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Leader-following consensus of second-order nonlinear multi-agent systems subject to disturbances*

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Abstract: In this study, we investigate the leader-following consensus problem of a class of heterogeneous second-order nonlinear multi-agent systems subject to disturbances. In particular, the nonlinear systems contain uncertainties that can be linearly parameterized. We propose a class of novel distributed control laws, which depends on the relative state of the system and thus can be implemented even when no communication among agents exists. By Barbalat's lemma, we demonstrate that consensus of the second-order nonlinear multi-agent system can be achieved by the proposed distributed control law. The effectiveness of the main result is verified by its application to consensus control of a group of Van der Pol oscillators.

Key words: Multi-agent systems; Leader-following consensus; Distributed control
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1 Introduction

The research on multi-agent systems has received considerable attention due to the broad application of cooperative control in engineering problems. Some typical examples of cooperative control include attitude consensus of multiple spacecrafts and formation flight of a group of aircrafts. Various cooperative control problems have been studied, such as consensus, formation, flocking, connectivity preservation, containment control, and cooperative output regulation (Jadbabaie et al., 2003; Moreau, 2004; Olfati-Saber and Murray, 2004; Ren and Beard, 2005; Olfati-Saber, 2006; Tuna, 2008; Wieland et al., 2011; Su and Huang, 2012; Deng et al., 2017). Among them, consensus is one of the most fundamental problems (Moreau, 2004; Olfati-Saber and Murray, 2004; Ren and Beard, 2005; Tuna, 2008; Cheng et al., 2010; Wieland et al., 2011; Lu and Liu, 2017).

The consensus problem of second-order nonlinear multi-agent systems has been extensively studied in the literature, for example, Song et al. (2010), Meng et al. (2013), Su and Huang (2013), Su (2015), Liu and Huang (2016), and Lu and Liu (2018). An interesting version of the consensus problem is the leader-following consensus problem. For this problem, the output of all systems (or called followers), needs to track some common trajectory generated by a system which is called the leader system. In particular, the leader-following consensus problem of a class of homogeneous second-order nonlinear multi-agent systems was first studied in Song et al. (2010) under the global Lipschitz condition. By proposing a pinning control law, it was shown that the consensus problem can be solved under general static directed networks. Then, the semi-global leader-

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following consensus problem of a class of second-order nonlinear multi-agent systems was addressed in Meng et al. (2013), where external disturbances were considered. Later, the leader-following consensus problem for a class of second-order nonlinear multi-agent systems without uncertainty was studied in Wang and Ji (2015) under static undirected networks and external disturbances. In addition, the leader-following consensus problem of second-order nonlinear multi-agent systems was addressed in Su (2015), where the system uncertain parameter belongs to some prescribed compact set. By the internal model approach, the disturbance can also be tackled (Su, 2015).

In this study, the leader-following consensus problem for second-order nonlinear multi-agent systems under disturbances is further addressed. In comparison with existing works, our result has some specific characteristics. First, all agents have nonidentical dynamics. Second, nonlinear systems in our work do not need to satisfy the global Lipschitz condition, and the initial conditions of the system can be arbitrary. Third, assuming that the system uncertain parameters can be linearly parameterized, we do not require the system uncertainty be in some prescribed compact set. In addition, communication among agents can be avoided. Specifically, we propose a class of distributed control laws that depends on the relative state of the system to solve the problem. Moreover, by the adaptive control approach, the system uncertainties can be accommodated. By means of Lyapunov analysis, it is shown that the proposed distributed control law can solve the leader-following consensus problem of a class of second-order nonlinear multi-agent systems subject to system uncertainties and disturbances under static directed networks. The result is demonstrated by one application to consensus control of a group of Van der Pol oscillators.

Notation: For $\boldsymbol{z}_i \in \mathbb{R}^{n_i \times q}$, $i = 1, 2, \ldots, m$, $\mathrm{col}(\boldsymbol{z}_1, \boldsymbol{z}_2, \ldots, \boldsymbol{z}_m) = [\boldsymbol{z}_1^{\mathrm{T}}, \boldsymbol{z}_2^{\mathrm{T}}, \ldots, \boldsymbol{z}_m^{\mathrm{T}}]^{\mathrm{T}}$. For $\boldsymbol{M} = \boldsymbol{M}^{\mathrm{T}} > 0$, $\lambda_m(\boldsymbol{M})$ denotes the smallest eigenvalue of $\boldsymbol{M}$.

2 Problem formulation and preliminaries

In this study, we consider the following second-order nonlinear multi-agent systems:

$$\begin{cases} \dot{\boldsymbol{q}}_i = \boldsymbol{p}_i, \\ \dot{\boldsymbol{p}}_i = \boldsymbol{f}_i(\boldsymbol{q}_i, \boldsymbol{p}_i, t) + \boldsymbol{d}_i + \boldsymbol{u}_i, \ i = 1, 2, \ldots, N, \end{cases} \tag{1}$$

where $\boldsymbol{q}_i \in \mathbb{R}^n$ and $\boldsymbol{p}_i \in \mathbb{R}^n$ are the system states, $\boldsymbol{u}_i \in \mathbb{R}^n$ is the control input, $\boldsymbol{f}_i(\boldsymbol{q}_i, \boldsymbol{p}_i, t) = \boldsymbol{g}_{i0}(\boldsymbol{q}_i, \boldsymbol{p}_i) + \boldsymbol{g}_i(\boldsymbol{q}_i, \boldsymbol{p}_i)\boldsymbol{\theta}_i$ for an unknown parameter vector $\boldsymbol{\theta}_i \in \mathbb{R}^{n_p}$ and some known matrices $\boldsymbol{g}_{i0}(\boldsymbol{q}_i, \boldsymbol{p}_i) \in \mathbb{R}^n$ and $\boldsymbol{g}_i(\boldsymbol{q}_i, \boldsymbol{p}_i) \in \mathbb{R}^{n \times n_p}$ which are continuous with respect to its arguments, and $\boldsymbol{d}_i$ is the disturbance.

