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Many-body scars in quantum Heisenberg *XY* models and analog simulations on superconducting circuits

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Abstract: Quantum many-body systems lie at the heart of modern fundamental physics. The study of these systems has revealed a plethora of fascinating phenomena, such as quantum thermalization, many-body localization, and quantum many-body scars. This review provides a comprehensive overview of the recent advances in understanding quantum many-body scars and non-ergodic dynamics in quantum systems on superconducting-circuit platforms, ranging from theoretical mechanisms and effective models to experimental observations.

Key words: Quantum many-body scars; Quantum chaos; Quantum simulation; Superconducting circuits

1 Introduction

Understanding the behavior of quantum many-body systems presents one of the most challenging frontiers in modern physics. These systems, comprising numerous interacting quantum particles, exhibit complex collective phenomena that often cannot be predicted from the properties of individual constituents. The fundamental question of quantum thermalization has driven significant theoretical and experimental advances in recent years. The eigenstate thermalization hypothesis (ETH) is a theoretical framework for understanding thermalization in isolated quantum systems, implying that individual energy eigenstates themselves act as thermal states, allowing for the emergence of thermal behavior in isolated systems without the external environments or classical approximations (Deutsch, 1991; Srednicki, 1994; Cazalilla and Rigol, 2010; Kaufman et al., 2016). However, in a number of quantum systems, ergodicity breaking occurs through various mechanisms that prevent the system from exploring its entire available state space. For example, quantum integrability (Sutherland, 2004) in finely tuned 1D systems can exhibit rich symmetries and thus strongly violate thermalization. Also, many-body localization (MBL) (Basko et al., 2006; Oganesyan and Huse, 2007; Bar Lev and Reichman, 2014; Huse et al., 2014; Nandkishore and Huse, 2015) represents a remarkable phenomenon where strong disorder combined with interactions leads to the violation of thermalization. Recently, more subtle forms of ergodicity breaking have been discovered, in which a few eigenstates violate the ETH while others remain thermalized. In particular, the discovery of quantum many-body scars (QMBSs) (Turner et al., 2018b; Serbyn et al., 2021)—rare non-thermalizing eigenstates embedded in otherwise thermal spectra—has challenged this paradigm, revealing mechanisms of weak ergodicity breaking with profound implications for quantum information (Turner et al., 2018b; Serbyn et al., 2021). These special states exhibit persistent coherent dynamics when quenched from specific initial conditions, evading rapid thermalization and preserving quantum correlations over extended timescales (Bernien et al., 2017; Zhang PF et al., 2023).

In general, direct observation of these quantum many-body states in naturally occurring physical

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systems, such as condensed matter systems, is quite challenging. Nevertheless, the advent of quantum simulators, including Rydberg atoms (Lin and Motrunich, 2019; Lin et al., 2020a; Ding et al., 2024), superconducting qubits (Lamata et al., 2018; Krantz et al., 2019; Babukhin et al., 2020), optical lattices (Hudomal et al., 2020; Zhao et al., 2020; Scherg et al., 2021; Su et al., 2023; Adler et al., 2024), and ion trapping systems (Davoudi et al., 2020; Kaplan et al., 2020; Monroe et al., 2021), provides a powerful analogy for simulating a wide range of their dynamical manifestations. Among varied quantum simulation platforms, superconducting processors, with their high-fidelity control and scalability, have emerged as an ideal platform for analog simulations of quantum many-body dynamics. Superconducting quantum processors had already proven their effectiveness as platforms for studying both fully thermal systems (Neill et al., 2016; Li et al., 2019) and localized quantum states (Xu et al., 2018; Orell et al., 2019; Guo et al., 2021a, 2021b), demonstrating precise control over quantum dynamics and measurement capabilities. These technical capabilities established superconducting circuits as an ideal platform for investigating the intermediate regime of weak ergodicity breaking represented by QMBS (Fig. 1a).

In this review, we summarize recent progress in the QMBS state in quantum Heisenberg *XY* models and their experimental realization on superconducting circuit platforms. We begin by outlining the theoretical framework of QMBS, emphasizing the fundamental mechanism within Hilbert space. Subsequently, we discuss experimental protocols for preparing and probing scarred states, including quantum state tomography and measurements of the local population. We then analyze key findings from superconducting platforms, such as disorder-tunable entanglement (Dong et al., 2023) and the observation of quantum many-body dynamics in 2D finite-size systems (Yao et al., 2023). Finally, we consider future research directions for superconducting platforms.

2 Background: quantum thermalization and its violation

2.1 Quantum thermalization and eigenstate thermalization hypothesis

The ETH provides a fundamental framework for understanding how isolated quantum systems reach thermal equilibrium (Deutsch, 1991). Originally proposed by Deutsch and Srednicki in the 1990s, ETH can be formulated mathematically through the matrix elements of local observables $\hat{O}$ in the energy eigenbasis $|E_n\rangle$ (Srednicki, 1999):

$$\langle E_m|\hat{O}|E_n\rangle = O(\bar{E})\delta_{mn}\mathrm{e}^{-\frac{S(\bar{E})}{2}} f_O(\bar{E},\omega)R_{mn}, \quad (1)$$

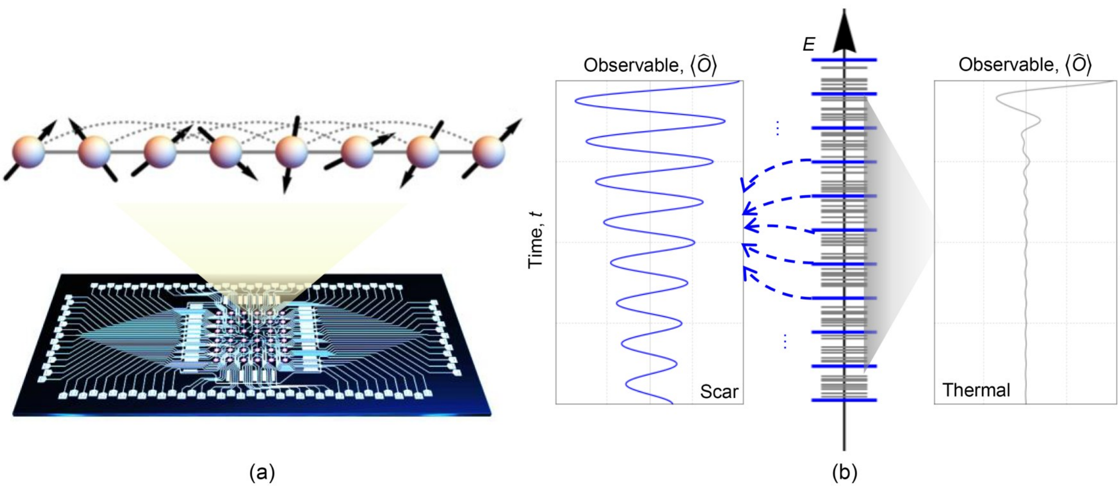

Fig. 1 (a) Illustration of a superconducting circuit system involving multi-qubits with tunable nearest-neighbor coupling strength (its effective model is a spin-1/2 *XY* Heisenberg model with weak long-range couplings); (b) diagram of energy spectrum (the blue lines represent scarred eigenstates, while the black lines are thermal eigenstates). The scarred eigenstates contribute a coherent dynamic of observables with strong oscillating behavior, and the thermal eigenstates result in thermalized dynamics. References to color refer to the online version of this figure

