1.25, & \frac{r}{D_i}\leqslant 0.08, \\ 1.46-2.6\left(\frac{r}{D_i}\right), & \frac{r}{D_i}>0.08, \end{cases}$$

$$R_e = \begin{cases} \frac{0.5D_i-r}{\cos(\beta_{th}-\phi_{th})}+r, & \phi_{th}<\beta_{th}, \\ 0.5D_i, & \phi_{th}\geqslant\beta_{th}, \end{cases}$$

$$\beta_{th} = \arccos\left(\frac{0.5D_i-r}{R_i-r}\right), \quad \phi_{th} = \frac{\sqrt{R_it}}{r},$$

where R_e is the stress radius at the point of maximum stress in the knuckle, and β_{th} and ϕ_{th} are parameters.

(c) Elastic buckling pressure P_e and yield pressure P_y

$$P_e = \frac{S_et}{C_2R_e\left(\frac{0.5R_e}{r}-1\right)}, \tag{10}$$

$$P_y = \frac{S_yt}{C_2R_e\left(\frac{0.5R_e}{r}-1\right)}. \tag{11}$$

(d) Buckling pressure of torispherical heads assembled from formed segments

$$P_b = \begin{cases} 0.6P_e, & \frac{P_e}{P_y}\leqslant 1.0, \\ 0.408P_y+0.192P_e, & 1.0<\frac{P_e}{P_y}\leqslant 8.29, \\ 2.0P_y, & \frac{P_e}{P_y}>8.29. \end{cases} \tag{12}$$

(e) Buckling pressure of the pressed and spun torispherical heads

$$P_b=\begin{cases}0.6P_e, & \dfrac{P_e}{P_y}\leqslant 1.667,\\ 0.748P_y+0.151P_e, & 1.667<\dfrac{P_e}{P_y}\leqslant 8.29,\\ 2.0P_y, & \dfrac{P_e}{P_y}>8.29.\end{cases} \quad (13)$$

Based on the formulas for calculating buckling pressure, ASME VIII-1 (ASME, 2023b), ASME VIII-2 (ASME, 2023c), and EN 13445-3 (CEN, 2021) provide design formulas for preventing the buckling of internal pressurized torispherical heads. The design formulas used in ASME VIII-1 and VIII-2 are based on the formulas of Miller (1999, 2001). The design formulas used in EN 13445-3 are based on the work by Galletly (1986a, 1986b).

As described above, the existing formulas were developed using elastic or perfectly plastic theories, in which material strain hardening is not considered. In addition, the formulas developed by Galletly et al. (1979, 1981, 1985) are not applicable to a sufficient range. Miller (1999, 2001)'s formulas are quite complicated and thus inconvenient to use. The objective of this paper is to develop a new formula to predict the buckling pressure of steel torispherical heads using elastic-plastic analysis and experimental results. First, the nonlinear FE method is established to calculate the buckling pressure of torispherical heads, taking into account the effects of stress hardening and geometrical nonlinearity. Second, the effects of geometrical parameters, material parameters, and restraint of head edge on the buckling pressure of torispherical heads are investigated. Third, a new formula for calculating plastic buckling pressure is developed by fitting the curve of FE results and introducing a reduction factor determined from experimental data. Finally, the predictions of the new formula and existing formulas are compared with experimental results.

2 Numerical simulation of buckling of torispherical heads

2.1 FE model

The nonlinear FE analysis using the arc-length method is used to simulate the buckling of torispherical heads under internal pressure. In this study, we used the ANSYS software to perform the FE buckling analysis including the effects of material and geometrical nonlinearities. Fig. 2 shows the geometrical model of a torispherical head with a cylinder. The model is assumed to have a perfect shape. A torispherical head is usually welded to a long cylinder in actual applications. For this model, it is assumed that the cylinder and the torispherical head have the same inside diameter and thickness. The cylinder has such a length that the effect of boundary stresses due to displacement constraint of the cylinder ends is negligible for torispherical heads.

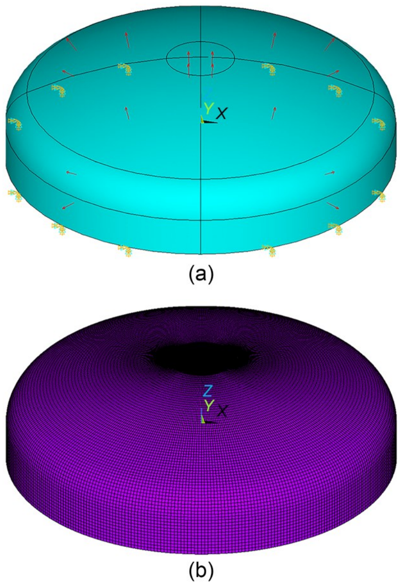

Fig. 2 Boundary condition (a) and meshes (b) of torispherical heads with cylinders

An increasing uniform pressure is applied to the inside surface of the model and the end of the cylinder is fully fixed, as shown in Fig. 2a. As buckling mainly occurs in a thin-walled torispherical head with a diameter-to-thickness ratio (D_i/t) greater than 100, the model is meshed by element SHELL181, as shown in Fig. 2b. The element SHELL181 is suitable for thin to moderately-thick shell structures and for linear, large deflection, and large strain nonlinear applications.

Most practical torispherical heads are constructed from steel. The true stress–strain curves, including the strain hardening characteristics, are used for the material model, and von Mises yield criterion and the flow rule are adopted. Table 1 presents the tensile properties of several typical sheets of steel commonly used for torispherical heads. These tensile properties, including the elastic modulus, engineering yield strength, and engineering tensile strength, are obtained from ASME 2023 BPV Code Section II, Part D (ASME, 2023a) and Chinese standards GB/T 150.2–2011 (AQSIQ and SA, 2011) and GB/T 24511–2017 (AQSIQ and SA, 2017). The true stress–strain curves of these steels are determined by Annex 3-D of ASME 2023 BPV Code Section VIII, Division 2 (ASME, 2023c), as shown in Fig. 3. When the curve exceeds the true tensile strength, the material is assumed to be perfectly plastic.

