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Review:

Research progress on and prospects for virtual brush modeling in digital calligraphy and painting*

Lei HUANG‡, Zeng-xuan HOU, You-hang ZHAO, Di-jing ZHANG

School of Mechanical Engineering, Dalian University of Technology, Dalian 116024, China

E-mail: 2856396202@qq.com; hou@dlut.edu.cn; 2914514353@qq.com; zxbdlut@163.com

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Abstract: As the crown of the scholar's four jewels, the brush has occupied a cherished position in the culture of traditional Chinese painting and calligraphy since its invention. In virtual painting, virtual brush modeling plays the most important role. A powerful virtual brush model can truly reflect the characteristics of a real brush and enhance the reality of virtual painting. By reviewing the state of the art in virtual brush modeling, we summarize the basic principles, merits, and drawbacks of typical modeling methods, and discuss simulation results based on empirical methods and physical methods separately. The influences of brush-paper, paper-ink, and human-computer interactive devices on virtual brush modeling are analyzed briefly. The main challenges and problems in virtual painting are analyzed, and the prospects for research on virtual brush modeling in the future are put forward.

Key words: Painting and calligraphy; Virtual brush; Human-computer interaction; Force feedback; Spring; Texture mapping
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1 Introduction

As a tool for traditional Chinese calligraphy and painting, the brush is offered by China to the treasure house of world art. As the crown of the scholar's four jewels, the brush has occupied a cherished position in the culture of traditional Chinese painting and calligraphy since its invention. Calligraphers and painters typically express their thoughts and feelings through the help of the expressive power of brushes that can produce rich visual effects. After the beginning of the digital era, the hope was to integrate the characteristics of the brush with modern science and technology to truly simulate the function of a brush and to promote the creation of calligraphy and paintings. "A workman must sharpen his tools if he is to do his work well" (Zhu et al., 2009). To simulate the creation of calligraphy and painting naturally and vividly with the help of modern computer technology, researchers have done a great deal of work researching models for a virtual brush. They have used different modeling methods to construct models for virtual brushes that possess the characteristics of brushes. Although some achievements have been made in this respect, there are still some areas in need of development to realize the ideal goal. In this paper, we review the state of the art regarding the modeling methods of virtual brushes to provide guidance for further development.

‡ Corresponding author

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ORCID: Lei HUANG, http://orcid.org/0000-0003-2504-0589

2 State of the art and classification of virtual brush modeling research

First, we will present the state of the art on virtual brush modeling, and the main classification of the techniques. The basic principles, merits, and drawbacks of typical modeling methods are also discussed.

2.1 Research on virtual brush modeling

Strassmann (1986) first proposed a simple two-dimensional (2D) virtual brush model, summarizing the objects involved in the process of brush painting into four elements: brush, stroke, ink dipping, and paper. This work successfully simulated Japanese "ashtray inks painting." Subsequently, with the research and development of non-photorealistic rendering technology, various virtual brush models have been released. Chua (1990) proposed a brush model based on the outline of brush strokes, which simulates the formation of brush strokes using Bezier curves connected back and forth. Hsu and Lee (1994) proposed a brush model based on vitalization skeleton strokes; the arbitrary patterns and their evolution based on a 2D deformation model were abstracted into "ink" to express the change of brush behavior.

Lee (1997) presented a three-dimensional (3D) virtual brush model with elastic brush hair based on the theory of elasticity, which successfully painted an orchid chart with the characteristics of traditional oriental art. Lee (1999) achieved a new brush model based on the original brush model (Strassmann, 1986) by adding a geometric model of the bristle bundle and its deformation algorithm, and it can better simulate the deformation of the brush. Saito and Nakajima (1999) constructed a brush model using a Bezier curve for the skeleton of the brush head. It uses a quasi-static energy optimization method to drive the Bezier curve to simulate the morphological changes of the brush head. Baxter et al. (2001) constructed a virtual brush model with a spring particle system as the brush skeleton and subdivided the surface mesh as the brush surface. They used the deformation of the tension spring to simulate the deformation of the bristles during the painting process, and used a force feedback device for the creation of interactive art.

Chan et al. (2002) constructed a brush model consisting of many brush hairs distributed randomly in a circular area. The circular area was divided into several small squares, and the position of each brush hair in the square was random. Each brush hair painted a line during the movement of the brush on paper, and all lines were assembled to form a stroke.