where δ_{mn} is the Kronecker delta symbol, $\bar{E}=(E_m+E_n)/2$ is the mean eigenenergy, $\omega=E_m-E_n$ is the eigenenergy difference, $S(\bar{E})$ is the thermodynamic entropy at energy $\bar{E}$, $O(\bar{E})$ and $f_O(\bar{E},\omega)$ are smooth functions of their arguments, and R_{mn} is a random variable with zero mean and unit variance (D'Alessio et al., 2016). This ansatz leads to several important predictions: (i) Diagonal ETH. For individual eigenstates, expectation values of local observables equal their microcanonical averages at the corresponding energy density (Garrison and Grover, 2018): $\langle E_m|\hat{O}|E_n\rangle \approx \mathrm{Tr}(\hat{O}\boldsymbol{\rho}_{\mathrm{mc}}(E_n))$. (ii) Off-diagonal ETH. The off-diagonal matrix elements are exponentially small in system size and exhibit random fluctuations, ensuring thermalization through dephasing (Mondaini et al., 2016). (iii) Eigenstate-to-eigenstate fluctuations of observables decrease exponentially with system size L as $\sigma^2(E)\sim\mathrm{e}^{-cL^d}$ (Beugeling et al., 2014), where d is the spatial dimension and c is a constant dependent on the model. For more details, see review articles (D'Alessio et al., 2016; Deutsch, 2018).

Considering the predictions above, the ETH provides sufficient criteria for ergodicity in quantum many-body systems, advancing the research area of quantum chaos. In classical systems, chaos manifests through exponential sensitivity to initial conditions, but quantum systems, governed by the linear Schrödinger equation, cannot exhibit such sensitivity in the same way (Haake, 1991). Instead, quantum chaos can be described by the predictions of random matrix theory (Wigner, 1955; Dyson, 1962; Mehta, 2004). Also, quantum chaos is characterized by the distinct signatures of ETH, which is observed in a wide variety of systems (Deutsch, 2018). In such chaotic systems, they naturally lead to thermal equilibrium without the need for external baths or reservoirs.

Furthermore, the ETH, which requires all eigenstates in a given energy window to show ergodicity, has been numerically verified for various non-integrable models (Rigol et al., 2008; LeBlond et al., 2019). However, some studies have identified key exceptions that result in non-ergodic behavior (Shiraishi and Mori, 2017; Buča, 2023).

2.2 Strong ergodicity breaking

In contrast to quantum chaos discussed above, quantum integrable systems represent a fundamentally different paradigm. Integrable quantum systems represent a special class of many-body systems characterized by an extensive number of local conservation laws (Grabowski and Mathieu, 1995), mainly in 1D systems. Mathematically, a quantum system with N degrees of freedom is considered integrable if it has N independent conserved quantities Q_i that satisfy: $[Q_i, Q_j]=0$, $[Q_i, H]=0$, where H is the system Hamiltonian (Caux and Mossel, 2011). These conservation quantities fundamentally constrain the dynamics and result in a breakdown of ergodicity. The presence of these constraints manifests in the system's long-time behavior through a generalized Gibbs ensemble (Rigol et al., 2007): $\rho_{\mathrm{GGE}}=(1/Z)\exp\left(-\sum_i\lambda_i Q_i\right)$, where λ_i represents Lagrange multipliers determined by initial conditions, and Z is the partition function. The exact solvability of quantum integrable systems comes with remarkable mathematical structures and leads to non-ergodic behavior (Caux and Mossel, 2011).

Another important ergodicity-breaking phenomenon is the MBL. Inspired by Anderson localization in single-particle systems, MBL is a phenomenon that represents perhaps the most dramatic failure of statistical mechanics in isolated many-body quantum systems (Basko et al., 2006; Oganesyan and Huse, 2007). In systems with MBL, the inter-particle interactions do not destroy localization. Instead, these systems maintain memory of their initial conditions indefinitely, failing to reach thermal equilibrium even after arbitrarily long times (Imbrie, 2016). It is a kind of strong ergodic breaking, exhibiting integrable behavior in interacting many-body quantum systems. Experimental observations of MBL have emerged across various quantum platforms (Schreiber et al., 2015; Smith et al., 2016; Wei et al., 2018; Silevitch et al., 2019). Particularly, Xu et al. (2018) demonstrated the MBL phenomenon in a spin-1/2 XY chain by introducing controlled disorder on a 10-qubit superconducting processor. Their measurements confirmed key MBL signatures: logarithmic entanglement growth, memory retention of initial states, and exponentially decaying distant interactions. These experimental realizations provide essential validation of theoretical frameworks while revealing the robustness of non-ergodic phenomena under realistic conditions with decoherence and control errors.

The transition between integrable and chaotic behavior presents a rich phenomenology. Generic perturbations to integrable many-body systems typically lead to quantum chaos. However, the approach to thermalization can be suppressed, involving various timescales and intermediate regimes (Friedman et al., 2019). Prethermalization, where the system temporarily behaves as if integrable before eventual thermalization, represents a particularly interesting phenomenon in nearly integrable systems (Bertini et al., 2015).

A variety of methods and metrics have been proposed to quantify the degree of quantum chaos. Level statistics of random matrix theory provide a quantitative distinction between quantum chaotic and MBL systems. One of the metrics is the average adjacent level spacing ratio $\langle r\rangle=\left\langle \min(s_n,s_{n+1})/\max(s_n,s_{n+1})\right\rangle$, where $s_n=E_{n+1}-E_n$ represents normalized eigenenergy gaps (Berry and Tabor, 1977; Bohigas et al., 1984). Quantum chaotic systems follow Wigner-Dyson statistics with $\langle r\rangle\approx0.53$ holding the time-reversal symmetry, indicating level repulsion (Atas et al., 2013), while MBL systems exhibit Poisson statistics with $\langle r\rangle\approx0.38$ and uncorrelated energy levels (Oganesyan and Huse, 2007). These statistical signatures provide a simple diagnostic for identifying non-ergodic phenomena in quantum many-body systems. Furthermore, the spectral form factor, defined as $K(t)=(1/D^2)\times\sum_{m,n}\exp[\mathrm{i}(E_n-E_m)t]$, offers a more precise diagnostic. Here, D is the dimension of Hilbert space. Recently, an experimentally feasible protocol for its measurement was proposed (Joshi et al., 2022) and subsequently implemented on a superconducting platform to study chaotic and localized phases (Dong et al., 2025).

The implications of these studies extend beyond fundamental physics. The controlled manipulation of integrability and chaos has potential applications in quantum information processing (Fogarty et al., 2021). Integrable systems, with their predictable dynamics and protected coherences, offer promising platforms for quantum memory and computation. Conversely, the rapid scrambling of information in chaotic systems could be harnessed for quantum randomization protocols and thermalization-based quantum devices (Harris et al., 2022).

2.3 Weak ergodicity breaking and quantum many-body scars

The discovery of QMBS represents a paradigm shift in our understanding of non-ergodic phenomena in quantum systems (Turner et al., 2018b; Serbyn et al., 2021). Unlike other mechanisms (such as MBL) that lead to complete breakdown of ergodicity, QMBSs manifest as special eigenstates scattered throughout the energy spectrum that violate the eigenstate thermalization hypothesis, while coexisting with thermal states. This subtle form of ergodicity breaking provides a fascinating bridge between quantum chaos and integrability (Shiraishi and Mori, 2017). The phenomenon of QMBS was first identified in a chain of Rydberg atoms, where certain initial states exhibit persistent oscillations instead of thermalizing (Bernien et al., 2017). These oscillations can be understood through the lens of special eigenstates that violate ETH. In the Rydberg atom chain model, the dynamics are governed by a constrained Hamiltonian, $H=\sum_i \Omega\sigma_x^i+\sum_{i<j}V_{ij}n_in_j$, where Ω represents the Rabi frequency, σ_x^i represents Pauli operators, V_{ij} describes the van der Waals interactions, and n_i represents number operators with nearest-neighbor constraints.