Table 1 Tensile properties of typical steels

Material	Elastic modulus, E (GPa)	Engineering yield strength, S_y (MPa)	Engineering tensile strength, S_u (MPa)
S30408	195	220	520
S22053	200	450	620
Q345R	201	345	510
SA-516 Gr.70	202	260	485
SA-738 Gr.B	202	415	585

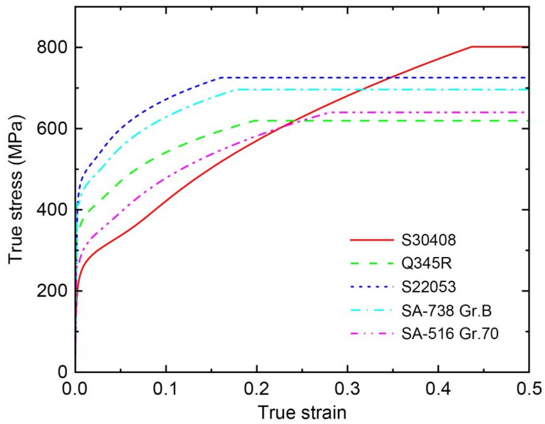

Fig. 3 True stress–strain curves of typical steels

2.2 FE results

2.2.1 Determination of the buckling pressure

A torispherical head with a crown radius-to-diameter ratio (R_i/D_i) of 1.0 and a knuckle radius-to-diameter ratio (r/D_i) of 0.1, is commonly used in engineering. For example, the head and cylinder have the same inside diameter of 5000 mm and the same thicknesses (t) of 5 mm, and the cylinder has a length (L) of 500 mm. The true stress–true strain curve of S30408 is used in the material model. The FE results on buckling of the torispherical head are shown in Fig. 4. It is shown that local buckles occur in the knuckle of the head. The radial displacements of points A, B, and C of a buckle were obtained to determine the buckling pressure. The pressure–displacement curves of points A, B, and C are nearly identical when the pressure is lower than 0.21 MPa. When it exceeds 0.21 MPa, the radial displacement of point B starts to develop differently from those of points A and C, which indicates the start of buckling. The buckling pressure of the model is thus determined as 0.21 MPa.

In addition, the pressure–displacement curves are linear when the pressure is lower than 0.12 MPa, indicating that the torispherical head is in elastic deformation. As the pressure increases, plastic deformation occurs and causes the pressure–displacement curves to become nonlinear. Buckles subsequently occur, as shown in Fig. 4. Therefore, the buckling of internally pressurized torispherical heads occurs in the plastic region of the material, as mentioned in (Galletly and Radhamohan, 1979; Galletly, 1981).

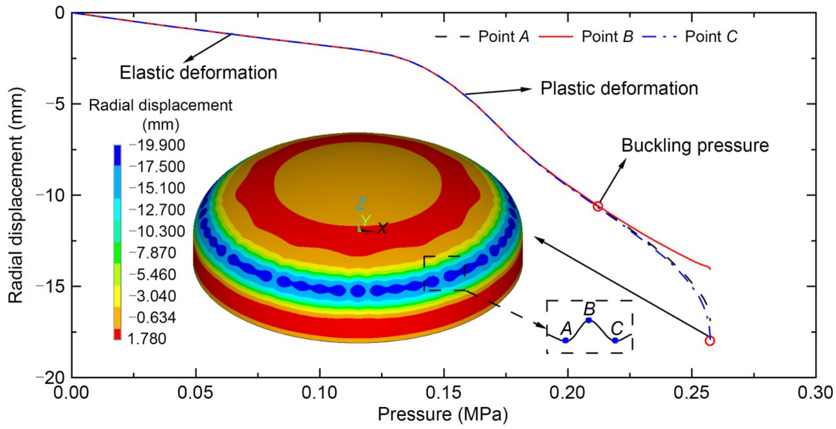

Fig. 4 FE results on buckling of torispherical heads

The FE method in this paper is compared with that in reference (Galletly and Radhamohan, 1979), which uses BOSOR5 to calculate the elastic-plastic bifurcation buckling pressure of torispherical heads. BOSOR5 is a computer program for the buckling of elastic-plastic complex shells of revolution including large deflections and creep (Bushnell, 1976). The pre-buckling and elastic-plastic bifurcation analyses of the program are based on the finite difference energy method. Bifurcation buckling load is computed corresponding to buckling modes. For the initial circumferential wavenumber of the shell (chosen by the user), BOSOR5 calculates the determinant of the global stability matrix for each load increment. The load increases until the sign of the stability determinant changes, which indicates the occurrence of bifurcation buckling. Once the bifurcation load is determined with the initial wavenumber, BOSOR5 computes the eigenvalues in wave number increments, and the minimum eigenvalue is used to decide on the proper wavenumber.

The nonlinear FE analysis in this paper is based on the arc-length method. This method is a highly efficient numerical method for structural nonlinear analysis. It has good adaptability and high efficiency in the pre- and post-buckling analyses of structures for tracing the whole load-deflection path and capturing the buckling load.

Three torispherical head models were used for comparison, using an elastic perfectly plastic material model with a yield strength of 207 MPa and an elastic modulus of 207 GPa. The comparison between the calculated buckling pressure in this paper and the literature (Galletly and Radhamohan, 1979) is shown in Table 2. The difference between them is within 5%. The buckling pressure results of torispherical heads calculated by the FE method in this paper are consistent with those of BOSOR5.

The BOSOR5 and FE methods in this paper are capable of calculating the buckling pressure and the number of circumferential waves of heads at buckling. For example, for a head with D_i/t=1000, r/D_i=0.15, and R_i/D_i=1, both BOSOR5 and the FE method calculate a circumferential wavenumber of 40 at buckling. However, the FE method can further simulate the post-buckling behavior of the heads, while BOSOR5 cannot.

