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A Chebyshev/Legendre polynomial interpolation approach for fingerprint orientation estimation smoothing and prediction

Ashkan TASHK, Mohammad Sadegh HELFROUSH, Mohammad Javad DEHGHANI

(*Faculty of Electrical Engineering, Shiraz University of Technology, Shiraz, Iran*)

E-mail: {a.tashk, ms_helfroush, dehghani}@sutech.ac.ir

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Abstract: We introduce a novel coarse ridge orientation smoothing algorithm based on orthogonal polynomials, which can be used to estimate the orientation field (OF) for fingerprint areas of no ridge information. This method does not need any base information of singular points (SPs). The algorithm uses a consecutive application of filtering- and model-based orientation smoothing methods. A Gaussian filter has been employed for the former. The latter conditionally employs one of the orthogonal polynomials such as Legendre and Chebyshev type I or II, based on the results obtained at the filtering-based stage. To evaluate our proposed method, a variety of exclusive fingerprint classification and minutiae-based matching experiments have been conducted on the fingerprint images of FVC2000 DB2, FVC2004 DB3 and DB4 databases. Results showed that our proposed method has achieved higher SP detection, classification, and verification performance as compared to competing methods.

Key words: Coherence, Consistency, Fingerprint orientation, Legendre/Chebyshev orthogonal polynomials

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1 Introduction

Identity management through biometrics like fingerprint recognition has long been popular because of its high distinctiveness, permanence, and performance. With the emergence of new and improved fraud and deceiving methods such as password hacking, the need for more reliable and immune fingerprint recognition systems has become essential. As always, critical issues in regards to fingerprint identification or verification systems are poor quality images and long processing time necessary for recognition procedure in large databases.

In fingerprint-based authentication systems, the ridge orientation estimation, defined by a local direction of ridge-valley structure (Jain *et al.*, 1997; Maio and Maltoni, 1997; Cappelli *et al.*, 1999; Jiang and Yau, 2000), is an inevitable procedure, since the orientation field (OF) is used for other processes like enhancement and also for detecting, describing, and matching fingerprint features such as minutiae and singular points (SPs).

There are many coarse ridge orientation estimation methods, including gradient-based (Kass and Witkin, 1987; Rao and Jain, 1992) and slit- and projection-based approaches (Stock and Swonger, 1969) or coarse ridge OF extraction directly from the image gray intensity changes (Bazen and Gerez, 2002). The gradient-based approach is more popular and can be computed using the gray-scale gradient $\nabla(x, y)$ at point $[x, y]$ of image I, as a 2D vector, where $\nabla_x(x, y)$ and $\nabla_y(x, y)$ components are the derivatives of I at coordinate (x, y) with respect to x and y directions, respectively. It is well known that the gradient phase angle denotes the direction of the maximum intensity changes. Thus, the direction θ of an edge that crosses the center of (x, y) is orthogonal to the gradient phase angle at this coordinate. Despite the simplicity and efficiency of the raw gradient-based method, this method has some drawbacks: first, nonlinearity and discontinuity around 90° due to classical convolutional masking, and second, severe noise sensitivity at too fine scale ridge-valley orientation. To deal with both of these deficiencies, the gradient-based method improves the situation according to

$$\begin{cases} \theta_{ij} = \dfrac{\pi}{2} + \dfrac{1}{2}\operatorname{atan2}(2G_{xy}, G_{xx} - G_{yy}), \\ G_{xy} = \sum\limits_{h=-N/2}^{N/2} \sum\limits_{k=-N/2}^{N/2} \nabla_x(x_i+h, y_j+k)\nabla_y(x_i+h, y_j+k), \\ G_{xx} = \sum\limits_{h=-N/2}^{N/2} \sum\limits_{k=-N/2}^{N/2} \nabla_x(x_i+h, y_j+k)^2, \\ G_{yy} = \sum\limits_{h=-N/2}^{N/2} \sum\limits_{k=-N/2}^{N/2} \nabla_y(x_i+h, y_j+k)^2, \end{cases} \tag{1}$$

where G_{xx}, G_{xy}, and G_{yy} are the dominant ridge directions in a local $N\times N$ block of image I to obtain a more robust OF estimation. In fact, by separate averaging of doubled ridge orientation directions computed through the gradient-based method, the noise sensitivity and nonlinearity problems are reduced in all but the case of low quality images. A fingerprint along with coarse orientation estimation is shown in Fig. 1. Note that the method fails to estimate the correct orientation in some areas of the image especially around high curvature areas.

(a) (b)

Fig. 1 Typical gradient-based ridge orientation estimation results for fingerprint images of FVC2004 DB3_A
(a) Blocked image; (b) Extracted coarse ridge orientation field

All coarse fingerprint ridge OF extraction methods, including gradient-based methods, are very sensitive to the noise issued by low quality fingerprint images. There are many unwanted factors that lead to poor quality fingerprint images, including extrinsic factors such as dust, oil, moisture, scars, breaks, creases, poor impression or excessive wet or dry fingers, and intrinsic factors such as pores, edge contours, and incipient ridges. It is difficult, therefore, to achieve a clean and suitable fingerprint ridge orientation pattern. Moreover, these deficiencies generate inherent limitations in the OF modeling, which are used for comprehensive fingerprint global feature descriptions. To counter these problems or at least reduce their undesirable effects, it is necessary to smooth the coarse ridge OF.

In recent years, to compensate for shortcomings of coarse ridge orientation estimation methods, many post-processing techniques have been proposed, in all of which the main goal is prevention of high curvature over-smoothing, especially in SP regions. The proposed smoothing methods have been divided into two general categories: (1) filtering-based methods such as Gaussian convolution (Jain *et al.*, 1997; Maio and Maltoni, 1997); (2) model-based methods such as phase portrait smoothing models (Sherlock and Monro, 1993; Vizcaya and Gerhardt, 1996; Gu *et al.*, 2004; Zhou and Gu, 2004a; 2004b). There is also a third, more recent group of polynomial interpolation methods.

