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A multi-functional dynamic state estimator for error validation: measurement and parameter errors and sudden load changes

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Abstract: We propose a new and efficient algorithm to detect, identify, and correct measurement errors and branch parameter errors of power systems. A dynamic state estimation algorithm is used based on the Kalman filter theory. The proposed algorithm also successfully detects and identifies sudden load changes in power systems. The method uses three normalized vectors to process errors at each sampling time: normalized measurement residual, normalized Lagrange multiplier, and normalized innovation vector. An IEEE 14-bus test system was used to verify and demonstrate the effectiveness of the proposed method. Numerical results are presented and discussed to show the accuracy of the method.

Key words: Dynamic state estimation, Kalman filter, Measurement errors, Branch parameter errors, Sudden load changes
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1 Introduction

State estimation (SE) is one of the fundamental functions of an Energy Management System (EMS). It estimates the bus voltage magnitudes and phase angles as state variables (Abur and Exposito, 2004; Bao *et al*., 2015). Electric power systems often show a dynamic performance and the state variables (or state vectors) are frequently varying. Hence, to achieve continuous monitoring of a power system, SE must be performed at short time intervals (Huang and Shih, 2002). Static SE cannot correctly follow the dynamic behavior of power systems. The concept of dynamic state estimation (DSE) has been introduced to solve this problem. DSE uses Kalman filtering techniques to improve the computational performance of the traditional estimation process in electric power systems (Debs and Larson, 1970; Gui *et al*., 2015; Karimipour and Dinavahi, 2015; Risso *et al*., 2015; Sharma *et al*., 2015; Tebianian and Jeyasurya, 2015). DSE not only estimates the current value of the state vector at time step k, but also provides a prediction of the state vector in the next sampling time k+1. This ability leads to many advantages in security analysis and provides more time to take control actions (Gu and Jirutitijaroen, 2015; Hu *et al*., 2015).

In recent years, there have been many studies on DSE. Prasad and Thakur (1998) proposed a new DSE method based on Kalman filter (KF) to increase computational accuracy. Valverde and Terzija (2011), Wang *et al.* (2012), and Qing *et al.* (2015) combined unscented transformation with KF theory to solve DSE. Shih and Huang (2002) described a robust DSE algorithm which formulates the absolute residual vector as the exponential weighted function, such that anomalous conditions can be taken into account. Application of enhanced fuzzy control to improve the estimation performance was also investigated (Lin *et al*., 2003). Qiu *et al.* (2013a) investigated the problem of robust H_∞ SE for a class of continuous-time nonlinear systems via Takagi-Sugeno (T-S) fuzzy affine dynamic models. Qiu *et al.* (2013b) addressed the problem of robust H_∞ output feedback control with parametric uncertainties and input constraints.

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Artificial neural networks have also been used to forecast state vector components (Glazunova, 2010).

It has been shown that errors in the measurements and branch parameters, as well as the sudden load changes, can have adverse impacts on SE results (Falcao *et al*., 1982; Silva *et al*., 1987; Zhu and Abur, 2010). Sudden changes in the power system operating conditions rarely occur, and are due mainly to predictable events such as the disconnection of a large industrial consumer or component outages.

From a practical point of view, these errors may exist in power systems at the same time. Obviously, the performance of SE depends on the accuracy of the measurements and branch parameters. To obtain a reliable SE, the simultaneous detection, identification, and estimation of measurement and branch parameter errors with high accuracy are challenging tasks.

We propose a DSE based on KF theory for the error processing problem. The proposed method can identify and correct errors in the measurements and branch parameters simultaneously, and take account of sudden changes in the loads by using Lagrangian multiplier theory. Note that the identification of sudden changes in a power system is impossible by normalized residual tests, and an innovative test needs to be proposed for this problem. Our proposed method uses three normalized vectors for error processing at each sampling time: normalized measurement residual, normalized Lagrange multiplier, and normalized innovation vectors. Furthermore, a new linear approximation approach is proposed to estimate and correct erroneous measurements and branch parameters at each sampling time.

Also, it is possible to extend the results from this study to the underlying systems under a network-based environment with time delays, packet dropouts, and quantization. An input-output (IO) approach is proposed for the delay-dependent stability analysis and H_∞ controller synthesis for a class of continuous time Markovian jump linear systems (MJLSs) (Qiu *et al*., 2015).

2 Dynamic state estimation based on Kalman filter

DSE uses the present and sometimes the previous states of a power system in addition to knowledge of the system's physical model to predict the state vector for the next step time. The following steps should be included in DSE (Filho and Souza, 2009; Filho *et al*., 2009).

2.1 Modeling

Different models for slow dynamic systems have been reported (Silva *et al*., 1987; Risso, 2015, Tebianian and Jeyasurya, 2015). To establish such a model, some important considerations of power system operations are usually assumed:

1. The timeframe of interest is small, and of the order of a few minutes;

2. A linear function properly represents the transition trajectory between consecutive states;

3. Control variables are not included, since their effects are much larger than the adopted time steps.

However, since the state trajectory is usually divided into small time intervals, a dynamic system can be represented by the following linear dynamic model to describe the system time evolution:

$$\begin{cases} \boldsymbol{x}_{k+1} = \boldsymbol{F}_k \boldsymbol{x}_k + \boldsymbol{g}_k + \boldsymbol{w}_k, \\ \boldsymbol{z}_k = \boldsymbol{h}_k(\boldsymbol{x}_k) + \boldsymbol{e}_k, \end{cases} \tag{1}$$

where $\boldsymbol{x}_k$ is an $n\times1$ dimensional state vector with voltage magnitudes and phase angles in all buses, $\boldsymbol{F}_k$ is a transition matrix representing the state transition between two instants of time, $\boldsymbol{g}_k$ is a vector associated with the trend behavior of the state trajectory, $\boldsymbol{w}_k$ is a modeling uncertainties vector, with a Gaussian noise with zero mean and covariance matrix $\boldsymbol{Q}_k$, $\boldsymbol{z}_k$ is an $m\times1$ dimensional measurement vector, $\boldsymbol{h}_k$ is a nonlinear function relating the measurements to the state variables, $\boldsymbol{e}_k$ is a measurement error vector, with a Gaussian noise with zero mean and diagonal covariance matrix $\boldsymbol{R}_k$, and k is a time sample.