It is assumed that the disturbance $\boldsymbol{d}_i$ and the reference signal q_0 are generated by a linear system as follows:

$$\begin{cases} \dot{\boldsymbol{v}} = \boldsymbol{S}\boldsymbol{v}, \\ \boldsymbol{d}_i = \boldsymbol{D}_i\boldsymbol{v}, \\ \boldsymbol{q}_0 = \boldsymbol{F}\boldsymbol{v}, \end{cases} \tag{2}$$

where $\boldsymbol{S} \in \mathbb{R}^{m \times m}$, $\boldsymbol{D}_i \in \mathbb{R}^{n \times m}$, and $\boldsymbol{F} \in \mathbb{R}^{n \times m}$ are constant matrices. It is assumed that $(\boldsymbol{F}, \boldsymbol{S})$ is observable.

Systems (1) and (2) constitute a multi-agent system, where system (2) is the leader and the N subsystems (1) are the followers. Associated with this multi-agent system, a nonnegative matrix $A = [a_{ij}] \in \mathbb{R}^{(N+1) \times (N+1)}$ can be defined, where for $i = 1, 2, \ldots, N, j = 0, 1, \ldots, N$, $a_{ij} > 0$ if and only if the relative state $(q_j - q_i)$ is accessible to agent i and $a_{ij} = 0$ otherwise. Associated with matrix A, a digraph $G = (V, E)$ can be defined (Godsil and Royle, 2001). Then, the node set is $V = \{0, 1, \ldots, N\}$, where node 0 denotes system (2) and nodes i, $i = 1, 2, \ldots, N$, denote system (1). The edge set is $E \subseteq V \times V$, where the edge $(i, j) \in E$ if and only if $a_{ji} > 0$. If the graph contains a sequence of edges (i_l, i_{l+1}), $l = 1, 2, \ldots, k-1$, then node i_k is said to be reachable from node i_1. The neighbor set of node i is denoted by $N_i = \{j \mid (j, i) \in E\}$. If $(i, j) \in E \Leftrightarrow (j, i) \in E$, the graph is called an undirected graph; otherwise, it is called a directed graph.

Consider the distributed control law as follows:

$$\begin{cases} \boldsymbol{u}_i = \boldsymbol{\psi}_i(\boldsymbol{\xi}_i, \boldsymbol{p}_i, \boldsymbol{q}_i, \dot{\boldsymbol{q}}_i, \boldsymbol{q}_j - \boldsymbol{q}_i, j \in N_i), \\ \dot{\boldsymbol{\xi}}_i = \boldsymbol{\varphi}_i(\boldsymbol{\xi}_i, \boldsymbol{p}_i, \boldsymbol{q}_i, \dot{\boldsymbol{q}}_i, \boldsymbol{q}_j - \boldsymbol{q}_i, j \in N_i), i = 1, 2, \ldots, N, \end{cases} \tag{3}$$

where $\boldsymbol{\xi}_i \in \mathbb{R}^{n_{\boldsymbol{\xi}_i}}$ for some positive integer $n_{\boldsymbol{\xi}_i}$, and ψ_i and φ_i are smooth functions defined later.

Now, we can describe the consensus problem of the second-order nonlinear multi-agent system:

Problem 1 Given the multi-agent system composed of followers (1) and the leader system (2), and a digraph G, design a distributed control law of the form (3) such that, for any initial condition $\boldsymbol{q}_i(0), \boldsymbol{p}_i(0), i = 1, 2, \ldots, N$, and $\boldsymbol{v}(0)$, the solution of system (1) satisfies

$$\lim_{t\to\infty}(\boldsymbol{q}_i(t) - \boldsymbol{q}_0(t)) = 0, \lim_{t\to\infty}(\boldsymbol{p}_i(t) - \dot{\boldsymbol{q}}_0(t)) = 0. \quad (4)$$

To solve the problem, we need some basic assumptions as follows:

Assumption 1 Matrix $\boldsymbol{S}$ is marginally stable.

Assumption 2 Digraph G contains a directed spanning tree with node 0 as its root.

Remark 1 Assumption 1 is used to guarantee the boundedness of the disturbance signals and reference signals described by system (2). Assumption 2 is a basic assumption on the static graph (Hu and Hong, 2007; Su and Huang, 2012).

Under Assumption 1, system (2) can still describe a large class of disturbance signals and reference signals, for example, signals that can be expressed as a combination of any finite number of sinusoidal signals.

3 Main result

In this section, we develop an adaptive distributed control law to solve the consensus problem. First, we introduce a distributed dynamic compensator as follows:

$$\dot{\boldsymbol{\eta}}_i = \boldsymbol{S}\boldsymbol{\eta}_i + \boldsymbol{L}\sum_{j\in N_i} a_{ij}(\boldsymbol{q}_j - \boldsymbol{q}_i), \quad (5)$$

where $\boldsymbol{\eta}_i \in \mathbb{R}^m$, and $\boldsymbol{L} \in \mathbb{R}^{m\times n}$ is a constant matrix to be determined.

Remark 2 It will be shown that the distributed dynamic compensator (5) is served as a distributed observer for the leader system. This is inspired by the distributed observer approach in Su and Huang (2012). The difference is that system (5) depends on the relative state of the multi-agent system. Thus, it can be implemented when no communication among agents exists.