The scarred eigenstates in this system form an approximately equally spaced tower in the energy spectrum as $E_k\approx E_0+k\Delta E$, $k=0, 1, \cdots, N$, where E_0 is the ground state energy and ΔE is a constant energy gap (Fig. 1b). This structure leads to coherent quantum dynamics and revivals of certain initial states (Turner et al., 2018a).

The theoretical understanding of QMBS reveals several key characteristics that distinguish them from other non-ergodic phenomena. First, scarred eigenstates exhibit sub-thermal entanglement entropy, typically following an area law or logarithmic scaling, while typical states show volume-law behavior (Khemani et al., 2019). Second, these states often have special algebraic structures, manifesting as representations of emergent symmetry algebras or quantum groups (Moudgalya et al., 2018b). Beyond the original Rydberg atom model, QMBSs have been discovered in a variety of systems through different mechanisms. One important class involves models with exact scars, where the scarred states can be constructed analytically through embedding theorems or projection techniques (Moudgalya et al., 2018a; Bull et al., 2019). Another mechanism involves dynamical symmetries and hidden conservation laws, leading to robust revivals and protected subspaces in the many-body dynamics (Chandran et al., 2023).

The phenomenon of quantum scarring shows remarkable connections to classical chaos theory (Ho et al., 2019; Michailidis et al., 2020; Pilatowsky-Cameo et al., 2021; Petrova et al., 2025; Ren et al., 2025). Just as classical scars represent enhanced probability density along unstable periodic orbits in chaotic billiards, QMBSs can be viewed as quantum signatures of periodic orbits in the exponentially large many-body Hilbert space (Hudomal et al., 2020). This analogy provides deep insights into the nature of quantum chaos and its classical correspondence.

The presence of QMBS has profound implications for quantum dynamics. Initial states with high overlap with scarred eigenstates exhibit persistent oscillations and slow thermalization (Lin et al., 2020b). This behavior shows a novel form of quantum memory, preserving information about initial conditions for exceptionally long times.

Recent theoretical developments have revealed unexpected connections between QMBS and other phenomena in quantum many-body physics. For example, the scarring in Luttinger liquid is related to the interplay of Fermi statistics and chirality (Schindler et al., 2022), while the construction of QMBS in the Sachdev-Ye-Kitaev model provides new perspectives on quantum gravity and holographic duality (Nandy et al., 2024). Furthermore, connections to quantum optimization (Ljubotina et al., 2022) and dynamical phase transitions (van Damme et al., 2023) suggest potential applications beyond fundamental physics.

The study of QMBS continues to yield surprises and new directions for research. Areas worthy of further investigation include the full classification of scarring mechanisms, the role of symmetries and topology in protecting scarred states, and the possibility of engineering robust quantum scars for practical applications (Dooley, 2021; Surace et al., 2023). The phenomenon of QMBS thus stands as a testament to the continuing richness of quantum many-body physics, challenging our understanding of thermalization and pointing toward new possibilities for quantum control and information processing.

3 Mechanisms underlying quantum many-body scars

3.1 Physical origin and spectral properties

The physical origin of QMBS can be traced to several fundamental mechanisms that prevent thermalization of specific eigenstates. The most thoroughly studied mechanism emerges in constrained quantum systems, exemplified by the PXP model describing Rydberg atom arrays (Bernien et al., 2017). In this system, the Hamiltonian is given by $H_{\mathrm{PXP}}=\sum_i \Omega P_{i-1}\sigma_x^i P_{i+1}$, where $P_i=1-n_i$ represents projection operators enforcing the Rydberg blockade constraint, and σ_x^i represents the atomic transition operator. The constraints imposed by the Rydberg blockade create an effective fragmentation of the Hilbert space, leading to the emergence of special trajectories that support persistent oscillations. The phenomenon of QMBS can be found in many models, including kinetically constrained models (Turner et al., 2018a; Bhattacharjee et al., 2022; Ivanov and Motrunich, 2025; Kerschbaumer et al., 2025), Heisenberg spin models (Schecter and Iadecola, 2019; Shibata et al., 2020; Wang JW et al., 2024), the Affleck-Kennedy-Lieb-Tasaki model (Shiraishi and Mori, 2017; Moudgalya et al., 2018b), the extended Hubbard model (Hudomal et al., 2020; Mark and Motrunich, 2020; Moudgalya et al., 2020; Kolb and Pakrouski, 2023), frustrated (McClarty et al., 2020) and topological (Ok et al., 2019) lattices, Floquet-driven systems (Mizuta et al., 2020; Mukherjee et al., 2020a; Huang et al., 2025), 2D systems (Lin et al., 2020a; Yao et al., 2023), and spin models with multi-body interaction (Sanada et al., 2023; Kaneko et al., 2024; Imai and Tsuji, 2025). Recently, how QMBSs respond to changes in the environment, such as disorder (Mondragon-Shem et al., 2021) and external fields (Yang et al., 2024), has also been extensively studied for the development of robust quantum technologies. Besides, some theoretical frameworks have been developed for studying QMBS, such as the embedding method (Shiraishi and Mori, 2017; Bull et al., 2019; Omiya and Müller, 2023a, 2023b), quasi-symmetry groups (Ren et al., 2021), and semiclassical unstable periodic orbit method (Ermakov et al., 2024; Evrard et al., 2024). Among the diverse models for studying QMBS, quantum Heisenberg spin *XY* models are important due to their rich symmetry structures and relevance to well-known condensed matter systems (Lerose et al., 2025), offering a fertile ground for exploring non-thermalizing dynamics in both integrable and non-integrable regimes.

From the viewpoint of the Hilbert-space graph, a new type of QMBS based on the Heisenberg spin *XY* model (Guo et al., 2023) and tilted Fermi-Hubbard model (Hudomal et al., 2020) was discovered, extending beyond the constraint model framework. This

'Hilbert-space quantum scar' originates from a special subspace with a hypercube or hypergrid geometry that weakly connects to thermalization regions in Hilbert space (Figs. 2a and 2b). Similar to previous studies, that kind of QMBS can also be directly visualized by the overlap between the eigenstates and the special collective state (Fig. 2c). Within this hypercube or hypergrid, certain collective Fock states show remarkable properties exhibiting periodic revival. The researchers demonstrated this phenomenon across various lattice configurations (1D Su-Schrieffer-Heeger (SSH) chain, comb lattice, and random dimer clusters) and extended it to higher dimensions using tetramers and octamers.

