Cooperative transport strategy for formation control of multiple mobile robots[*#]

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Abstract: This paper addresses a box transport problem that requires the cooperation of multiple mobile robots. A geometric-based distributed formation control strategy is proposed for robots to push the box to the target, which might be static or dynamic. Velocity and hardware constraints are considered in the advanced planning of the trajectory. Information sharing is included because the robots used as box pushers cannot acquire the required environmental information from their local sensors. Simulation results show the effectiveness of the proposed distributed cooperation strategy.

Key words: Multiple mobile robots, Cooperative transport, Formation control, Information sharing, Box transport
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1 Introduction

The cooperative control problem of multi-robot systems has been studied extensively over the past decade. Several new issues, such as consensus (Ren and Sorensen, 2008), flocking (Tanner, 2004), rendezvous (Cortes *et al*., 2006), and formation control (Balch and Arkin, 1998; Consolini *et al*., 2008), have been formulated and solved. Because of its wide range of applicability, formation control has received a great deal of attention and has become one of the most popular topics in multi-robot cooperative control.

In many applications, such as transporting goods (Ota and Arai, 1999; Wang *et al*., 2004) and patrolling (Mas *et al*., 2009), mobile robots are required to maintain a given shape of formation while executing their tasks. This kind of formation control problem can be converted into the problem of controlling the relative positions and orientations of the robots in a group while they are moving as a whole. Robots that act as pushers in such a task formation are tightly coupled. Take a large object transport task for example: to keep the object moving to its destination or target, robots that are holding the object need to keep strict relative positions and orientations to their neighbors (Ota and Arai, 1999; Gerkey and Mataric, 2002; Wang *et al*., 2004). Furthermore, the executive robots equipped with vision and distance sensors might become blind in this kind of task, as a result of obstruction by the object being transported.

A pusher-watcher system was introduced by Gerkey and Mataric (2002) to deal with this large box pushing problem. In the system, a reactive control strategy was proposed based on the visual information of the watcher. A stigmergic cooperation strategy for the same problem was reported by Zielinski and Trojanek (2009), in which robots also cooperate in the pusher-watcher mode. The proposed strategy partitions the task into separate bio-inspired elementary behaviors, and then composes those behaviors into robot actions. Furthermore, robots observe only the

effects of the activities of other robots in the environment. A geometric approach was introduced for robot formation control by Consolini *et al.* (2009), considering velocity and curvature constraints, and asymptotic stabilization was proved. Implicit communication was studied to deal with the box handling problem (Pereira *et al.*, 2002). The development of the follower controller relies on its local information such as visual or force sensors. In relation to the box pushing problem, a global feasible path for transporting goods has been generated in a complex indoor environment (Parra-Gonzalez *et al.*, 2009), and humanoid robots have been studied in cooperation using the classifier system and Q-learning (Inoue *et al.*, 2007).

However, most of the previous studies of the box transport problem have assumed a static environment. In some applications such as harbor handling or highway transport, the environment including the target might be dynamic. In this situation, the movement of the target should be acquired for the path generation of robots that act as pushers. However, some simple behavioral control strategies require robots to move repeatedly. This action may occur intelligently in a static environment, but may be less efficient in catching up with a moving target.

This paper discusses mobile robot formation navigation in a pusher-watcher system, with application to a cooperative box pushing task. The system is composed of a watcher and a formation which contains two pushers and a box being pushed. A distributed geometric-based navigation strategy is proposed for this problem. The main contributions with respect to the existing literature are as follows. The robots in the pusher-watcher system have to push the box to its target, either static or dynamic. The box being pushed acts as a virtual 'leader' of the formation. The watcher robot travels ahead of the formation and shares the information of the environment with the pusher robots. The design of the navigation strategy for each robot in the formation is based on shared information, and considers robot velocity constraints and limitations on both the actuator and local sensors. Obstacle avoidance is also included when the tightly coupled formation is moving. A characteristic of the proposed cooperative transport strategy is that robots which act as pushers can accomplish the task without repetition of their simple actions.

2 Basic definition and problem formulation

The box transport task in this paper can be converted into a relative position and orientation maintenance problem for mobile robots in formation. Consider the following differential drive robot as a velocity-controlled model.