Uncertainties $\boldsymbol{w}_k$ and $\boldsymbol{e}_k$ are assumed to be uncorrelated. The state vector components are the phase angles and magnitudes of the bus voltages.

One of the most common approaches for online calculation and adjustment of $\boldsymbol{F}_k$ and $\boldsymbol{g}_k$ is Holt's two-parameter exponential smoothing technique (Silva and Filho, 1983). In this method, elements of $\boldsymbol{Q}_k$ remain approximately constant or are estimated offline (Filho *et al*., 1989).

Based on Holt's exponential smoothing technique, $\boldsymbol{F}_k$ and $\boldsymbol{g}_k$ are updated based on the following equation:

$$\begin{cases} \boldsymbol{F}_k = \alpha_k(1+\beta_k)\boldsymbol{I}, \\ \boldsymbol{g}_k = (1+\beta_k)(1-\alpha_k)\tilde{\boldsymbol{x}}_k - \beta_k \boldsymbol{a}_{k-1} + (1-\beta_k)\boldsymbol{b}_{k-1}, \end{cases} \tag{2}$$

where $\boldsymbol{I}$ is the identity matrix, α_k and β_k are parameters lying in the range from 0 to 1, $\hat{\boldsymbol{x}}_{k+1}$ and $\tilde{\boldsymbol{x}}_k$ is the estimated and predicted state vector at time k, respectively, and vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ at time k are obtained by

$$\begin{cases} \boldsymbol{a}_k = \alpha_k \hat{\boldsymbol{x}}_k + (1-\alpha_k)\tilde{\boldsymbol{x}}_k, \\ \boldsymbol{b}_k = \beta_k(\boldsymbol{a}_k - \boldsymbol{a}_{k-1}) + (1-\beta_k)\boldsymbol{b}_{k-1}. \end{cases} \tag{3}$$

2.2 Forecasting

In the forecasting step, the forecasted state vector in the next time sample ($\tilde{\boldsymbol{x}}_{k+1}$) is obtained using the estimated state vector in time sample k. $\tilde{\boldsymbol{x}}_{k+1}$ and its error covariance matrix $\boldsymbol{M}_{k+1}$ are given by

$$\begin{cases} \tilde{\boldsymbol{x}}_{k+1} = \boldsymbol{F}_k \hat{\boldsymbol{x}}_k + \boldsymbol{g}_k, \\ \boldsymbol{M}_{k+1} = \boldsymbol{F}_k \boldsymbol{P}_k \boldsymbol{F}_k^{\mathrm{T}} + \boldsymbol{Q}_k. \end{cases} \tag{4}$$

Therefore, the forecasted measurement vector $\tilde{\boldsymbol{z}}_{k+1}$ and its error covariance matrix $\boldsymbol{T}_{k+1}$ can be obtained by

$$\begin{cases} \tilde{\boldsymbol{z}}_{k+1} = \boldsymbol{h}_{k+1}(\tilde{\boldsymbol{x}}_{k+1}), \\ \boldsymbol{T}_{k+1} = \boldsymbol{H}_{k+1}\boldsymbol{M}_{k+1}\boldsymbol{H}_{k+1}^{\mathrm{T}}, \end{cases} \tag{5}$$

where $\boldsymbol{H}_{k+1} = \partial \boldsymbol{h}_{k+1} / \partial \boldsymbol{x}\,|_{\boldsymbol{x}=\tilde{\boldsymbol{x}}_{k+1}}$ is the measurement Jacobian matrix. The standard deviation of the error for the ith forecasted measurement is expressed by $\sigma_T(i) = \sqrt{T_{k+1}(i,i)}$.

2.3 Innovation analysis

By using dynamic estimators in power system monitoring, it is possible to determine in advance if a set of gathered information that will be processed by the estimator contains anomalies. This is achieved by exploiting the forecasting capability of the dynamic estimators.

Let $\boldsymbol{v}_{k+1}$ be the innovation vector at time sample k+1. The ith component of this vector is defined as the difference between the received $z_{k+1}(i)$ and forecasted $\tilde{z}_{k+1}(i)$ measurements:

$$v_{k+1}(i) = z_{k+1}(i) - \tilde{z}_{k+1}(i), \tag{6}$$

where the innovation vector $\boldsymbol{v}_{k+1}$ is a Gaussian vector with zero mean and covariance matrix $\boldsymbol{N}_{k+1}$, stated as follows:

$$\boldsymbol{N}_{k+1} = \boldsymbol{R}_{k+1} + \boldsymbol{H}_{k+1}\boldsymbol{M}_{k+1}\boldsymbol{H}_{k+1}^{\mathrm{T}} = \boldsymbol{R}_{k+1} + \boldsymbol{T}_{k+1}. \tag{7}$$

Thus, the innovation vector $\boldsymbol{v}_{k+1}$ can be normalized as follows:

$$v_{k+1}^N(i) = \frac{|v_{k+1}(i)|}{\sqrt{\sigma_N(i)}}, \tag{8}$$

where the error standard deviation of the ith innovation $\sigma_N(i)$ is expressed by $\sigma_N(i) = \sqrt{N_{k+1}(i,i)}$.

If at least one innovation exceeds the threshold, then an error is detected and should be identified.

