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An LDPC coded cooperative MIMO scheme over Rayleigh fading channels with unknown channel state information*

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Abstract: This paper describes a coded cooperative multiple-input multiple-output (MIMO) scheme, where structured low-density parity-check (LDPC) codes belonging to a family of repeat-accumulate (RA) codes are employed. The outage probability of the scheme over Rayleigh fading channels is deduced. In an unknown channel state information (CSI) scenario, adaptive transversal filters based on a spatio-temporal recursive least squares (ST-RLS) algorithm are adopted in the destination to realize receive diversity gain. Also, a joint 'Min-Sum' iterative decoding is effectively carried out in the destination. Such a decoding algorithm agrees with the bilayer Tanner graph that can be used to fully characterize two distinct structured LDPC codes employed by the source and relay. Simulation results verify the effectiveness of the adopted filter in the coded cooperative MIMO scheme. Theoretical analysis and numerical simulations show that the LDPC coded cooperative MIMO scheme can well combine cooperation diversity, multi-receive diversity, and channel coding gains, and clearly outperforms coded noncooperation schemes under the same conditions.

Key words: Cooperative multiple-input multiple-output (MIMO), Repeat-accumulate (RA) codes, Bilayer Tanner graph, Spatio-temporal recursive least squares (ST-RLS) algorithm, Joint 'Min-Sum' iterative decoding
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1 Introduction

Multiple-input multiple-output (MIMO) is an effective technique for combating fading and improving channel capacity by offering spatial diversity. Conventional MIMO systems require both the transmitter and receiver of a communication link to be equipped with multiple antennas. In practice, however, many mobile devices may not be able to support multiple antennas due to size, cost, etc. The cooperation technique (Hunter, 2004) is one of the most effective ways to solve this crucial problem. The technique enables single-antenna mobiles to share the use of their antennas to form a virtual antenna array. Recently, many schemes have been proposed to support cooperative communications, including amplify-and-forward (AF) (Laneman *et al.*, 2001), detect-and-forward (DF) (Cover and Gamal, 1979), coded cooperation (Sendonaris *et al.*, 2003), and network coding cooperation (Xiao *et al.*, 2007). In a network coding cooperation scheme, which is well suited to wireless networks, the transmitting node firstly combines the multiple data flows from other nodes in the wireless network, then transmits that combination to the destination, rather than simply routing the data flows individually. In an AF scheme, the relay node retransmits only scaled versions of signals received from the source to the destination. In a DF scheme, the relay node first detects the noise-corrupted received signals by hard decisions and then forwards the estimated signals to the destination. Generally, both AF and DF modes may not be suitable for systems in which a low error rate is strictly required in the destination.

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To pursue further transmission reliability, many studies (Janani *et al.*, 2004; Zhang and Duman, 2005; Li *et al.*, 2008a; Bao, 2009) concentrated on the attractive topic of high performance coded cooperations. In particular, the use of low-density parity-check (LDPC) codes (Gallager, 1963) and turbo codes (Berrou and Glavieux, 1996) has been explored. Compared to turbo codes, LDPC codes have the advantages of low decoding complexity and delay in practice. Chakrabarti *et al.* (2007) derived the exact relationships that the component LDPC code profiles must satisfy in coded relay cooperation. In the destination, the source transmission is decoded with the help of side information in the form of additional parity-check bits from the relay. The design of LDPC codes for coded relay cooperation over additional white Gaussian noise (AWGN) channels was considered by Li *et al.* (2008b). The half-duplex relay code design problem is transformed into a problem of LDPC code design where different code segments experience different signal-to-noise ratios (SNRs). Two bilayer-LDPC codes structures (Razaghi and Yu, 2007), known as bilayer-expurgated and bilayer-lengthened, were proposed as optimized for coded relay channels. Li *et al.* (2011b) investigated the LDPC code design for a multisource single relay system, where network coding is used in the relay. However, such LDPC codes designed for the relay cooperation introduce new problems: the encoding complexity is high for the source and relay, and it is difficult to ensure systematic codewords.

In this paper, relay cooperation with multiple receiving antennas in the destination is formulated as a cooperative MIMO system (Wang *et al.*, 2010; Cheng *et al.*, 2012). The system can achieve receive diversity and cooperation diversity gains. Simple-encoding structured LDPC codes belonging to a family of repeat-accumulate (RA) codes (ten Brink and Kramer, 2004; Sun *et al.*, 2011) are applied to the cooperative MIMO system. Most existing schemes assume that the perfect channel state information (CSI) is available in wireless communications. However, in practice the CSI is usually unknown to the receiver. Assuming Rayleigh fading channels and an unknown CSI scenario, we propose a spatio-temporal recursive least squares (ST-RLS) algorithm based on a training sequence, to detect the received signals in the destination. A joint iterative decoding algorithm is also introduced to decode multiple detected signals coming from the source and relay in the destination based on the bilayer Tanner graph, which can fully characterize double structured LDPC codes designed for the source and relay. The contributions of this paper can be summarized as follows: (1) The outage probability of the coded cooperative MIMO system over Rayleigh fading channels is analyzed. (2) For the coded cooperative MIMO system with unknown CSI in the destination, an ST-RLS algorithm based on a training sequence is applied to detect the received signals.

In this paper, the superscripts ‘T’ and ‘*’, and the overbar ‘¯’ denote the transpose, Hermitian transpose, and conjugation of a matrix or vector, respectively; $\mathbf{0}_{M\times M}$ is an $M\times M$ zero matrix.

2 System descriptions

A one-relay LDPC coded cooperative MIMO system is depicted in Fig. 1. As mobile users, the source node (S) and relay node (R) have single antennas; as the base station, the destination node (D) is equipped with multiple receiving antennas. In the S node, a codeword $\boldsymbol{c}_1$ conveying information bits encoded by the first LDPC encoder (LDPC-1) is sent simultaneously to R and D over a broadcast channel. The relay decodes the incoming signal to recover the information bits, which are then encoded into another distinct codeword $\boldsymbol{c}_2$ by the second LDPC encoder (LDPC-2). As $\boldsymbol{c}_1$ and $\boldsymbol{c}_2$ have the same information bits, the relay sends only its check bits to D over the R-D channel so as to retain a highly efficient coded transmission.