Baxter and Lin (2004) simulated the deformation of real brushes, for example, bifurcation and the plastic deformation of brushes, by improving the original brush model (Baxter et al., 2001). They used the energy minimization method to enable a novel geometric representation of the brush head. Their model can generate brushes with differently shaped brush heads, and simulate different strokes. By observing the movement of real brushes, Girshick (2004) discovered that the overall movement of the brush is closely related to the movement of a single bristle. On this basis, a new parametric brush model was constructed. The deformation of the brush was simulated by adjusting the parameters during the painting process. Adams et al. (2004) built a brush model using a point sampling surface to wrap the skeleton of the spring based on Baxter et al. (2001)'s model. The spring skeleton was used to simulate the dynamic behavior of the brush, and the point sampling surface was used to describe the change in the brush surface, and to store painting information.

Yin et al. (2005) proposed a brush model based on a pressure sensitive model. This model consists of a pressure-sensing model and a contact model, used to describe the relationship between pressure sensitivity and stroke width, and the interaction between the brush tip and the paper surface, respectively. It improved the authenticity of the user's writing with the brush, and the fluency in painting complex strokes.

van Laerhoven and van Reeth (2007) proposed a novel design method for a brush model. The geometric model of the brush constructed uses a motion chain as the spine of the brush head and a polygonal mesh as the bristle cluster. The free-form deformable grids associated with the geometric model were used to deal with various shapes of brush heads in the dynamic model. According to user input actions, an energy-optimization method was used to describe the deformation of the brush head. Baxter and Govindaraju (2010) proposed a data-driven 3D virtual brush model in which the measurement data of the actual brush deformation was stored in tables. The deformation of the virtual brush was simulated dynamically using the data in the table. This model can effectively simulate the complex behavior of the brush, and had higher computational efficiency and good numerical stability.

We can find that the above systems either model a brush using a deformable model without individual bristles (Chu and Tai, 2002; Baxter and Lin, 2004), or model a bristle brush but ignore the simulation of 3D paint completely (Sun et al., 2009; Zhu et al., 2015).

Chen et al. (2015) presented a real-time painting system that can simulate the interactions among the brush, paint, and canvas at the bristle level. They developed a variety of techniques to ensure the performance and robustness of the simulator under large time steps, including brush and particle simulations in non-inertial frames, and a fixed-point method for accelerating the Jacobi iterations. The whole system was implemented on a graphics processing unit (GPU) by the Compute Unified Device Architecture (CUDA), and overcame several challenges including the large time step, inaccuracy, and liquid transfer. However, it places a limit on the stroke speed, so artists need more time to finish paintings in their system than in the real world.

Meyer et al. (2016) proposed a tactile paintbrush to generate spatial haptic texture. Their brush can provide a method for creating stochastic friction patterns by drawing samples from a Weibull distribution for each fine-texture wavelength.

Otsuki et al. (2018) developed a mixed reality (MR) painting system, and used a visual and haptic feedback to provide the sensation of painting on virtual 3D objects using a new brush device called the "MAI Painting Brush++." They developed a mechanism that can simulate the effect of touch and movement when the brush device is used to paint on virtual objects in the MR space, and proposed an extended model that consists of two sub-models: friction-force changing model and brush-tip spring model. They confirmed the effectiveness of the device and proposed a brush model through various user studies. However, the real-time performance of the rendering system needs to be further improved.

The research on virtual brushes in China is comparable to that in foreign countries. Researchers from many universities and research institutes in China carried out research on virtual brushes from different perspectives in the mid and late 1990s. Yu et al. (1996) proposed a skeleton-based brush model. Based on the actual brush and actual painting process, a scatter point set was used to simulate the stroke of the brush. By changing the guiding parameters of the defined skeleton attribute function to control the distribution shape, color, and density of scatters, a more realistic painting effect can be achieved.