2.4 Filtering

In the filtering step, the estimated state vector in the next time sample ($\hat{\boldsymbol{x}}_{k+1}$) is obtained using the forecasted state vector in current sample ($\tilde{\boldsymbol{x}}_{k+1}$), calculated as

$$\hat{\boldsymbol{x}}_{k+1} = \tilde{\boldsymbol{x}}_{k+1} + \boldsymbol{K}_{k+1}\boldsymbol{v}_{k+1}, \tag{9}$$

which leads to the following calculation process:

$$\begin{cases} \boldsymbol{K}_{k+1} = \boldsymbol{P}_{k+1}\boldsymbol{H}_{k+1}^{\mathrm{T}}\boldsymbol{R}_{k+1}^{-1}, \\ \boldsymbol{P}_{k+1} = (\boldsymbol{H}_{k+1}^{\mathrm{T}}\boldsymbol{R}_{k+1}^{-1}\boldsymbol{H}_{k+1})^{-1}, \\ \boldsymbol{v}_{k+1} = \boldsymbol{z}_{k+1} - \boldsymbol{h}_{k+1}(\tilde{\boldsymbol{x}}_{k+1}). \end{cases} \tag{10}$$

2.5 Residual analysis

The residual vector $\boldsymbol{r}_{k+1}$ is defined as the difference between the received and estimated measurement vectors (stated as $\boldsymbol{z}_{k+1}$ and $\hat{\boldsymbol{z}}_{k+1}$, respectively):

$$\boldsymbol{r}_{k+1} = \boldsymbol{z}_{k+1} - \hat{\boldsymbol{z}}_{k+1}. \tag{11}$$

$\boldsymbol{r}_{k+1}$ is a Gaussian vector with zero mean and covariance matrix $\boldsymbol{\Omega}_{k+1}$, which is calculated as follows:

$$\boldsymbol{\Omega}_{k+1} = \boldsymbol{R}_{k+1} - \boldsymbol{H}_{k+1}\boldsymbol{P}_{k+1}\boldsymbol{H}_{k+1}^{\mathrm{T}}. \tag{12}$$

Thus, $\boldsymbol{r}_{k+1}$ can be normalized as follows:

$$r_{k+1}^{N}(i)=\frac{|r_{k+1}(i)|}{\sqrt{\sigma_{\Omega}(i)}}, \tag{13}$$

where $\sigma_{\Omega}(i)=\sqrt{\boldsymbol{\Omega}_{k+1}(i,i)}$ is the standard deviation of the ith component of vector $\boldsymbol{r}_{k+1}$.

If at least one residual exceeds the threshold, an error is detected and should be identified.

3 Proposed algorithm

The proposed algorithm employs Lagrange multiplier analysis to identify and correct errors in branch parameters. In this section, the proposed Lagrange multiplier analysis is presented. Then the flowchart of the proposed algorithm is explained in detail.

3.1 Proposed algorithm using Lagrange multiplier analysis

The following equations are given at sampling time k+1. The mathematical model which relates the measurements to the state variables and the branch parameter errors can be formulated as follows:

$$\boldsymbol{z}_{k+1}=\boldsymbol{h}_{k+1}(\boldsymbol{x}_{k+1},\boldsymbol{P}_{\text{error}})+\boldsymbol{e}_{k+1}, \tag{14}$$

where $\boldsymbol{P}_{\text{error}}$ specifies the vector of power system branch parameter errors. The branch parameter vector is modeled as

$$\boldsymbol{P}=\boldsymbol{P}_{\text{true}}+\boldsymbol{P}_{\text{error}}, \tag{15}$$

where $\boldsymbol{P}_{\text{true}}$ and $\boldsymbol{P}_{\text{error}}$ signify the true and incorrect branch parameter vectors, respectively.

If there are no errors in the branch parameters, the error vector of the power system parameters, $\boldsymbol{P}_{\text{error}}$, will be zero. Therefore, the conventional weighted least squares (WLS) SE approach in the presence of parameter errors can be formulated as an optimization problem:

$$\begin{aligned}\min \boldsymbol{J}(\boldsymbol{x}_{k+1})=&[\boldsymbol{z}_{k+1}-\boldsymbol{h}_{k+1}(\boldsymbol{x}_{k+1},\boldsymbol{P}_{\text{error}})]^{\mathrm{T}}\\ &\cdot\boldsymbol{R}_{k+1}^{-1}[\boldsymbol{z}_{k+1}-\boldsymbol{h}_{k+1}(\boldsymbol{x}_{k+1},\boldsymbol{P}_{\text{error}})]\\ &\text{s.t.}\quad \boldsymbol{P}_{\text{error}}=\boldsymbol{0}.\end{aligned} \tag{16}$$

If the analysis of Lagrange multipliers is applied to solve this problem, the objective function $\boldsymbol{J}(\boldsymbol{x}_{k+1})$ can be written as follows:

$$\begin{aligned}\mathrm{L}(\boldsymbol{x}_{k+1},\boldsymbol{P}_{\text{error}},\boldsymbol{\lambda}_{k+1})=&[\boldsymbol{z}_{k+1}-\boldsymbol{h}_{k+1}(\boldsymbol{x}_{k+1},\boldsymbol{P}_{\text{error}})]^{\mathrm{T}}\\ &\cdot\boldsymbol{R}_{k+1}^{-1}[\boldsymbol{z}_{k+1}-\boldsymbol{h}_{k+1}(\boldsymbol{x}_{k+1},\boldsymbol{P}_{\text{error}})]-\boldsymbol{\lambda}_{k+1}^{\mathrm{T}}\boldsymbol{P}_{\text{error}},\end{aligned} \tag{17}$$

which can be solved using the Krush-Kuhn-Tucker (KKT) first-order optimality conditions. The resulting equations are

$$\begin{aligned}&\frac{\partial\,\mathrm{L}(\boldsymbol{x}_{k+1},\boldsymbol{P}_{\text{error}},\boldsymbol{\lambda}_{k+1})}{\partial\boldsymbol{x}_{k+1}}\\ &=\boldsymbol{H}_{k+1}^{\mathrm{T}}\boldsymbol{R}_{k+1}^{-1}[\boldsymbol{z}_{k+1}-\boldsymbol{h}_{k+1}(\boldsymbol{x}_{k+1},\boldsymbol{P}_{\text{error}})]=\boldsymbol{0},\end{aligned} \tag{18}$$