Table 2 Comparison of buckling pressure results calculated by the FE method and BOSOR5

D_i/t	r/D_i	R_i/D_i	Buckling pressure, P_b (MPa)		Difference (%)
			FE method (this paper)	BOSOR5	
1000	0.15	1	0.255	0.258	−1.2
1500	0.10	1	0.099	0.095	4.2
1000	0.08	1.25	0.122	0.117	4.3

2.2.2 Effect of geometrical parameters

The effects of the head diameter-to-thickness ratio (D_i/t), crown radius-to-head diameter ratio (R_i/D_i), and knuckle radius-to-head diameter ratio (r/D_i) on the buckling pressure (P_b) of torispherical heads are studied using the FE model. Fig. 5 shows the FE results of the buckling pressure for some heads with different geometrical parameters. According to the FE results, the buckling pressure of heads decreases with the increases of D_i/t and R_i/D_i, and increases with the decrease of r/D_i.

Taking torispherical heads (D_i=5000 mm, D_i/t=2000) with different knuckles and crown radii as an example, the FE method was used to conduct stress analysis of the head. The heads were attached to cylinders with the same diameter and thickness as the head, and the head and cylinder were assumed to be made of S22053. Fig. 6 shows the distribution of circumferential stress in the middle surface of the head and cylinder under an internal pressure of 0.09 MPa. The circumferential stress in the crown of the head is constant. However, the circumferential stresses fluctuate greatly in the head knuckle because of the curvature discontinuity in the junctions between the crown, knuckle, and cylinder. There are significant compressive stresses in the head knuckle, which is the cause of buckling. The maximum compression stresses increase with the increase of R_i/D_i and decrease of r/D_i, which results in a decrease of buckling pressure.

2.2.3 Effect of material parameters

In order to study the effect of material parameters on the buckling pressure of torispherical heads, the true stress–strain curves of five kinds of typical steels are used in the FE model, as shown in Fig. 3. The FE results show that buckling occurs in the plastic region of materials for the proposed heads. Thus, the effect of yield strength (S_y) on buckling pressure (P_b) is studied, as shown in Fig. 7. It is shown that the buckling pressure increases approximately linearly with the increase of yield strength.

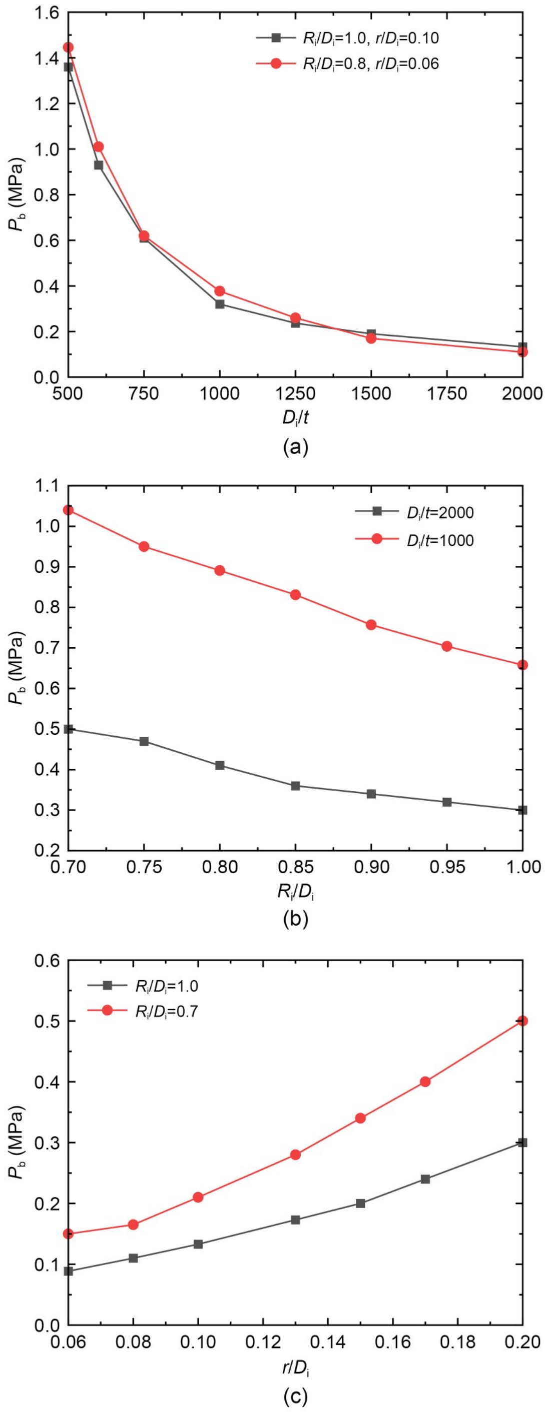

Fig. 5 Effects of geometrical parameters D_i/t, R_i/D_i, and r/D_i on buckling pressure of torispherical heads: (a) D_i/t; (b) R_i/D_i; (c) r/D_i

2.2.4 Effect of restraint forms of head edge

Torispherical heads are mainly attached to cylinders, but in some applications, they may also be connected to bolting flanges. When connecting with the

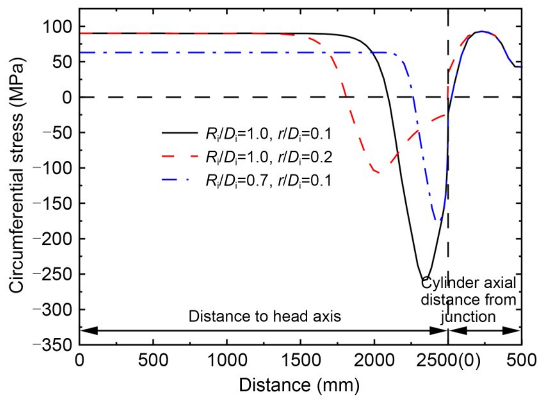

Fig. 6 Circumferential stresses of torispherical heads under internal pressure

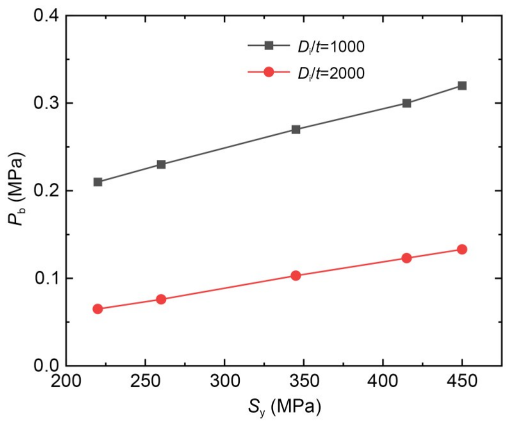

Fig. 7 Effect of yield strength on buckling pressure of torispherical heads under internal pressure