Ip and Wong (1997) proposed a virtual brush that uses the parametric method to synthesize how real calligraphy works. The appearance of real calligraphy can be simulated well by controlling the physical parameters related to the writing process. Wong and Ip (2000) simulated different styles of calligraphic writing and realized the design and generation of scalable fonts. Yeh et al. (2002) constructed a brush model using a bending spring as the skeleton. The brush hairs consisted of many particles connected to the bending spring and were arranged on the eight directions of the skeleton model, and the particle system was used to simulate the change of the skeleton of the brush head. Chu and Tai (2002) proposed a more accurate and effective 3D virtual brush model, by using an energy minimization method to simulate the change in the behavior of the brush, and designing a new input device, which can better simulate the different effects of the brush such as splitting of the brush head, flat head, and half-dry stroke. Xu et al. (2002) proposed a virtual brush model based on solid modeling technology, defining the primitive element as the smallest working unit of the brush model and using the aggregation of several primitives to represent the geometric shape of the brush head. The instantaneous interaction between all primitives and the paper with the cumulative effect of time forms the painting effect. Guo et al. (2002) proposed a pressure-sensitive brush model based on experimental experience and actual writing experience. Based on improvements in commercial software such as Photoshop and Elf Brush, and using a circle model as closed graphics to simulate strokes, the Bessel method was used to connect control points to form a closed curve. Then the closed curve was changed by zooming and translation to simulate the brush strokes in real time. Mi et al. (2003) proposed an experience-based brush model, which considers only the change in the contact area between the brush and the paper surface and simulates the stroke by defining the parametric "raindrop" model driven by this basic action. In the same year, Xu et al. (2003) improved the original brush model (Xu et al., 2002) and constructed a hierarchical brush model that can express the scattering effect. Sun et al. (2005) combined the empirical model with the physical model and proposed a new 3D brush model. In the painting process, this model avoids the complexity of calculating the deformation of each bristle, greatly improving the computational efficiency and real-time interaction, and better simulating the deformation and painting effect of the brush. Bai et al. (2008) proposed a virtual brush similar to Wong and Ip (2000)'s brush model. In the process of simulating the deformation of the brush, the impression produced by the interaction between the brush and paper was used to simulate the brush stroke considering the influence of the internal and external forces acting on the brush. At the same time, according to the force acting on the brush, a set of spring meshes was used to calculate the deformation of the brush accurately. Zhu et al. (2009) proposed a virtual brush model based on statistical analysis, using the statistical analysis model to simulate the contact area between the brush and the paper surface. This model does not need to consider complicated situations such as brush modeling and brush deformation, has high computational efficiency and good real-time performance, and can achieve a better writing effect. Zhang et al. (2010) presented a virtual brush model based on triangular mesh, using a connected line segment and Bezier curve as the central axis of the brush head, and using the triangular mesh model to simulate the bristle bundle. During the writing process, the triangular mesh is driven to change by adjusting the central axis to simulate different morphological changes of the brush. Zhang et al. (2014) used statistical methods to construct a parametric brushstroke model to simulate the shape of different types of pens. Combined with actual calligraphy experience, it can better generate writing animation with different styles of calligraphy.

Tang et al. (2015) modified the tactile brush algorithm to support the smooth motion of rectangular contact areas, which can be used to convey complex touch gestures involving several simultaneous points of contact.

Yhang et al. (2015) presented a model to simulate a virtual Chinese hair brush using a camshaft curve, a novel analytic curve, to define the shape of the brush stroke. Using the camshaft curve, the shape of the stroke can be expressed flexibly via few adjustments of the parameters instead of simulation of a huge amount of brush tuft.

Li et al. (2005) proposed a calligraphy imitation system based on a virtual brush and the scale-invariant feature transform (SIFT). In their system, a virtual brush simulation algorithm was proposed. According to the rate and strength of movement of the finger on the touch screen, the trajectory of a virtual brush can be simulated, the thickness of which can be set dynamically. The system can imitate and match Chinese calligraphy based on the excellent performance of SIFT with rotation, scale, viewpoint, and illumination change, and achieves high accuracy and fast response.

Guo et al. (2017b) proposed a novel 3D interactive painting method for Chinese painting and calligraphy via real-time force feedback. The relationship between the force exerted on the brush head and the resulting brush deformation has been analyzed for the first time, and a virtual spring-mass model was applied to construct a model of the 3D brush. The 3D brush footprint can be obtained via projecting the 2D brush footprint onto the surface of the 3D virtual object in real time. The proposed methods have been applied in a virtual 3D interactive painting system via haptic feedback technology, which can effectively enhance the reality of the virtual painting process. However, their brush model does not take into account the mechanical properties of the variable stiffness that exists in a real brush.

2.2 Classification of virtual brush modeling research

Researchers have carried out various types of research on brush modeling from different perspectives, and many important results have been obtained. According to the simulation results, virtual brush modeling can be divided into empirical brush modeling and physical simulation based brush modeling.