$$\begin{aligned}&\frac{\partial\,\mathrm{L}(\boldsymbol{x}_{k+1},\boldsymbol{P}_{\text{error}},\boldsymbol{\lambda}_{k+1})}{\partial\boldsymbol{P}_{\text{error}}}=\boldsymbol{H}_{\mathrm{p},k+1}{}^{\mathrm{T}}\boldsymbol{R}_{k+1}^{-1}\\ &\cdot[\boldsymbol{z}_{k+1}-\boldsymbol{h}_{k+1}(\boldsymbol{x}_{k+1},\boldsymbol{P}_{\text{error}})]+\boldsymbol{\lambda}_{k+1}=\boldsymbol{0},\end{aligned} \tag{19}$$

$$\frac{\partial\,\mathrm{L}(\boldsymbol{x}_{k+1},\boldsymbol{P}_{\text{error}},\boldsymbol{\lambda}_{k+1})}{\partial\boldsymbol{\lambda}_{k+1}}=\boldsymbol{P}_{\text{error}}=\boldsymbol{0}, \tag{20}$$

where $\boldsymbol{H}_{\mathrm{p},k+1}=\partial\boldsymbol{h}_{k+1}/\partial\boldsymbol{P}_{\text{error}}\big|_{\boldsymbol{x}=\hat{\boldsymbol{x}}_{k+1}}$ and $\boldsymbol{\lambda}_{k+1}$ are the Jacobian matrix of parameters and Lagrange multiplier vector for the parameter errors, respectively. Note that $\boldsymbol{\lambda}_{k+1}$ can be now expressed in terms of $\boldsymbol{r}_{k+1}$, stated as follows:

$$\boldsymbol{\lambda}_{k+1}=\boldsymbol{S}_{\mathrm{p},k+1}\boldsymbol{r}_{k+1}, \tag{21}$$

where $\boldsymbol{S}_{\mathrm{p},k+1}=-\boldsymbol{H}_{\mathrm{p},k+1}^{\mathrm{T}}\cdot\boldsymbol{R}_{k+1}^{-1}$ is the parameter sensitivity matrix, $\boldsymbol{r}_{k+1}=\boldsymbol{z}_{k+1}-\boldsymbol{h}_{k+1}(\boldsymbol{x}_{k+1},\boldsymbol{P}_{\text{error}})$, and $\boldsymbol{r}_{k+1}$ represents the measurement residual vector.

It is assumed that all Lagrange multipliers are distributed according to a normal distribution with zero mean value and a non-zero covariance. The covariance matrix $\boldsymbol{\Lambda}_{k+1}$ can be derived from the relationship between Lagrange multipliers and measurement residuals:

$$\begin{cases} \boldsymbol{\Lambda}_{k+1} = \operatorname{cov}(\boldsymbol{\lambda}_{k+1}) = \boldsymbol{S}_{\mathrm{p},k+1} \operatorname{cov}(\boldsymbol{r}_{k+1}) \boldsymbol{S}_{\mathrm{p},k+1}{}^{\mathrm{T}} \\ \qquad = \boldsymbol{S}_{\mathrm{p},k+1} \boldsymbol{\Omega}_{k+1} \boldsymbol{S}_{\mathrm{p},k+1}{}^{\mathrm{T}}, \\ \boldsymbol{\Omega}_{k+1} = \boldsymbol{I} - \boldsymbol{H}_{k+1} (\boldsymbol{H}_{k+1}^{\mathrm{T}} \boldsymbol{R}_{k+1}^{-1} \boldsymbol{H}_{k+1})^{-1} \boldsymbol{H}_{k+1}^{\mathrm{T}} \boldsymbol{R}_{k+1}^{-1}. \end{cases} \tag{22}$$

Thus, the Lagrange multiplier vector can be normalized as follows:

$$\lambda_{k+1}^{N}(i) = \frac{|\lambda_{k+1}(i)|}{\sqrt{\sigma_{\Lambda}(i)}}, \quad i = 1, 2, \ldots, n_{\mathrm{p}}, \tag{23}$$

where n_{p} is the total number of power system branch parameters, and $\sigma_{\Lambda}(i) = \sqrt{\Lambda_{k+1}(i,i)}$ is the standard deviation of the ith component of vector $\boldsymbol{r}_{k+1}$. When at least one Lagrange multiplier exceeds the threshold, the presence of a branch parameter error is identified, and should be corrected.

Erroneous measurements and branch parameters are corrected using a proposed new linear approximation approach, explained in detail as follows.

The measurements are related to the state variables and branch parameters by

$$\boldsymbol{z}_{k+1} = \boldsymbol{h}_{k+1}(\boldsymbol{x}_{k+1}, \boldsymbol{P}) + \boldsymbol{e}_{k+1}, \tag{24}$$

which can be rewritten as

$$\begin{aligned} \boldsymbol{z}_{k+1} &= \boldsymbol{h}_{k+1}(\boldsymbol{x}_{k+1}, \boldsymbol{P}_{\mathrm{true}}) \\ &\quad + [\boldsymbol{h}_{k+1}(\boldsymbol{x}_{k+1}, \boldsymbol{P}) - \boldsymbol{h}_{k+1}(\boldsymbol{x}_{k+1}, \boldsymbol{P}_{\mathrm{true}})] + \boldsymbol{e}_{k+1}, \end{aligned} \tag{25}$$

where the term in the square brackets can be linearized as

$$\begin{aligned} &\boldsymbol{h}_{k+1}(\boldsymbol{x}_{k+1}, \boldsymbol{P}) - \boldsymbol{h}_{k+1}(\boldsymbol{x}_{k+1}, \boldsymbol{P}_{\mathrm{true}}) \\ &\simeq \frac{\partial \boldsymbol{h}_{k+1}(\boldsymbol{x}_{k+1}, \boldsymbol{P})}{\partial \boldsymbol{P}} \boldsymbol{e}_{\mathrm{p},k+1} = \boldsymbol{H}_{\mathrm{p},k+1} \boldsymbol{e}_{\mathrm{p},k+1}, \end{aligned} \tag{26}$$

where $\boldsymbol{e}_{\mathrm{p},k+1}$ is the vector of branch parameter errors in time sample k+1, considering a random Gaussian variable with zero mean value and covariance matrix $\boldsymbol{R}_{\mathrm{p}}$.