To develop the control law, we define

$$\dot{\boldsymbol{q}}_{ri} = \boldsymbol{F}\boldsymbol{S}\boldsymbol{\eta}_i - \alpha(\boldsymbol{q}_i - \boldsymbol{F}\boldsymbol{\eta}_i), \quad (6)$$

where α is a positive constant to be defined. It follows that

$$\ddot{\boldsymbol{q}}_{ri} = \boldsymbol{F}\boldsymbol{S}\dot{\boldsymbol{\eta}}_i - \alpha(\dot{\boldsymbol{q}}_i - \boldsymbol{F}\dot{\boldsymbol{\eta}}_i). \quad (7)$$

Furthermore, let

$$\boldsymbol{s}_i = \dot{\boldsymbol{q}}_i - \dot{\boldsymbol{q}}_{ri}. \quad (8)$$

Now, we can design the adaptive distributed control law as follows:

$$\begin{cases} \boldsymbol{u}_i = -\boldsymbol{K}_i\boldsymbol{s}_i - \boldsymbol{g}_{i0}(\boldsymbol{q}_i, \boldsymbol{p}_i) - \boldsymbol{g}_i(\boldsymbol{q}_i, \boldsymbol{p}_i)\hat{\boldsymbol{\theta}}_i - \boldsymbol{D}_i\boldsymbol{\eta}_i + \ddot{\boldsymbol{q}}_{ri}, \\ \dot{\hat{\boldsymbol{\theta}}}_i = -\boldsymbol{\Lambda}_i\boldsymbol{g}_i^{\mathrm{T}}(\boldsymbol{q}_i, \boldsymbol{p}_i)\boldsymbol{s}_i, \\ \dot{\boldsymbol{\eta}}_i = \boldsymbol{S}\boldsymbol{\eta}_i + \boldsymbol{L}\sum_{j\in N_i} a_{ij}(\boldsymbol{q}_j - \boldsymbol{q}_i), \ i = 1, 2, \ldots, N, \end{cases} \quad (9)$$

where $\hat{\boldsymbol{\theta}}_i \in \mathbb{R}^{n_p}$, $\boldsymbol{K}_i \in \mathbb{R}^{n\times n}$ and $\boldsymbol{\Lambda}_i \in \mathbb{R}^{n_p\times n_p}$ are positive definite matrices.

Under the distributed control law (9), the closed-loop system can be put as follows:

$$\begin{cases} \dot{\boldsymbol{q}}_i = \boldsymbol{p}_i, \\ \dot{\boldsymbol{p}}_i = -\boldsymbol{K}_i\boldsymbol{s}_i - \boldsymbol{g}_i(\boldsymbol{q}_i, \boldsymbol{p}_i)\tilde{\boldsymbol{\theta}}_i - \boldsymbol{D}_i\tilde{\boldsymbol{\eta}}_i + \ddot{\boldsymbol{q}}_{ri}, \\ \dot{\hat{\boldsymbol{\theta}}}_i = -\boldsymbol{\Lambda}_i\boldsymbol{g}_i^{\mathrm{T}}(\boldsymbol{q}_i, \boldsymbol{p}_i)\boldsymbol{s}_i, \\ \dot{\boldsymbol{\eta}}_i = \boldsymbol{S}\boldsymbol{\eta}_i + \boldsymbol{L}\sum\limits_{j\in N_i} a_{ij}(\boldsymbol{q}_j - \boldsymbol{q}_i), \ i = 1, 2, \ldots, N, \end{cases} \quad (10)$$

where $\tilde{\boldsymbol{\eta}}_i = \boldsymbol{\eta}_i - \boldsymbol{v}$ and $\tilde{\boldsymbol{\theta}}_i = \hat{\boldsymbol{\theta}}_i - \boldsymbol{\theta}_i$, $i = 1, 2, \ldots, N$.

We are ready to present the solution to the consensus problem. The main result is given in the following theorem:

Theorem 1 Given Assumptions 1 and 2, leader-following consensus for the multi-agent system consisting of Eqs. (1) and (2) can be achieved by the adaptive distributed control law of the form (9).

Proof By Eqs. (1) and (8), we have

$$\begin{aligned} \dot{\boldsymbol{s}}_i &= \boldsymbol{f}_i(\boldsymbol{q}_i, \boldsymbol{p}_i, t) + \boldsymbol{d}_i + \boldsymbol{u}_i - \ddot{\boldsymbol{q}}_{ri} \\ &= -\boldsymbol{K}_i\boldsymbol{s}_i - \boldsymbol{g}_i(\boldsymbol{q}_i, \boldsymbol{p}_i)\tilde{\boldsymbol{\theta}}_i - \boldsymbol{D}_i\tilde{\boldsymbol{\eta}}_i, \ i = 1, 2, \ldots, N. \end{aligned} \quad (11)$$

Let

$$V_1 = \frac{1}{2}\sum_{i=1}^{N}(\boldsymbol{s}_i^{\mathrm{T}}\boldsymbol{s}_i + \tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}\boldsymbol{\Gamma}_i^{-1}\tilde{\boldsymbol{\theta}}_i). \quad (12)$$

Then, the time derivative of V_1 satisfies

$$\begin{aligned} \dot{V}_1 &= \sum_{i=1}^{N}(\boldsymbol{s}_i^{\mathrm{T}}\dot{\boldsymbol{s}}_i + \tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}\boldsymbol{\Gamma}_i^{-1}\dot{\tilde{\boldsymbol{\theta}}}_i) \\ &= \sum_{i=1}^{N}(\boldsymbol{s}_i^{\mathrm{T}}(-\boldsymbol{K}_i\boldsymbol{s}_i - \boldsymbol{g}_i(\boldsymbol{q}_i, \boldsymbol{p}_i)\tilde{\boldsymbol{\theta}}_i - \boldsymbol{D}_i\tilde{\boldsymbol{\eta}}_i) \\ &\quad + \tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}\boldsymbol{\Gamma}_i^{-1}(-\boldsymbol{\Gamma}_i\boldsymbol{g}_i^{\mathrm{T}}(\boldsymbol{q}_i, \boldsymbol{p}_i)\boldsymbol{s}_i)) \\ &= \sum_{i=1}^{N}(-\boldsymbol{s}_i^{\mathrm{T}}\boldsymbol{K}_i\boldsymbol{s}_i - \boldsymbol{s}_i^{\mathrm{T}}\boldsymbol{D}_i\tilde{\boldsymbol{\eta}}_i). \end{aligned} \quad (13)$$