2.2.1 Empirical brush modeling

Empirical brush modeling uses the change in the contact area between the brush and the paper surface to simulate the brush painting effect based on summing up the practical experience in creation of calligraphy and painting and much experimental experience. By adjusting the parameters obtained via experience to change the shape of the contact area, the simulation of different effects can be realized, so this approach has simple and fast calculation and good real-time performance. However, this model does not consider the characteristics of the actual brush and some other factors such as the fickle nature of brushwork, which inevitably leads to a certain discrepancy between the simulated and real effects.

2.2.2 Physical simulation based brush modeling

Physical simulation based brush modeling considers physical factors such as the shape of the brush head, elasticity, and deformation of the brush hairs to construct the physical model of a brush. On this basis, this model uses physical laws or rules to control the dynamic change in the brush during the painting process. It not only makes the creative process more realistic and intuitive, but also makes the simulation effect rival the real effect. However, the calculation process for the physical model is generally complicated, leading to inefficiency, and sometimes it cannot guarantee a real-time interaction.

3 Detailed analysis of modeling methods for virtual brushes

The two main types of modeling methods for virtual brushes are discussed here in detail.

3.1 Brush modeling based on empirical simulation

We detail various methods used in empirical brush modeling in this subsection.

3.1.1 Empirical parametric brush model

Mi et al. (2003) proposed a parametric virtual brush model based on experience, which simulated the change in the contact area between the brush and the paper surface through the "raindrop" model. The basic interactive actions and parameters of the brush are defined in this model. The basic interactive actions include dipping the brush in ink, pressing the brush, brush movement, and brush rotation; the parameters of the brush include not only the basic parameters such as length, diameter, and number of bristles, but also parameters such as speed, humidity, and pressure based on experience. During the painting process, the basic interactive actions are used to change the parameters of the brush in real time; the change in the parameters of the brush leads to heteromorphosis of the "raindrop" model. The morphological changes of the brush are simulated by the merging and splitting of "raindrops" to realize the simulation of different painting effects. This method does not construct the physical model of the brush, avoids the complicated situation of modeling and calculating the brush deformation, reduces the amount of calculation, improves the real-time interaction, and can better simulate the effect of a half-dry stroke. However, some factors such as the characteristics of the actual brush and the fickle nature of brush writing are not considered, which inevitably leads to a certain discrepancy between the simulated and real effects.

3.1.2 Brush model based on the pressure-sensitive model

Guo et al. (2002) constructed a brush model based on a pressure-sensitive model according to experimental experience and practical writing experience. In the painting process, according to the relationship between pressure and stroke width, the Bessel method is used to connect control points to form a closed curve based on improving the previous methods that use the circle as closed graphics to simulate strokes. The closed curve is changed by zooming and translation to simulate brush strokes in real time. Using relevant knowledge of graphics to optimize the processing, a more realistic stroke effect can be achieved. This model can simulate some painting effects of brushes and make users feel the realism of using actual brushes in the painting process. However, the pressure-sensitive brush model is based on limited experience, which inevitably leads to a large discrepancy between the simulated and real effects. In addition, the brush model based on the pressure-sensitive model includes the model proposed by Yin et al. (2005).

3.2 Brush modeling based on physical simulation

We detail various methods used in physical simulation based brush modeling in this subsection.

3.2.1 Strassmann's brush model

Strassmann (1986) designed a painting system that includes four representative elements: brush, strokes, dipping in ink, and paper. A brush is regarded as a collection of one-dimensional (1D) arrays of brush hair; each brush hair has its own information such as ink volume, relative position, and color. The stroke is defined as the set of parameters including position and pressure, whose shape is determined by the fitting curve with control points specified by the user, and whose width is determined by the pressure function at the interpolation nodes between control points. Dipping in ink is used to express rich and varied stroke effects, restoring the changed brush to its original state. Paper is used to receive information from the brush hairs, map its texture onto the stroke, and display the painting effect. The information contained in the brush hairs is updated with the movement of the brush. If the brush hairs are fully in contact with the paper surface and contain a certain amount of ink, then the updated information contained in the brush hair will be transmitted to the paper through ink-wash. When the paper receives the information, it will display the corresponding ink marks drawn by the brush. Although this model can simulate some painting effects of the brush, the amount of calculation is too large, and it takes 1–2 min to generate a stroke, so the real- time interaction is poor.