All of the previously proposed smoothing methods suffer from different deficiencies such as low speed, high complexity, and topological information requirements. Moreover, if some parts of the OF are not available in the fingerprint images due to environmental or intrinsic aspects, then ideally the algorithms should be able to use known data to predict the lost data. In early model-based methods, there were many topological feature dependencies like predetermination of SPs. Several recent attempts have been devoted to fingerprint ridge orientation estimation smoothing based on ridge information. Although this obviates many of the requirements of previous methods, attempts devoted to fingerprint ridge orientation suffer from other limitations, such as a need for following the ridge curvature. More on these methods is reviewed in Section 2. According to what is known about other methods, we have proposed a method to improve coarse ridge orientation estimation even for low quality fingerprint images. Our proposed method considers deficiencies, and improves and even predicts coarse ridge orientation estimation in non-ridge regions. The algorithm employs a combination of filtering- and model-based orientation smoothing methods. The model-based method employs either Legendre or Chebyshev polynomials, according to the results of the filtering-based stage.

2 Overview on fingerprint ridge orientation estimation smoothing

According to coarse ridge orientation estimation results shown in Fig. 1, the root of gradient-based method deficiencies is the high noise sensitivity, which depends on the fingerprint image quality. To solve this problem, it is better to model ridge OF mathematically. The advantages of this model have been improved through many mathematical methods. The early model-based methods attempted to improve coarse ridge OF estimations and even to predict them in the very noisy regions of fingerprint images. In one method suggested by Stock and Swonger (1969) and Sherlock and Monro (1993), rational polynomial functions were used to model the OF:

$$O_m(z)=O_0+\frac{1}{2}\left(\sum_{i=1}^{I}\arg(z-z_{d_i})-\sum_{j=1}^{J}\arg(z-z_{c_j})\right), \quad (2)$$

where z_{d_i} and z_{c_j} are the positions of testing fingerprint images deltas and cores, respectively. This constraint, however, makes the modeling method unable to handle fingerprints lacking SPs and even unable to model non-ridge areas of fingerprints. Vizcaya and Gerhardt (1996) revised Sherlock and Monro (1993)'s model by adding a nonlinear term:

$$O_m(z)=O_0+\frac{1}{2}\left(\sum_{i=1}^{I}g_{d_i}(\arg(z-z_{d_i}))-\sum_{j=1}^{J}g_{c_j}(\arg(z-z_{c_j}))\right), \quad (3)$$

where g coefficients are correction terms necessary for singularity preservation at given points. Then, Gu *et al.* (2004) proposed a combination model to smooth ridge flows, but the model still had difficulty near an SP where high curvature and discontinuity were present. To solve the remaining problem, the authors combined a previous polynomial model with a point-charge model, and introduced a new global orientation approximation and smoothing model. Yet, the proposed method had a limitation and that was the requirement for prior knowledge of an SP. In fact, the more accurate the SP information, the more reliable the OF estimation, and vice versa. This relationship between SP data information and ridge estimation generated an inherent error-prone restriction in fingerprint global feature description.

As mentioned before, in addition to mathematical model-based OF smoothing methods, there are also filtering-based ones (Rao and Schunck, 1989; Jain *et al.*, 1997; Maio and Maltoni, 1997). It is common to name these methods as the local constraint methods. Certainly, it is an incorrect viewpoint to recognize filtering-based methods to be the same as post-processing techniques like averaging, which are used to reduce noise effects. In other words, these methods are mostly similar to post-processing techniques, such as averaging. One of the most popular filtering-based methods is Gaussian convolution with the optimal variance values σ=5, 12, 25 as in Bazen and Gerez (2002), in which a pixel-wise smoothing of coarse ridge orientation data takes place. Some other model-based methods, which are not directly related to fingerprints, have been introduced recently. One of these works was introduced by Rao and Jain (1992), as a phase portrait model-based method. The consequent orientation achieved from first-order phase portrait modeling has a form as shown below:

$$\theta(x,y)=\arctan\left(\frac{\mathrm{d}x}{\mathrm{d}y}\right)=\arctan\left(\frac{ax+by+c}{cx+dy+f}\right), \quad (4)$$

where dx and dy are the x and y components of the orientation respectively, (x, y) is the coordinate, and a, b, c, d, e, and f are the first-order phase-portrait coefficients.

This approach was expanded and also improved by Ford and Strickland (1995). All of these model-based OF smoothing works can approximate only OF according to both global ridge flow field pattern and local SP information. One of the newest phase-portrait fingerprint orientation modeling approaches was presented by Li *et al.* (2006), having a nonlinear constrained prediction ability. The most important shortcoming of this method was its dependency on SP data information and OF smoothing failure due to the poor quality of fingerprint images. Wang *et al.* (2007) proposed another model-based method for expressing fingerprint ridge orientation estimation function by a series of 2D Fourier expansion basis functions in the following general form:

$$f(x,y)=\sum_{m=0}^{k}\sum_{n=0}^{k}\Psi(m\nu x,n\omega y;\beta_{mn})+\varepsilon(x,y), \quad (5)$$

where

$$\Psi(m\nu x, n\omega y; \beta_{mn}) = \lambda_{mn}[a_{mn}\cos(m\nu x)\cos(n\omega y) + b_{mn}\sin(m\nu x)\cos(n\omega y) + c_{mn}\cos(m\nu x)\sin(n\omega y) + d_{mn}\sin(m\nu x)\sin(n\omega y)].$$

This approach also uses local theory (Perko, 1991) that topologically determines the local behavior of the dynamic system $f(x)$ at the critical point x_0. This constitutes the basic idea of the fingerprint orientation model based on 2D Fourier expansion (FOMFE) SP extraction. Of course, the introduced fingerprint orientation estimation smoothing methods suffer from the shifting of SPs after applying smoothing methods to the ridge OF. To overcome this problem, another OF smoothing method based on Legendre polynomial approximation has been proposed by Ram *et al.* (2008). According to the authors, the most important advantage and preference of Legendre polynomials over trigonometric ones employed in the FOMFE method are related to the orthogonality of Legendre basis functions. Yet, they span a Euclidean parameter space. In general, model-based methods can be viewed like the global constraint smoothing methods, except for high ridge curvature regions of fingerprint images like SPs. In this work, an interpolation smoothing method is proposed where special orthogonal polynomials are the basis function constituents for smoothing coarse ridge orientation estimation. The main contributions of our proposed method in this work are:

1. Smoothing coarse estimation of ridge OF without preliminary knowledge of global or local fingerprint features.

2. Using coherence and consistency information for considering reliability and confidentiality of coarse ridge orientation estimation.

3. Setting up a novel relation between filtering-based and proposed model-based methods for improving smoothing OF.

4. Selected adaptation of the orthogonal polynomials (Legendre, Chebyshev type I or II), according to the coarse and smoothed coherence and consistency information.