Let

$$\begin{cases} \boldsymbol{\eta}_0 = \boldsymbol{v}, \\ \tilde{\boldsymbol{q}}_i = \boldsymbol{q}_i - \boldsymbol{F}\boldsymbol{\eta}_i, i = 0, 1, \ldots, N. \end{cases} \tag{14}$$

Then, we have $\tilde{\boldsymbol{\eta}}_0 = 0$, $\tilde{\boldsymbol{q}}_0 = 0$, and

$$\begin{aligned} \dot{\tilde{\boldsymbol{\eta}}}_i =& \dot{\boldsymbol{\eta}}_i - \dot{\boldsymbol{v}} \\ =& \boldsymbol{S}\boldsymbol{\eta}_i + \boldsymbol{L}\sum_{j\in N_i} a_{ij}(\boldsymbol{q}_j - \boldsymbol{q}_i) - \boldsymbol{S}\boldsymbol{v} \\ =& \boldsymbol{S}\tilde{\boldsymbol{\eta}}_i + \boldsymbol{L}\sum_{j\in N_i} a_{ij}(\boldsymbol{q}_j - \boldsymbol{q}_i) \\ =& \boldsymbol{S}\tilde{\boldsymbol{\eta}}_i + \boldsymbol{L}\boldsymbol{F}\sum_{j\in N_i} a_{ij}(\tilde{\boldsymbol{\eta}}_j - \tilde{\boldsymbol{\eta}}_i) \\ &+ \boldsymbol{L}\sum_{j\in N_i} a_{ij}(\tilde{\boldsymbol{q}}_j - \tilde{\boldsymbol{q}}_i). \end{aligned} \tag{15}$$

Associated with the graph G, define a matrix $\boldsymbol{H} = [h_{ij}] \in \mathbb{R}^{n\times n}$, where $h_{ij} = -a_{ij}$, $i \neq j$, and $h_{ii} = \sum_{j=0,1}^{N} a_{ij}$.

Denote $\tilde{\boldsymbol{\eta}} = \mathrm{col}(\tilde{\boldsymbol{\eta}}_1, \tilde{\boldsymbol{\eta}}_2, \ldots, \tilde{\boldsymbol{\eta}}_N)$, $\tilde{\boldsymbol{q}} = \mathrm{col}(\tilde{\boldsymbol{q}}_1, \tilde{\boldsymbol{q}}_2, \ldots, \tilde{\boldsymbol{q}}_N)$. Then, Eq. (15) can be put in compact form as follows:

$$\dot{\tilde{\boldsymbol{\eta}}} = (\boldsymbol{I}_N \otimes \boldsymbol{S} - \boldsymbol{H} \otimes \boldsymbol{L}\boldsymbol{F})\tilde{\boldsymbol{\eta}} - (\boldsymbol{H} \otimes \boldsymbol{L})\tilde{\boldsymbol{q}}. \tag{16}$$

By Eq. (8), we have

$$\begin{aligned} \boldsymbol{s}_i =& \dot{\boldsymbol{q}}_i - \boldsymbol{F}\boldsymbol{S}\boldsymbol{\eta}_i + \alpha(\boldsymbol{q}_i - \boldsymbol{F}\boldsymbol{\eta}_i) \\ =& \dot{\boldsymbol{q}}_i - \boldsymbol{F}\Big(\dot{\boldsymbol{\eta}}_i - \boldsymbol{L}\sum_{j\in N_i} a_{ij}(\boldsymbol{q}_j - \boldsymbol{q}_i)\Big) + \alpha(\boldsymbol{q}_i - \boldsymbol{F}\boldsymbol{\eta}_i). \end{aligned} \tag{17}$$

Thus,

$$\begin{aligned} &\dot{\boldsymbol{q}}_i - \boldsymbol{F}\dot{\boldsymbol{\eta}}_i + \alpha(\boldsymbol{q}_i - \boldsymbol{F}\boldsymbol{\eta}_i) \\ =& -\boldsymbol{F}\boldsymbol{L}\sum_{j\in N_i} a_{ij}(\boldsymbol{q}_j - \boldsymbol{q}_i) + \boldsymbol{s}_i \\ =& -\boldsymbol{F}\boldsymbol{L}\sum_{j\in N_i} a_{ij}(\tilde{\boldsymbol{q}}_j - \tilde{\boldsymbol{q}}_i) - \boldsymbol{F}\boldsymbol{L}\boldsymbol{F}\sum_{j\in N_i} a_{ij}(\tilde{\boldsymbol{\eta}}_j - \tilde{\boldsymbol{\eta}}_i) \\ &+ \boldsymbol{s}_i,\ i = 1, 2, \ldots, N. \end{aligned} \tag{18}$$

It follows that

$$\begin{aligned} \dot{\tilde{\boldsymbol{q}}}_i =& -\alpha\tilde{\boldsymbol{q}}_i - \boldsymbol{F}\boldsymbol{L}\sum_{j\in N_i} a_{ij}(\tilde{\boldsymbol{q}}_j - \tilde{\boldsymbol{q}}_i) \\ &- \boldsymbol{F}\boldsymbol{L}\boldsymbol{F}\sum_{j\in N_i} a_{ij}(\tilde{\boldsymbol{\eta}}_j - \tilde{\boldsymbol{\eta}}_i) + \boldsymbol{s}_i, i = 1, 2, \ldots, N. \end{aligned} \tag{19}$$