Definition 1 $q=[x(t), y(t), \theta(t)]^{\mathrm{T}} \in \mathbb{R}^3$ is called a differential drive robot, with an initial state $q_{\mathrm{init}} \in \mathbb{R}^3$, in a global coordinate system *xoy* (all postures are in the global coordinate system *xoy*). The robot model is defined as follows:

$$\dot{q}(t) \triangleq \begin{bmatrix} r\cos\theta(t)/2 & r\cos\theta(t)/2 \\ r\sin\theta(t)/2 & r\sin\theta(t)/2 \\ r/d & -r/d \end{bmatrix} \begin{bmatrix} \dot{\varphi}_{\mathrm{r}}(t) \\ \dot{\varphi}_{\mathrm{l}}(t) \end{bmatrix}, \quad (1)$$

where r is the radius of the differential drive wheel, and d stands for the distance between two differential drive wheels. $\dot{\varphi}_{\mathrm{r}}(t)$ is the rotation angular velocity of the right driven wheel, while $\dot{\varphi}_{\mathrm{l}}(t)$ stands for the left one. Eq. (1) represents the differential kinematics of the robot as the relationship between the rotation angular velocity of the drive wheel $\dot{\boldsymbol{\varphi}}(t) = [\dot{\varphi}_{\mathrm{l}}(t), \dot{\varphi}_{\mathrm{r}}(t)]^{\mathrm{T}} \in ([0,+\infty), \mathbb{R}^2)$ and $\dot{q}(t)=[\dot{x}(t), \dot{y}(t), \dot{\theta}(t)]^{\mathrm{T}} \in \mathbb{R}^3$. $\dot{\boldsymbol{\varphi}}(t)$ is chosen as the input of the robot model instead of the linear and angular velocities $[v, \omega]^{\mathrm{T}} \in \mathbb{R}^2$, considering velocity constraints in task execution. The constraint $\dot{x}(t)\sin\theta(t) - \dot{y}(t)\cos\theta(t) = 0$ is included in Eq. (1).

The box transport formation is formulated as the transformation for the postures of the whole system including the robot formation, the box, and the transport destination in the environment by

$$F\big(q_{\mathrm{b}}(t), q_{\mathrm{D}}(t), q_{\mathrm{o}}(t)\big) = \big[q_1(*), q_2(*), \dots, q_n(*)\big], \quad (2)$$

$$q_i(t) = q_i(t-\tau) + \int_{t-\tau}^{t} \dot{q}_i(\tau)\mathrm{d}\tau, \quad (3)$$

where '*' is the abbreviation for the input arguments $q_{\mathrm{b}}(t)$, $q_{\mathrm{D}}(t)$, and $q_{\mathrm{o}}(t)$, n is the number of robots in the formation, and τ is the time interval. $q_{\mathrm{D}}(t)$ and $q_{\mathrm{o}}(t)$ stand for the postures of the transport destination and the obstacles, respectively. Both postures can be acquired by the watcher robot equipped with vision and

distance sensors. $\boldsymbol{q}_b(t)$ is the posture of the box needed to be transported. A set of constraints are defined to formulate the problem of the box transport task in a multi-robot system:

　　1. The pusher robots can sense only the box on their way to the destination, and the environment including obstacles cannot be perceived because of obstruction by the box.

　　2. The pushers are smaller than the box. They need to cooperate and can move the box only by pushing it.

　　3. Robots postures are known in task initialization. The friction between the box and the ground is low such that the box will not cause the pushers to slip.

　　4. The environment with obstacles is unknown to the robots but at least one feasible path is maintained for the formation to move to the destination.

　　With these four constraints, a pusher-watcher formation transport environment is set up, which is similar to that of Gerkey and Mataric (2002). The actual formation in this study can be represented by Definition 2.

Definition 2　　The formation is composed of two pushers and the box which acts as a 'leader'. Formation control should deal with the following relation among robots and the box:

$$E = < X, Y, \Phi >, \tag{4}$$

where X, Y stand for the relative distances in directions X and Y, respectively. Φ stands for the orientation among the pushers and the box. All the parameters can be easily acquired from Fig. 1. In this study, these three parameters are defined as follows:

$$X = \begin{bmatrix} 0 & x_{b,2} & \cdots & x_{b,n} \\ x_{2,b} & 0 & \cdots & x_{2,n} \\ \vdots & \vdots & & \vdots \\ x_{n,b} & x_{n,2} & \cdots & 0 \end{bmatrix}, Y = \begin{bmatrix} 0 & y_{b,2} & \cdots & y_{b,n} \\ y_{2,b} & 0 & \cdots & y_{2,n} \\ \vdots & \vdots & & \vdots \\ y_{n,b} & y_{n,2} & \cdots & 0 \end{bmatrix},$$

$$\Phi = \begin{bmatrix} 0 & \phi_{b,2} & \cdots & \phi_{b,n} \\ \phi_{2,b} & 0 & \cdots & \phi_{2,n} \\ \vdots & \vdots & & \vdots \\ \phi_{n,b} & \phi_{n,2} & \cdots & 0 \end{bmatrix}.$$