cylinder, a cylinder with the same thickness as the head and sufficient length is considered in the FE model. When connecting to the bolting flange, the cylinder is not considered in the FE model, and the edge of the torispherical head is fully fixed. The effects of restraint forms on the buckling pressure of torispherical heads are shown in Table 3. When the head diameter-to-thickness ratio (D_i/t) is large, the restraint form of the head edge has little effect on the buckling pressure. When the head diameter-to-thickness ratio (D_i/t) decreases, the fixed constraint of the head edge will enhance the buckling resistance of the torispherical heads.

As shown in Table 3, for a torispherical head with D_i/t=2000, r/D_i=0.06, and R_i/D_i=0.7, the buckling pressure is 0.14 MPa without a cylinder, and it decreases to 0.11 MPa when attached to a sufficiently long cylinder with $L=5\sqrt{D_i t/2}$. To further investigate the influence of the cylinder length on the buckling pressure,

Table 3 Influence of restraint forms on the buckling pressure of the torispherical head

D_i/t	Buckling pressure, P_b (MPa)					
	r/D_i=0.06, R_i/D_i=1		r/D_i=0.20, R_i/D_i=1		r/D_i=0.06, R_i/D_i=0.7	
	With cylinder	No cylinder	With cylinder	No cylinder	With cylinder	No cylinder
500	0.91	NB	NB	NB	NB	NB
2000	0.07	0.07	0.22	0.22	0.11	0.14

NB: no buckling

additional calculations were performed for cylinder lengths of $0.5\sqrt{D_it/2}$, $\sqrt{D_it/2}$, and $2\sqrt{D_it/2}$, and the buckling pressures were 0.12, 0.11, and 0.11 MPa, respectively. It is evident that the buckling pressure increases as the cylinder length L decreases; however, beyond a certain value of L, the buckling pressure is no longer affected.

3 Development of formulas for the buckling pressure of torispherical heads

3.1 Formula by curve fitting of FE results

To develop a formula for predicting the buckling pressure of torispherical heads under internal pressure, more FE models were generated for further parametric studies. As discussed in Section 2.2.4, such heads attached to long cylinders have lower buckling pressures than those with a fixed edge, and the restraint form of the head edge has little effect on the buckling pressure when D_i/t is large. Therefore, a long cylinder attached to a torispherical head is considered in the FE models to obtain conservative results. The ranges of the geometric parameters in this study are determined as $200 \leqslant D_i/t \leqslant 2000$, $0.7 \leqslant R_i/D_i \leqslant 1.0$, and $0.06 \leqslant r/D_i \leqslant 0.2$, covering the range of applicability of the torispherical heads commonly used in engineering. The five kinds of typical steels are considered in the FE models, and their true stress–true strain curves are shown in Fig. 3.

A total of 149 FE models for torispherical heads with different geometrical parameters and material properties were generated to determine the buckling pressures. The FE results are plotted in Fig. 8. According to the FE results, the buckling pressure has a positive linear correlation with S_y, an exponential decay relationship with D_i/t and R_i/D_i, and an exponential growth relationship with r/D_i. Based on these relationships, a formula is obtained by nonlinear curve fitting of these FE results, as shown in Fig. 8. The coefficient of determination is 0.94, showing a good fitting. Therefore, a new formula for prediction of buckling pressure of perfect torispherical heads under internal pressure is given as

$$P_b = 305 S_y \left(\frac{D_i}{t}\right)^{-1.67} \left(\frac{R_i}{D_i}\right)^{-1.32} \left(\frac{r}{D_i}\right)^{0.47}. \quad (14)$$

According to the parameters of the heads used in the FE simulation, Eq. (14) is applicable for torispherical heads with $200 \leqslant D_i/t \leqslant 2000$, $0.7 \leqslant R_i/D_i \leqslant 1.0$, and $0.06 \leqslant r/D_i \leqslant 0.2$.

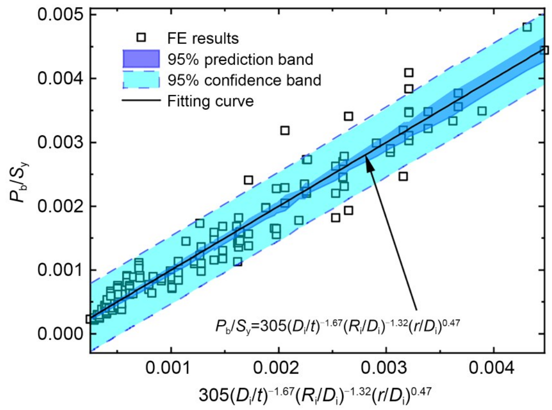

Fig. 8 Curve fitting of FE results for the buckling of torispherical heads

3.2 Formula modified by experimental results

Shape imperfections are inevitable during the manufacturing process of the head, especially for large-scale thin-walled heads. Li et al. (2017) calculated the buckling pressure of a large-scale thin-walled head by nonlinear FE analysis considering the initial measured and the initial perfect shapes. The experimental buckling pressure of the head agrees well with that predicted by the analysis considering the initial measured shape. Zheng et al. (2018) found that shape imperfections have significant effects on the buckling pressure of large-scale thin-walled heads assembled from formed segments. Wagner et al. (2021) validated a wide variety of empirical designs and numerical geometric imperfection approaches for torispherical shells under external pressure and developed new design factors for them. Therefore, to develop a new formula for the buckling pressure of actual torispherical heads, Eq. (14) should be modified to take into account shape imperfections.