System (19) can be written in the following compact form:

$$\dot{\tilde{\boldsymbol{q}}} = -(\alpha\boldsymbol{I} - \boldsymbol{H} \otimes \boldsymbol{F}\boldsymbol{L})\tilde{\boldsymbol{q}} + (\boldsymbol{H} \otimes \boldsymbol{F}\boldsymbol{L}\boldsymbol{F})\tilde{\boldsymbol{\eta}} + \boldsymbol{s}. \tag{20}$$

Denote $\tilde{\boldsymbol{x}} = \mathrm{col}(\tilde{\boldsymbol{\eta}}, \tilde{\boldsymbol{q}})$. Then, the system composed of Eqs. (16) and (20) can be put into the form

$$\dot{\tilde{\boldsymbol{x}}} = \tilde{\boldsymbol{A}}\tilde{\boldsymbol{x}} + \tilde{\boldsymbol{B}}\boldsymbol{s}, \tag{21}$$

where

$$\begin{cases} \tilde{\boldsymbol{A}} = \begin{bmatrix} (\boldsymbol{I}_N \otimes \boldsymbol{S} - \boldsymbol{H} \otimes \boldsymbol{L}\boldsymbol{F}) & -(\boldsymbol{H} \otimes \boldsymbol{L}) \\ (\boldsymbol{H} \otimes \boldsymbol{F}\boldsymbol{L}\boldsymbol{F}) & -(\alpha\boldsymbol{I} - \boldsymbol{H} \otimes \boldsymbol{F}\boldsymbol{L}) \end{bmatrix}, \\ \tilde{\boldsymbol{B}} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \end{cases} \tag{22}$$

Let

$$\boldsymbol{L} = \nu\boldsymbol{Q}\boldsymbol{F}^{\mathrm{T}}, \tag{23}$$

where $\nu \geq \frac{1}{\delta}$ with $\delta = \min_{i=1,2,\ldots,N}\{\mathrm{Re}\{\lambda_i(\boldsymbol{H})\}\}$, and Q is the solution to the Riccati equation:

$$\boldsymbol{S}\boldsymbol{Q} + \boldsymbol{Q}\boldsymbol{S}^{\mathrm{T}} + \boldsymbol{I} - \boldsymbol{Q}\boldsymbol{F}^{\mathrm{T}}\boldsymbol{F}\boldsymbol{Q} = \boldsymbol{0}. \tag{24}$$

In view of the observability of $(\boldsymbol{F}, \boldsymbol{S})$, Eq. (24) indeed has a unique solution $\boldsymbol{Q}$ and it satisfies $\boldsymbol{Q} = \boldsymbol{Q}^{\mathrm{T}} > 0$.

Let $\boldsymbol{P}$ be such that

$$\begin{aligned} &(\boldsymbol{I}_N \otimes \boldsymbol{S} - \boldsymbol{H} \otimes \boldsymbol{L}\boldsymbol{F}) \\ =& (\boldsymbol{P}^{-1} \otimes \boldsymbol{I}_m)(\boldsymbol{I}_N \otimes \boldsymbol{S} - \boldsymbol{J}_H \otimes \boldsymbol{L}\boldsymbol{F})(\boldsymbol{P} \otimes \boldsymbol{I}_m), \end{aligned} \tag{25}$$

where $\boldsymbol{J}_H$ is the Jordan form of $\boldsymbol{H}$.

Under Assumption 2, by Lemma 1 of Su and Huang (2012), the matrix $-\boldsymbol{H}$ is Hurwitz. Then, by Lemma 1 of Tuna (2008), for $i = 1, 2, \ldots, N$, $(\boldsymbol{S}^{\mathrm{T}} - \lambda_i(\boldsymbol{H})\nu\boldsymbol{F}^{\mathrm{T}}\boldsymbol{F}\boldsymbol{Q})$ are all Hurwitz and thus $(\boldsymbol{S} - \lambda_i(\boldsymbol{H})\nu\boldsymbol{Q}\boldsymbol{F}^{\mathrm{T}}\boldsymbol{F})$ are all Hurwitz. Therefore, by Eq. (25), we have that $(\boldsymbol{I}_N \otimes \boldsymbol{S} - \boldsymbol{H} \otimes \boldsymbol{L}\boldsymbol{F})$ is Hurwitz.

Furthermore, it can be verified that the matrix $\tilde{\boldsymbol{A}}$ is Hurwitz for a sufficiently large positive real number α. Thus, there exists a unique $\tilde{\boldsymbol{P}} \in \mathbb{R}^{N(m+n)}$, $\tilde{\boldsymbol{P}} = \tilde{\boldsymbol{P}}^{\mathrm{T}} > 0$ such that

$$\tilde{\boldsymbol{A}}^{\mathrm{T}}\tilde{\boldsymbol{P}} + \tilde{\boldsymbol{P}}\tilde{\boldsymbol{A}} = -2\boldsymbol{I}. \tag{26}$$

Define $V_2 = \tilde{\boldsymbol{x}}^{\mathrm{T}}\tilde{\boldsymbol{P}}\tilde{\boldsymbol{x}}$. Then, we have

$$\dot{V}_2 = -2\tilde{\boldsymbol{x}}^{\mathrm{T}}\tilde{\boldsymbol{x}} + 2\tilde{\boldsymbol{x}}^{\mathrm{T}}\tilde{\boldsymbol{P}}\tilde{\boldsymbol{B}}\boldsymbol{s}. \tag{27}$$