In this system, the tuple $<X, Y, \Phi>$ has taken the box as a 'leader', with the subscript 'b' in $x_{i,b}$, $y_{i,b}$, and $\phi_{i,b}$.
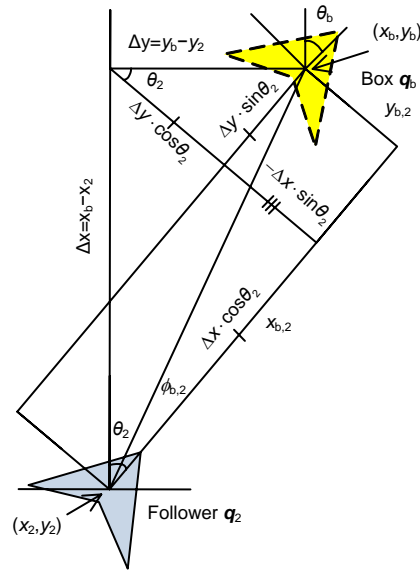

Fig. 1　Illustration of $x_{b,2} \in X$, $y_{b,2} \in Y$, and $\phi_{b,2} \in \Phi$
The shape with the dashed edge represents the box $\boldsymbol{q}_b$, which is the 'leader' in the formation. The shape with the solid edge represents the follower $\boldsymbol{q}_2$ in the formation

　　In this study, the robot system contains three differential drive robots. The actual task of the box transport problem is to keep all elements in $E = <X, Y, \Phi>$ constant when the box is moving, which can be formulated as Problem 1.

Problem 1　　Let $<X, Y, \Phi>$ be an admissible tuple, indicating the position and orientation relations between the box and the pushers. Let $\boldsymbol{q}_1$, $\boldsymbol{q}_2$, and $\boldsymbol{q}_3$ be three differential drive robots and $\boldsymbol{q}_b$ be the box being pushed. $\boldsymbol{q}_1$ is a watcher and $\boldsymbol{q}_2$ and $\boldsymbol{q}_3$ are pushers. If a formation navigation strategy can be found to keep all elements in $<X, Y, \Phi>$ constant when the box is being pushed to its target, then robots $\boldsymbol{q}_2$ and $\boldsymbol{q}_3$ will push the box $\boldsymbol{q}_b$ to any destination which can be detected by the watcher $\boldsymbol{q}_1$. This problem will be studied in Section 3.

　　Unlike simple behavior-based transport approach, the proposed cooperation strategy contains local path programming, with information shared by both the watcher and the pushers. Since the pushers cannot sense the obstacles and the way to their destination, they need the information from the watcher robot to calculate their movements. Thus, the robots communicate with each other using wireless network.

　　Furthermore, the constraints show that the pushers have no permanent connection to the box. That is, the box moves if and only if the pushers move with non-negative rotation velocities. Once a pusher has its drive wheel velocity set negatively, the box

might be slipped by other pushers. However, a maximum rotation velocity is required both for the hardware restriction of the robot and to prevent inertia motion of the box.

3 Cooperative transport strategy design

A geometric approach is presented in this section for the cooperative transport problem described as Problem 1. Fig. 2 shows a schematic diagram of the path generation strategy and navigation for the formation.
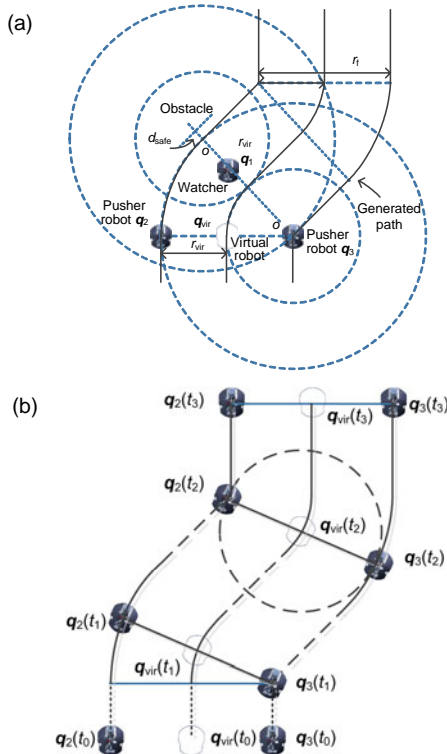