By combining Eqs. (25) and (26), a linear relationship can be established between the residual measurement vector $\boldsymbol{r}_{k+1}$ and the parameter errors vector $\boldsymbol{e}_{\mathrm{p},k+1}$:

$$\boldsymbol{r}_{k+1} = \boldsymbol{z}_{k+1} - \boldsymbol{h}_{k+1}(\hat{\boldsymbol{x}}_{k+1}, \boldsymbol{P}_{\mathrm{true}}) = \boldsymbol{H}_{\mathrm{p},k+1} \boldsymbol{e}_{\mathrm{p},k+1}. \tag{27}$$

By using Eqs. (21) and (27), parameter error vector $\boldsymbol{e}_{\mathrm{p}}$ can be written as follows:

$$\boldsymbol{e}_{\mathrm{p},k+1} = (\boldsymbol{S}_{\mathrm{p},k+1} \boldsymbol{H}_{\mathrm{p},k+1})^{-1} \boldsymbol{\lambda}_{k+1} = \boldsymbol{G}_{\mathrm{p},k+1}^{-1} \boldsymbol{\lambda}_{k+1}, \tag{28}$$

where $\boldsymbol{G}_{\mathrm{p},k+1}$ is the parameter gain matrix ($n_{\mathrm{p}} \times n_{\mathrm{p}}$). Suppose that the ith branch parameter is identified | as erroneous. Then it can be denoted that $\lambda_{k+1}^{\mathrm{bad}}(i) = G_{\mathrm{p},k+1}(i,i) e_{\mathrm{p},k+1}(i)$.

The covariance matrix of $\boldsymbol{\lambda}_{k+1}$ can be obtained as follows:

$$\begin{aligned} \boldsymbol{\Lambda}_{k+1} &= \operatorname{cov}(\boldsymbol{\lambda}_{k+1}) = \operatorname{cov}[(\boldsymbol{G}_{\mathrm{p},k+1} \boldsymbol{e}_{\mathrm{p},k+1})(\boldsymbol{G}_{\mathrm{p},k+1} \boldsymbol{e}_{\mathrm{p},k+1})^{\mathrm{T}}] \\ &= \boldsymbol{G}_{\mathrm{p},k+1} \operatorname{cov}[\boldsymbol{e}_{\mathrm{p},k+1} \boldsymbol{e}_{\mathrm{p},k+1}^{\mathrm{T}}] \boldsymbol{G}_{\mathrm{p},k+1}{}^{\mathrm{T}} \\ &= \boldsymbol{G}_{\mathrm{p},k+1} \boldsymbol{R}_{\mathrm{p},k+1} \boldsymbol{G}_{\mathrm{p},k+1}{}^{\mathrm{T}} = \boldsymbol{G}_{\mathrm{p},k+1} \boldsymbol{R}_{\mathrm{p},k+1}. \end{aligned} \tag{29}$$

Consequently, the parameter error vector $\boldsymbol{e}_{\mathrm{p},k+1}$ for the ith branch parameter in Eq. (28) can be written as follows:

$$e_{\mathrm{p},k+1}(i) = \frac{\lambda_{k+1}^{\mathrm{bad}}(i)}{G_{\mathrm{p},k+1}(i,i)} = \frac{R_{\mathrm{p},k+1}(i,i)}{\Lambda_{k+1}(i,i)} \cdot \lambda_{k+1}^{\mathrm{bad}}(i). \tag{30}$$

So, the actual branch parameter value can be estimated as

$$\begin{aligned} p_{k+1}^{\mathrm{correct}}(i) &= p_{k+1}^{\mathrm{bad}}(i) - e_{\mathrm{p},k+1}(i) \\ &= p_{k+1}^{\mathrm{bad}}(i) - \frac{R_{\mathrm{p},k+1}(i,i)}{\Lambda_{k+1}(i,i)} \cdot \lambda_{k+1}^{\mathrm{bad}}(i), \end{aligned} \tag{31}$$

where $p_{k+1}^{\mathrm{bad}}(i)$ and $p_{k+1}^{\mathrm{correct}}(i)$ are the erroneous value and the estimated (corrected) value, respectively, of the identified branch parameter in time sample k+1. Also, $R_{\mathrm{p},k+1}(i,i)$ is the ith diagonal element of $\boldsymbol{R}_{\mathrm{p}}$.

The redundancy index is an important index of the accuracy of SE results in power systems. All parameter error detection and identification approaches need high redundancy. If a system has many measurement errors, removing erroneous measurements will reduce the redundancy and observation ability of SE. In this study, not only the erroneous measurements are not deleted, but their corrected values are estimated by a corrective method. A similar corrective equation for parameter errors can be used to calculate measurement errors as follows:

$$\begin{cases} z_{k+1}^{\text{correct}}(i) = z_{k+1}^{\text{bad}}(i) - \dfrac{R_{k+1}(i,i)}{\Omega_{k+1}(i,i)} \cdot r_{k+1}^{\text{bad}}(i), \\ r_{k+1}^{\text{bad}}(i) = z_{k+1}^{\text{bad}}(i) - h(\hat{x}_{k+1}^{\text{bad}}(i)). \end{cases} \tag{32}$$

3.2 Flowchart of the proposed algorithm

The proposed DSE algorithm applies the normalized residual, normalized innovation, and normalized Lagrange multiplier vectors for error processing. The first step in error processing is error detection (measurement errors, branch parameter errors, and sudden load changes) using these normalized vectors. Variations in the maximum elements of these vectors are presented in Table 1, in which c is the threshold for error processing.

Table 1 shows that, for all types of error, $\max\{\boldsymbol{v}_{k+1}^N\} > c$. So, this normalized vector can be used as the first processor in error detection. If $\max\{\boldsymbol{v}_{k+1}^N\} < c$, then no errors are detected in the power system. Otherwise, one of the three errors has occurred. In sudden load change error, each of the two normalized vectors $\boldsymbol{r}_{k+1}$ and $\boldsymbol{\lambda}_{k+1}$ is lower than the threshold. However, these vectors are higher than the threshold when measurement or branch parameter errors occur. If there are measurement errors, $\max\{\boldsymbol{r}_{k+1}^N\} > \max\{\boldsymbol{\lambda}_{k+1}^N\}$; if there are parameter errors, $\max\{\boldsymbol{r}_{k+1}^N\} < \max\{\boldsymbol{\lambda}_{k+1}^N\}$.