Let $V = V_1 + \gamma V_2$, where γ is a positive real number. Then, we have

$$\begin{aligned}\dot{V} =&\sum_{i=1}^{N}(-\boldsymbol{s}_i^{\mathrm{T}}\boldsymbol{K}_i\boldsymbol{s}_i - \boldsymbol{s}_i^{\mathrm{T}}\boldsymbol{D}_i\tilde{\boldsymbol{\eta}}_i) - 2\gamma\tilde{\boldsymbol{x}}^{\mathrm{T}}\tilde{\boldsymbol{x}} + 2\gamma\tilde{\boldsymbol{x}}^{\mathrm{T}}\tilde{\boldsymbol{P}}\tilde{\boldsymbol{B}}\boldsymbol{s}\\ \leq&\sum_{i=1}^{N}(-\boldsymbol{s}_i^{\mathrm{T}}\boldsymbol{K}_i\boldsymbol{s}_i + \boldsymbol{s}_i^{\mathrm{T}}\boldsymbol{s}_i + \tilde{\boldsymbol{\eta}}_i^{\mathrm{T}}\boldsymbol{D}_i^{\mathrm{T}}\boldsymbol{D}_i\tilde{\boldsymbol{\eta}}_i) - 2\gamma\tilde{\boldsymbol{x}}^{\mathrm{T}}\tilde{\boldsymbol{x}}\\ &+ \gamma\tilde{\boldsymbol{x}}^{\mathrm{T}}\tilde{\boldsymbol{x}} + \gamma\boldsymbol{s}^{\mathrm{T}}\tilde{\boldsymbol{B}}^{\mathrm{T}}\tilde{\boldsymbol{P}}^{\mathrm{T}}\tilde{\boldsymbol{P}}\tilde{\boldsymbol{B}}\boldsymbol{s}\\ \leq&\sum_{i=1}^{N}(-\boldsymbol{s}_i^{\mathrm{T}}\boldsymbol{K}_i\boldsymbol{s}_i + \boldsymbol{s}_i^{\mathrm{T}}\boldsymbol{s}_i + \tilde{\boldsymbol{\eta}}_i^{\mathrm{T}}\boldsymbol{D}_i^{\mathrm{T}}\boldsymbol{D}_i\tilde{\boldsymbol{\eta}}_i) - \gamma\tilde{\boldsymbol{x}}^{\mathrm{T}}\tilde{\boldsymbol{x}}\\ &+ \gamma||\tilde{\boldsymbol{P}}||^2||\boldsymbol{s}||^2\\ =&\sum_{i=1}^{N}(-\boldsymbol{s}_i^{\mathrm{T}}\boldsymbol{K}_i\boldsymbol{s}_i + \boldsymbol{s}_i^{\mathrm{T}}\boldsymbol{s}_i + \tilde{\boldsymbol{\eta}}_i^{\mathrm{T}}\boldsymbol{D}_i^{\mathrm{T}}\boldsymbol{D}_i\tilde{\boldsymbol{\eta}}_i - \gamma\tilde{\boldsymbol{\eta}}_i^{\mathrm{T}}\tilde{\boldsymbol{\eta}}_i\\ &- \gamma\tilde{\boldsymbol{q}}_i^{\mathrm{T}}\tilde{\boldsymbol{q}}_i + \gamma||\tilde{\boldsymbol{P}}||^2\boldsymbol{s}_i^{\mathrm{T}}\boldsymbol{s}_i)\\ =&\sum_{i=1}^{N}((-\boldsymbol{s}_i^{\mathrm{T}}\boldsymbol{K}_i\boldsymbol{s}_i + \boldsymbol{s}_i^{\mathrm{T}}\boldsymbol{s}_i + \gamma||\tilde{\boldsymbol{P}}||^2\boldsymbol{s}_i^{\mathrm{T}}\boldsymbol{s}_i)\\ &+ (-\gamma\tilde{\boldsymbol{\eta}}_i^{\mathrm{T}}\tilde{\boldsymbol{\eta}}_i + \tilde{\boldsymbol{\eta}}_i^{\mathrm{T}}\boldsymbol{D}_i^{\mathrm{T}}\boldsymbol{D}_i\tilde{\boldsymbol{\eta}}_i) - \gamma\tilde{\boldsymbol{q}}_i^{\mathrm{T}}\tilde{\boldsymbol{q}}_i)\\ \leq&\sum_{i=1}^{N}(-(\lambda_m(\boldsymbol{K}_i) - 1 - \gamma||\tilde{\boldsymbol{P}}||^2)||\boldsymbol{s}_i||^2\\ &- (\gamma - ||\boldsymbol{D}_i||^2)||\tilde{\boldsymbol{\eta}}_i||^2 - \gamma\tilde{\boldsymbol{q}}_i^{\mathrm{T}}\tilde{\boldsymbol{q}}_i).\end{aligned} \tag{28}$$

Choosing

$$\begin{cases}\gamma \geq ||\boldsymbol{D}_i||^2 + 1,\\ \lambda_m(\boldsymbol{K}_i) \geq 2 + \gamma||\tilde{\boldsymbol{P}}||^2,\end{cases} \tag{29}$$

yields

$$\begin{aligned}\dot{V} \leq&\sum_{i=1}^{N}(-||\boldsymbol{s}_i||^2 - ||\tilde{\boldsymbol{\eta}}_i||^2 - \gamma||\tilde{\boldsymbol{q}}_i||^2)\\ \leq&0.\end{aligned} \tag{30}$$

Since $V(t) \geq 0$, inequality (30) means that $V(t)$ is lower bounded, which implies the boundedness of $\boldsymbol{s}_i$, $\tilde{\boldsymbol{\theta}}_i$, $\tilde{\boldsymbol{\eta}}_i$, and $\tilde{\boldsymbol{q}}_i$. Under Assumption 1, $\boldsymbol{v}$ is bounded. Thus, $\boldsymbol{\eta}_i$ is bounded, which implies that $\boldsymbol{q}_i$ is bounded. By Eq. (5), $\dot{\boldsymbol{\eta}}_i$ is bounded. Furthermore, $\dot{\tilde{\boldsymbol{\eta}}}_i$ is bounded. By Eq. (19), $\dot{\tilde{\boldsymbol{q}}}_i$ is bounded, which together with the boundedness of $\dot{\boldsymbol{\eta}}_i$ implies that $\dot{\boldsymbol{q}}_i$ is bounded. By Eq. (11), $\dot{\boldsymbol{s}}_i$ is bounded.