Fig. 2 Geometric based navigation and path generation strategy and a detailed path for robot formation
(a) The pushers turn right to avoid obstacle point detected by the watcher. When q_2 reaches the point o, the formation will execute a 'Forward' movement until they pass the detected obstacle point. Finally the formation will return to move to the destination with Eq. (15). (b) The pushers turn right to avoid obstacle by the time t_1, and restore to move to the destination by the time t_2

Considering the box transport problem, two robots are needed in our cooperation strategy to push the box, namely the pusher robots q_2 and q_3 in Fig. 2a. A watcher is included as q_1. All the robots are

required to satisfy the drive wheel rotation velocity constraint

$$\dot{\varphi}_i \in [0, \dot{\varphi}_{\text{MAX}}], \tag{5}$$

where the predefined $\dot{\varphi}_{\text{MAX}}$ stands for the maximum rotation velocity of the robot differential drive wheel, considering the robot hardware constraint and preventing inertia motion of the box.

The pusher robots are tightly coupled, and a virtual robot is assumed to be at the midpoint of the line connecting the two pushers. The virtual robot is instructed to help the robots keep a static distance and orientation when they are moving, as illustrated by the hollow shape in Fig. 2. The detailed strategy is described as follows.

The box pushing task has two parts: movement to the destination and obstacle avoidance. In the first part, robots track the target by heading adjustment and moving forward. The watcher moves to the destination by minimizing $e_{1,\text{D}}=<x_{1,\text{D}},\ y_{1,\text{D}}>$, which represents the position bias between the target and the watcher robot.

As shown in Fig. 1, when the position bias $y_{1,\text{D}}(e_{1,\text{D}})$ in the watcher's local coordinates meets

$$\left|y_{1,\text{D}}(e_{1,\text{D}})\right| \ge \delta_{\text{P}} \text{ or } x_{1,\text{D}}(e_{1,\text{D}}) < -\delta_{\text{P}}\left(y_{1,\text{D}}(e_{1,\text{D}})=0\right), \tag{6}$$

each robot will adjust its heading to minimize $y_{1,\text{D}}(e_{1,\text{D}})$ until

$$\lim_{t\to\infty}\left|y_{1,\text{D}}(e_{1,\text{D}})\right| < \delta_{\text{P}},\ \ x_{1,\text{D}}(e_{1,\text{D}}) \ge \delta_{\text{P}}, \tag{7}$$

where δ_{P} is the given position bias constant.

Eq. (8) could easily be perceived to be another form of Eq. (7):

$$\left|y_{1,\text{D}}(e_{1,\text{D}})\right| < \delta_{\text{P}},\ \ x_{1,\text{D}}(e_{1,\text{D}}) \ge \delta_{\text{P}}. \tag{8}$$

When Eq. (8) has been satisfied, which means that $y_{1,\text{D}}(e_{1,\text{D}})\to 0$, each robot will move forward to minimize the position bias $x_{1,\text{D}}(e_{1,\text{D}})$.

With the minimization of $y_{1,\text{D}}(e_{1,\text{D}})$ and $x_{1,\text{D}}(e_{1,\text{D}})$, Eq. (9) can be satisfied:

$$\lim_{t\to\infty}\sqrt{x_{1,\text{D}}(e_{1,\text{D}})^2 + y_{1,\text{D}}(e_{1,\text{D}})^2} < \delta_{\text{P}},\ \left|y_{1,\text{D}}(e_{1,\text{D}})\right| < \delta_{\text{P}}, \tag{9}$$

and each robot will arrive at its destination.

The watcher's drive wheel rotation velocity for moving forward is set as $\dot{\varphi}_{l1} = \dot{\varphi}_{r1}$. In heading adjustment, the rotation velocity of the wheel in the turning direction is set to 0. The pusher robot has the same drive wheel rotation velocity as the watcher in moving forward, while the rotation velocity of heading adjustment is decided by the virtual robot, which is illustrated by obstacle avoidance.