Moreover, the strokes generated are too rigid, regular, and inflexible. From the point of view of brush modeling, this model does not consider the characteristics of the actual brush or variability of the brush-motion style. On the construction of a paper model, only the texture mapping of paper is considered, and the properties of paper such as absorptivity and humidity are not considered. Therefore, the simulation of ink diffusion and stroke superposition needs to be further improved. In addition, there is a certain difficulty in controlling the pressure acting on the brush and stroke rotation during the painting process. Strassmann's work is seen as setting a precedent in brush modeling, and a model providing inspiration for follow-up researchers.

3.2.2 Wong et al.'s brush model

Considering the shortcomings of Strassmann's model in stroke generation, Ip and Wong (1997) and Wong and Ip (2000) proposed a virtual brush model consisting of a brush geometry model and a brush dynamic model. When constructing the geometric model of a brush, the emphasis is on modeling the brush head. By specifying parameters such as the radius of the circle of the brush root, the length of the brush head, and the number of bristles, they constructed a geometric model of the brush head with an inverted conical shape (Fig. 1).

In the dynamic model of the brush, the shape of the intersection area between the brush and paper is described by an ellipse, which is considered the shape

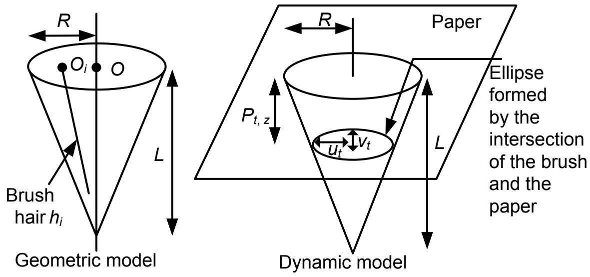

Fig. 1 Wong et al.'s virtual brush model

of the footprint. The elliptical footprint changes as the brush is moved. To better control the change in footprint shape, the long axis and short axis of the ellipse need to be multiplied by a change factor separately. In addition, to show the phenomenon of brush dispersion and rotation of specific strokes caused by some brush movements, it is necessary to increase the dispersion factor on the long and short axes of the ellipse and add a specified rotation increment on the default rotation angle. Considering the influence of the various factors above, a series of closures of elliptic clusters with different shapes and orientations is used to fill the stroke trajectory in the painting process. Combined with the ink on paper deposition model, the stroke generated can better reflect the effects of pen-lifting, pen-pressing, and pen-spinning in calligraphy. Wong and Ip (2000) presented a parametric geometric model and dynamic model for virtual brushes, making them more flexible. It can not only simulate some of the stroke effects, but also vividly express the characteristics of brush writing. However, Wong et al.'s brush modeling is based on bristles, which are characterized by a large amount of calculation and poor real-time performance. The model also requires manual setting and adjustment of complex parameters, so the operation is complex and inefficient. In addition, considering the soft and changeable brush head as an inverted cone leads to a phenomenon where the brush head penetrates the paper surface in the painting process. It cannot simulate the effects of the side-front, and is inconsistent with the actual painting situation. Bai et al. (2008) improved Wong et al.'s model using a set of vertices instead of a single bristle as the basic unit for brush modeling, and they changed the bristles from being inclined to the main axis of the cone in the geometric model of the brush to be parallel to the main axis of the cone. This approach simplifies the model and the calculation becomes more manageable.

Considering the impact of internal and external brush forces on bristle deformation, the brush deformation is simulated using the imprint produced by the interaction between the brush and paper during the painting process. This method reduces the simulation complexity and improves real-time interaction. At the same time, according to the force acting on the brush, a set of spring meshes can be used to accurately calculate the deformation of the brush to obtain a more realistic painting effect.

3.2.3 Brush model based on solid modeling technology

Xu et al. (2002, 2003) built a virtual brush using solid modeling techniques similar to Wong et al.'s modeling approach. In the brush modeling process, the concept of a "primitive element" (the basic painting unit) is proposed to reduce the modeling complexity and the amount of calculation. The primitive element is defined as a geometric entity that is composed mainly of a bottom control circle C_i, a middle control ellipse E_i, a central control axis A_i, and a tip control line L_i (Fig. 2).