By (28), we have

$$\begin{aligned}\ddot{V} =&\sum_{i=1}^{N}(-2\boldsymbol{s}_i^{\mathrm{T}}\boldsymbol{K}_i\dot{\boldsymbol{s}}_i - \dot{\boldsymbol{s}}_i^{\mathrm{T}}\boldsymbol{D}_i\tilde{\boldsymbol{\eta}}_i - \boldsymbol{s}_i^{\mathrm{T}}\boldsymbol{D}_i\dot{\tilde{\boldsymbol{\eta}}}_i) - 4\gamma\tilde{\boldsymbol{x}}^{\mathrm{T}}\dot{\tilde{\boldsymbol{x}}}\\ &+ 2\gamma\dot{\tilde{\boldsymbol{x}}}^{\mathrm{T}}\tilde{\boldsymbol{P}}\tilde{\boldsymbol{B}}\boldsymbol{s} + 2\gamma\tilde{\boldsymbol{x}}^{\mathrm{T}}\tilde{\boldsymbol{P}}\tilde{\boldsymbol{B}}\dot{\boldsymbol{s}}.\end{aligned} \tag{31}$$

Since $\boldsymbol{s}_i$, $\dot{\boldsymbol{s}}_i$, $\tilde{\boldsymbol{\eta}}_i$, $\dot{\tilde{\boldsymbol{\eta}}}_i$, $\tilde{\boldsymbol{q}}_i$, and $\dot{\tilde{\boldsymbol{q}}}_i$ are all bounded, $\ddot{V}$ is bounded. Therefore, by Barbalat's lemma (Slotine and Li, 1991), we obtain $\lim_{t\to\infty}\dot{V}(t) = 0$. Thus, by inequality (30), $\lim_{t\to\infty}\boldsymbol{s}_i(t) = 0$, $\lim_{t\to\infty}\tilde{\boldsymbol{\eta}}_i(t) = 0$, and $\lim_{t\to\infty}\tilde{\boldsymbol{q}}_i(t) = 0$.

Since $\lim_{t\to\infty}\tilde{\boldsymbol{\eta}}_i(t) = 0$, we have

$$\lim_{t\to\infty}(\boldsymbol{F}\boldsymbol{\eta}_i(t) - \boldsymbol{q}_0(t)) = 0. \tag{32}$$

Note that

$$\boldsymbol{q}_i - \boldsymbol{q}_0 = (\boldsymbol{q}_i - \boldsymbol{F}\boldsymbol{\eta}_i) + (\boldsymbol{F}\boldsymbol{\eta}_i - \boldsymbol{q}_0) = \tilde{\boldsymbol{q}}_i + (\boldsymbol{F}\boldsymbol{\eta}_i - \boldsymbol{q}_0). \tag{33}$$

By Eq. (32), we obtain $\lim_{t\to\infty}(\boldsymbol{q}_i(t) - \boldsymbol{q}_0(t)) = 0$.

By Eq. (18), it is easy to verify that

$$\lim_{t\to\infty}(\dot{\boldsymbol{q}}_i(t) - \boldsymbol{F}\dot{\boldsymbol{\eta}}_i(t)) = 0. \tag{34}$$

By Eqs. (2) and (5),

$$\begin{aligned}\boldsymbol{F}\dot{\boldsymbol{\eta}}_i - \dot{\boldsymbol{q}}_0 =&\boldsymbol{F}\boldsymbol{S}\boldsymbol{\eta}_i + \boldsymbol{F}\boldsymbol{L}\sum_{j\in N_i}a_{ij}(\boldsymbol{q}_j - \boldsymbol{q}_i) - \boldsymbol{F}\boldsymbol{S}\boldsymbol{v}\\ =&\boldsymbol{F}\boldsymbol{S}\tilde{\boldsymbol{\eta}}_i + \boldsymbol{F}\boldsymbol{L}\sum_{j\in N_i}a_{ij}(\boldsymbol{q}_j - \boldsymbol{q}_i)\\ =&\boldsymbol{F}\boldsymbol{S}\tilde{\boldsymbol{\eta}}_i + \boldsymbol{F}\boldsymbol{L}\sum_{j\in N_i}a_{ij}((\tilde{\boldsymbol{q}}_j - \tilde{\boldsymbol{q}}_i) + (\tilde{\boldsymbol{\eta}}_j - \tilde{\boldsymbol{\eta}}_i)).\end{aligned} \tag{35}$$

Thus, we have $\lim_{t\to\infty}(\boldsymbol{F}\dot{\boldsymbol{\eta}}_i(t) - \dot{\boldsymbol{q}}_0(t)) = 0$.

Note that

$$\boldsymbol{p}_i - \dot{\boldsymbol{q}}_0 = (\boldsymbol{p}_i - \boldsymbol{F}\dot{\boldsymbol{\eta}}_i) + (\boldsymbol{F}\dot{\boldsymbol{\eta}}_i - \dot{\boldsymbol{q}}_0). \tag{36}$$

In view of Eqs. (34) and (35), we conclude $\lim_{t\to\infty}(\boldsymbol{p}_i(t) - \dot{\boldsymbol{q}}_0(t)) = 0$, $i = 1, 2, \ldots, N$. The proof is thus completed.

4 An example

In this section, one example is given to illustrate the effectiveness of our main result. In particular, consider four Van der Pol oscillators in the following form:

$$\begin{cases}\dot{q}_i = p_i,\\ \dot{p}_i = \theta_i(1 - q_i^2)p_i - q_i + d_i + u_i,\ i = 1,2,3,4,\end{cases} \tag{37}$$

where q_i, $p_i \in \mathbb{R}$ are the system states, $u_i \in \mathbb{R}$ is the control input, $\theta_i \in \mathbb{R}$ is the unknown system parameter, and d_i is the disturbance.