When the watcher robot detects a nearby obstacle on its way to the destination, it will calculate the distance to the box with the information from its distance sensor by

$$\eta = \text{dist} \cdot \left| \sin \phi_{\text{vir},o} \right| - \frac{1}{2} \sqrt{(x_2 - x_3)^2 + (y_2 - y_3)^2}, \quad (10)$$

where dist is the closest distance measured by the sensors, and $\phi_{\text{vir},o}$ stands for the direction of the obstacle on the virtual robot's local coordinates. The position of the target is calculated with a distance d_{safe}. If $\eta \le 0$, which means that the obstacle is in front of the pusher robots on course to the destination, information of the obstacle position will be stored and shared by the pusher robots. The parameters in Eq. (10) can be acquired by the information from the watcher robot. The pusher robots will then select the side of the obstacle in their way, comparing the slopes of the lines connecting the obstacle and themselves, and choose the direction in which they are going to turn. The obstacle avoidance starts at the time when the distance between the obstacle and the pusher farthest from it is $r_f + d_{\text{safe}}$ (the turning radius r_f is the distance between two robot centers). A complete description is given as

$$\begin{cases} \text{TurnR}, & \text{if } \left| k_{2,o} \right| \le k_{3,o}, \\ \text{TurnL}, & \text{if } \left| k_{2,o} \right| > k_{3,o}, \end{cases} \quad (11)$$

where

$$\begin{cases} k_{2,o} < 0 < k_{3,o}, \\ k_{2,o} = (y_o - y_2)/(x_o - x_2), \\ k_{3,o} = (y_o - y_3)/(x_o - x_3). \end{cases}$$

$k_{i,o}$ stands for the slope of the line connecting robot i and the obstacle o. The motion 'Forward' is the motion of moving forward, with the same rotation velocities of two differential drive wheels. TurnR and TurnL stand for the motions of turning right and

turning left, respectively, and will be finished separately in both pusher robots. The watcher decides whether to change its motion to avoid the obstacle by the turning decision results of the pusher robots:

$$\begin{cases} \text{Forward}, & \text{if } \eta \le 0, \ \text{dist} \cdot \left| \sin \phi_{1,o} \right| > d / 2, \\ \text{TurnR}, & \text{if } \eta \le 0, \ \text{dist} \cdot \left| \sin \phi_{1,o} \right| < d / 2, \ \left| k_{2,o} \right| \le k_{3,o}, \\ \text{TurnL}, & \text{if } \eta \le 0, \ \text{dist} \cdot \left| \sin \phi_{1,o} \right| < d / 2, \ \left| k_{2,o} \right| > k_{3,o}, \end{cases}$$

$$(12)$$

where d is the width of the robot, and $\phi_{1,o}$ stands for the direction of the obstacle on the virtual robot's local coordinates. The turning velocity of the pusher robots is defined in consideration of Eq. (5) as

$$\begin{cases} \begin{bmatrix} \dot{\varphi}_{l2} \\ \dot{\varphi}_{r2} \end{bmatrix} = \begin{bmatrix} \dot{\varphi}_{\text{MAX}} \\ \dot{\varphi}_{\text{MAX}} \cdot (r_f + d)/(r_f + 2d) \end{bmatrix}, \\ \begin{bmatrix} \dot{\varphi}_{l3} \\ \dot{\varphi}_{r3} \end{bmatrix} = \begin{bmatrix} \dot{\varphi}_{\text{MAX}} \cdot d/(r_f + 2d) \\ 0 \end{bmatrix}, \quad \text{if TurnR}, \\ \begin{bmatrix} \dot{\varphi}_{l2} \\ \dot{\varphi}_{r2} \end{bmatrix} = \begin{bmatrix} 0 \\ \dot{\varphi}_{\text{MAX}} \cdot d/(r_f + 2d) \end{bmatrix}, \\ \begin{bmatrix} \dot{\varphi}_{l3} \\ \dot{\varphi}_{r3} \end{bmatrix} = \begin{bmatrix} \dot{\varphi}_{\text{MAX}} \cdot (r_f + d)/(r_f + 2d) \\ \dot{\varphi}_{\text{MAX}} \end{bmatrix}, \quad \text{if TurnL}. \end{cases} \quad (13)$$