The above results can be used to develop an algorithm to detect, identify, and correct the errors in a power system. The flowchart of our proposed algorithm is shown in Fig. 1. Note that the DSE process will start from sampling time k+1 in Fig. 1.

3.3 Performance indices of dynamic state estimation

The performance indices (Qing *et al.*, 2015) at different stages are described as follows:

Prediction index: At the state prediction stage, the following calculated formula is employed to evaluate performance:

$$\text{Index}_k^{\text{pre}} = \left| \frac{\tilde{\boldsymbol{x}}_k - \boldsymbol{x}_k^{\text{true}}}{\boldsymbol{x}_k^{\text{true}}} \right| \times 100\%. \tag{33}$$

Estimation index: At the state filtering stage, the following formula is applied to assess the performance index:

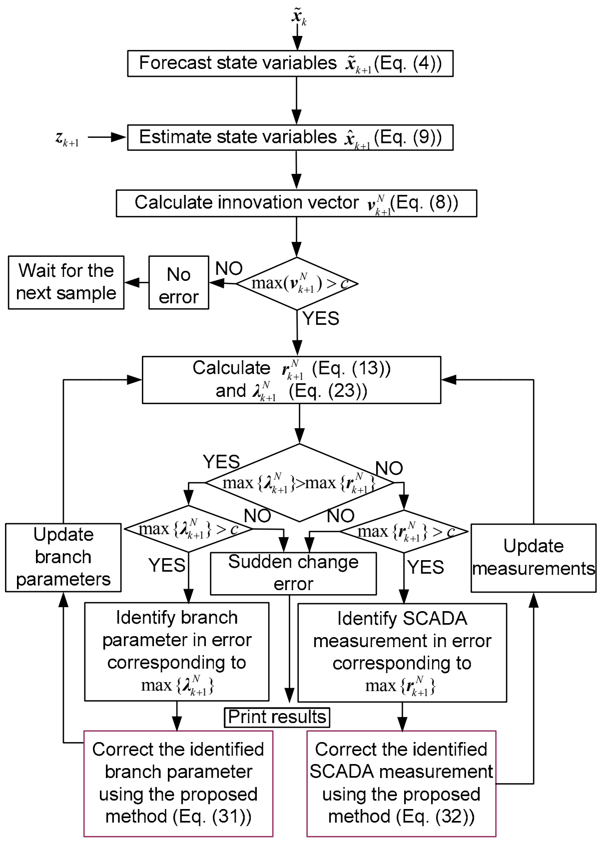

Fig. 1 Flowchart of the proposed algorithm (SCADA: supervisory control and data acquisition)

Table 1 Variation in the maximum elements of normalized vectors by various errors

Vector	Measurement error	Branch parameter error	Sudden load change
$\boldsymbol{v}_{k+1}$	$\max\{\boldsymbol{v}_{k+1}^N\} > c$	$\max\{\boldsymbol{v}_{k+1}^N\} > c$	$\max\{\boldsymbol{v}_{k+1}^N\} > c$
$\boldsymbol{r}_{k+1}$	$\max\{\boldsymbol{r}_{k+1}^N\} > c$	$\max\{\boldsymbol{r}_{k+1}^N\} > c$	$\max\{\boldsymbol{r}_{k+1}^N\} < c$
$\boldsymbol{\lambda}_{k+1}$	$\max\{\boldsymbol{\lambda}_{k+1}^N\} > c$	$\max\{\boldsymbol{\lambda}_{k+1}^N\} > c$	$\max\{\boldsymbol{\lambda}_{k+1}^N\} < c$

c is the threshold for error processing

$$\mathrm{Index}_k^{\mathrm{est}} = \left| \frac{\hat{\boldsymbol{x}}_k - \boldsymbol{x}_k^{\mathrm{true}}}{\boldsymbol{x}_k^{\mathrm{true}}} \right| \times 100\%. \quad (34)$$

Performance index: For overall achievement, the following performance index is used as the comparison benchmark:

$$\mathrm{Index}_k^{\mathrm{performance}} = \frac{\sum_{i=1}^{m} \left| \hat{z}_k^i - z_k^{\mathrm{ture},i} \right|}{\sum_{i=1}^{m} \left| z_k^i - z_k^{\mathrm{ture},i} \right|}, \quad (35)$$

where i and k indicate the ith measurement and the kth time sample, respectively.

4 Simulation results

The validity and performance of our proposed algorithm were evaluated on an IEEE 14-bus test system. Performance was evaluated under a scenario of simultaneous measurement and branch parameter errors plus a sudden load change.

The unique features and main advantages of the proposed method are fourfold:

1. The proposed method enhances the sensitivity and reliability of SE by successful processing of anomalies, and the identification and correction of errors with high accuracy.

2. The method is able to detect and correct errors in measurements and parameters, even when they occur simultaneously, as well as under sudden load change conditions.

3. The paradigm of the method is easy to understand and implement. It can also be developed as a didactic tool to help engineers and students become better acquainted with power system operations.

4. The method can be used in practical systems, and is suitable for applications under different operating scenarios according to the results obtained.

To simulate the slow dynamic performance of a system, a linear trend (1%) along with a random fluctuation was added to the load curve, and 40 time sample intervals were obtained through successful running of load flows under different loading conditions. The outputs of the load flow were used as true states ($\boldsymbol{x}^{\mathrm{true}}$) and true values of measurements ($\boldsymbol{z}^{\mathrm{true}}$), including bus voltage magnitudes, bus injection powers, and line flow powers. Then, measurements of voltage magnitudes, power injections, and power flows were corrupted with a random Gaussian noise with zero mean and standard deviation of 1% for voltages and 2% for powers of actual values.