3 Fingerprint orientation modeling based on selective Chebyshev/Legendre polynomial expansions

In our proposed method, the fingerprint ridge OF smoothing is accomplished in three distinct stages to use a mixture of filtering- and model-based methods, so that the resulting OF can be smoothed even in corrupted areas of fingerprint images. To do so, a gradient-based approach is employed (Tashk *et al.*, 2009) for coarse ridge orientation extraction, which results in the original orientation field of the fingerprint image, $\mathrm{OF}_{\mathrm{orig}}$. After estimating the coarse OF using the gradient based method, it is time to smooth the OF using the proposed smoothing methods. All the stages of the proposed method are depicted in Fig. 2. With these shown, the main steps of our proposed method can be dealt with in detail.

3.1 Filtering-based smoothing by Gaussian convolution

To improve coarse OF ($\mathrm{OF}_{\mathrm{orig}}$) simply, it is better to first smooth it by a filtering-based method. The

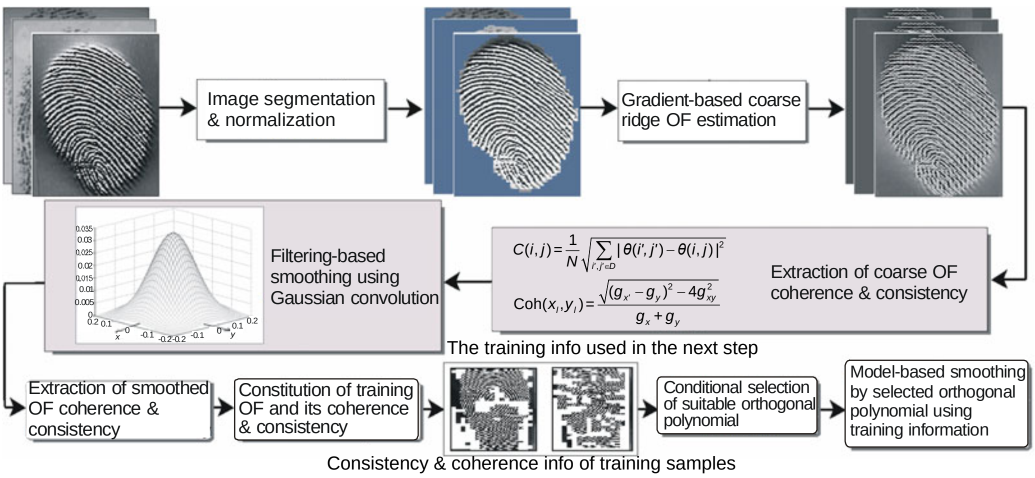

Fig. 2 Stages of the proposed method

benefit of the filter smoothing method is reduction of noise effects by averaging over local directions of fingerprint ridges. Therefore, a Gaussian mask (GM) is convolved with the coarse OF image. A 2D GM plot and its referred mask are shown in Fig. 3. It is important to note that the greater the GM standard deviation σ, the smoother OF will be achieved. The GM filtered OF is named $\mathrm{OF}_{\mathrm{smooth}}$. Of course, because of pixel-wise effect of Gaussian filtering, the smoothed ridge orientation has been shifted along SPs toward the low curvature areas, and this causes an undesirable shift of detected SPs in comparison with the manual SP position labeling. To overcome this problem, new stages are established (which will be described in the next sections). Note that the most popular SP detection method used here is Poincare indexing.

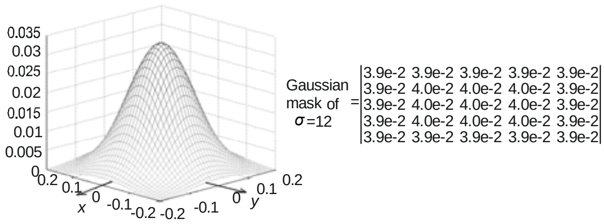

Fig. 3 Schematic of Gaussian filtering function and its respective mask

3.2 Computation of coarse and smoothed orientation field coherence and consistency matrices

As shown in Fig. 4, coherence and consistency matrices comprise significant information on the reliability of local ridge orientations inside each block of global OF. In fact, coherence contains the fidelity of local OF estimations while consistency includes their clarity. In this method, the coherence matrix acts as the weighting factor for determining the overall orientation values of the coarse ridge, smoothed by a filtering-based method and the final estimated ridge orientation resulting in smoothed OFs. To obtain a reasonable judgment of this information, it is suggested one compute the coarse and smoothed coherence ($\mathrm{Coh}_{\mathrm{orig}}$ and $\mathrm{Coh}_{\mathrm{smooth}}$) and consistency matrices ($\mathrm{Con}_{\mathrm{orig}}$ and $\mathrm{Con}_{\mathrm{smooth}}$) before and after filtering-based smoothing, respectively.

3.3 Fingerprint orientation smoothing as an optimization problem

This subsection describes the significant advantages of orthogonal polynomials, which are employed in our proposed method as a model-based OF smoothing. To develop an innovative combination of orthogonal polynomial basis functions such as Chebyshev types I & II and Legendre polynomials, it is essential that local information achieved by coherence ($\mathrm{Coh}_{\mathrm{orig}}$ and $\mathrm{Coh}_{\mathrm{smooth}}$) and consistency ($\mathrm{Con}_{\mathrm{orig}}$ and $\mathrm{Con}_{\mathrm{smooth}}$) matrices be used. The preference of orthogonal polynomials is that, unlike other polynomials, orthogonal polynomials do not result in poor conditions for linear equation systems. In our method, to approximate a discrete 2D function $f(x, y)$, a series of Legendre polynomials as the basis functions are used in a general form of

$$f(x,y) \approx \sum_{k=0}^{n} a_k \boldsymbol{\Phi}_k(x,y), \tag{6}$$

where k is the sigma index and changes from 0 to n, and n is the maximum order of the selected polynomial, a_k's are the approximation coefficients, and $\boldsymbol{\Phi}_k$'s are the row vectors containing the set of basis functions (Ram *et al.*, 2008):

$$\boldsymbol{\Phi}_k(x,y) = \left[\varphi_0(x,y),\ \varphi_1(x,y),\ ....,\ \varphi_n(x,y)\right], \tag{7}$$