A deeper understanding of QMBS formation comes from analyzing the underlying symmetry structures. This 'Hilbert-space quantum scar' is classified as a SU(2) symmetrical structure, which forms an approximately equally spaced tower in the energy spectrum (Fig. 2c). This regular spacing arises from the presence of SU(2) symmetries, characterized by ladder operators $Q^{\pm}$ that approximately satisfy $[H, Q^{\pm}]\approx \pm\omega Q^{\pm}$. Starting from the spectrum generating algebra (SGA), the exact scar can be described through ladder operators that satisfy $Q^{\pm}|\psi_{\text{th}}\rangle=0$, where $|\psi_{\text{th}}\rangle$ represents thermal eigenstates, which ensures that ladder operators act coherently only on scarred states, leaving thermal states unaffected. Scarred states are constructed as $|S_k\rangle\propto(Q^{+})^k|S_0\rangle$ with energies $E_k=E_0+k\omega$, creating the characteristic equally spaced tower. Then, the connection between the scarred subspace and thermal subspace is described using the non-equilibrium Green's function and hypercube decay approximation (Fig. 2b), which gives more subtle information about the energy spectrum (Guo et al., 2023). A similar method, as restricted SGA, explains the equally spaced towers of eigenstates in other well-known models with emergent SU(2) symmetry, including the PXP model (Choi et al., 2019) and the Affleck-Kennedy-Lieb-Tasaki model (Moudgalya et al., 2018b).

QMBSs caused by other symmetries, such as SU(3) (O'Dea et al., 2020) and U(N) (Pakrouski et al., 2020), can also be studied using SGA. Note that some

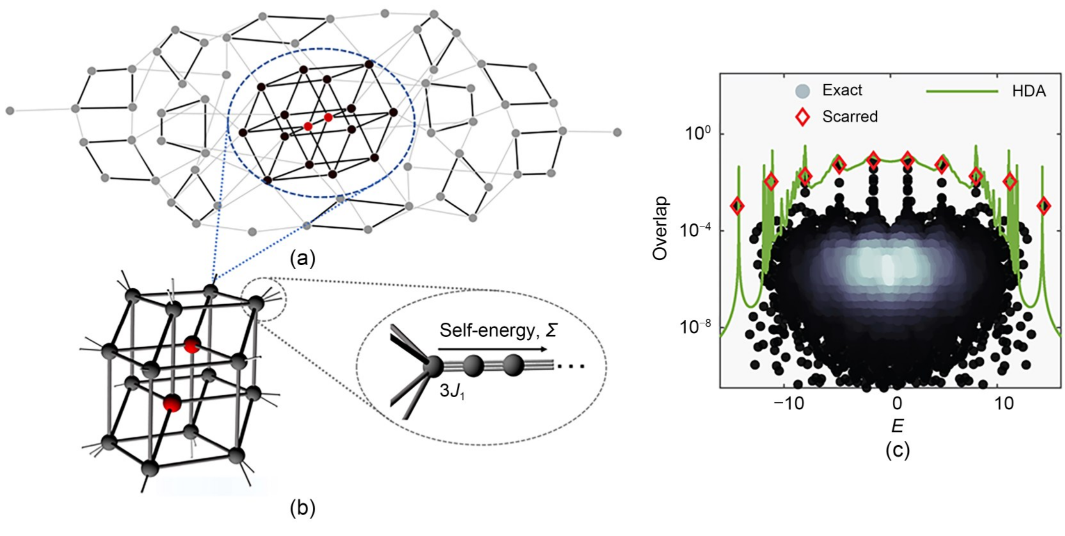

Fig. 2 (a) Adjacency graph in Hilbert space of an 8-site spin-1/2 *XY* model (the whole system is divided into the hypercube and the thermal environment, which are inside and outside the blue circle, respectively; red bullets represent two scar states; bullets belonging to the hypercube denote Fock states $|z_{a_1}z_{b_1}z_{a_2}z_{b_2}z_{a_3}z_{b_3}z_{a_4}z_{b_4}\rangle$ with $z_{a_\alpha}+z_{b_\alpha}=1$ for α=1, 2, $\cdots$, *N*, while grey bullets do not meet this requirement; *a* and *b* denote the sites within a dimer; black and grey lines represent the intra-dimer J_a and inter-dimer J_e couplings, respectively); (b) from the perspective of a vertex in the hypercube, the thermal environment considered a semi-infinite chain, represented by the self-energy Σ (a set of semi-infinite chains leads to an approximate decay rate of the Hilbert hypercube for a Fock state in the Hilbert hypercube directly connecting to the thermalization region); (c) scattering plot of the overlap of many-body eigenstates with the special collective state as a function of energy for an 18-site spin-1/2 *XY* model (red diamonds represent the scar states, while the other dots represent the thermal states; the green line is the energy spectrum predicted by the hypercube decay approximation method illustrated in (b)). HDA represents the hypercube decay approximation. Reprinted from (Guo et al., 2023), Copyright 2023, with permission from American Physical Society. References to color refer to the online version of this figure

methods used in the past to deal with high-energy physics, such as lattice gauge theory, are now also used to construct symmetry-based QMBS (Budde et al., 2024; Osborne et al., 2024; Calajó et al., 2025). Symmetry-based scars tend to be robust, and perturbation theory shows that the scarred subspace remains isolated up to corrections of order λ^2, where λ is the perturbation strength (Lin et al., 2020b).

It is important to distinguish QMBS from other forms of ergodicity breaking, namely Hilbert space fragmentation (Moudgalya et al., 2022), where the Hilbert space is partitioned into dynamically disconnected sectors. Unlike QMBS, these systems have a complex structure in which specific initial states become 'trapped' within certain sectors of the Hilbert space, leading to restricted dynamics and a breakdown of ergodicity. In a seminal study, de Tomasi et al. (2019) showed how this fragmentation can naturally emerge in systems with conservation laws, resulting in robust non-thermalizing dynamics that persist even in the thermodynamic limit.

3.2 Dynamics and entanglement properties

The distinctive features of QMBS manifest most prominently in their non-equilibrium dynamics and unconventional entanglement properties. Unlike generic many-body systems that rapidly thermalize following the ETH, QMBS exhibit persistent oscillations and maintain low entanglement over extended time periods (Turner et al., 2018b). In this section, we examine these remarkable properties and their underlying physical mechanisms.

The most striking signature of QMBS appears in their non-equilibrium dynamics. When initialized in specific states with high overlap with scarred eigenstates, these systems show robust oscillatory behavior that persists for unusually long times. The time evolution of local observables shows oscillation behavior with an anomalously low decay rate (Serbyn et al., 2021). This behavior stands in stark contrast to the exponential approach to equilibrium expected in generic thermalizing systems.

The origin of these persistent oscillations can be traced to the special structure of scarred eigenstates. In the PXP model, for example, the many-body spectrum contains an approximately equally spaced tower of scarred states. The regular energy spacing ΔE determines the oscillation frequency $\omega\approx\Delta E/\hbar$. The fidelity revival amplitude can be written as

$$F(t)=|\langle\psi(0)|\psi(t)\rangle|^2=\left|\sum_k c_k^2 \mathrm{e}^{-\frac{\mathrm{i}E_k t}{\hbar}}\right|^2, \quad (2)$$

where $c_k=\langle E_k|\psi(0)\rangle$ is the overlap between eigenstates and the initial state. However, for an open system, the mixed-state fidelity $F(t)=\mathrm{Tr}\left[\sqrt{\sqrt{\boldsymbol{\rho}(t)}\,\boldsymbol{\rho}(0)\sqrt{\boldsymbol{\rho}(t)}}\right]$ is used as a metric to quantify the persistence of quantum correlations under decoherence, where $\boldsymbol{\rho}(t)$ is the density matrix at time t. The entanglement structure of QMBS provides another crucial signature of their non-ergodic nature. While typical high-energy eigenstates exhibit volume-law entanglement entropy following the Page value (Page, 1993), i.e., $S^{\mathrm{thermal}}(L)\sim L^d$, scarred states show dramatically reduced entanglement that typically follows a logarithmic or area-law scaling: $S^{\mathrm{scar}}(L)\sim\log(L)$ or $\sim L^{d-1}$. This sub-thermal entanglement has been rigorously demonstrated in various QMBS systems (Bull, 2022).