Experimental results on the buckling of 49 internally pressurized torispherical heads, including 30 heads formed by spinning and pressing and 19 heads assembled from formed segments, are summarized in Section S1 of the electronic supplementary materials. The ranges of the geometric parameters of these tested heads are $238 \leqslant D_i/t \leqslant 2304$, $0.722 \leqslant R_i/D_i \leqslant 1.093$, and $0.04 \leqslant r/D_i \leqslant 0.21$. These heads are constructed in carbon steel, low alloy steel, and stainless steel. We have conducted a series of internally pressurized buckling experiments for heads (Li et al., 2017, 2019a, 2019b; Zheng et al., 2018; Zheng and Li, 2021). The tested heads were welded to reusable test vessels, and the test vessels were pressurized with water until the heads burst. Four heads formed by cold spinning (ZJU-X1–ZJU-X4) and four heads assembled from formed segments (ZJU-CAP1, ZJU-CAP2 and ZJU-STD1, ZJU-STD2) are torispherical heads which are equivalent to ellipsoidal heads. For heads ZJU-X1–ZJU-X4, the experimental buckling pressure was determined by the occurrence of the first buckle using video monitoring. For large-scale heads (ZJU-CAP1, ZJU-CAP2 and ZJU-STD1, ZJU-STD2), the strain in the knuckle was measured by strain gauges, and the deformation was measured by 3D laser scanners (Li et al., 2017; Zheng et al., 2018). The buckling pressure was determined by the circumferential strain–pressure curves of the first buckle, and the initiation of waves were detected by the 3D laser scanners. The experimental results show that buckles form in the knuckles of heads and gradually evolve with increasing pressure. Some typical buckles of tested heads are presented in Fig. 9. For example, for head ZJU-STD2, two buckles were formed at a pressure of 0.72 MPa. Subsequently, as the pressure reached 0.95 MPa, the number of buckles increased to 10 and, finally, 14 buckles occurred on this head (Li et al., 2019a). For head ZJU-X3, a total of 13 buckles were finally formed. The rest of the experimental results are obtained from the literature reported by Miller (1999). The experimental buckling pressure of most heads is the pressure at which the first buckle is detected visually. However, for some heads, such as T11 and T13, it is the pressure at which the first buckle is detected by a pressure drop.

Table S1 summarizes the experimental results for torispherical heads formed by spinning and pressing, and gives a comparison of the experimental results and predictions of Eq. (14). It can be seen that the relative error between the experimental and computational results ranged from −33% to 38%, which is in relatively good agreement.

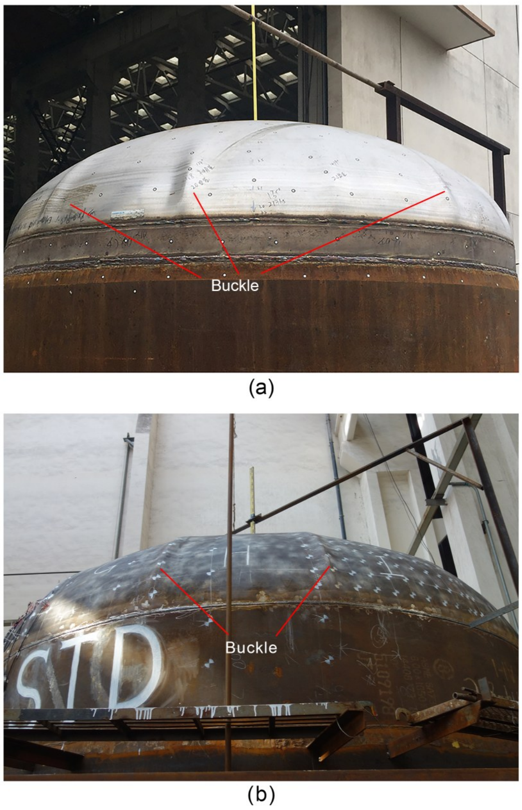

Fig. 9 Photograph of typical buckles in torispherical heads under internal pressure: (a) ZJU-X3; (b) ZJU-STD2

Table S2 summarizes the experimental results for torispherical heads assembled from formed segments and gives the comparison of buckling pressures between the experimental results and predictions of Eq. (14). The relative error between the experimental and computational results of Eq. (14) ranges from −14% to 57%. The relative error of the heads with small diameters is small, but for large heads with a diameter of 4–5 m, the relative error is relatively large, and mostly above 40%. This is because large shape imperfections occur in torispherical heads assembled from formed segments due to welding residual deformation, which causes a decrease of buckling pressure (Li et al., 2019a). Therefore, a formula for predicting the buckling pressure of torispherical heads assembled from formed segments is obtained by introducing a reduction factor of 0.8 into Eq. (14), as given by

$$P_b = 244 S_y \left(\frac{D_i}{t}\right)^{-1.67} \left(\frac{R_i}{D_i}\right)^{-1.32} \left(\frac{r}{D_i}\right)^{0.47}. \quad (15)$$

As shown in Table S2, the relative error between the experimental and computational results of Eq. (15) ranged from −31% to 24%, which is in relatively good agreement.