The reference signal and the disturbance are generated by the leader system as follows:

$$\begin{cases} \begin{bmatrix} \dot{v}_1 \\ \dot{v}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}, \\ d_1 = 2v_2, \\ d_2 = v_2, \\ d_3 = v_1 + v_2, \\ d_4 = v_1 - 2v_2, \\ q_0 = v_1. \end{cases} \tag{38}$$

It can be found that Assumption 1 holds. Fig. 1 depicts the network topology of the five agents. It can be verified that Assumption 2 is satisfied. Then, the distributed control law (9) can be designed as follows:

$$\begin{cases} \boldsymbol{u}_i = -K_i \boldsymbol{s}_i - g_{i0}(q_i, p_i) - g_i(q_i, p_i)\hat{\boldsymbol{\theta}}_i - \boldsymbol{D}_i \boldsymbol{\eta}_i + \ddot{\boldsymbol{q}}_{ri}, \\ \dot{\hat{\boldsymbol{\theta}}}_i = -\Gamma_i \boldsymbol{g}_i^{\mathrm{T}}(\boldsymbol{q}_i, \boldsymbol{p}_i)\boldsymbol{s}_i, \\ \dot{\boldsymbol{\eta}}_i = \boldsymbol{S}\boldsymbol{\eta}_i + \boldsymbol{L} \sum_{j \in N_i} a_{ij}(q_j - q_i), \end{cases} \tag{39}$$

where $g_{i0}(q_i, p_i) = -q_i$, $g_i(q_i, p_i) = (1 - q_i^2)p_i$, $\boldsymbol{D}_1 = [0 \ \ 2]$, $\boldsymbol{D}_2 = [0 \ \ 1]$, $\boldsymbol{D}_3 = [1 \ \ 1]$, $\boldsymbol{D}_4 = [1 \ \ -2]$, $\boldsymbol{F} = [1 \ \ 0]$, $\alpha = 30$, $\boldsymbol{L} = [5.4088 \ 1.6569]$, $K_i = 40$, and $\Gamma_i = 1$, $i = 1, 2, 3, 4$.

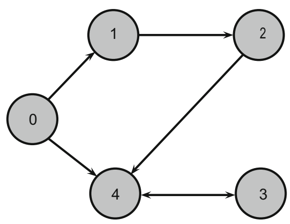

Fig. 1 Network topology G

A simulation was conducted, where the actual values of θ_i were $\theta_1 = 4$, $\theta_2 = 1$, $\theta_3 = 2$, and $\theta_4 = 4.5$, and they were unknown to the controller. The initial condition for the leader system was chosen as $\boldsymbol{v}_0(0) = \mathrm{col}(1, -1)$ and all the other initial conditions were chosen randomly from the interval $[-2, 2]$. From Figs. 2 and 3, it can be seen that the state of the dynamic compensator tends to the state of the leader system asymptotically. Furthermore, the tracking errors of all followers are shown in Figs. 4 and 5, respectively. From the simulation results, we can conclude that the leader-following consensus problem of this example is solved under the proposed adaptive distributed control law.

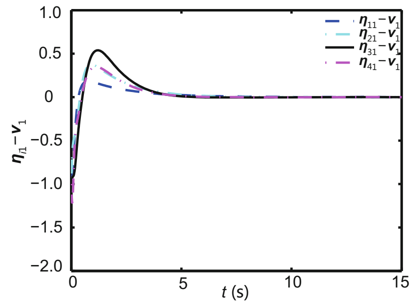

Fig. 2 Observer error ($\boldsymbol{\eta}_{i1} - \boldsymbol{v}_1$) of all followers

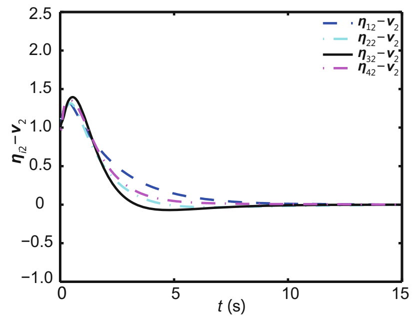

Fig. 3 Observer error ($\boldsymbol{\eta}_{i2} - \boldsymbol{v}_2$) of all followers

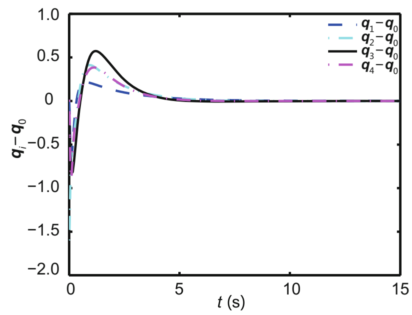

Fig. 4 Tracking error ($\boldsymbol{q}_i - \boldsymbol{q}_0$) of all followers

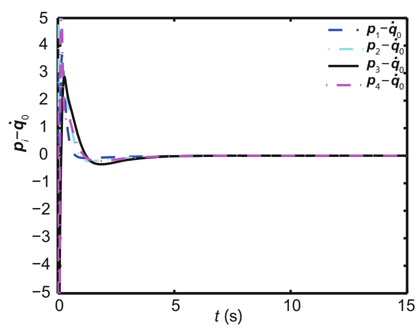

Fig. 5 Tracking error ($\boldsymbol{p}_i - \dot{\boldsymbol{q}}_0$) of all followers

5 Conclusions

In this study, we have investigated the leader-following consensus problem of heterogeneous multi-agent systems with second-order nonlinear uncertain dynamics and disturbances. With the system uncertainty being linearly parameterized, we have proposed a distributed control law by adopting the adaptive control approach. We have shown that the consensus problem can be solved by the proposed distributed control law. The results have been demonstrated by one application to the consensus problem of multiple Van der Pol oscillators. In the future, we will focus on the similar problem for high-order nonlinear multi-agent systems under switching networks.

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