This turning velocity is also the velocity of both pusher robots in destination tracking. The termination condition of turning is given by

$$\text{Turnend}, \ \text{if } k_{o2} = k_{o3} = \pm\infty. \quad (14)$$

The turning termination criterion shows that pushers will stop their turning movement if the point coordinates of the obstacle are ±90 degrees of the robots' heading direction. Fig. 2b shows the path generated in detail. After the turning step of obstacle avoidance is over, the robots continue to take a forward movement until the first turning robot q_2 passes the whole obstacle, or the virtual robot q_{vir} passes the first obstacle point coordinates received by two pusher robots, in their local positive x-direction. This condition can be illustrated as $x_{i\,\text{local}} > x_{o\,\text{lastlocal}}$ or $x_{v\,\text{local}} > x_{o\,\text{local}}$. Here $x_{i\,\text{local}}$, $x_{o\,\text{local}}$, and $x_{v\,\text{local}}$ are the x-coordinates of pusher robot i, the obstacle, and the virtual robot in the local coordinates of robot i, respectively. $x_{o\,\text{lastlocal}}$ is the local x-value of the obstacle $q_{o\,\text{last}}$ in coordinates of q_i, and $q_{o\,\text{last}}$ can be considered

to be one obstacle with q_o detected by the watcher q_1. The pusher robots then continue to move to the destination with Eq. (15), comparing the direction of the destination in local coordinates of the virtual robot:

$$\begin{cases} \text{Forward, if } |y_{\text{vir,D}}| \leq \delta, \\ \text{TurnR, \quad if } y_{\text{vir,D}} < -\delta, \\ \text{TurnL, \quad if } y_{\text{vir,D}} > \delta, \end{cases} \quad (15)$$

where $y_{\text{vir,D}}$ stands for the relation between the destination and the virtual robot, as illustrated in Fig. 1. The turning velocities of the robots are set according to Eq. (13). A robot will choose to move forward if $|y_{\text{vir,}D}|$ is no longer than δ, a predefined constant. The motion 'Forward' is necessary, for other parts of the obstacle like $x_{o \text{ lastlocal}}$ may still be in front of the formation if the pusher robots turn back in the direction of the destination once Eq. (14) has been satisfied. As shown in Fig. 2b, the left solid curve in the path from t_0 to t_2 is the trajectory generated by the first turning robot, while the right curve from t_0 to t_2 is the trajectory of the last turning robot. The dashed lines from t_1 to t_2 are the forward movement of the robot formation to avoid oscillation against the obstacle.

Proposition 1 Let $<X, Y, \Phi>$ be an admissible tuple. Consider a multi-robot system in the box transport problem with three differential drive robots q_1, q_2, and q_3. Let $0 \leq \dot{\varphi}_i \leq \dot{\varphi}_{\text{MAX}}$ be the drive wheel rotation velocity constraint. The tuple $<X, Y, \Phi>$ can be kept static and the box can be moved to its destination if the linear velocity of the box $v_b>0$.

Proof Suppose robots q_2, q_3 have relations $<x_{\text{b},2}, y_{\text{b},2}, \phi_{\text{b},2}>$, $<x_{\text{b},3}, y_{\text{b},3}, \phi_{\text{b},3}>$ with the box q_b, respectively. q_2 and q_3 also have static relations $<x_{\text{vir},2}, y_{\text{vir},2}, \phi_{\text{vir},2}>$, $<x_{\text{vir},3}, y_{\text{vir},3}, \phi_{\text{vir},3}>$ with the virtual robot. With the assumption in Section 2, the box can be moved only by the push power of pusher robots. This means that the box needs a positive power to drive. The linear velocity of the box can be transformed into the velocity of the virtual robot. First the linear velocity of virtual robot can be acquired by

$$v_{\text{vir}} = \begin{cases} \dfrac{r}{2}(\dot{\varphi}_{\text{r}2} + \dot{\varphi}_{\text{l}2}) = \dfrac{r}{2}(\dot{\varphi}_{\text{r}3} + \dot{\varphi}_{\text{l}3}), \quad \text{Forward,} \\ \dfrac{r}{4}(\dot{\varphi}_{\text{r}2} + \dot{\varphi}_{\text{l}2} + \dot{\varphi}_{\text{r}3} + \dot{\varphi}_{\text{l}3}), \text{ TurnR / TurnL.} \end{cases} \quad (16)$$

Based on Eq. (16), when the formation moves forward, the linear velocity of the box can be easily obtained from the geometric relationship, $v_b=v_{\text{vir}}=v_2=v_3 \in [0, r\dot{\varphi}_{\text{MAX}}]$, while $v_b=0$ means there is no push power and the box is stationary.