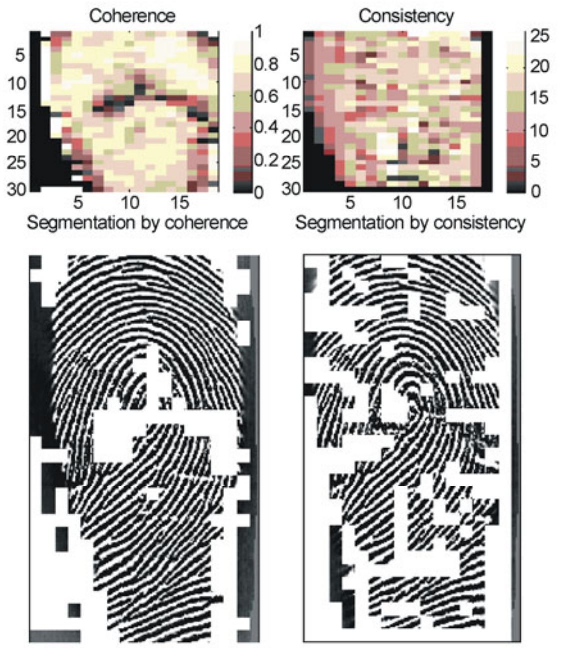

Fig. 4 Schematic of coarse orientation field (OF) coherence and consistency and their effects on image segmentation
Areas of low coherence or consistency are removed during segmentation

where each element of the above vector, $\varphi_k(x, y)$, is a Legendre or Chebyshev type I or II polynomial. For instance, the Legendre polynomial univariate basis functions can be computed using the Rodriguez formula. Computation of the first six Legendre basis functions leads to the following polynomials:

$$\begin{cases} \varphi_{l0}(x)=1, \quad \varphi_{l1}(x)=x, \\ \varphi_{l2}(x)=\dfrac{3x^2-1}{2}, \quad \varphi_{l3}(x)=\dfrac{5x^3-3x}{2}, \\ \varphi_{l4}(x)=\dfrac{35x^4-30x^2+3}{8}, \\ \varphi_{l5}(x)=\dfrac{63x^5-70x^3+15x}{8}. \end{cases} \tag{8}$$

Another kind of orthogonal polynomial is Chebyshev types I and II. Chebyshev polynomials play an important role in optimization problems. Eqs. (9) and (10) show the first six basis functions of Chebyshev polynomials types I and II, respectively:

$$\begin{cases} \varphi_{l0}(x)=1, \quad \varphi_{l1}(x)=x, \\ \varphi_{l2}(x)=2x^2-1, \quad \varphi_{l3}(x)=4x^3-3x, \\ \varphi_{l4}(x)=8x^4-8x^2+1, \\ \varphi_{l5}(x)=16x^5-20x^3+5x. \end{cases} \tag{9}$$

$$\begin{cases} \varphi_{l0}(x)=1, \quad \varphi_{l1}(x)=2x, \\ \varphi_{l2}(x)=4x^2-1, \quad \varphi_{l3}(x)=8x^3-4x, \\ \varphi_{l4}(x)=16x^4-12x^2+1, \\ \varphi_{l5}(x)=32x^5-32x^3+6x. \end{cases} \tag{10}$$

It can be seen that Legendre and Chebyshev type II polynomials have very similar basis functions, but with different scaling and coefficients.

Smoothing of fingerprint ridge orientation estimation based on orthogonal polynomials is also an optimization problem. In almost all recent model-based methods, especially in the proposed orthogonal polynomial interpolation, we encounter an optimization problem. In fact, the optimization scheme being used minimizes a specific and predetermined cost function. The benefit of the optimization scheme is its ability for approximating high curvature areas such as SPs without discontinuity modeling requirements. Another significant preference of such an optimization scheme to other interpolation methods is modeling SPs by exploiting the zero-poles of orthogonal polynomials. Orthogonal polynomials approximate the sine and cosine data of coarse OF and compute the optimum model parameters as

$$\min_{\boldsymbol{a},\boldsymbol{b}} \sum_{j=1}^{n} w_n \left[\sin\left(\frac{1}{2}\arctan\left(\frac{\boldsymbol{\Phi}(x_j)\boldsymbol{a}^{\mathrm{T}}}{\boldsymbol{\Phi}(x_j)\boldsymbol{b}^{\mathrm{T}}} \right) - O(x_j) \right) \right]^2, \tag{11}$$

where the sine-function is used to determine the distinction between zero and π directions and w_n's are the elements of the diagonal weighting matrix containing the weights for every coordinate. Of course, there are many ways to optimize expression (11), such as sum of squares of these distances (Ford and Strickland, 1995), or any other linear or nonlinear technique. In agreement with Ram *et al.* (2008), single nonlinear optimization methods are time consuming and depend on local minima. Therefore, hybrid optimization techniques such as quasi-Newton (QN) or Levenberg-Marquardt (LM) are more acceptable and efficient (Nocedal and Wright, 2006). Treating the optimization problem with the best hybrid nonlinear method, the model's orientation estimation can then be computed as

$$O(x_j) = \frac{1}{2}\arctan\left(\frac{\boldsymbol{\Phi}(x_j)\boldsymbol{a}^{\mathrm{T}}}{\boldsymbol{\Phi}(x_j)\boldsymbol{b}^{\mathrm{T}}} \right). \tag{12}$$

4 Summary of the implementation procedure

In this section, the proposed approach is described step by step.

Step 1: Compute the coarse OF and its local information, e.g., coherence and consistency, using Eqs. (13)–(15):

$$O_{\mathrm{orig}}(x_i, y_i) = \frac{1}{2}\arctan\left(\frac{\sum_r 2G_xG_y}{\sum_r (G_x^2 - G_y^2)} \right) + \frac{\pi}{2}, \tag{13}$$

$$\mathrm{Coh}_{\mathrm{orig}}(x_i, y_i) = \frac{\left(\sum_r (G_x^2 - G_y^2)\right)^2 + 4\left(\sum_r G_xG_y\right)^2}{\left(\sum_r (G_x^2 + G_y^2)\right)^2}, \tag{14}$$

$$\begin{cases} \mathrm{Con}_{\mathrm{orig}}(i, j) = \dfrac{1}{N}\sqrt{\displaystyle\sum_{(i',j')\in D} \left|\theta(i',j') - \theta(i,j)\right|^2}, \\ \left|\theta' - \theta\right| = \begin{cases} d, & \text{if } d = (|\theta' - \theta| + 2\pi) \bmod (2\pi) < \pi, \\ d - \pi, & \text{otherwise,} \end{cases} \end{cases} \tag{15}$$

where G_x and G_y are the gradient magnitudes along x and y directions, respectively (Jain and Pankanti, 2000).