Combining Eqs. (14) and (15), a new Eq. (16) for calculating the buckling pressure of torispherical heads is proposed as follows:

$$P_b = 305 \beta_t S_y \left(\frac{D_i}{t}\right)^{-1.67} \left(\frac{R_i}{D_i}\right)^{-1.32} \left(\frac{r}{D_i}\right)^{0.47}, \quad (16)$$

where

$$\beta_t = \begin{cases} 1.0, & \text{for steel torispherical heads fabricated by spinning and pressing,} \\ 0.8, & \text{for steel torispherical heads assembled from formed segments.} \end{cases}$$

Eq. (16) is applicable for steel torispherical heads with $200 \leqslant D_i/t \leqslant 2000$, $0.7 \leqslant R_i/D_i \leqslant 1.0$, and $0.06 \leqslant r/D_i \leqslant 0.2$. For the tested heads beyond that scope, the relative error between the calculated buckling pressure of Eq. (16) and the experimental results is analysed. For head Nos. Fino (D_i/t=2304), ZJU-X2 (R_i/D_i=1.01), T1–T4 (R_i/D_i=1.093), and ZJU-CAP1 and CAP2 (r/D_i=0.21), the relative errors range from −13% to 16%, as shown in Tables S1 and S2. For head Nos. T1–T4 (r/D_i=0.04), and SC12 (r/D_i=0.056), the relative errors range from −33% to 38%, as shown in Table S1. Thus, for tested heads with D_i/t>2000, R_i/D_i>1.0, and r/D_i>0.2, Eq. (16) is still relatively accurate, but for heads with r/D_i<0.06, the calculation error is slightly high.

Tables S1 and S2 and Fig. 10 show the comparison between the new Eq. (16) and experimental results. Stripping out heads (r/D_i=0.04) beyond the range of the formula, the relative error between the prediction of the new formula and the experimental results ranges from −31% to 24%, and the average absolute relative error is 14%, which shows that the predicted buckling pressures of the proposed new formula for calculating the buckling pressure of the torispherical head agree well with the experimental results.

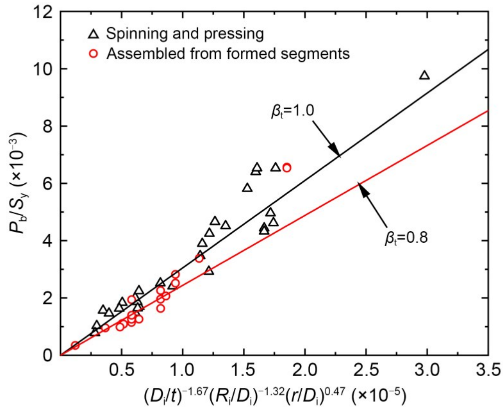

Fig. 10 Comparison of experimental results and Eq. (16)

4 Comparison with other formulas

Table 4 lists the formulas for predicting the buckling pressure of torispherical heads subjected to internal pressure. Aylward and Galletly (1979)'s formula is used for elastic buckling although the buckling type of commonly used steel heads is usually plastic buckling. Therefore, the calculated buckling pressures of Aylward and Galletly (1979)'s formula are 2.9–7.1 times the experimental results for steel heads, as shown in Table S3.

Table S4 presents a comparison of the buckling pressure between the existing formulas and the experimental results. Galletly et al. (1979, 1981, 1985) developed formulas for the plastic buckling pressures of torispherical heads, as described in Section 1. Eq. (7) has a limitation in the scope of applicability (it is only

Table 4 Summary of formulas for the buckling of torispherical heads under internal pressure

Formula	Eq. No.	Limitation	Buckling type
Aylward and Galletly (1979)'s formula	(1)	$500 \leqslant D_i/t \leqslant 2000$, $0.75 \leqslant R_i/D_i \leqslant 1.5$, $0.06 \leqslant r/D_i \leqslant 0.15$	Elastic buckling
Galletly (1981)'s formula	(7)	$500 \leqslant D_i/t \leqslant 1250$, R_i/D_i=1.0, $0.05 \leqslant r/D_i \leqslant 0.2$	Plastic buckling
Galletly (1986a)'s formula	(4)	$500 \leqslant D_i/t \leqslant 1500$, $0.75 \leqslant R_i/D_i \leqslant 1.5$, $0.06 \leqslant r/D_i \leqslant 0.18$	Plastic buckling
Galletly and Błachut (1985)'s formulas	(5), (6)	$300 \leqslant D_i/t \leqslant 1500$, $0.8 \leqslant R_i/D_i \leqslant 1.0$, $0.05 \leqslant r/D_i \leqslant 0.2$	Plastic buckling
Miller (1999, 2001)'s formulas	(8)–(13)	$20 \leqslant D_i/t \leqslant 2806$, $0.72 \leqslant R_i/D_i \leqslant 1.82$, $0.04 \leqslant r/D_i \leqslant 0.35$	Elastic/plastic buckling
Proposed in this paper	(16)	$200 \leqslant D_i/t \leqslant 2000$, $0.7 \leqslant R_i/D_i \leqslant 1.0$, $0.06 \leqslant r/D_i \leqslant 0.2$	Plastic buckling

applicable to torispherical heads with R_i/D_i=1.0); thus, it is not discussed here.

The relative error range between the prediction of Galletly (1986a)'s Eq. (4) and the experimental results is −54% to 53%. As for heads formed by spinning and pressing, the predicted buckling pressure of the formula is 6% to 54% lower than the experimental results, and it is 40% to 53% higher for some heads assembled from formed segments. In addition, this formula is not applicable to heads with D_i/t<500 and D_i/t>1500.

The relative error between the predicted buckling pressure of Galletly and Błachut (1985)'s formula Eq. (5) and experimental results ranges from −59% to 37%. For heads formed by spinning and pressing, the predicted buckling pressure of the formula ranges from 21% to 59% lower than the experimental results, and the relative error is −37% to 37% for heads assembled from formed segments. In addition, this formula is not applicable to heads with D_i/t>1500.

The relative error between the predicted buckling pressure of Galletly and Błachut (1985)'s formula Eq. (6) and the experimental results ranges from −63% to 34%. For heads formed by spinning and pressing, the predicted buckling pressure of the formula is 22% to 63% lower than the experimental results, and the relative error is −44% to 34% for heads assembled from formed segments. In addition, this formula does not apply to heads with D_i/t>1500.

The relative error between the predicted buckling pressure of Miller (1999, 2001)'s formulas and the experimental results ranges from −57% to 23%. The predicted buckling pressure of Miller's formulas is much lower than the experimental results for most tested heads. The reason is that Miller's formulas provide a conservative prediction for experimental results in order to further develop a safe design method for preventing the buckling of torispherical heads. However, for heads ZJU-X3, ZJU-CAP2, SC7, and K1−K4, the predicted buckling pressures of Miller's formulas are higher than the experimental results.