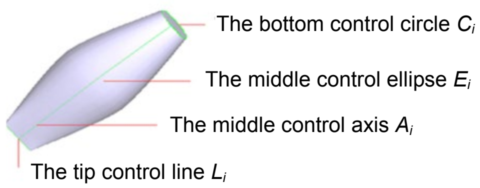

Fig. 2 Primitive element in Xu et al. (2002)'s brush model
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Among the four characteristic parameters of the primitive element, C_i is the state feature parameter that will not be changed after the initialization; A_i, E_i, and L_i are dynamic feature parameters which will be adjusted in real time to reflect the morphological changes of the primitives. During the creative painting and calligraphy process, the primitives are the smallest units of work: they are independent of each other. A primitive represents a bunch of brush hairs; several primitives are gathered to describe the geometric shape of the brush head (Fig. 3). The instantaneous interaction between all primitives and the paper over time is built into the painting effect.

In the whole painting process, the brush model corresponds to three states: initial state, dipping state, and working state. In the initial state, A_i, E_i, and L_i are reduced into straight lines, circles, and points, respectively; the primitive is given related information such as color and humidity of ink when dipping the ink, and the bifurcated brush tip can be reset to the initial state. The working state is the brush deformation process such that different painting effects can be obtained by controlling the three dynamic feature parameters.

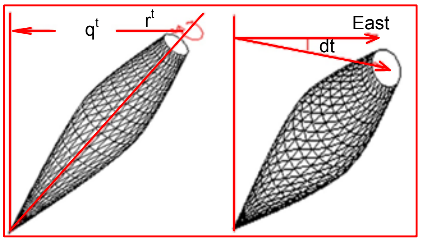

Fig. 3 The geometric shape of the brush head in Xu et al. (2002)'s model
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This model can simulate very realistic calligraphy and painting effects, and it does not require users specify a large number of parameters manually, so it is easy to operate. However, operations for the representation and intersection of complex surfaces are required during the painting process. Thus, it is computationally complex, inefficient, and slow. Furthermore, the brush head cannot be automatically restored after the bifurcation phenomenon occurs, and it can only be reset to the initial state by dipping ink.

3.2.4 Girshick's brush model

By observing the movement of real brushes, Girshick (2004) found that the overall motion of the brush is closely related to the movement of a single bristle. Inspired by Xu et al. (2002)'s brush model, a new parameterized brush model was proposed that simulates the deformation of the brush during writing by adjusting the parameters. This model defines a hierarchical structure consisting of a control volume and its "child" brush bundle (Fig. 4).

The brush bundle is defined as a Bezier curve with four control points, and is the smallest unit in the brush model. Depending on the resolution of the rendering, a brush bundle can be either a single bristle or a collection of adjacent bristles. The control volume is intuitively considered a shell formed by sweeping a defined geometric model along the control curve. Specifically, it consists of a control curve CA, a control line CL, a control ellipse CE, and two control circles CC_1 and CC_2, where CA is a Bezier curve for defining the central axis of the control volume, CL and CE are used to define the last two control points of each bristle bundle, and CC_1 and CC_2 are used to define the first two control points of each bristle bundle. Girshick (2004) used a simple approximate geometric model to simulate the deformation of the control volume based on an intuitive method: when the brush was moved to a certain distance on the paper, the brush holder would be moved to the same distance, while the brush tip was moved to only a part of the distance, which was proportional to the pressure of the brush acting on the paper. The pressure was proportional to the speed of the brush, the elasticity of the brush hair, the curvature of the brush hair, and the roughness of the paper.