Step 2: Smooth the coarse OF estimation using Gaussian convolution described in Section 3. In addition, the smoothed OF coherence and consistency data are evaluated.

Step 3: Form the training data $\mathrm{OF}_{\mathrm{trn}}$, $\mathrm{Coh}_{\mathrm{trn}}$, and $\mathrm{Con}_{\mathrm{trn}}$ by combining the coarse and Gaussian smoothed orientation information according to the following criteria:

$$\begin{cases} \mathrm{OF}_{\mathrm{trn}} = \begin{cases} \mathrm{OF}_{\mathrm{smooth}}, & \mathrm{Coh}_{\mathrm{orig}} < 0.5, \\ \mathrm{OF}_{\mathrm{orig}}, & \mathrm{Coh}_{\mathrm{orig}} \geq 0.5, \end{cases} \\ \mathrm{Coh}_{\mathrm{trn}} = \begin{cases} 0.3, & \mathrm{Coh}_{\mathrm{orig}} < 0.5, \\ \mathrm{Coh}_{\mathrm{orig}}, & \mathrm{Coh}_{\mathrm{orig}} \geq 0.5, \end{cases} \\ \mathrm{Con}_{\mathrm{trn}} = \begin{cases} \mathrm{Con}_{\mathrm{smooth}}, & \mathrm{Coh}_{\mathrm{orig}} < 0.5, \\ \mathrm{Con}_{\mathrm{orig}}, & \mathrm{Coh}_{\mathrm{orig}} \geq 0.5. \end{cases} \end{cases} \tag{16}$$

Note that the reason for replacing $\mathrm{Coh}_{\mathrm{trn}}$ with 0.3 for $0<\mathrm{Coh}_{\mathrm{orig}}<0.5$ was suggested by Li *et al.* (2006).

Step 4: Select the best orthogonal polynomial (OP) for approximating OF using its basis function according to the following criteria:

$$\mathrm{OP}_{\mathrm{selected}} = \begin{cases} \text{Chebyshev type I, if} \\ \quad |\mathrm{Con}_{\mathrm{trn}} \cdot \mathrm{Coh}_{\mathrm{trn}}| \geq \mathrm{UB}, \\ \text{Chebyshev type II, if} \\ \quad \mathrm{UB} > |\mathrm{Con}_{\mathrm{trn}} \cdot \mathrm{Coh}_{\mathrm{trn}}| \geq \mathrm{LB}, \\ \text{Legendre polynomial, if} \\ \quad |\mathrm{Con}_{\mathrm{trn}} \cdot \mathrm{Coh}_{\mathrm{trn}}| < \mathrm{LB}. \end{cases} \tag{17}$$

The explanation for Eq. (17) is as follows. If multiplication of coherence and consistency levels for a fingerprint block is in a certain interval, then the local orientation around this region should be smoothed by a suitable polynomial interpolation until $\mathrm{Con}_{\mathrm{trn}} \cdot \mathrm{Coh}_{\mathrm{trn}}$ lies below the lower bound of that interval. Indeed, the information of both coherence and consistency is considered simultaneously like a logical AND.

Note that the lower bound (LB) and the upper bound (UB) in Eq. (17) have been determined through trial and error by experimenting on series B fingerprint image databases of FVC2000 DB2 (Maio and Jain, 2002) and FVC2004 DB3 (Maio *et al.*, 2004). For instance, the LB and UB computed for FVC2000 DB2_B are 0.5 and 0.7, respectively.

5 Experimental results and performance evaluation

Three different types of experiment were conducted. Experiment 1 was carried out to show the effectiveness of the proposed method qualitatively, while Experiments 2 and 3 were aimed to test the smoothing performance of the proposed method quantitatively. Experiment 1 shows the ability of the proposed method both in improving coarse OF even in low coherence or ridge information areas of fingerprint images, and in extracting SPs accurately and reliably in comparison with other methods. In Experiment 2 the mean absolute error (MAE) was compared between the final smoothed OF and the original one. Finally, in Experiment 3 an indirect method of OF smoothing was used to show the performance improvement when used as an element of a fingerprint verification system. Note that all the fingerprint images used in the experiments come from the FVC2000 DB2 and FVC2004 DB3 and DB4 databases.

5.1 Experiment 1: discrete classification and singular point detection

For fingerprint images that do not contain full ridge information, it is difficult to estimate or predict ridge orientations. Our proposed method has solved this problem without the need for SP detection, unlike Li *et al.* (2006). Some resultant images are provided here with their reconstructed orientations in the regions of missed ridges, using the proposed method, to show its effectiveness (Fig. 5). Another qualitative advantage of the proposed method was tested via SP extraction ability. The SP detection and position extraction for the proposed method were more reliable and accurate compared with other methods (Fig. 6). Fig. 7 presents the manual classification of the topologic map prototypes of FVC2004 DB4_A. The smoothing methods being compared are Gaussian convolution as a filtering-based and FOMFE (Wang *et al.*, 2007), M-FOMFE (Tashk *et al.*, 2009), and Legendre polynomial (Ram *et al.*, 2008) as model-based. To prove these qualitative claims, a discrete classification based on the characteristic matrix achieved by their local properties of extracted OF were employed on the FVC2004 DB4_A database with 800 images of size 384×288, belonging to 100 individuals.