In the turning movement, the linear velocity of the virtual robot $v_{\text{vir}} = r\dot{\varphi}_{\text{MAX}} / 2$ can be acquired from the turning velocity illustrated in Eq. (13). The linear velocity of the box is $v_b = v_{\text{vir}} \sqrt{a^2 + r_{\text{vir}}^2} / r_{\text{vir}} \in (r\dot{\varphi}_{\text{MAX}}, +\infty)$, which is also positive. Accordingly, in both situations, $v_b>0$ can be satisfied, and thus the tuple $<X, Y, \Phi>$ can be kept static.

The steps of the cooperative box transport problem can be summarized briefly as follows.

Step 1: Robots in the formation cooperatively push the box to the destination.

Step 2: Once the watcher detects an obstacle, it will share the information with the pushers.

Step 3: The pushers avoid collision with the obstacle by following Eqs. (11)–(15).

Step 4: The pushers continue a forward movement to pass the whole obstacle, and return to Step 1.

The proposed geometric strategy takes account of the drawbacks of a simple behavior-based strategy in generating a more effective path that is theoretically feasible from geometric theory. When the formation turns, the box will turn at the point of contact with the pusher. No negative velocity is generated in the strategy, and the box can move with a positive power given by the pushers.

4 Simulation results

The cooperative box transport task experiments were executed in the simulation environment of Microsoft Robotics Developer Studio. The task needs the robot formation to push a box to a target truck, with the robot system and environmental constraints detailed in Section 2. Simulations were separated into two types: one with a static target, the other with a dynamic target.

Both simulations contained a group of three robots, a box needed to be transported, and a truck set to be the target. The box had a size of 1.6 m×0.8 m×1 m. The box center was set as the origin of the world coordinates $q_b(t_0)=[0, 0, 0]^{\text{T}}$ (unit: x and y, m; θ, rad),

with heading to the positive direction of the *x*-axis. All robots in the formation acquired their initial postures from the sensed distance to the box. In the following two simulations, the initial postures of the three robots were $q_1(t_0)=[1.5, 0, 0]^T$, $q_2(t_0)=[-0.55, 0.6, 0]^T$, and $q_3(t_0)=[-0.55, -0.6, 0]^T$, where q_1 is the watcher robot, and q_2, q_3 are the pushers. The termination condition was that the box was close to the target truck, within a distance of 1 m.

4.1 Cooperative box transport with a static target

In this simulation the target truck had an initial posture of $q_D(t_0)=[5, 15, 0]^T$, which was static during task execution. The box transport process is shown in Fig. 3.

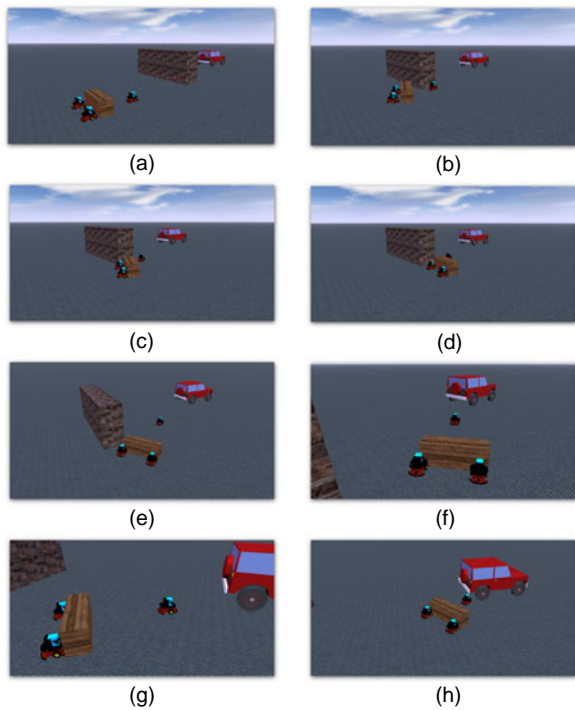Fig. 4 shows the path generated by the box and the robot formation. The box has been transported to

Fig. 3 Process of the box transport in a static target situation
(a) At the beginning, the watcher and the formation moved forward with a static velocity. The watcher robot detected an obstacle and made a decision to move forward to avoid the obstacle based on Eq. (12). (b) The pusher robots turned at an angle according to Eqs. (11)–(15), avoiding the obstacle. (c) A forward behavior was taken, as illustrated in Step 4, Section 3. (d) The pushers pushed the box dependent on Eqs. (6)–(9). (e)–(g) The formation moved to the target truck. (h) The task was finished, as the distance between the box and the truck was less than 1 m

the desired target truck without any collision with the obstacle. Fig. 5 shows the deviation between the box and the virtual robot. During the task process, the pusher robots were supposed to keep a constant distance from the box, represented as a distance between the virtual robot and the box. The average deviation value of the static distance was 0.0316 m during the simulation process in a static target situation.