Fig. 11 shows a comparison of the prediction accuracies of existing formulas and the new formula proposed in this paper. The new formula has the smallest average absolute relative error of 14%, which is approximately half that of the other formulas. The root mean square error (RMSE) of the new formula is also the smallest. In general, the prediction accuracy of the new formula is the best among these formulas.

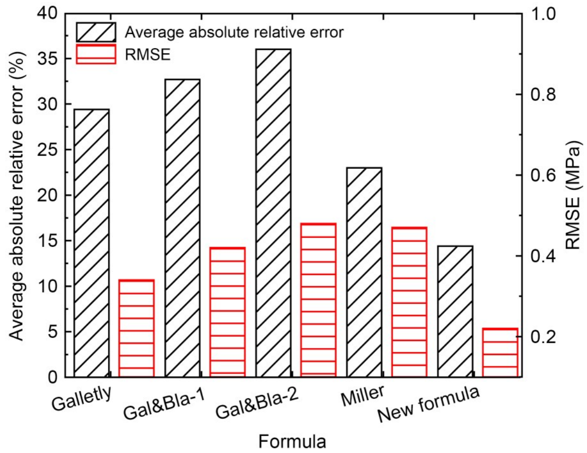

Fig. 11 Comparison of different formulas for buckling pressure of torispherical heads

5 Conclusions

This paper investigates the formulas of plastic buckling pressure for steel torispherical heads under internal pressure. A nonlinear FE method based on the arc-length method is established to calculate the buckling pressure of torispherical heads, taking into account the effects of stress hardening and geometrical nonlinearity. The buckling pressure results calculated by the FE method are consistent with those of BOSOR5, which is a program for calculating the elastic-plastic bifurcation buckling pressure based on the finite difference energy method. The effects of the geometry, material parameters, and restraint forms of the head edge on the buckling pressure were investigated. It shows that the buckling pressure of the torispherical heads decreases with the increase of D_i/t and R_i/D_i, and increases with the decrease of r/D_i. Buckling pressure increases approximately linearly with the increase of yield strength. The fixed constraint of head edge will enhance the buckling resistance of torispherical heads, but has little effect on heads with large D_i/t.

By fitting the curve of elastic-plastic FE results and introducing a reduction factor determined from experimental results, a new formula of plastic buckling pressure for steel torispherical heads under internal pressure is proposed. This formula is applicable for steel torispherical heads with 200≤D_i/t≤2000, 0.7≤R_i/D_i≤1.0, and 0.06≤r/D_i≤0.2. Compared with Galletly's formula, Galletly and Błachut's formulas, and Miller's formulas, the new formula has a comprehensive advantage in terms of its accuracy and applicability.

Acknowledgments

This work is supported by the National Natural Science Foundation of China (No. 52105161).

Author contributions

Jinyang ZHENG led the research project. Jinyang ZHENG and Keming LI designed the research. Sheng YE, Keming LI and Shan SUN processed the corresponding data. Sheng YE wrote the first draft of the manuscript. Shan SUN helped to organize the manuscript. Keming LI and Sheng YE revised and edited the final version.

Conflict of interest

Sheng YE, Keming LI, Jinyang ZHENG, and Shan SUN declare that they have no conflict of interest.

References

AQSIQ (General Administration of Quality Supervision, Inspection and Quarantine of the People's Republic of China), SA (Standardization Administration of the People's Republic of China), 2011. Pressure Vessels–Part 2: Materials, GB/T 150.2–2011. National Standards of the People's Republic of China (in Chinese).

AQSIQ (General Administration of Quality Supervision, Inspection and Quarantine of the People's Republic of China), SA (Standardization Administration of the People's Republic of China), 2017. Stainless Steel and Heat Resisting Steel Plate, Sheet and Strip for Pressure Equipment, GB/T 24511–2017. National Standards of the People's Republic of China (in Chinese).

ASME (American Society of Mechanical Engineers), 2023a. ASME Boiler and Pressure Vessel Code, Section II: Materials, Part D, ASME BPVC.II.D.C-2023. ASME.

ASME (American Society of Mechanical Engineers), 2023b. ASME Boiler and Pressure Vessel Code, Section VIII: Rules for Construction of Pressure Vessels, Division 1, ASME BPVC.VIII.2-2023. ASME.

ASME (American Society of Mechanical Engineers), 2023c. ASME Boiler and Pressure Vessel Code, Section VIII: Rules for Construction of Pressure Vessels, Division 2, ASME BPVC.VIII.2-2023. ASME.

Aylward RW, Galletly GD, 1979. Elastic buckling of, and first yielding in, thin torispherical shells subjected to internal pressure. *International Journal of Pressure Vessels and Piping*, 7(5):321-336.
https://doi.org/10.1016/0308-0161(79)90025-5

Błachut J, 2020. Impact of local and global shape imperfections on buckling of externally pressurised domes. *International Journal of Pressure Vessels and Piping*, 188:104178.
https://doi.org/10.1016/j.ijpvp.2020.104178

Błachut J, 2023. Buckling behaviour of auxetic domes under external pressure. *Thin-Walled Structures*, 182:110262.
https://doi.org/10.1016/j.tws.2022.110262

Bushnell D, 1976. B0S0R5—program for buckling of elastic-plastic complex shells of revolution including large deflections and creep. *Computers & Structures*, 6(3):221-239.
https://doi.org/10.1016/0045-7949(76)90034-1

Bushnell D, 1977a. BOSOR4–program for stress, buckling, and vibration of complex shells of revolution. *In*: Perrone N, Pilkey W (Eds.), Structural Mechanics Software Series. University Press of Virginia, Charlottesville, USA, p.11-143.