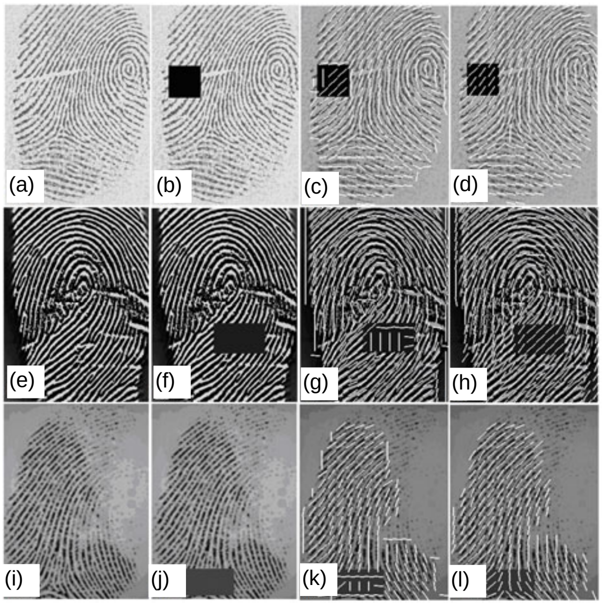

Fig. 5 Sampling performance of the proposed method over three different fingerprints of FVC2000 DB2_A, FVC2004 DB3_A, and FVC2004 DB4_A

(a), (e), and (i) are the fingerprint image samples from FVC2000 DB2_A, FVC2004 DB3_A, and FVC2004 DB4_A, respectively. (b), (f), and (j) are the manually corrupted fingerprint images for (a), (e), and (i), respectively. (c), (g), and (k) are the fingerprint images combined with the coarse orientation fields (OFs). (d), (h), and (l) are the fingerprint images combined with the OF extracted based on the proposed OF extraction smoothing method. Clearly, the method corrects the coarse ridge orientation estimations in the dark area shown in the third column

L	L	W	W	W	R	R	L	L	W
W	R	L	L	R	W	W	R	L	R
R	W	R	R	R	W	L	L	R	L
W	R	R	L	L	L	R	L	L	W
L	L	A	R	L	R	A	L	L	R
R	L	R	W	W	W	W	W	R	W
R	L	L	R	R	R	A	R	R	R
R	R	R	W	A	L	W	L	R	R
W	L	A	L	R	L	L	R	L	L
L	L	A	L	R	R	W	L	R	A

Fig. 7 Manual classification of prototypes for FVC2004 DB4_A

As there is no recognized fingerprint with tented arch pattern in that database, the classification is one of four patterns: left loop, right loop, whorl, and plain arch. The classification confusion matrices are depicted in Fig. 8, and the computed values of validation (accuracy or A) and correctness (reliability or R) parameters are listed in Table 1 for all competitive methods. Note that these parameters are defined as

$$A=\frac{S_{\mathrm{d}}}{S}, \quad R=\frac{1}{5}\sum_{i=1}^{5}\frac{D_i}{S_i}, \tag{18}$$

where S_{d} is the sum of diagonal elements of the confusion matrix, S is the summation of all the elements of the confusion matrix, D_i is the diagonal element of the ith column of the confusion matrix, and S_i is the summation of the ith column of the confusion matrix.

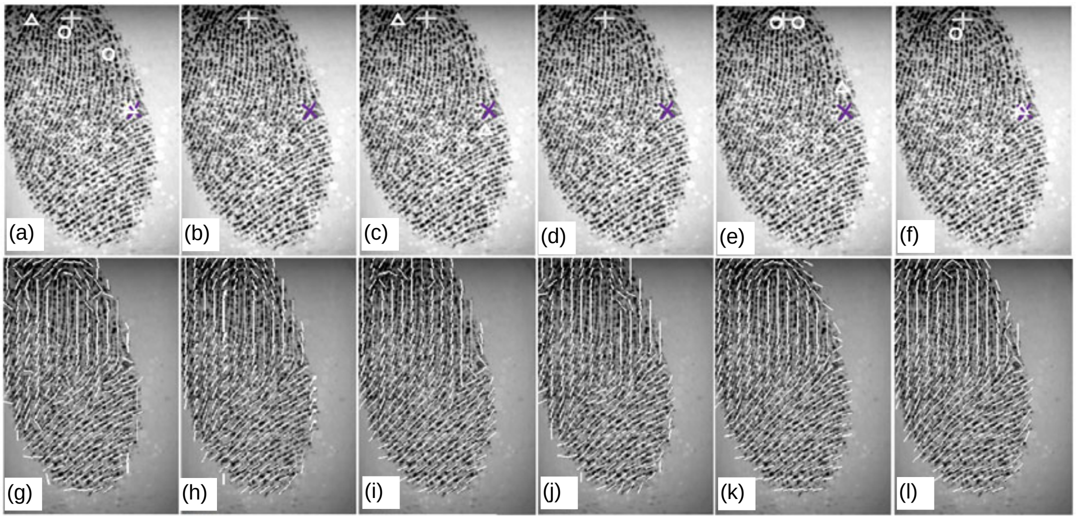

Fig. 6 A typical low quality FVC2004 DB4_A fingerprint image, its different ridge orientation estimation, and corresponding singular point (SP) extraction using the Poincare-indexing method

(a, g) The coarse orientation field (OF) and its SP extraction. The rest are coarse OF smoothing and SP extraction results based on: (b, h) Gaussian filtering; (c, i) M-FOMFE; (d, j) FOMFE; (e, k) Legendre polynomial; (f, l) the proposed method. Circular and triangular symbols are those SPs detected by Poincare indexing, while the crosses are real SPs highlighted manually. Clearly, in some smoothing methods such as Gaussian filtering and FOMFE, no SP is extracted

As. / Tr.	L	R	W	A
L	**148**	11	117	4
R	21	**155**	107	5
W	10	7	**158**	1
A	11	2	13	**30**

(a)

As. / Tr.	L	R	W	A
L	**152**	32	90	6
R	15	**154**	109	10
W	7	6	**162**	1
A	4	3	10	**39**

(b)

As. / Tr.	L	R	W	A
L	**151**	16	98	15
R	20	**160**	102	6
W	7	5	**164**	0
A	2	5	4	**45**

(c)

As. / Tr.	L	R	W	A
L	**160**	17	89	14
R	14	**177**	90	7
W	6	2	**169**	0
A	2	3	2	**49**

(d)

Fig. 8 Confusion matrices of FVC2004 DB4_A fingerprint images classification based on the proposed indexing method for coarse (a), FOMFE 162p (b), Legendre polynomial (c), and the proposed method (d)

As.: assigned classes; Tr.: true classes. L: left loop; R: right loop; W: whorl; A: plain arch. FOMFE: fingerprint orientation model based on 2D Fourier expansion

Table 1 Comparison of accuracy and reliability values acquired from their related confusion matrices between the proposed method and competing methods

OF smoothing method	Accuracy (%)	Reliability (%)
Coarse OF	61.38	62.51
FOMFE 162p	63.38	67.36
Legendre polynomial	65.00	70.76
Proposed	**69.38**	**75.53**