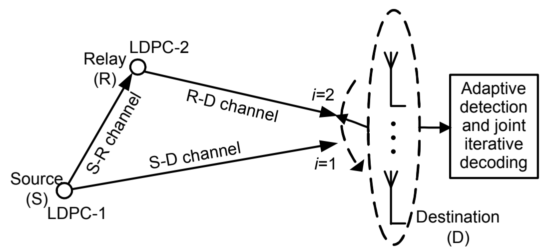

Fig. 1 One relay low-density parity-check (LDPC) coded cooperative multiple-input multiple-output (MIMO) system

In the destination, the two incoming signals of multiple receiving antennas, which are transmitted in two time slots, do not interfere with each other. First, they are detected by adaptive transversal filters based on channel estimation; then the decoder performs the joint iterative decoding algorithm with respect to the two detected signals. Assuming that the relay can correctly decode the signals received from the source, the coded ideal cooperative MIMO scheme is formulated in this paper.

3 LDPC coded cooperative MIMO scheme

3.1 Encoding scheme for the LDPC coded cooperative MIMO scheme

Assume a sparse parity-check matrix $\boldsymbol{H}$ corresponding to the structured LDPC, which belongs to a family of RA codes, has the form below:

$$\boldsymbol{H}=[\boldsymbol{A}\ \ \boldsymbol{D}] =\begin{bmatrix} 1&0&0&1&0&1&0&1& &1&0&0&0\\ 0&1&0&1&0&1&1&0& &1&1&0&0\\ 0&1&1&0&1&0&1&0& &0&1&1&0\\ 1&0&1&0&1&0&0&1& &0&0&1&1 \end{bmatrix}, \tag{1}$$

where $\boldsymbol{A}$ is a regular matrix that has the numbers of 1's in each column as $d_{\rm v}$ and each row as $d_{\rm c}$. In Eq. (1), $d_{\rm v}$=2 and $d_{\rm c}$=4. $\boldsymbol{D}$ is a quasi-diagonal matrix in which all elements are zero except the elements of the principal diagonal and the elements immediately below this diagonal. The structured LDPC codes defined by a similar such quasi-diagonal structure are also adopted by the DVB-S2 standard. They are also referred to as extended irregular repeat-accumulate (eIRA) codes (Jin et *al.*, 2000; Yang et *al.*, 2004). This kind of codes has not only a differential encoding character but also a sparse parity-check matrix structure. Hence, it combines simple encoding with low complexity decoding, while ensuring good performance. These structured LDPC codes can be applied directly to encoding without using the Gaussian elimination algorithm, which guarantees systematic codewords. Hence, the relay can extract the information bits easily to re-encode them. Also, compared to the common systematic LDPC codes, called generator-based LDPC (G-LDPC) (Bao, 2009), the structured LDPC codes in general exhibit a lower error floor and a slightly better water-fall region.

Xiao and Skoglund (2010) proposed diversity network codes (DNCs) over non-binary finite fields for multiuser cooperative communications, where each user transmits independent messages. These codes and the encoding matrices are designed such that the destination is able to rebuild the user information from a minimum possible set of coded blocks. An algorithm was proposed to design low-complexity encoders of binary frame-wise network coding (BFNC) schemes by exploiting the parity-check matrices of quasi-cyclic LDPC codes. The aim was to provide both network diversity and coding gain for the multisource multirelay networks, where channel coding is not considered in sources (Li *et al.*, 2011a; 2012). Here, the encoding scheme for the coded cooperative MIMO employing the structured LDPC codes is implemented. The relay only assists the source without generating messages of its own, and channel coding is considered in both the source and relay.

1. A block of information bits $\boldsymbol{s}=[s_1, s_2, \cdots, s_{N-M_1}]^{\rm T}$ is encoded into a codeword

$$\boldsymbol{c}_1=[s_1, s_2,\cdots, s_{N-M_1}, p_1^{(1)}, p_2^{(1)},\cdots, p_{M_1}^{(1)}]^{\rm T} \tag{2}$$

by LDPC-1 defined by its parity-check matrix as

$$\boldsymbol{H}_1=\left[\boldsymbol{A}_{M_1\times(N-M_1)}\ \ \boldsymbol{D}_{M_1\times M_1}\right]. \tag{3}$$

The codeword is sent simultaneously to the relay and destination over a broadcast channel.

2. Assume that the noise-corrupted signal received from the source can be correctly decoded by the relay. The block of recovered information bits is encoded again, resulting in a distinct codeword

$$\boldsymbol{c}_2=[s_1, s_2, \cdots, s_{N-M_1}, p_1^{(2)}, p_2^{(2)}, \cdots, p_{M_2}^{(2)}]^{\rm T} \tag{4}$$

by LDPC-2 with the parity-check matrix as

$$\boldsymbol{H}_2=\left[\boldsymbol{B}_{M_2\times(N-M_1)}\ \ \boldsymbol{D}_{M_2\times M_2}\right]. \tag{5}$$

The relay sends only the parity-check bits $[p_1^{(2)}, p_2^{(2)}, \cdots, p_{M_2}^{(2)}]$ to the destination since the information bits $\boldsymbol{s}=[s_1, s_2, \cdots, s_{N-M_1}]$ have already been transmitted to the destination by the source over the broadcast channel.

3. From the viewpoint of the decoder in the destination, the overall parity-check matrix $\boldsymbol{H}$ of the coded cooperative system satisfies

$$\boldsymbol{Hc}=\boldsymbol{0}, \tag{6}$$

where the whole codeword

$$\boldsymbol{c}=[s_1,\ s_2,\cdots,\ s_{N-M_1},\ p_1^{(1)},\ p_2^{(1)},\cdots,\\ p_{M_1}^{(1)},\ p_1^{(2)},p_2^{(2)},\cdots,\ p_{M_2}^{(2)}]^{\mathrm{T}}, \tag{7}$$

has a block length of $N+M_2$, corresponding to the overall parity-check matrix as

$$\boldsymbol{H}=\begin{bmatrix} \boldsymbol{A}_{M_1\times(N-M_1)} & \boldsymbol{D}_{M_1\times M_1} & \boldsymbol{0}_{M_1\times M_2} \\ \boldsymbol{B}_{M_2\times(N-M_1)} & \boldsymbol{0}_{M_2\times M_1} & \boldsymbol{D}_{M_2\times M_2} \end{bmatrix}. \tag{8}$$

Fig. 2 illustrates the bilayer Tanner graph (Razaghi and Yu, 2007) corresponding to the overall parity-check matrix for the LDPC-coded cooperative MIMO scheme from the viewpoint of the decoder in the destination.