The initialization of Holt's technique was carried out using the first two samples at times k=0 and k=1 of state variables taken from the load flow executions. This means that the estimation process ran from time instant k=2 up to k=40 and α_k=0.8 and β_k=0.5 were used during the whole simulation time interval. The elements of diagonal matrix $\boldsymbol{Q}_k$ were kept constant at 10^{-6} during the whole simulation (Valverde and Terzija, 2011). Also, the threshold for error processing was selected as c=4. The proposed algorithm deals with electrical parameters in the classical steady-state π-equivalent model of branches, which consists of series and shunt admittances. In the test system, we assumed that the measurement system was highly redundant. In this regard, we also assumed that all power injections and power flows were available.

Table 2 summarizes the simulated errors in the test system, including measurement error, branch parameter error, and a sudden load change in various time samples.

In Table 2, P_{13} and Q_{13} are the active and reactive power injections in bus 13, respectively. Note that these measurements are closely related, and thus could easily be missed as a sudden load change at bus 13. Also, $g_{i\text{–}j}$ is the series conductance of the π-equivalent model of the branch connecting buses i and j.

Simulation results are presented and discussed below. To demonstrate the effectiveness of the proposed algorithm, Fig. 2 shows the variation in evaluation indexes as defined by Eqs. (33)–(35).

Table 2 Simulated errors on the IEEE 14-bus test system

Time step k	Error type	Error information
30	Sudden load change	Bus 13 (load completely cut off)
20–30	Measurement error	P_{13}=0 (true value=−0.135 p.u.) Q_{13}=0 (true value=−0.058 p.u.)
30–40	Branch parameter error	$g_{1\text{–}2}$=7.5 (true value=−4.990 p.u.)

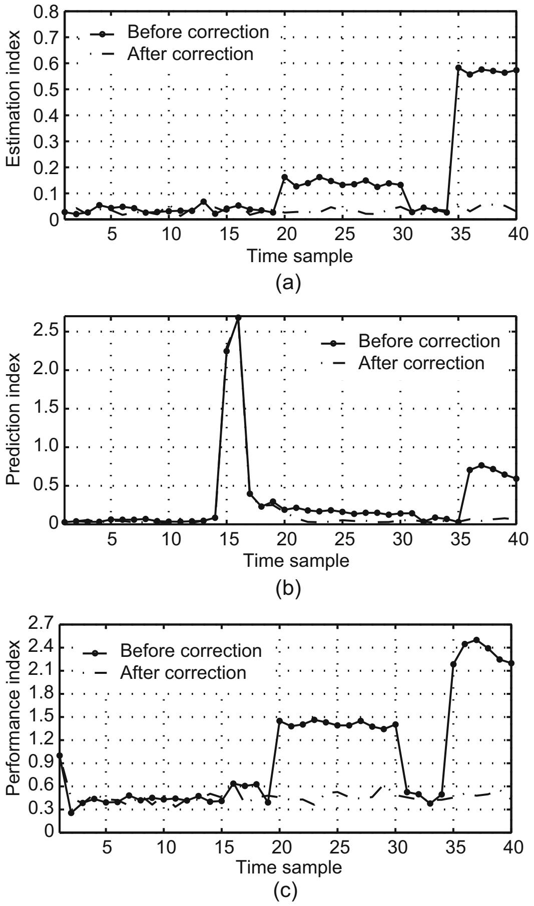

Fig. 2 Variation in evaluation indexes on the IEEE 14-bus test system: (a) estimation index; (b) prediction index; (c) performance index

Figs. 2a–2c also represent the variations in evaluation indexes before and after applying the proposed methodology.

The indexes from sampling times k=20 to k=30 and from k=35 to k=40 increased before using the proposed correction method. Evaluation indexes decreased in the following application of the proposed algorithm, which demonstrates the effectiveness of the proposed method.

To show the capability of the proposed correction algorithm, the measured, predicted, and estimated values for active and reactive power injections at bus 13 (P_{13} and Q_{13}, respectively) are shown in Figs. 3a–3d. The proposed DSE algorithm performed appropriately and provided suitable corrections.

Furthermore, the proposed method identified and corrected the measurement and branch parameter errors as well as the sudden load change error.

To provide a comprehensive comparison and analysis, the variations in normalized vectors ($\boldsymbol{v}_{k+1}$, $\boldsymbol{r}_{k+1}$, and $\boldsymbol{\lambda}_{k+1}$) are presented in Figs. 4a–4f.

In Fig. 4, after successful error identification and correction, all elements of the normalized innovation, normalized residual, and normalized Lagrange multiplier vectors are lower than the threshold of four.

The proposed method can be used in practical applications in view of the high accuracy of the simulation results. Note that in practical applications, the