OF: orientation field; FOMFE: fingerprint orientation model based on 2D Fourier expansion

The results confirmed the improved SP detection ability of the proposed method. Moreover, one may consider another performance measurement based on the accuracy and validity of SP correct position detection:

$$\begin{cases} \text{Recall} = \dfrac{\text{TP}}{\text{TP+FN}}, \quad \text{Precision} = \dfrac{\text{TP}}{\text{TP+FP}}, \\ F\text{-measure} = \dfrac{2 \times \text{Recall} \times \text{Precision}}{\text{Recall+Precision}}, \end{cases} \tag{19}$$

where TP, FP, and FN stand for true positive (number of correctly detected SPs), false positive (number of spuriously detected SPs), and false negative (number of lost SPs), respectively. As can be seen, these parameters judge the SP detection accuracy and reliability based on counting the number of correct and spurious SP detections in different vicinities. The measurement results of these parameters shown in Fig. 9 revealed the higher smoothing ability and greater robustness of the proposed method in comparison to other methods.

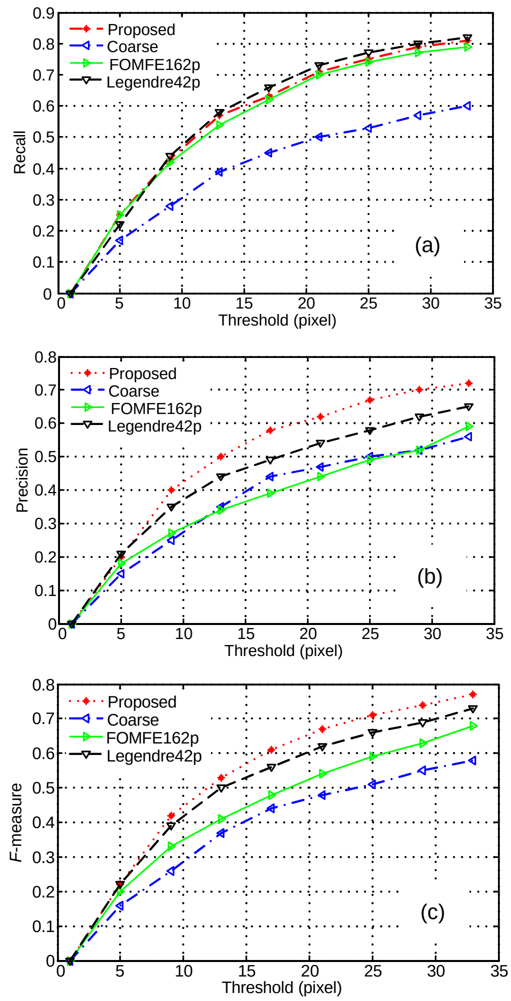

Fig. 9 Singular point position detection measurements for FVC2004 DB4_A to compare the proposed method with other competitive orientation field smoothing ones

(a) Recall; (b) Precision; (c) *F*-measure. FOMFE: fingerprint orientation model based on 2D Fourier expansion

5.2 Experiment 2: mean absolute error computation

For this experiment, 50 images from FVC2004 DB3_A were selected randomly, with the fingerprint images being of relatively large size (480×300 pixels).

To evaluate the effectiveness of the proposed ridge OF estimation smoothing method quantitatively, a random region was removed in all selected fingerprint images. Then, the mean absolute error (MAE) between the smoothed and original orientations at local removed areas of images was calculated as

$$\mathrm{MAE}=\frac{1}{N}\sum_{(x,y)\in\Omega} d(\mathrm{OF}_{\mathrm{smoothed}}(x,y)-\mathrm{OF}_{\mathrm{orig}}(x,y)), \quad (20)$$

where the orientation function d is defined as

$$d(\theta)=\begin{cases}|\theta|, & |\theta|<\pi/2,\\ |\theta|-\pi, & \text{otherwise,}\end{cases} \quad \theta\in(-\pi/2, \pi/2]. \quad (21)$$

Here Ω is the removed region of fingerprint images used for comparison of the coarse OF result with the smoothed one. The resultant MAE mean and standard deviation of test images were 5.52° and 4.63°, respectively. On the other hand, the global difference between coarse and smoothed OFs had an MAE with an average of 6.9° and a standard deviation of 4.0°. Considering the resulting MAE here and those implied in Li *et al*. (2006), our results showed significantly better OF smoothing, even at defective fingerprint regions.

5.3 Experiment 3: minutiae-based matching as an indirect evaluation method

To demonstrate the higher performance of our proposed fingerprint ridge orientation estimation smoothing, an indirect method is devised. This method is based on a revised minutiae-based fingerprint matching algorithm, which is available in DVD, included in Maltoni *et al*. (2009) and can be downloaded from Jilli (2003). The reason for establishing this type of algorithm is that the final stage of each trustworthy fingerprint identification system is fingerprint matching, especially for minutiae-based systems. Among the essential data for minutiae matching identification, the orientation of ridge ending and ridge bifurcation has a very important role. Therefore, it is possible to check the efficiency of a proposed OF smoothing method by implementing it for minutiae-based matching. Here, the minutiae-based matching algorithm was evaluated on fingerprint images from FVC2000 DB2_A and FVC2004 DB3_A. To evaluate the performance, four measurements were used.

$$\mathrm{FAR}=\frac{\text{Number of accepted imposter claims}}{\text{Total number of imposter accesses}}, \quad (22)$$

$$\mathrm{FRR}=\frac{\text{Number of rejected genuine claims}}{\text{Total number of genuine accesses}}, \quad (23)$$

where FAR and FRR represent the false acceptance rate and false rejection rate, respectively.

When FRR=FAR, the equal error rate (EER) is preferred:

$$\mathrm{EER}=(\mathrm{FAR}+\mathrm{FRR})/2. \quad (24)$$

And the total success rate is defined as

$$\mathrm{TSR}=\left(1-\frac{2\times\mathrm{EER}}{\text{Total number of accesses}}\right). \quad (25)$$

To determine the most suitable lower and upper bounds used in the proposed method, we used series B of the reference databases. These database series contain 80 fingerprint images belonging to 10 individuals with 8 impressions for each. In this way, the EERs of various bounds were computed by implementing the minutiae-based matching algorithm on the fingerprint images of series B databases. The corresponding bounds to the minimum EER were selected for use for fingerprint images of series A. The results of this evaluation experiment are shown in Fig. 10. The resulting LB and UB for FVC2000 DB2 were about 0.5 and 0.7, respectively. Note that the numbers of imposter and genuine claims were 9900 (99 imposter claims, 100 individuals) for FAR and 700 (7 genuine claims, 100 individuals) for FRR, respectively.