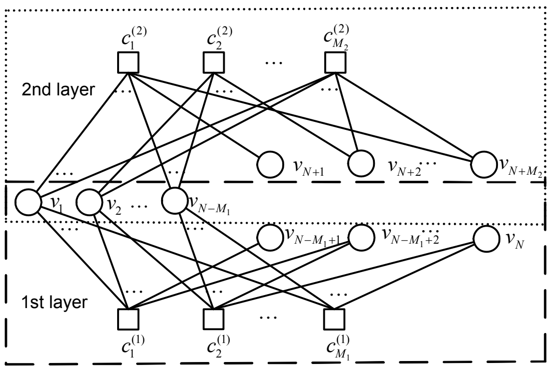

Fig. 2 Bilayer Tanner graph used to characterize the overall parity-check relationship of the one-relay LDPC coded cooperative MIMO scheme (Razaghi and Yu, 2007)

The first and second layers correspond to parity-check matrices $\boldsymbol{H}_1$ and $\boldsymbol{H}_2$, respectively. Clearly, v_n (n=1, 2, …, $N-M_1$) are the common variable nodes of $\boldsymbol{H}_1$ and $\boldsymbol{H}_2$, while the variable nodes v_n ($n=N-M_1+1$, $N-M_1+2$, …, N) are related only to $\boldsymbol{H}_1$, and v_n ($n=N+1$, $N+2$, …, $N+M_2$) are related only to $\boldsymbol{H}_2$. The check nodes $c_m^{(1)}$ (m=1, 2, …, M_1) and $c_m^{(2)}$ (m=1, 2, …, M_2) correspond to $\boldsymbol{H}_1$ and $\boldsymbol{H}_2$, respectively.

3.2 Outage probability analysis of the coded cooperative MIMO system

Suppose all signals over the S-D and R-D channels suffer from Rayleigh fading and AWGN, simultaneously. i=1 and 2 correspond to the S-D and R-D channels, respectively. For L receiving antennas in the destination, the received signal $\boldsymbol{r}^{(i)}=[r_1^{(i)}, r_2^{(i)}, ..., r_L^{(i)}]^{\mathrm{T}}$ is

$$\boldsymbol{r}^{(i)}=A\boldsymbol{h}^{(i)}+\boldsymbol{n}^{(i)}, \tag{9}$$

where A is the modulated codeword symbol sent from the source or relay, and $\boldsymbol{n}^{(i)}$ and $\boldsymbol{h}^{(i)}$ are two complex column random vectors for AWGN and Rayleigh fading, respectively:

$$\begin{cases} \boldsymbol{n}^{(i)}=[n_1^{(i)}, n_2^{(i)}, ..., n_L^{(i)}]^{\mathrm{T}}, \\ \boldsymbol{h}^{(i)}=[h_1^{(i)}, h_2^{(i)}, ..., h_L^{(i)}]^{\mathrm{T}}, \end{cases} \tag{10}$$

where $n_j^{(i)}$ (j=1, 2, …, L) is a zero-mean complex Gaussian random variable (RV) with variance $E[|n_j^{(i)}|^2]=N_0$. For Rayleigh fading, $h_j^{(i)}$ (j=1, 2, …, L) is also a zero-mean complex Gaussian RV with unit variance $E[|h_j^{(i)}|^2]=1$.

An outage event occurs when the instantaneous channel capacity falls below the date transmission rate. The probability of an outage event occurring is defined as the outage probability (Zhou *et al.*, 2009).

Here, we investigate the outage probability for the coded cooperative MIMO scheme, where the relay forwards only the parity-check bits. For comparison, let us consider firstly a noncooperation transmission scheme.

1. Outage probability analysis of a noncooperation scheme.

In a noncooperation scheme, there are links only between S and D, and the superscript 'i' can be removed. The instantaneous channel capacity can be calculated as

$$C_{\mathrm{non}}=\log_2\left(1+\sum_{j=1}^{L}\left|h_j\right|^2\gamma\right), \tag{11}$$

where γ is the average SNR per bit per antenna, which is the same for each antenna. The outage probability of the noncooperation scheme is given by

$$\begin{aligned}\mathrm{Pr}_{\mathrm{non}}&=\Pr\left\{\log_2\left(1+\sum_{j=1}^{L}\left|h_j\right|^2\gamma\right)<R\right\}\\ &=\Pr\left\{\sum_{j=1}^{L}\left|h_j\right|^2<\frac{2^R-1}{\gamma}\right\},\end{aligned} \tag{12}$$

where R is the data transmission rate, which is regarded as the code rate in this paper, and $|h_j|^2$ follows an exponential distribution $|h_j|^2 – \varepsilon(1)$. We can further deduce that $X=\sum_{j=1}^{L}|h_j|^2$ follows the distribution

$$f_X(x)=\begin{cases}\dfrac{x^{L-1}}{(L-1)!}\mathrm{e}^{-x}, & x>0,\\ 0, & x\le 0.\end{cases} \tag{13}$$

Hence, Eq. (12) can be calculated as

$$\mathrm{Pr}_{\mathrm{non}}=\int_0^{\frac{2^R-1}{\gamma}}\frac{x^{L-1}}{(L-1)!}\mathrm{e}^{-x}\mathrm{d}x. \tag{14}$$

2. Outage probability analysis of the coded cooperative MIMO scheme.

In the ideal cooperative MIMO scheme, the messages are transmitted over the S-D and R-D channels as described in Section 3.1. The instantaneous channel capacity can be calculated as

$$\begin{aligned}C_{\mathrm{coop}}=&\frac{N}{N+M_2}\log_2\left(1+\sum_{j=1}^{L}\left|h_j^{(1)}\right|^2\gamma^{(1)}\right)\\&+\frac{M_2}{N+M_2}\log_2\left(1+\sum_{j=1}^{L}\left|h_j^{(2)}\right|^2\gamma^{(2)}\right),\end{aligned} \tag{15}$$

where $\gamma^{(1)}$ and $\gamma^{(2)}$ are the average SNR per bit per antenna for the S-D and R-D channels, respectively. The outage probability of the ideal cooperative MIMO scheme is given by

$$\begin{aligned}\mathrm{Pr}_{\mathrm{coop}}=\mathrm{Pr}\Bigg\{&\frac{N}{N+M_2}\log_2\left(1+\sum_{j=1}^{L}\left|h_j^{(1)}\right|^2\gamma^{(1)}\right)\\&+\frac{M_2}{N+M_2}\log_2\left(1+\sum_{j=1}^{L}\left|h_j^{(2)}\right|^2\gamma^{(2)}\right)<R\Bigg\},\end{aligned} \tag{16}$$

which is deduced in the Appendix.