Bushnell D, 1977b. Nonsymmetric buckling of internally pressurized ellipsoidal and torispherical elastic-plastic pressure vessel heads. *Journal of Pressure Vessel Technology*, 99(1):54-63.
https://doi.org/10.1115/1.3454520

Bushnell D, Galletly GD, 1977. Stress and buckling of internally pressurized, elastic-plastic torispherical vessel heads—comparisons of test and theory. *Journal of Pressure Vessel Technology*, 99(1):39-53.
https://doi.org/10.1115/1.3454519

CEN (European Committee for Standardization), 2021. Unfired Pressure Vessels—Part 3: Design, BS EN 13445-3-2021. CEN.

Fino A, Schneider RW, 1961. Wrinkling of a Large Thin Code Head Under Internal Pressure. Welding Research Council Bulletin No. 69, Welding Research Council, New York, USA.

Galletly GD, 1959. Torispherical shells—a caution to designers. *Journal of Manufacturing Science and Engineering*, 81(1): 51-62.
https://doi.org/10.1115/1.4008245

Galletly GD, 1981. Plastic buckling of torispherical and ellipsoidal shells subjected to internal pressure. *Proceedings of the Institution of Mechanical Engineers*, 195(1):S39-S46.
https://doi.org/10.1243/PIME_PROC_1981_195_054_02

Galletly GD, 1986a. Design equations for preventing buckling in fabricated torispherical shells subjected to internal pressure. *Proceedings of the Institution of Mechanical Engineers, Part A: Journal of Power and Energy*, 200(2): 127-139.
https://doi.org/10.1243/PIME_PROC_1986_200_018_02

Galletly GD, 1986b. A simple design equation for preventing buckling in fabricated torispherical shells under internal pressure. *Journal of Pressure Vessel Technology*, 108(4): 521-526.
https://doi.org/10.1115/1.3264824

Galletly GD, Radhamohan SK, 1979. Elastic-plastic buckling of internally pressurized thin torispherical shells. *Journal of Pressure Vessel Technology*, 101(3):216-225.
https://doi.org/10.1115/1.3454626

Galletly GD, Błachut J, 1985. Torispherical shells under internal pressure—failure due to asymmetric plastic buckling or axisymmetric yielding. *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science*, 199(3):225-238.
https://doi.org/10.1243/PIME_PROC_1985_199_117_02

Li K, Zheng J, Zhang Z, et al., 2017. Experimental investigation on buckling of ellipsoidal head of steel nuclear containment. *Journal of Pressure Vessel Technology*, 139(6): 061206.
https://doi.org/10.1115/1.4038013

Li K, Zheng J, Liu S, et al., 2019a. Buckling behavior of large-scale thin-walled ellipsoidal head under internal pressure.

Thin-Walled Structures, 141:260-274.
https://doi.org/10.1016/j.tws.2019.04.031

Li K, Zheng J, Zhang Z, et al., 2019b. A new formula to predict buckling pressure of steel ellipsoidal head under internal pressure. *Thin-Walled Structures*, 144:106311.
https://doi.org/10.1016/j.tws.2019.106311

Miller CD, 1999. Buckling Criteria for Torispherical Heads Under Internal Pressure. Welding Research Council Bulletin No. 444, Welding Research Council, New York, USA.

Miller CD, 2001. Buckling criteria for torispherical heads under internal pressure. *Journal of Pressure Vessel Technology*, 123(3):318-323.
https://doi.org/10.1115/1.1360692

Sowiński K, 2022. Stress distribution optimization in dished ends of cylindrical pressure vessels. *Thin-Walled Structures*, 171:108808.
https://doi.org/10.1016/j.tws.2021.108808

Sowiński K, 2023a. Experimental and numerical verification of stress distribution in additive manufactured cylindrical pressure vessel–a continuation of the dished end optimization study. *Thin-Walled Structures*, 183:110336.
https://doi.org/10.1016/j.tws.2022.110336

Sowiński K, 2023b. Application and accuracy of shell theory in the analysis of stress and deformations in cylindrical pressure vessels. *Thin-Walled Structures*, 188:110826.
https://doi.org/10.1016/j.tws.2023.110826

Wagner HNR, Niewöhner G, Pototzky A, et al., 2021. On the imperfection sensitivity and design of tori-spherical shells under external pressure. *International Journal of Pressure Vessels and Piping*, 191:104321.
https://doi.org/10.1016/j.ijpvp.2021.104321

Yang LF, Zhu YM, Yu JF, et al., 2021. Design of externally pressurized ellipsoidal heads with variable wall thicknesses. *International Journal of Pressure Vessels and Piping*, 191: 104330.
https://doi.org/10.1016/j.ijpvp.2021.104330

Zhang J, Hu HF, Wang F, et al., 2022. Buckling of externally pressurized torispheres with uniform and stepwise thickness. *Thin-Walled Structures*, 173:109045.
https://doi.org/10.1016/j.tws.2022.109045

Zheng J, Li K, 2021. New Theory and Design of Ellipsoidal Heads for Pressure Vessels. Springer, Singapore, Singapore.
https://doi.org/10.1007/978-981-16-0467-6

Zheng J, Li K, Liu S, et al., 2018. Effect of shape imperfection on the buckling of large-scale thin-walled ellipsoidal head in steel nuclear containment. *Thin-Walled Structures*, 124:514-522.
https://doi.org/10.1016/j.tws.2018.01.001

Zheng J, Zhang Z, Li K, et al., 2020. A simple formula for prediction of plastic collapse pressure of steel ellipsoidal heads under internal pressure. *Thin-Walled Structures*, 156: 106994.
https://doi.org/10.1016/j.tws.2020.106994

Zheng J, Yu Y, Chen Y, et al., 2021. Comparison of ellipsoidal and equivalent torispherical heads under internal pressure: buckling, plastic collapse and design rules. *Journal of Pressure Vessel Technology*, 143(2):021301.
https://doi.org/10.1115/1.4048099

Zhu YM, Liu W, Guan W, et al., 2022. Buckling pressure of imperfect spherical shell damaged by concentrated impact load. *International Journal of Pressure Vessels and Piping*, 200:104810.
https://doi.org/10.1016/j.ijpvp.2022.104810

Electronic supplementary materials

Section S1, Tables S1–S4