The results of Experiment 3 implemented on FVC2000 DB2_A and FVC2004 DB3_A are shown in Figs. 11a and 11b and Tables 2 and 3. These results confirmed the greater ability of the proposed OF smoothing method in fingerprint minutiae-based matching compared to other methods. Note that the LB and UB in choosing a suitable polynomial were derived from series B of the above databases using the same fingerprint minutiae-based matching system.

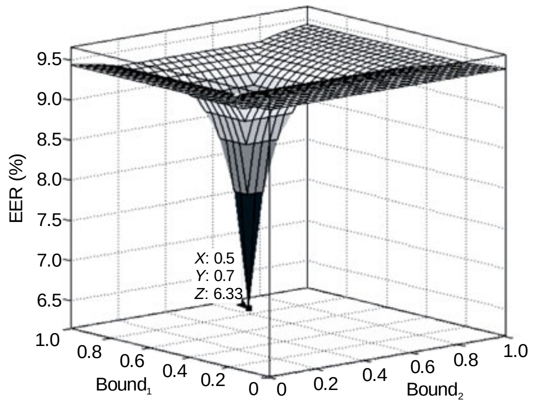

Fig. 10 Equal error rate (EER) versus different lower and upper bounds used in the proposed method for FVC2000 DB2_B

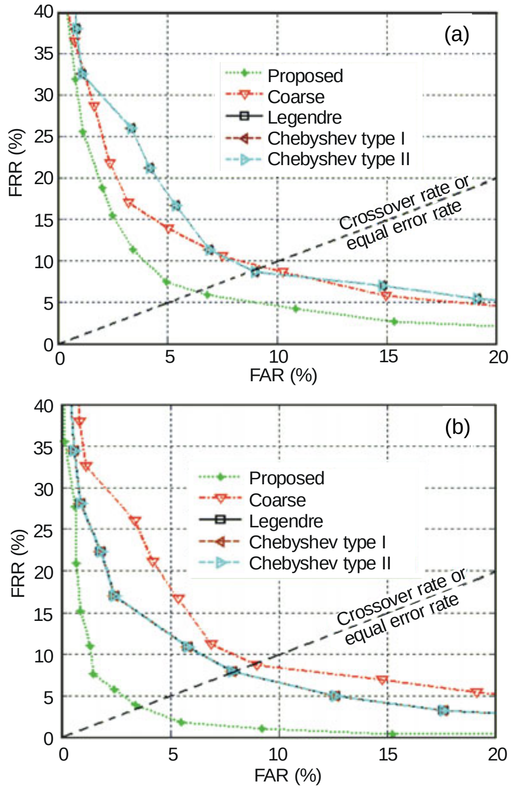

Fig. 11 Receiver operating characteristic (ROC) comparison between the proposed method and other competitive orientation field smoothing methods for FVC2000 DB2_A (a) and FVC2004 DB3_A (b)

5.4 Computational complexity and timing issues

The computational complexity of our proposed method is the same as that of OF smoothing based on Legendre polynomial, except that the former consumes more time for execution. Computational issues and runtime of the proposed and competitive methods are listed in Table 4, for Matlab 7.5 implemented on a 1.86 GHz Pentium IV (Celeron M) machine. The results showed that the proposed method had the same implementation time as FOMFE 162p and Legendre polynomial 42p smoothing. It also had a computational complexity of $2O(MK')$ for both training and reconstruction procedures, and this cost is less than that needed for FOMFE, $O(2MK^2)+O(MK^2)$, where M is the number of blocks and K is the number of FOMFE parameters. K=2k+1, and k is the order of the FOMFE fingerprint ridge orientation estimation smoothing method.

Table 2 Identification matching results for FVC2000 DB2_A

Orientation field estimation or smoothing method	TSR (%)	EER (%)
Proposed	93.26	6.33
Coarse	89.85	9.48
Legendre	91.03	8.85
Chebyshev type I	91.04	8.80
Chebyshev type II	90.97	8.90

TSR: total success rate; EER: equal error rate

Table 3 Identification matching results for FVC2004 DB3_A

Orientation field estimation or smoothing method	TSR (%)	EER (%)
Proposed	96.58	3.63
Coarse	91.03	8.85
Legendre	92.11	7.95
Chebyshev type I	92.19	7.92
Chebyshev type II	92.14	7.94

TSR: total success rate; EER: equal error rate

Table 4 Computational complexity and runtime comparison between the proposed method and other competitive methods*

OF smoothing method	Computational complexity	Average runtime (s)
Coarse OF	$O(M)$	1.75
FOMFE 162p	$O(2MK^2)+O(MK^2)$	8.36
Legendre 42p	$O(2MK')+O(MK')$	7.47
Proposed	$2O(MK')$	7.53

* Dimension of the typical fingerprint images of the used fingerprint databases is 256 in 384 pixels. $M=L\times H$, K=2k+1, $K'=(k'+1)(k'+2)$, where L=48=[384/8], H=36=[288/8], k=4, k'=5, $2(K+1)^2$=number of FOMFE parameters, K'=number of polynomial parameters. OF: orientation field; FOMFE: fingerprint orientation model based on 2D Fourier expansion

6 Conclusions

In this paper, an effective fingerprint orientation estimation smoothing method has been proposed. This enables fingerprint authentication systems to estimate ridge orientation field more accurately and reliably. The proposed method makes a balance between filtering- and model-based OF smoothing methods. The singular point (SP) extraction results confirmed the efficiency of the proposed smoothing method. The proposed method also predicted ridge orientation field (OF) at defective areas of low quality fingerprint images. It should be noted that the proposed method is independent of not only SP detection, but also training data, such that the proposed ridge OF smoothing can be used even with low quality fingerprint images. The equal error rate (EER) of the proposed method was at least 2.47% less than those of other OF smoothing methods, such as Chebyshev type I polynomial. There are parameters for conditional selection of Chebyshev/Legendre orthogonal polynomials in our proposed method. Future work may be conducted on estimating suitable values for these parameters. Since selection of polynomials in the proposed depends on the lack of constant bounds, development of an optimum bound detection algorithm will be included in our future proposals.

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