4 Adaptive detection and joint iterative decoding for an LDPC coded cooperative MIMO with unknown CSI

In an LDPC coded cooperative MIMO scheme over Rayleigh block fading channels with unknown CSI, two adaptive transversal filters corresponding to S-D and R-D channels are proposed (Fig. 3) based on the ST-RLS algorithm (Sayed and Kailath, 1994; Arablouei and Dogancay, 2011; Li and de Lamare, 2012). The transversal filter can adaptively detect the blocks of signals over various Rayleigh fading coefficients based on training sequences. Two detected sequences with soft information are multiplexed by the joint iterative decoding to achieve the decoded bits.

4.1 Adaptive detection based on the spatio-temporal recursive least squares (ST-RLS) algorithm

Suppose the CSI of S-D and R-D channels is unknown to the destination. The ST-RLS algorithm adopted by D is shown in Fig. 4. $\boldsymbol{c}=[c_1, c_2, \ldots, c_L]^{\mathrm{T}}$ is the transversal filter vector. A_k and y_k are the transmitted bit and the output of the filter, respectively.

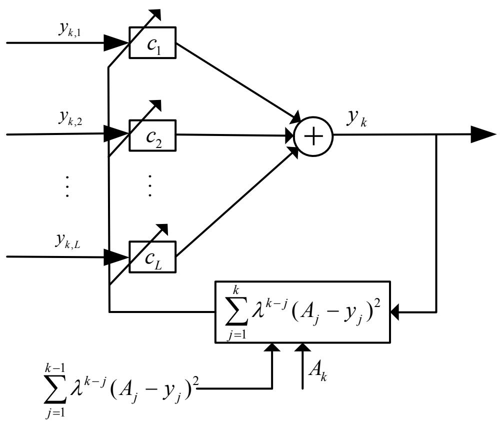

Fig. 4 Diagram of the adaptive spatio-temporal recursive least squares (ST-RLS) transversal filter

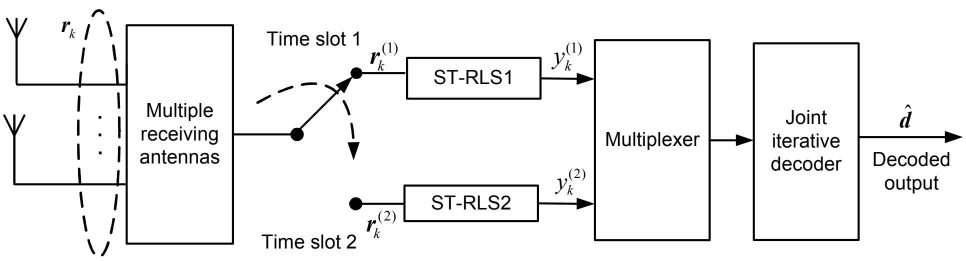

Fig. 3 Diagram of adaptive detection and the joint iterative decoder in the destination

The S-D and R-D channel models are assumed to be the same as in Section 3.2. Note that the S-D and R-D channels are multiplexed by time division to avoid mutual interference. Two ST-RLS filters perform similar detection functions for the signals received from the source and relay. Hence, the superscript 'i' can be removed in the sequel without causing any confusion. For one time slot, the jth received signal becomes

$$\boldsymbol{r}_j = \boldsymbol{h}A_j + \boldsymbol{n}_j. \tag{17}$$

Define E_j as the difference between the transmitted bit and the output of the filter:

$$E_j = A_j - \boldsymbol{c}^{\mathrm{T}}\boldsymbol{r}_j, \tag{18}$$

$$\begin{aligned}\xi_k &= \sum_{j=1}^{k}\lambda^{k-j}E_j^2 \\ &= \sum_{j=1}^{k}\lambda^{k-j}\left(|A_j|^2 - 2\mathrm{Re}\left(A_j(\boldsymbol{c}^*\overline{\boldsymbol{r}}_j)\right) + \boldsymbol{c}^*(\overline{\boldsymbol{r}}_j\boldsymbol{r}_j^{\mathrm{T}})\boldsymbol{c}\right),\end{aligned} \tag{19}$$

where λ ($0<\lambda\leq 1$) is defined as the forgetting factor. As shown in Eq. (19), the exponential weighting and λ are used to limit the effective size of E_j^2 (j=1, 2, …, k). For a fast varying system, a small λ is required to reduce the weighting corresponding to the much earlier time slots. However, for a system which is time-invariant during the transmission period, λ=1 is the optimal value. When λ=1, the criterion in Eq. (19) is the minimum mean square error (MMSE) criterion, and the adopted filter is equivalent to a spatio-temporal MMSE filter.

To minimize ξ_k based on $\boldsymbol{c}$, the gradient of ξ_k is achieved as

$$\nabla_c\xi_k = -2\sum_{j=1}^{k}\lambda^{k-j}(A_j - \boldsymbol{c}^{\mathrm{T}}\boldsymbol{r}_j)\overline{\boldsymbol{r}}_j. \tag{20}$$

Let $\nabla_c\xi_k = 0$, the minimum ξ_k can be obtained by

$$\boldsymbol{c} = \left(\sum_{j=1}^{k}\lambda^{k-j}\overline{\boldsymbol{r}}_j\boldsymbol{r}_j^{\mathrm{T}}\right)^{-1}\left(\sum_{j=1}^{k}\lambda^{k-j}A_j\overline{\boldsymbol{r}}_j\right). \tag{21}$$

Note that

$$\boldsymbol{\Phi}_k = \sum_{j=1}^{k}\lambda^{k-j}\overline{\boldsymbol{r}}_j\boldsymbol{r}_j^{\mathrm{T}}, \tag{22a}$$

$$\boldsymbol{P}_k = \sum_{j=1}^{k}\lambda^{k-j}A_j\overline{\boldsymbol{r}}_j. \tag{22b}$$