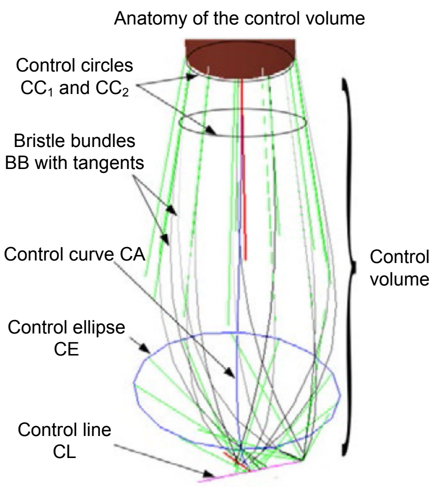

Fig. 4 Girshick (2004)'s brush model

In the painting process, the shape of the control volume was changed by adjusting these parameters to achieve deformation of the brush. The deformation of the control volume would cause the change of the control points on each brush bundle, causing the change of the brush shape. Adjusting the parameters of the control points on the brush bundle according to certain rules can also cause the changes of the inner bristles such as aggregation of the bristles and bifurcation of the brush head. This model uses simple and flexible geometric models to represent the internal structure of real brushes, which simplifies the model and improves the computational efficiency. Users can achieve very realistic artistic effects without manually configuring complex and difficult parameters. Yet, Girshick (2004) concentrated only on constructing the parametric brush model and cannot carry out further research on other aspects. For instance, the rendering to painting result is obtained using only a simple texture mapping method, without considering factors such as ink diffusion, shade change, and stroke change, and without simulation of special effects such as a half-dry stroke.

3.2.5 Brush model based on elasticity

Lee (1999) constructed an inverted conical brush model with elastic brush hair (Fig. 5) based on Strassmann's brush model and elasticity theory.

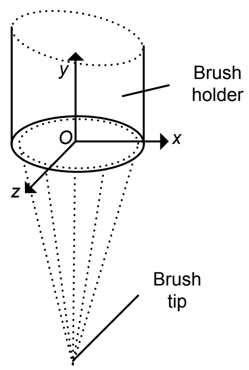

Fig. 5 Lee (1999)'s virtual brush model

When describing the bending deformation model of a single bristle, Lee (1999) regarded the bristle as an elastic rod (Fig. 6) with isotropic elements, and used the elastic deformation of the elastic rod to simulate the deformation of the bristle.

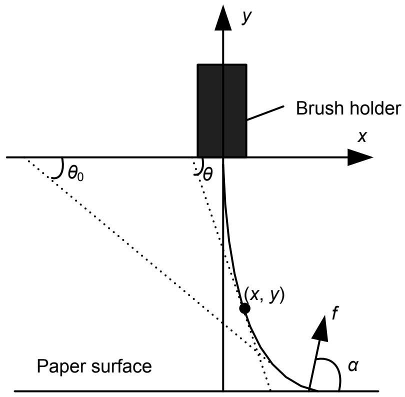

Fig. 6 Bending deformation of a single brush hair

If one end of the bristle is fixed on the brush holder, and a force with a certain angle is applied on the other end, the coordinates of a point on the bristle and the expressions of the general bending shape of the bristle can be obtained according to the pure bending equation of the bristle, when the bending plane of the bristle is assumed. Because a single bristle is used to simulate the effect of a brush, it is necessary to calculate the deformation of each bristle, which results in a large amount of calculation, low efficiency, and unsatisfactory painting effect. Therefore, Lee (1999) studied the geometric model and deformation of the brush bundle, and used the brush bundle to simulate the painting effect of the brush. The geometric model of the brush bundle was related to the length, quantity, and distribution of the bristles, so the number of bristles was quantified, and each bristle was marked. To simplify the calculation, the central and surface bristles were defined. The central bristles were used to determine the deformation of the brush bundle, the stroke center-line, and other parameters. The surface bristles were used to obtain the stroke contour at any time. Due to the interaction between the bristle and the paper surface, the deformation of the brush bundle was different from that of a single bristle. Therefore, the bending deformation model of a single bristle was used to calculate the shape of the brush bundle, and then the shape of the brush head was regulated by the deformation generated inside the brush bundle (Fig. 7). Finally, an orchid diagram with the characteristics of traditional oriental art was successfully painted. This model can make users experience a more authentic real-time painting process, and provide rich visual feedback. It can simulate some characteristics of the actual brush behavior. Different painting effects can be simulated by adjusting the bristle parameters, such as bristle hardness, bristle length, bristle quantity, and texture selection of the paper model.

However, this model also has many shortcomings: the number of bristles on the brush head is small, which is not consistent with real brushes, and this would affect artistic expression; when increasing the number of bristles, the simulation efficiency and real-time performance would be degraded. Although the brush is regarded as an elastic rod with isotropic elements that can show the soft characteristics of brush hair, it was not consistent with the characteristics of a real brush which has different areas of softness and hardness (from the root to the tip). This reduces the expressive effect of the brush model. This model can only create paintings with strokes perpendicular to the stroke path, so it lacks a tilt effect.