The simulation results show that the proposed strategy is effective for a box transport task with a static destination.
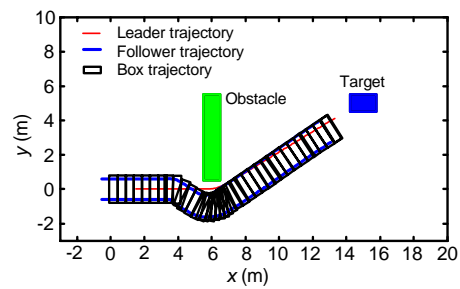

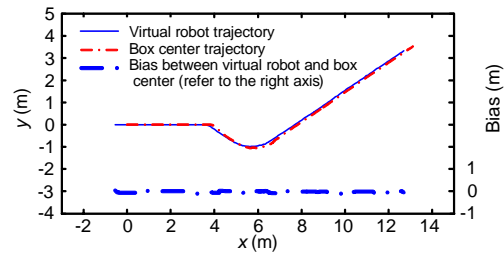**Fig. 4 Trajectory of the robot and box to a static target**

Fig. 5 Trajectory of the virtual robot and box in a static target situation

4.2 Cooperative box transport with a dynamic target

The simulation of a box transport problem with a dynamic target had the same initial state as the previous static simulation. The process of the task has been omitted. The result trajectory is shown in Fig. 6 and the deviation in Fig. 7.

During the execution process of the box transport task, the velocity of the target truck was static. The single hollow rectangle indicates the initial position while the solid rectangle represents the final position when the task was over. The box was pushed close to the target truck while successfully avoiding collisions with the obstacle (Fig. 6). Fig. 7 shows the trajectories for both the virtual robot and the box. The average value of the deviation in this simulation with a dynamic target was 0.0211 m. The deviations in

both experiments were generated by inertia movements of both the robots and the box when the robots changed their behaviors.
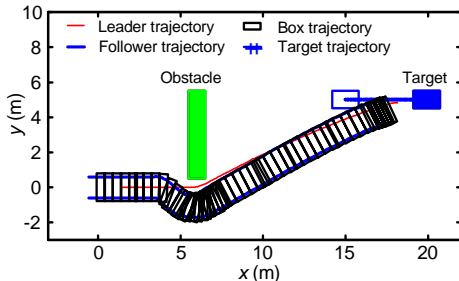

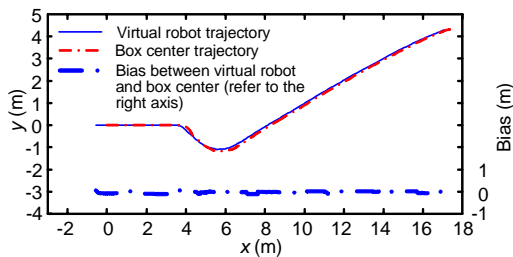**Fig. 6 Trajectory of the robot and box to a dynamic target**

Fig. 7 Trajectory of the virtual robot and box in a dynamic target situation

These simulation results demonstrate the validity of our proposed cooperative formation transport strategy, applied successfully to a box transport problem with both static and dynamic targets. The trajectories also show that the proposed strategy can prevent robots from repeating their movements.

5 Conclusions and future work

This paper provides a geometric navigation strategy for multiple mobile robot formations in box transport problems. Advanced planning of the robots' trajectories is achieved using shared information from all robots, helping the robots to maintain a given formation shape without movement repetition while they are executing tasks. Constraints are considered, including the robots' local sensors and velocities. The proposed strategy can work in situations of either static or dynamic targets, with negligible deviation. Two types of box transport simulations were successfully performed using our cooperation strategy, demonstrating the effectiveness of the strategy. Further study will focus on physical experiments with uneven pavements in outdoor environments, especially when there are disturbances and sensor errors.

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