The dynamical evolution of entanglement provides additional insights into the nature of QMBS. For a bipartition of the system into regions A and B, the time-dependent entanglement entropy $S(t)$ is given by $S(t)=-\mathrm{Tr}_{\mathrm{A}}[\boldsymbol{\rho}_{\mathrm{A}}(t)\log\boldsymbol{\rho}_{\mathrm{A}}(t)]$, where $\boldsymbol{\rho}_{\mathrm{A}}(t)$ is the reduced density matrix of subsystem A. In QMBS systems, this quantity exhibits several remarkable features, such as slow growth, oscillatory behavior, and a special structural pattern.

Beyond the entanglement entropy, other quantum information measures provide valuable insights into QMBS physics. The out-of-time-order correlator (OTOC), $C(t)=\langle[W(t), V(0)]^2\rangle$, where $W(t)$ and $V(0)$ are local operators, shows distinctive behavior in QMBS systems, reflecting the restricted spread of quantum information (Maldacena et al., 2016; Yuan et al., 2022; Xiang et al., 2024; Liang et al., 2025). Similarly, the quantum Fisher information $F_{\mathrm{Q}}(t)=4(\langle\psi(t)|H^2|\psi(t)\rangle-\langle\psi(t)|H|\psi(t)\rangle^2)$ provides a sensitive probe of the quantum coherence maintained in scarred dynamics (Desaules et al., 2022).

4 Experimental observations and advances

4.1 Quantum simulation techniques

The experimental investigation of QMBS represents one of the most challenging frontiers in quantum simulation. The observation and characterization of these unique non-ergodic states require unprecedented precision in quantum control and measurement, presenting formidable challenges that approach the current boundaries of experimental capabilities. The successful study of QMBS phenomena requires a careful orchestration of multiple experimental techniques, each operating at the boundary of what is technically feasible. At the heart of QMBS experiments lies the crucial requirement for precise quantum state preparation and control. The observation of scarred dynamics typically begins with the initialization of a many-body system in a specific state that has substantial overlap with the scarred subspace. This initialization process requires single-site addressability with low error rates, a technical capability that has been demonstrated in advanced quantum simulation platforms. Also, it is easy to control quantum states by periodically driving single qubits (Bluvstein et al., 2021).

The engineering of interactions presents another critical challenge in QMBS studies. The characteristic van der Waals interaction between Rydberg atoms is given by $V_{ij}=C_6/|\boldsymbol{r}_i-\boldsymbol{r}_j|^6$, where $\boldsymbol{r}_i$ and $\boldsymbol{r}_j$ are the position vectors of atom i and atom j, respectively, and C_6 is the van der Waals interaction coefficient. This kind of interaction must be controllable over several orders of magnitude. This tunability is essential for accessing different dynamical regimes and investigating the robustness of scarred states. Moreover, the interaction engineering must be achieved with spatial resolution better than that of lattice spacing, requiring sophisticated optical control techniques (Semeghini et al., 2021).

Traditional quantum state tomography becomes exponentially complex with system size, necessitating the development of more efficient measurement protocols. Time-resolved correlation measurements, essential for characterizing the coherent oscillation characteristic of QMBS, require temporal resolution better than $\hbar/J$, where J represents the typical interaction strength. The measurement of entanglement properties, crucial for distinguishing scar from thermal states, often relies on accessing Renyi entropies through sophisticated interference protocols (Lanyon et al., 2017).

System size presents another crucial consideration in QMBS experiments. While theoretical proposals often assume large systems, practical implementations must balance the competing demands of coherence time, control precision, and measurement capability. Current experiments typically operate with 20–30 qubits, a regime where QMBS signatures become discernible but finite-size effects remain significant. Scaling to larger systems while maintaining high-fidelity control and measurement capabilities represents one of the key obstacles in the field (Ebadi et al., 2021). To address these limitations, quantum error mitigation strategies (Takagi et al., 2022) such as probabilistic error cancellation and measurement error tomography have proven effective in suppressing readout noise.

4.2 Advantages of superconducting circuit plateforms

Superconducting circuits have emerged as one of the main platforms for studying QMBS, offering unique advantages that have been further solidified by recent advancements. Scalability also remains a key strength of superconducting circuits. In 2021, IBM unveiled its 127-qubit 'Eagle' processor, showcasing the ability to control and measure large numbers of interacting qubits. This was followed by the 433-qubit 'Osprey' processor in 2022, demonstrating rapid progress in scaling up these systems (di Meglio et al., 2024). Recent experiments have highlighted the flexibility of superconducting circuits in implementing various quantum Hamiltonians.

Superconducting circuits continue to excel in fast operation times with high fidelity. In 2023, single-qubit gate fidelities have been reported exceeding 99.99% with coherence times surpassing 1 ms, demonstrating unprecedented control capabilities for quantum simulation experiments (Somoroff et al., 2023). Two-qubit gates have also seen improvements, with fidelities now reaching 99.9% and operation times as low as 50 ns (Ding et al., 2023). Integration with classical control electronics continues to improve, allowing for increasingly precise qubit control and readout. State-of-the-art readout fidelities now exceed 99.5% with measurement times as short as 140 ns (Chen et al., 2023). This high-fidelity, rapid measurement capability is crucial for studying quantum trajectories and implementing quantum feedback protocols in QMBS experiments.

Hence, recent advances in superconducting circuits have further solidified their position as a powerful platform for studying QMBS dynamics and related applications. Superconducting circuits present several competitive advantages over alternative platforms for QMBS research and offer programmable connectivity patterns ranging from nearest-neighbor to complex geometries (Kounalakis et al., 2018), enabling exploration of how scarred dynamics depends on lattice topology. The nanosecond gate times (compared to microseconds in ion traps (Wright et al., 2019)) allow for observation of multiple quantum revivals before decoherence intervenes. Furthermore, superconducting platforms excel in individual qubit addressability and high-fidelity readout, which is critical for precise state preparation and measurement of scarred states. Although cold atom systems can achieve larger sizes (Ebadi et al., 2021), they typically lack the fine individual control and measurement precision of superconducting circuits. Superconducting platforms also allow in situ parameter tuning during experiments, facilitating protocols such as dynamical decoupling to improve scar coherence (Dong et al., 2023). This combination of high-precision control, rapid operation, and scalability has made superconducting circuits an ideal testbed for investigating the intricate mechanisms underlying QMBSs.

4.3 Frontier of QMBS experiment on superconducting circuit

Recent experimental advances in superconducting quantum processors have opened up new possibilities for investigating QMBS phenomena. These platforms offer unique advantages, including high-precision control, tunable interactions, and site-resolved measurements, enabling the observation and manipulation of novel non-ergodic dynamics in quantum many-body systems. Specifically, the success of superconducting circuits in simulating quantum many-body models relies on their ability to precisely map theoretical Hamiltonians onto physical hardware. For instance, the spin-1/2 *XY* model, central to many QMBS studies, can be directly implemented using capacitively coupled transmons.