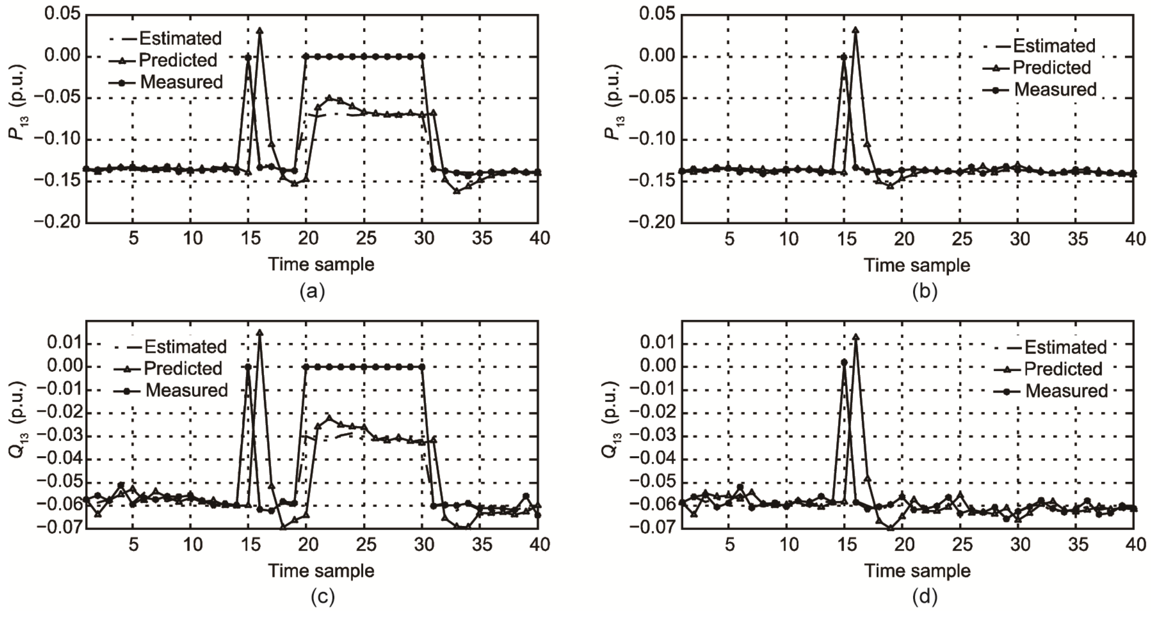

Fig. 3 Active and reactive power injection at bus 13: (a) P_{13} before correction; (b) P_{13} after correction; (c) Q_{13} before correction; (d) Q_{13} after correction

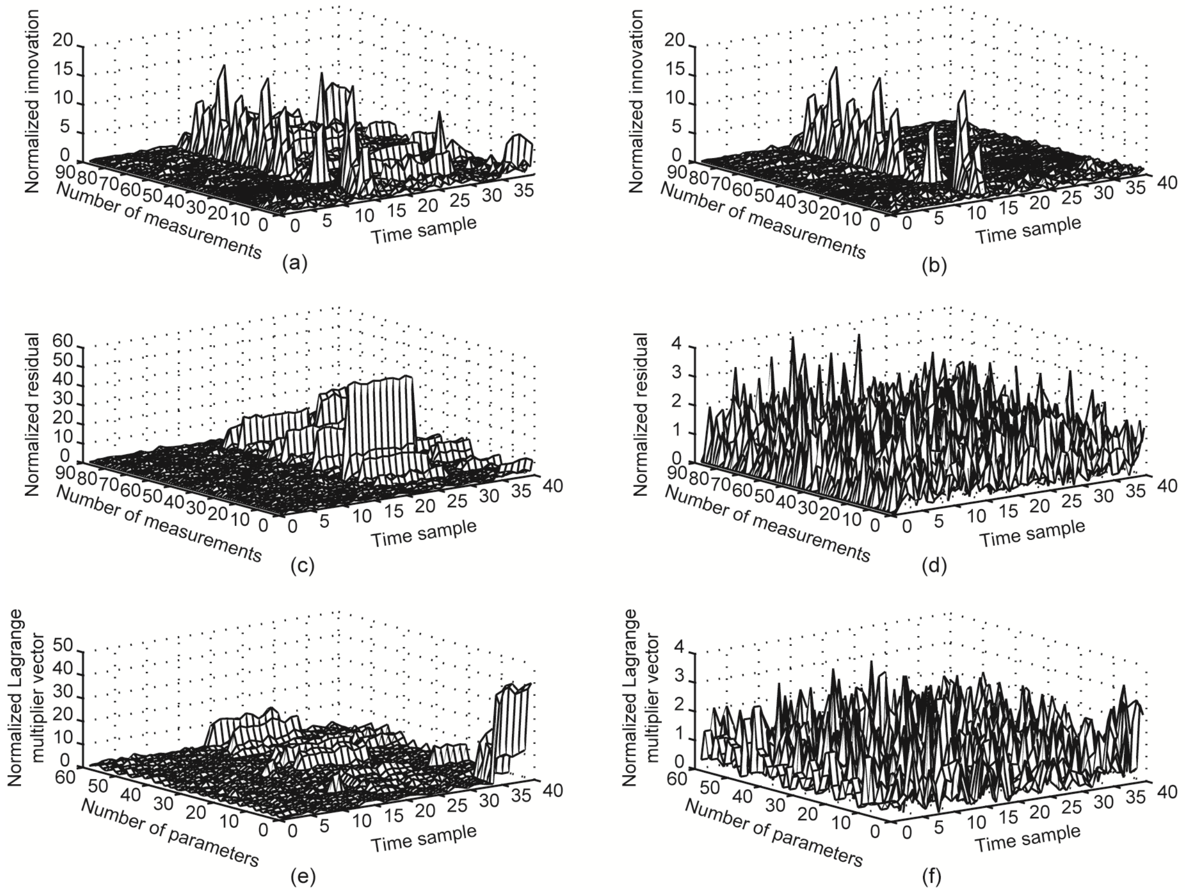

Fig. 4 Variations in normalized vectors: (a) v_{k+1} before correction; (b) v_{k+1} after correction; (c) r_{k+1} before correction; (d) r_{k+1} after correction; (e) λ_{k+1} before correction; (f) λ_{k+1} after correction

errors in measurement sets are inherent, natural, and depend on the measurement accuracy. In real power systems, SE is carried out at time intervals of a few minutes (five minutes, as time samples). The proposed methodology can be simulated in less than this time.

5 Conclusions

A novel and efficient algorithm for simultaneous detection, identification, and correction of measurement and branch parameter errors has been proposed based on a dynamic state estimation algorithm and Kalman filter theory. The proposed correction methodology also successfully detected and identified sudden load changes. The proposed methodology applies three normalized vectors for error processing in each time sample, namely normalized measurement residual, normalized Lagrange multiplier, and normalized innovation vectors. It was shown that the normalized innovations are highly suitable for error processing. An IEEE 14-bus system was used as the test system to verify and demonstrate the effectiveness of the proposed algorithm by introducing various indices. The performance of the proposed DSE algorithm was illustrated through these tests. Suitable results were obtained and the proposed method successfully processed the anomalies, and identified and corrected the errors with high accuracy.

Future work will focus on applying extended Kalman filter (EKF) and unscented Kalman filter (UKF) techniques to error processing, and might include the following topics: development of estimation methods, incorporation of phasor measurement unit (PMU) measurements, computational efficiency for large-scale power system applications, and forecasting methods based on nonlinear models.

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