To obtain the transversal filter vector $\boldsymbol{c}$ by Eq. (21), $\boldsymbol{\Phi}_k^{-1}$ needs to be calculated. Firstly, $\boldsymbol{\Phi}_k$ is achieved by accumulating, and then the inverse matrix of $\boldsymbol{\Phi}_k$ is computed by a matrix inversion algorithm. It is very complex to directly compute $\boldsymbol{\Phi}_k^{-1}$ using a matrix inversion algorithm via this approach, when the number of receiving antennas is large. It will introduce time-delay for the channel estimation. To reduce the computational complexity, $\boldsymbol{\Phi}_k^{-1}$ is calculated by recursion without accumulation or the direct matrix inversion algorithm (Sayed and Kailath, 1994). Similarly, $\boldsymbol{P}_k$ is computed by recursion:

$$\boldsymbol{\Phi}_k^{-1} = \frac{1}{\lambda}\left(\boldsymbol{\Phi}_{k-1}^{-1} - \frac{\boldsymbol{\Phi}_{k-1}^{-1}\overline{\boldsymbol{r}}_k\boldsymbol{r}_k^{\mathrm{T}}\boldsymbol{\Phi}_{k-1}^{-1}}{\lambda + \boldsymbol{r}_k^{\mathrm{T}}\boldsymbol{\Phi}_{k-1}^{-1}\overline{\boldsymbol{r}}_k}\right), \tag{23a}$$

$$\boldsymbol{P}_k = \lambda\boldsymbol{P}_{k-1} + A_k\overline{\boldsymbol{r}}_k, \tag{23b}$$

where the starting values for the recursion are assigned as $\boldsymbol{\Phi}_k^{-1}=\delta^{-1}\boldsymbol{I}$ (δ is a small positive number), $\boldsymbol{P}_k=\boldsymbol{0}$ (Haykin, 2002).

If the length of a training sequence is K, the adaptive ST-RLS transversal filter vector $\boldsymbol{c}_K$ is obtained by Eq. (21). Hence, the input of the joint iterative decoder is

$$y^{(i)} = (\boldsymbol{c}_K^{(i)})^{\mathrm{T}}\boldsymbol{r}^{(i)}. \tag{24}$$

If i=1, the S-D channel is adaptively estimated by ST-RLS1 and $y^{(1)}$ is achieved; otherwise, i=2, the R-D channel is adaptively estimated by ST-RLS2 and $y^{(2)}$ is achieved.

4.2 Joint 'Min-Sum' iterative decoding based on the bilayer Tanner graph

Due to the absence of perfect CSI, a standard belief-propagation (BP) algorithm is not applicable to the LDPC coded cooperative MIMO over Rayleigh fading channels. Fortunately, a simplified BP-based algorithm, known as the 'Min-Sum' algorithm (Chen *et al.*, 2005), can be used, which greatly reduces the implementation complexity without much degradation in decoding performance. Moreover, it does not acquire perfect CSI.

In this paper, we adopt a joint 'Min-Sum' iterative decoding based on the bilayer Tanner graph, which is an incorporated Tanner graph associated with the double structured LDPC codes used by the source and relay. During the iterative decoding process, the extrinsic information resulting from the variable and check nodes in the bilayer Tanner graph is exchanged sufficiently in each iteration. Hence, it can accelerate the decoding convergence and achieve a better performance compared to the traditional approach, where two received signals are decoded separately.

In the bilayer Tanner graph (Fig. 2), the set $C(v_n)$ contains all the check nodes in both layers related to the variable node v_n, and $V(c_m^{(i)})$ (i=1, 2) is the set of all variable nodes in both layers associated with $c_m^{(i)}$. Let the output sequences of the adaptive transversal filters in the destination associated with the source and relay be $\boldsymbol{y}^{(1)}=(y_1, y_2, \ldots, y_N)$ and $\boldsymbol{y}^{(2)}=(y_{N+1}, y_{N+2}, \ldots, y_{N+M_2})$, respectively. Let $\boldsymbol{y}=(\mathrm{Re}(\boldsymbol{y}^{(1)}), \mathrm{Re}(\boldsymbol{y}^{(2)}))$, which is directly applied to the joint iterative decoding. The whole codeword related to $\boldsymbol{y}$ is $\boldsymbol{d}=(d_1, d_2, \ldots, d_{N+M_2})$. The codeword is modulated by binary phase shift keying (BPSK).

Define $\mathrm{Lq}_{m,n}^{(i)}$ as extrinsic information from a variable node v_n to an incident check node $c_m^{(i)}$, and $\mathrm{Lr}_{m,n}^{(i)}$ as extrinsic information from a check node $c_m^{(i)}$ to an incident variable node v_n.

Following the bilayer Tanner graph, the joint 'Min-Sum' iterative decoding algorithm is summarized as follows.

Preparations: Initially, the decoder in the destination obtains only the received signals and does not have any prior information from the check nodes. Each bit n is assigned a log-likelihood ratio (LLR):

$$\mathrm{Lp}_n = \log\frac{P(d_n=0\,|\,y_n)}{P(d_n=1\,|\,y_n)},\quad n=1, 2, \cdots, N+M_2. \quad (25)$$

Step 1 (Initialization): Before commencing the iterative decoding, $\mathrm{Lq}_{m,n}^{(i)}$ can be initialized as Lp_n in Eq. (25).

Step 2 (Horizontal process): The extrinsic information $\mathrm{Lr}_{m,n}^{(i)}$ sent from a check node $c_m^{(i)}$ to an incident variable node v_n is evaluated as

$$\mathrm{Lr}_{m,n}^{(i)} = \left(\prod_{v_{n'}\in V(c_m^{(i)})\backslash v_n} \mathrm{sign}(\mathrm{Lq}_{m,n'}^{(i)})\right)\left(\min_{v_{n'}\in V(c_m^{(i)})\backslash v_n} |\,\mathrm{Lq}_{m,n'}^{(i)}\,|\right), \quad (26)$$

where sign() and min() are the sign function and minimal function, respectively, and $V(c_m^{(i)})/v_n$ is the remaining set after $V(c_m^{(i)})$ excluding the element v_n. This results in the updated extrinsic information $\mathrm{Lr}_{m,n}^{(i)}$ from the check nodes $c_m^{(i)}$ in the first (i=1) or second (i=2) layer of the bilayer Tanner graph.

Step 3 (Vertical process): Update the extrinsic information $\mathrm{Lq}_{m,n}^{(i)}$ sent from a variable node v_n to an incident check node $c_m^{(i)}$.