The experimental implementation of quantum many-body models on superconducting circuits begins with the Hamiltonian of circuit-quantum electrodynamics. Each superconducting qubit can be described as an anharmonic oscillator with Hamiltonian $H_q = 4E_C(\hat{n} - n_g) - E_J\cos\hat{\phi}$, where E_C is the charging energy, E_J is the Josephson energy, $\hat{n}$ is the Cooper pair number operator, n_g is the effective gate charge, and $\hat{\phi}$ is the phase operator. When the anharmonicity is sufficiently large ($E_C/E_J \ll 1$), we can restrict our analysis to the two lowest energy levels, effectively treating each transmon as a qubit. Tunable couplers between qubits are typically implemented as flux-biased superconducting quantum interference devices with effective Josephson energy $E_J(\Phi_{ext})=(E_J)_0\cos(\pi\Phi_{ext}/\Phi_0)$, where Φ_{ext} is the external magnetic flux threading the device loop, and Φ_0 is the magnetic flux quantum. By adjusting this external flux, researchers can continuously tune the effective coupling strength g_{ij} (g_{ic}) between qubits (qubit and coupler). After applying the rotating wave approximation and considering only the computational subspace, the coupled system of transmons and tunable couplers is described by the following Hamiltonian:

$$\frac{\mathcal{H}}{\hbar} = \sum_i \omega_i S_i^+ S_i^- + \sum_c \omega_c S_c^+ S_c^- + \sum_{\langle i,j\rangle} g_{ij}(S_i^- S_j^+ + S_j^- S_i^+) + \sum_{\langle i,c\rangle} g_{ic}(S_i^+ S_c^- + S_i^- S_c^+), \tag{3}$$

where ω_i (ω_c) denotes the frequency of qubit Q_i (coupler c), S^+ (S^-) represents the creation (annihilation) operator for qubit Q_i, and g_{ij} (g_{ic}) quantifies the coupling strength between qubits Q_i and Q_j (qubit Q_i and coupler c). Through the Schrieffer-Wolff transformation and in the dispersive regime ($|\Delta_{ij}|=|\omega_i-\omega_j| \gg g_{ij}$), this system reduces to an effective *XY* model:

$$\frac{\mathcal{H}_{\text{eff}}}{\hbar} \approx \sum_{\langle i,j\rangle} J_{ij}(S_i^- S_j^+ + S_j^- S_i^+) + \sum_i \Omega_i S_i^+ S_i^-, \tag{4}$$

with $J_{ij} \approx g_{ij} g_{jc} g_{ci}(1/\Delta_{ic} + 1/\Delta_{jc})$. The flexibility of superconducting circuits allows researchers to tune these parameters with high precision by adjusting flux-biased couplers and qubit frequencies, enabling the implementation of the *XY* models and beyond.

The first experimental observation of QMBS on superconducting circuits was achieved by Zhang PF et al. (2023) through realizing a distinct kind of scarring mechanism by approximately decoupling part of the many-body Hilbert space in the computational basis. Using a programmable processor with 30 qubits and tunable couplings, they implemented a tunable spin-1/2 *XY* Hamiltonian (Eq. (4)). This layout exploits the structure of the SSH chain, where the intra-dimer coupling $J_{i,i+1}=J_a$ with i being odd is slightly stronger than the inter-dimer coupling $J_{i,i+1}=J_e$ with i being even.

On the superconducting multi-qubit processor, the perturbation term is structured by the cross couplings and is given by

$$\frac{\mathcal{H}_x}{\hbar} \approx \sum_{R_{ij}=\sqrt{2}a_0} J_{ij}(S_i^- S_j^+ + S_j^- S_i^+), \tag{5}$$

where $R_{ij}=|\boldsymbol{r}_i-\boldsymbol{r}_j|$ is the separation distance of a qubit pair $\{i, j\}$, and a_0 is the separation distance of two nearest neighbor qubits.

QMBS states were demonstrated by measuring qubit population dynamics (Fig. 3), quantum fidelity $F_A(t)=\mathrm{Tr}[\boldsymbol{\rho}_A(0)\boldsymbol{\rho}_A(t)]$ (Fig. 4), and entanglement entropy (Fig. 4) after quenching from initial unentangled states. The observations revealed coherent oscillations with periods of about 50 ns and slow entropy growth, characteristic signatures of QMBS.

Dong et al. (2023) experimentally realized disorder-tunable entanglement at infinite temperature. Using a custom-built superconducting qubit ladder with coupling ranges of [−8, 8] and [−8, −2] MHz for parallel and vertical couplings, respectively, they demonstrated non-thermalizing states with rich entanglement structures. They measured the fidelity dynamics (Fig. 5), showing that the scar they found was disorder-tunable.

Recently, experimental studies have also extended to the observation of Hilbert space fragmentation phenomena on the superconducting circuit platform (Zhao et al., 2025), which revealed how certain Hamiltonian structures can create dynamically disconnected sectors in the Hilbert space. The result was persistent quantum revivals with well-defined frequencies and characteristically slow thermalization, even in the absence of explicit conservation laws (Wang et al., 2025). This revealed another mechanism for weak ergodicity breaking.

Furthermore, investigations into ergodicity-breaking phenomena in higher dimensions are underway. Such purely numerical investigations are quite challenging.