1. For i=1, this implies that the extrinsic information $\mathrm{Lq}_{m,n}^{(1)}$ is sent from a variable node v_n to an incident check node in the first layer of the bilayer Tanner graph.

$$\mathrm{Lq}_{m,n}^{(1)} = \mathrm{Lp}_n + \sum_{c_k^{(1)}\in C(v_n)\backslash c_m^{(1)}} \mathrm{Lr}_{k,n}^{(1)} + \sum_{c_l^{(2)}\in C(v_n)} \mathrm{Lr}_{l,n}^{(2)}. \quad (27)$$

2. For i=2, this means that the extrinsic information $\mathrm{Lq}_{m,n}^{(2)}$ is sent from a variable node v_n to an incident check node in the second layer of the bilayer Tanner graph. Similarly,

$$\mathrm{Lq}_{m,n}^{(2)} = \mathrm{Lp}_n + \sum_{c_k^{(1)}\in C(v_n)} \mathrm{Lr}_{k,n}^{(1)} + \sum_{c_l^{(2)}\in C(v_n)\backslash c_m^{(2)}} \mathrm{Lr}_{l,n}^{(2)}. \quad (28)$$

Step 4 (Final decision): Repeat Steps 2 and 3 until the maximum number of decoding iterations is reached. The a posterior LLR associated with each codeword bit is calculated as

$$R_n = \mathrm{Lp}_n + \sum_{c_k^{(1)}\in C(v_n)} \mathrm{Lr}_{k,n}^{(1)} + \sum_{c_l^{(2)}\in C(v_n)} \mathrm{Lr}_{l,n}^{(2)}, \quad n=1, 2, \cdots, N+M_2. \quad (29)$$

Therefore, the final decoded block of $N+M_2$ bits is resulted as

$$\hat{d}_n = \begin{cases} 0, & R_n \geq 0, \\ 1, & R_n < 0, \end{cases} \quad n=1, 2, \ldots, N+M_2. \quad (30)$$

Note that for AWGN channels,

$$\mathrm{Lp}_n = \log\frac{P(d_n = 0 \mid y_n)}{P(d_n = 1 \mid y_n)} = \frac{2y_n}{N_0/2} = \frac{4}{N_0} y_n, \tag{31}$$

where y_n is the received sequences and N_0 is the variance of the complex additive noise.

For the joint 'Min-Sum' algorithm described above, the factor $4/N_0$ is a fixed positive number which can be ignored during the joint iteration. For simplicity, Lp_n in Eq. (25) can be further evaluated as

$$\mathrm{Lp}_n = y_n. \tag{32}$$

Hence, the joint 'Min-Sum' iterative decoding algorithm does not need knowledge of complex additive noise, which is also effective in fading channels scenarios.

5 Simulation results

In this section, we describe numerical simulations performed to investigate the performance of an LDPC coded ideal cooperative MIMO system over Rayleigh block fading channels. The fading coefficient for each channel remains constant over each codeword. The average received SNRs per bit per antenna of the signals from S and R are the same. BPSK modulation, adaptive ST-RLS transversal filters, and the joint 'Min-Sum' iterative decoding algorithm are adopted in the destination.

The structured LDPC codes employed in the coded cooperative MIMO and noncooperation schemes are given in Table 1. In the noncooperation scheme, only S-D links transmit data and thus it can be regarded as a single-input multiple-output (SIMO) system. For comparison under the same conditions, LDPC codes from the viewpoint of the destination equal to those of the cooperative MIMO scheme are employed.

5.1 Outage probability of the cooperative MIMO versus that of a noncooperation scheme

We present the theory and simulation results of the outage probability of the cooperative MIMO scheme. Let the data transmission rate referred to as the overall code rate $R=(N-M_1)/(N+M_2)=(1500-500)/(1500+500)=1/2$. For comparison, the code rate for the noncooperation scheme is the same as 1/2. Fig. 5 illustrates the outage probability versus SNR of the cooperative MIMO and noncooperation with one, two, or three receiving antennas. The outage probability of the cooperative MIMO is much lower than that of the noncooperation scheme with the same number of receiving antennas, showing the merits of the cooperative MIMO. When the number of receiving antennas increases, the outage probability of the cooperative MIMO decreases much faster than that of the noncooperation scheme.

Table 1 Structured LDPC codes employed in simulations

Scheme	Code
Coded cooperative MIMO	
LDPC-1	$\boldsymbol{H}_1=[\boldsymbol{A}_{500\times1000}\ \ \boldsymbol{D}_{500\times500}]$, for $\boldsymbol{A}$, $d_v=2$, $d_c=4$, rate=2/3, length=1500
LDPC-2	$\boldsymbol{H}_2=[\boldsymbol{B}_{500\times1000}\ \ \boldsymbol{D}_{500\times500}]$, for $\boldsymbol{B}$, $d_v=2$, $d_c=4$, rate=2/3, length=1500
Noncooperation LDPC	$\boldsymbol{H}=\begin{bmatrix}\boldsymbol{A}_{500\times1000} & \boldsymbol{D}_{500\times500} & \boldsymbol{0}_{500\times500}\\ \boldsymbol{B}_{500\times1000} & \boldsymbol{0}_{500\times500} & \boldsymbol{D}_{500\times500}\end{bmatrix}$, rate=1/2, length=2000

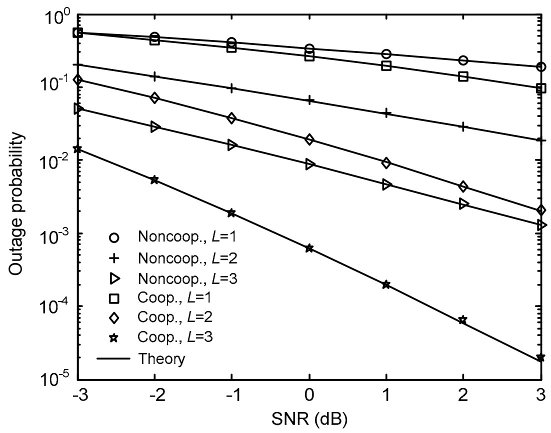

Fig. 5 Outage probability comparison of the cooperative MIMO and noncooperation schemes with one, two, or three receiving antennas in the destination
Symbols indicate the simulation values and solid lines indicate the theoretical values. L: number of receiving antennas

5.2 Bit error rate (BER) performance of the LDPC coded cooperative MIMO versus that of a noncooperation scheme

Assume perfect CSI is available in the destination.

Fig. 6 compares the BER performances of LDPC coded cooperative MIMO and coded noncooperation under the identical conditions of two or three receiving antennas and 10 decoding iterations. The results show that with the same number of receiving antennas, the LDPC coded cooperative scheme can achieve a higher diversity order compared with the coded noncooperation scheme. Also, the coded cooperative MIMO clearly outperforms the coded noncooperation scheme in terms of BER performance, in all ranges of SNR with the same number of receiving antennas. For instance, at a BER of 10^{-4} with three receiving antennas, the cooperative MIMO scheme achieves about 1.2 dB gain over the coded noncooperation scheme. The significant gain can be attributed to the fact that two detected signals from S and R, which are sent through two independent fading channels, are jointly decoded by the proposed high-efficiency joint 'Min-Sum' iterative decoding algorithm. Hence, it can dramatically overcome signal fading to achieve the diversity gain.

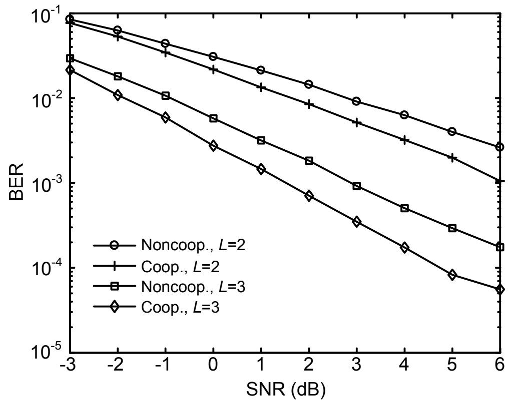

Fig. 6 BER comparison of LDPC coded cooperative MIMO and noncooperation schemes with two or three receiving antennas and perfect CSI in the destination
L: number of receiving antennas

5.3 Validity of ST-RLS algorithms in LDPC coded cooperative MIMO

In the scenario where CSI is unknown to the destination, adaptive ST-RLS transversal filters are adopted by the destination, where K=50, λ=1, $\boldsymbol{P}_0$=$\mathbf{0}$, and δ=0.01 ($\boldsymbol{P}_0$ and δ are assigned these same values in the following comparison). Fig. 7 compares the BERs of ST-RLSs and perfect CSI scenarios under identical conditions with three receiving antennas, and decoding iterations of one and 10. The BER performance gap between the ST-RLSs and the perfect CSI scenarios is narrow (Fig. 7). This indicates that the proposed adaptive ST-RLS transversal filters can accurately estimate the CSI to combine the received signals from the multiple antennas in the destination. At an SNR of 6 dB, the BER performances of ST-RLS and perfect CSI scenarios are nearly equal for the same number of decoding iterations. The gap can be further narrowed by increasing the length of the training sequence at the cost of some transmission efficiency.

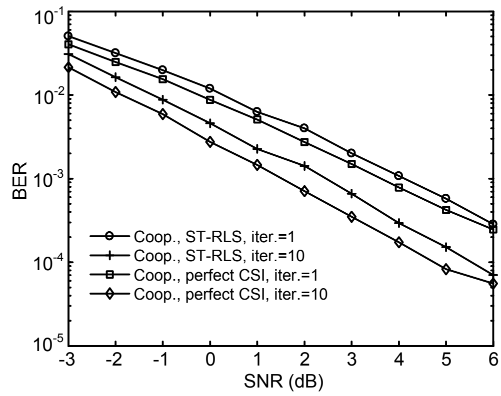

Fig. 7 BER performances of LDPC coded cooperative MIMO in ST-RLSs and perfect CSI scenarios with three receiving antennas in the destination

5.4 BER performance of LDPC coded cooperative MIMO with various receiving antennas

To study the receive diversity of multiple antennas, various receiving antennas are applied in the destination. Suppose the CSI is unavailable and ST-RLSs are employed with K=50 and λ=1. The BER performances of the cooperative MIMO are shown in Fig. 8, where the number of receiving antennas is two, three, or four. Significant diversity gains can be obtained as the number of receiving antennas increases. For instance, at a BER of 10^{-2} with 10 decoding iterations, about 2, −1, and −3 dB of SNR are required for two, three, and four receiving antennas, respectively. This is because with more receiving antennas, more receive spatial diversity gain can be achieved in the coded cooperative MIMO.

5.5 Effect of ST-RLSs parameters on the LDPC coded cooperative MIMO

The effect of the length of training sequences K and the forgetting factor λ employed by ST-RLSs in the LDPC cooperative MIMO was investigated.

Fig. 9 depicts the BER performances of the cooperative MIMO with four receiving antennas and 10 decoding iterations in the destination where ST-RLSs with various K and λ are adopted. For the same K, the smaller is λ, the worse is the BER performance. For the same λ, the larger is K, the better is the BER performance that can be achieved. However, it is known that transmission efficiency decreases with increasing K. The BER performance and the transmission efficiency need to be balanced in a practical system.

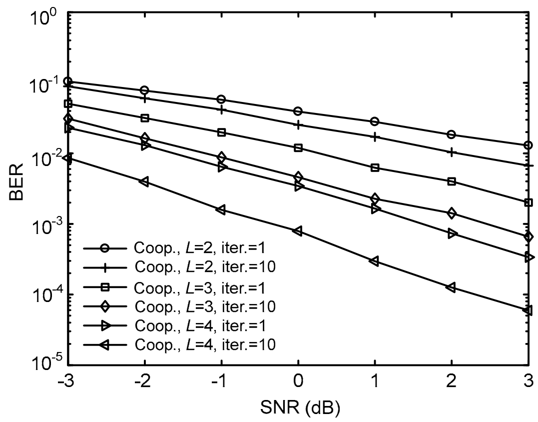

Fig. 8 BER performances of LDPC coded cooperative MIMO with two, three, and four receiving antennas and ST-RLSs in the destination

L: number of receiving antennas

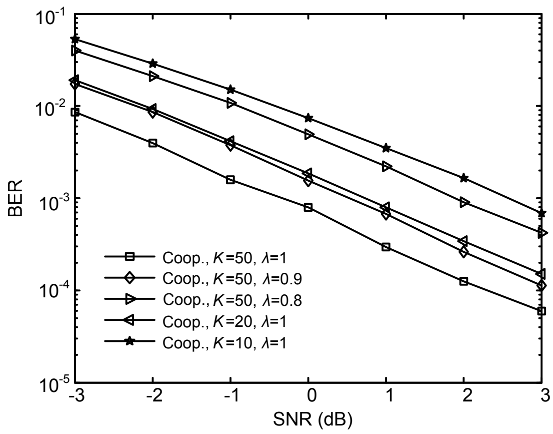

Fig. 9 BER performances of LDPC coded cooperative MIMO with four receiving antennas and 10 decoding iterations where ST-RLSs with various K and λ are employed in the destination