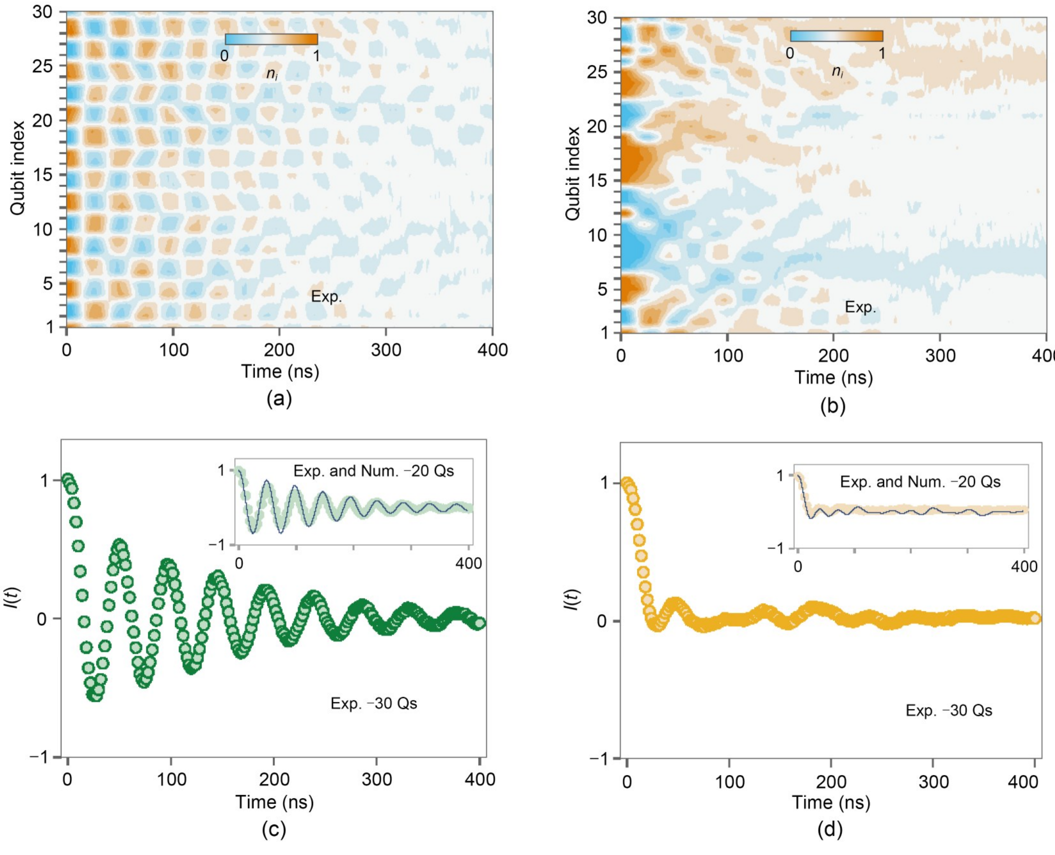

Fig. 3 (a) Contour diagrams of the experimental qubit population as a function of interaction time for a QMBS state; (b) contour diagrams for a rapidly thermalizing state; (c and d) generalized imbalance $I(t)$ extracted from the plots in (a) and (b), respectively, as a function of interaction time. The insets show the imbalance dynamics from experiments (dots or circles) and numerical simulations (solid curves) in a 20-qubit chain. The simulation parameter values are intra-dimer coupling $J_a/(2\pi)=-9.3$ MHz, inter-dimer coupling $J_e/(2\pi)=-6.1$ MHz, and cross coupling $J_{ij}/(2\pi)\in[0.3, 1.2]$ MHz for $R_{ij}=\sqrt{2}a_0$. The scar states are defined as $|\Pi\rangle=|d_+d_-d_+d_-\cdots\rangle$ and $|\Pi'\rangle=|d_-d_+d_-d_+\cdots\rangle$. Each dimer holds for four states: $|d_0\rangle=|00\rangle$, $|d_1\rangle=|11\rangle$, $|d_+\rangle=|10\rangle$, and $|d_-\rangle=|01\rangle$, where $|1\rangle$ ($|0\rangle$) represents the excited (ground) state. Reprinted from (Zhang PF et al., 2023), Copyright 2023, with permission from Springer Nature

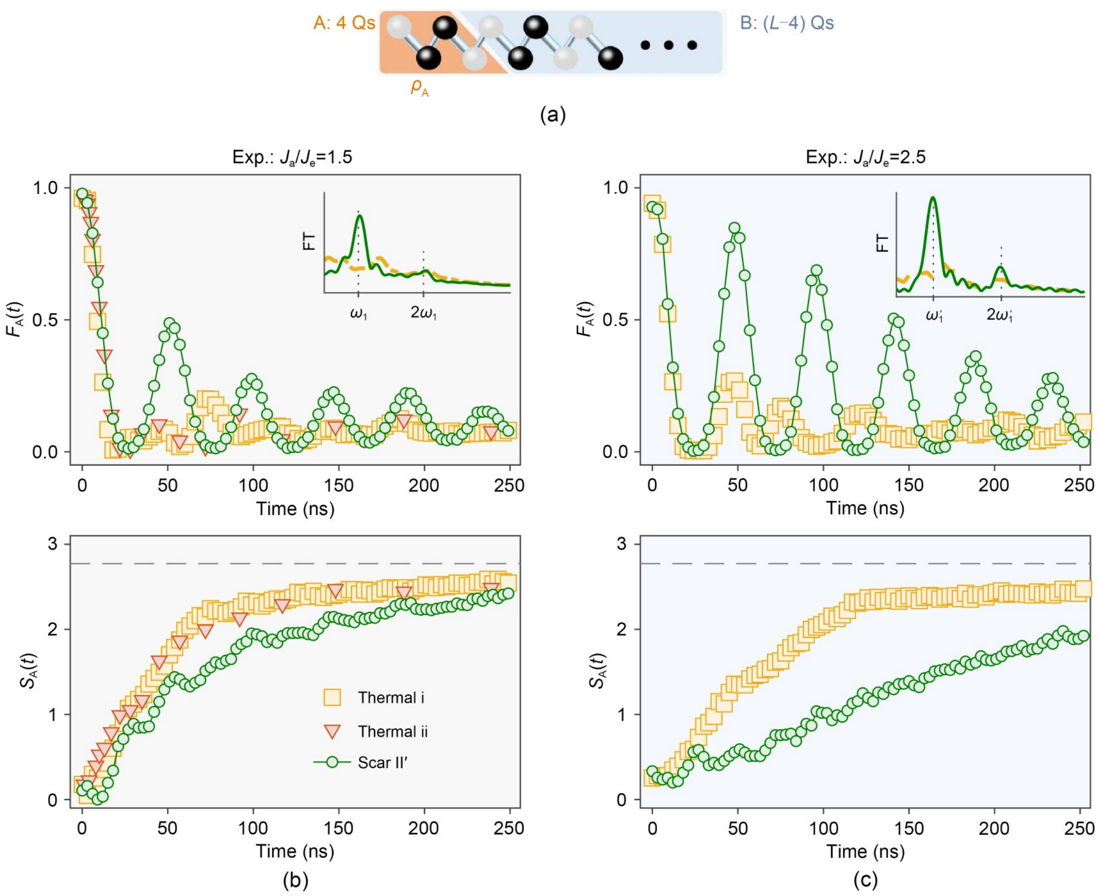

Fig. 4 (a) Schematic illustrating the bipartitioning of the system; (b) quantum state tomography for the four-qubit fidelity $F_A(t)$ and entanglement entropy $S_A(t)$ in a 30-qubit chain, showing thermalizing initial states (red and yellow) and the QMBS state (green) (the coupling strengths are $J_a/(2\pi)=1.5J_e/(2\pi)\approx-9$ MHz, the inset shows the Fourier transform (FT) of the four-qubit fidelity with a peak at $\omega_1/(2\pi)\approx21$ MHz, and the dashed gray line in the bottom panel represents the maximum thermal entropy for the subsystem, approaching 4ln(2)); (c) similar measurements with different coupling strengths: $J_a/(2\pi)=2.5J_e/(2\pi)\approx-10$ MHz and $\omega_1'/(2\pi)\approx22$ MHz. Reprinted from (Zhang PF et al., 2023), Copyright 2023, with permission from Springer Nature. References to color refer to the online version of this figure

In experiments, characterizations beyond local measurements, such as fidelity and entanglement entropy, also present difficulties. However, alternative approaches offer promising solutions. For example, Yao et al. (2023) have shown signatures of 2D QMBS and localized phenomena on a 4×6 superconducting qubit array, with a Hilbert space dimension of N=2704156. First, this work introduced a dynamical radial probability distribution:

$$\Pi(d,t)=\sum_{s\in\{D(s,s_0)=d\}}\left|\langle s|e^{-\frac{i\mathcal{H}t}{\hbar}}|s_0\rangle\right|^2, \qquad (6)$$

where $D(s, s_0)$ represents the Hamming distance between $|s_0\rangle$ and $|s\rangle$ (Fig. 6a). Then, they directly visualized wave-packet propagation in the many-body state space (Fig. 6b), enabling observation of distinct dynamical behaviors including thermalization, localization, and scarring.

As these experimental platforms continue to evolve with improved coherence times and larger qubit counts, with current records reaching 100+ coherently coupled qubits, scaling up to larger systems will enable exploration of thermodynamic limits and potentially reveal new physics at the boundary between quantum and classical behaviors.

5 Conclusions and outlook

In this review, we have synthesized recent advances in QMBS, spanning fundamental mechanisms,

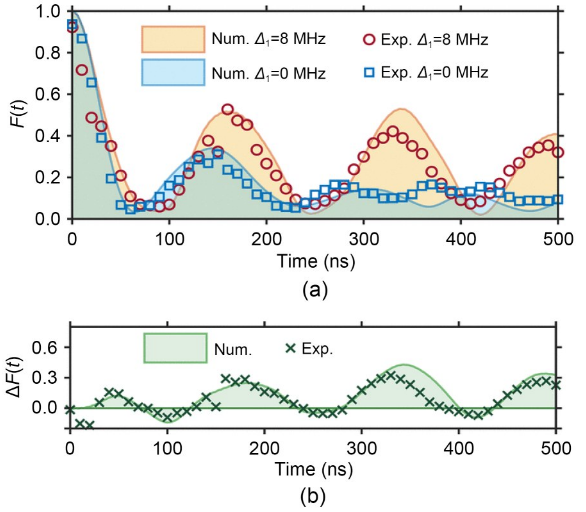

Fig. 5 (a) Fidelity $F(t)$ for a tunability of Δ_1=0 (blue square) or Δ_1=8 MHz (red circles) over the first inter-dimer coupling $J_{e,1}=J_e+\Delta_1$ (the experimental data (markers) are for N=8 qubits, J_a=3.0 MHz, and $J_{e,k}\in$[2.0, 3.0] MHz, drawn from a uniform distribution, while the curves are from the numerical simulations based on the experimental model); (b) fidelity difference $\Delta F(t)$ between the two cases in (a), illustrating the revival enhancement by Δ_1 (regions with negative $\Delta F(t)$ are due to a slight shift of the revival peaks). Reprinted from (Dong et al., 2023), Copyright 2023, with permission from American Association for the Advancement of Science. References to color refer to the online version of this figure

experimental realizations, and technological applications. The experimental observation of persistent revivals in superconducting quantum processors, demonstrated in systems up to 30 qubits (Zhang PF et al., 2023), has established QMBS as a robust platform for exploring non-ergodic dynamics.

Theoretical insights have also unveiled profound connections between QMBS and fundamental physics. The discovery of scarred states in Sachdev-Ye-Kitaev models links non-ergodic dynamics to quantum gravity and holographic duality (Maldacena and Stanford, 2016; Nandy et al., 2024), with recent proposals suggesting scarred dynamics as analogs of quantum wormhole traversability (Wildeboer et al., 2022; Liska et al., 2023). These connections bridge condensed matter physics and high-energy theory, positioning QMBS as a testbed for black hole information paradox analogs.

With increasing theoretical research on QMBS, many open problems have been well investigated (Zhang SY et al., 2023; Larsen and Nielsen, 2024; Shen et al., 2024; Wang HR et al., 2024; Burke and Dooley, 2025; Ma et al., 2025; Omiya, 2025; Ren et al., 2025). For example, recent studies proposed more effective methods to identify the periodic trajectories of QMBS (Hallam et al., 2023; Petrova et al., 2025; Ren et al., 2025). Those studies have led to the development of a new algorithm based on the time-dependent variational principle that facilitates the identification and optimization of coherent dynamics in quantum many-body systems. This approach has the potential to significantly enhance the QMBS states at a much larger scale. Also, it establishes a closer

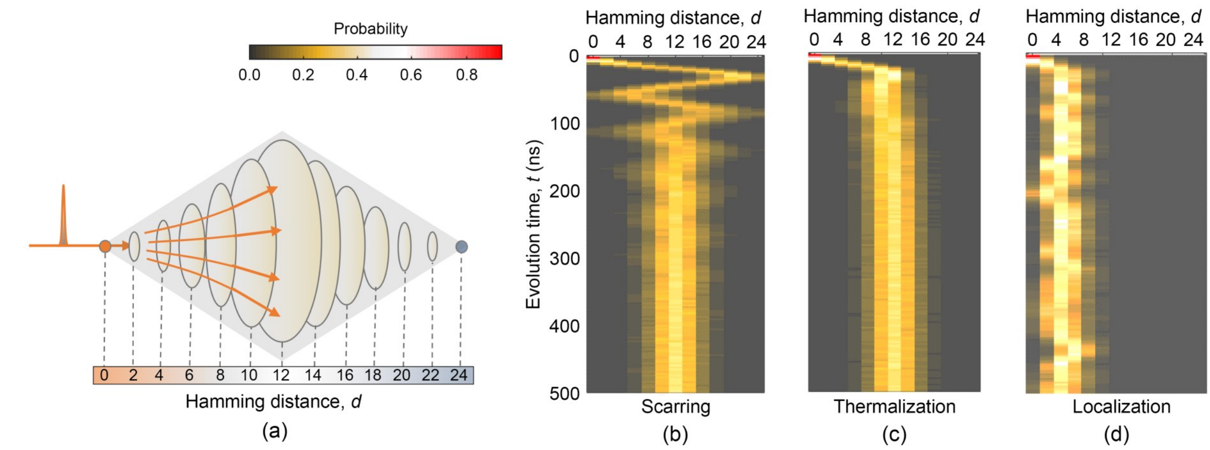

Fig. 6 (a) Fock-space visualization of the typical dynamics (a far-from-equilibrium initial state $|s_0\rangle$ is prepared as a Fock state, whose wave packet is a unit impulse function located at $D(s_0, s_0)$=0, and the unitary dynamics result in its propagation and diffusion in the Fock-space network over time, where the long-time behaviour describes the equilibration properties); (b–d) plots of experimental dynamical radial probability distribution $\Pi(d, t)$ in Fock space (color bar on the far left), which characterize the dynamics of scarring (b), thermalization (c), and localization (d). Reprinted from (Yao et al., 2023), Copyright 2023, with permission from Springer Nature

connection between quantum and classical chaos, offering valuable insights into the dynamics of complex quantum systems.

Another intriguing area of research is understanding how QMBSs respond to decoherence, a phenomenon that is unavoidable in all realistic quantum systems. Recent work by Ma et al. (2025) demonstrated that the scarred eigenmodes of the Liouvillean can exhibit a transition reminiscent of spontaneous parity-time-symmetry breaking as the dephasing strength increases. This study revealed that the critical dephasing rate shows only a weak dependence on the system size, suggesting an unexpected robustness of scarred dynamics in noisy environments. This insight is crucial for the development of robust quantum technologies, as it indicates that QMBS can maintain coherence longer than previously anticipated. Based on the tunability of decoherence of the superconducting platform (Krantz et al., 2019), particularly for the pure dephasing, this phenomenon is worthy of verification in the near future.

The controlled non-ergodic dynamics of QMBS systems offer computational advantages in quantum sensing (Dooley, 2021; Dooley et al., 2023), quantum optimization, and machine learning (Schmitt and Heyl, 2020; Cao et al., 2024; Ye and Lai, 2025). Recent work demonstrates a potential speedup in quantum algorithms by leveraging scar-induced coherent oscillations (Mukherjee et al., 2020b), while hybrid architectures integrating QMBS with error correction protocols show promise for fault-tolerant quantum memory (Feng et al., 2025). These applications, however, face scalability challenges, as cross-talk errors grow combinatorially beyond 50 qubits (Ebadi et al., 2021). Critical challenges persist in three domains: (1) scaling implementations to 100+ qubits while suppressing state leakage errors, (2) developing decoherence-insensitive control protocols for scarred subspaces (Takagi et al., 2022), and (3) unifying fragmented theoretical frameworks for higher-dimensional systems. Future progress demands synergistic advances in topological protection schemes, dissipation engineering, and hybrid quantum-classical algorithms optimized for scar-state dynamics.

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Author contributions

Lei YING led this work. Zexian GUO and Yu GAO wrote the first draft of the manuscript. Lei YING and Jinlou MA helped to organize the manuscript. Zexian GUO, Jinlou MA, and Lei YING revised and edited the final version.

Conflict of interest

Zexian GUO, Jinlou MA, Yu GAO, and Lei YING declare that they have no conflict of interest.

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