K: length of training sequency; λ: forgetting factor

6 Conclusions

In this paper, we have investigated a kind of LDPC coded cooperative MIMO scheme that employs structured LDPC codes belonging to a family of RA codes. On account of Rayleigh fading channels and the unavailability of CSI in the destination, adaptive ST-RLS-based transversal filters are adopted by the destination to estimate cooperative channels and realize receive diversity gain. Without knowledge of fading coefficients and complex additive noise, an efficient joint 'Min-Sum' iterative decoding algorithm is carried out following the bilayer Tanner graph. We have also studied the outage probability and BER performances of the LDPC coded cooperative MIMO scheme by theoretical analysis and numerical simulations. Further work may focus on extending the one-relay cooperation to multi-relay cooperation. It may also be interesting to study the performance of an LDPC coded non-ideal cooperative MIMO scheme, where the relay R may decode received signals incorrectly.

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Appendix: Deduction of Eq. (16)

For simplicity, let $N/(N+M_2)=a$, $M_2/(N+M_2)=b$, $\gamma^{(1)}=t_1$, $\gamma^{(2)}=t_2$, $\sum_{j=1}^{L}\left|h_j^{(1)}\right|^2=X_1$, $\sum_{j=1}^{L}\left|h_j^{(2)}\right|^2=X_2$, $Y_1=a\log_2(1+t_1X_1)$, $Y_2=b\log_2(1+t_2X_2)$, and $Z=Y_1+Y_2$.

We have the probability density function (PDF) of X_1 as

$$f_{X_1}(x)=\begin{cases}\dfrac{x^{L-1}}{(L-1)!}\mathrm{e}^{-x}, & x>0,\\ 0, & x\le 0.\end{cases} \tag{A1}$$

Y_1 is a monotonic increasing function of X_1. The PDF of Y_1 can be deduced by

$$f_{Y_1}(y_1)=\begin{cases}f_{X_1}\left(h(y_1)\right)\cdot\left|h'(y_1)\right|, & y_1>0,\\ 0, & y_1\le 0,\end{cases} \tag{A2}$$

where $x_1=h(y_1)$ is the inverse function of $y_1=g(x_1)$ and $|h'(y_1)|$ is the absolute value of derived function $h'(y_1)$. This gives

$$f_{Y_1}(y_1)=\begin{cases}\dfrac{\ln 2}{at_1(L-1)!}2^{\frac{y_1}{a}}\mathrm{e}^{\frac{-(2^{\frac{y_1}{a}}-1)}{t_1}}\left(\dfrac{2^{\frac{y_1}{a}}-1}{t_1}\right)^{L-1}, & y_1>0,\\ 0, & y_1\le 0.\end{cases} \tag{A3}$$

Similarly,

$$f_{Y_2}(y_2)=\begin{cases}\dfrac{\ln 2}{bt_2(L-1)!}2^{\frac{y_2}{b}}\mathrm{e}^{\frac{-(2^{\frac{y_2}{b}}-1)}{t_2}}\left(\dfrac{2^{\frac{y_2}{b}}-1}{t_2}\right)^{L-1}, & y_2>0,\\ 0, & y_2\le 0.\end{cases}\tag{A4}$$

If Y_1 and Y_2 are independent, the PDF of Z can be deduced as Eq. (A5).

If $t_1=t_2=t$, Eq. (A5) can be further rewritten as Eq. (A6).

Hence, for $\gamma^{(1)}=\gamma^{(2)}=t$, $\Pr_{\text{coop}}$ can be calculated using Eq. (A7). Note that the definite integral in Eq. (A7) can be calculated by the function 'int' in MATLAB.

$$\begin{aligned}f_Z(z)&=f_{Y_1}*f_{Y_2}(z)=\int_0^z f_{Y_1}(y_1)\cdot f_{Y_2}(z-y_1)\mathrm{d}y_1\\&=\begin{cases}\displaystyle\int_0^z\frac{1}{abt_1t_2}\left(\frac{\ln 2}{(L-1)!}\right)^2 2^{\frac{y_1}{a}}\mathrm{e}^{\frac{-(2^{\frac{y_1}{a}}-1)}{t_1}}\left(\frac{2^{\frac{y_1}{a}}-1}{t_1}\right)^{L-1}\left(2^{\frac{z-y_1}{b}}\mathrm{e}^{\frac{-(2^{\frac{z-y_1}{b}}-1)}{t_2}}\left(\frac{2^{\frac{z-y_1}{b}}-1}{t_2}\right)^{L-1}\right)\mathrm{d}y_1, & z>0,\\ 0, & z\le 0.\end{cases}\end{aligned}\tag{A5}$$

$$f_Z(z)=\begin{cases}\displaystyle\frac{1}{ab}\left(\frac{\ln 2}{t(L-1)!}\right)^2\int_0^z 2^{\left(\frac{y_1}{a}+\frac{z-y_1}{b}\right)}\mathrm{e}^{\frac{-1}{t}\left(2^{\frac{y_1}{a}}+2^{\frac{z-y_1}{b}}-2\right)}\left(\frac{(2^{\frac{y_1}{a}}-1)(2^{\frac{z-y_1}{b}}-1)}{t^2}\right)^{L-1}\mathrm{d}y_1, & z>0,\\ 0, & z\le 0.\end{cases}\tag{A6}$$

$$\begin{aligned}\Pr(Z<R)&=\int_{-\infty}^R f_Z(z)\mathrm{d}z\\&=\frac{1}{ab}\left(\frac{\ln 2}{t(L-1)!}\right)^2\int_0^R\int_0^z 2^{\left(\frac{y_1}{a}+\frac{z-y_1}{b}\right)}\mathrm{e}^{\frac{-1}{t}\left(2^{\frac{y_1}{a}}+2^{\frac{z-y_1}{b}}-2\right)}\left(\frac{(2^{\frac{y_1}{a}}-1)(2^{\frac{z-y_1}{b}}-1)}{t^2}\right)^{L-1}\mathrm{d}y_1\mathrm{d}z.\end{aligned}\tag{A7}$$