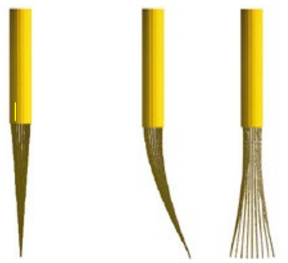

Fig. 7 Shape of the bristle bundle under different states
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3.2.6 Chu's brush model

Inspired by Baxter et al. (2001)'s model, Chu and Tai (2004) proposed a more accurate and effective 3D brush model to simulate the force and deformation of the brush head using energy minimization methods. They constructed the geometric model of a brush with a hierarchical structure including two layers: brush skeleton and brush surface (Fig. 8).

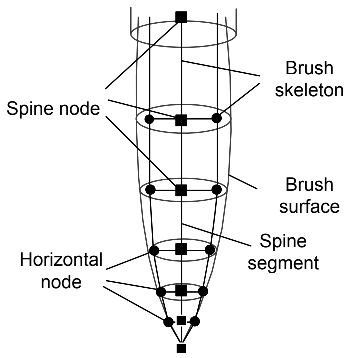

Fig. 8 Chu et al.'s brush model (Chu and Tai, 2004)

The brush skeleton is composed of the spine and transverse nodes. The spine is formed by successively connecting the spine nodes, simulating the bending deformation and rotation of the brush head. The length of the spine segment becomes shorter and shorter along the direction (from the brush root to the brush tip), and the spine nodes are denser and denser, showing the characteristics of the brush tip and belly, which are often used and whose brush hairs are softer. At the same time, two transverse nodes are defined for each spine node, and the connecting line of the two transverse nodes passes through the spine node, and is perpendicular to the spine segment. It is used mainly to simulate the deformation and bifurcation of brush tufts. The brush surface is formed by sweeping different elliptical cross sections (Fig. 9) along the spine, which is used to simulate the contact trajectory between the brush and the paper surface, according to the position of the spine nodes and the transverse nodes in the painting process.

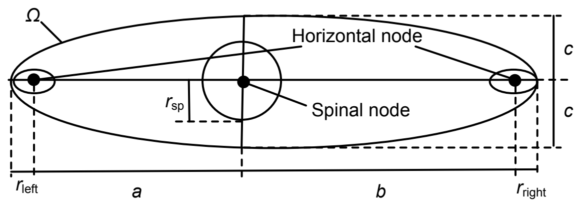

Fig. 9 Tuft cross section for Chu et al.'s brush model (Chu and Tai, 2004)

In the painting process, the energy to deform a brush head includes the deformation energy of the brush hair cluster and the friction energy between the brush hair cluster and the paper surface. These two types of energy are determined by the bending angle of the spine and the positive pressure between the brush head and the paper surface, respectively. In the dynamic model, the stable state of the brush in a given time step is determined using energy minimization methods, and then the dynamic change of the brush is simulated.

In addition, Chu and Tai (2004) constructed a unique input device to install ultrasonic devices and micro-gyroscopes on real brushes to detect the position and posture of the brush (Fig. 10). In this way, users can paint with real brushes in hand.

This model can simulate the deformation of the brush very well, especially the extension and bifurcation of brush hairs. However, the numerical integration using the energy minimization method is time-consuming, and this greatly limits the number of bifurcations of brush hairs (no more than five). In addition, the number of bifurcations needs to be set manually in advance according to the painting conditions, and in their brush mode the multiple bifurcations of the brush head after the ink in the brush head gradually dries out cannot be simulated. Although the input device constructed allows users to hold a real brush to paint, the gyroscope is too heavy and inconvenient to operate, which affects the painting process.

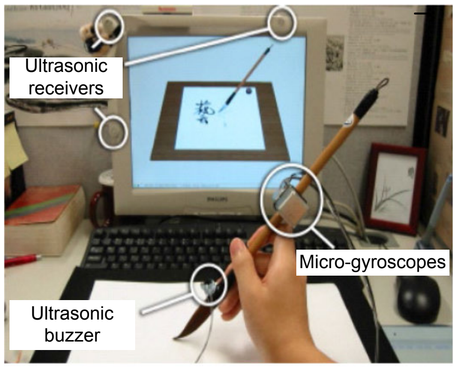

Fig. 10 Chu et al.'s input device for a virtual brush
Reprinted from Chu and Tai (2002), Copyright 2002, with permission from IEEE

Sun et al. (2010) improved Chu and Tai (2005)'s model by applying an empirical method to the construction of the physical brush model, and constructed a geometric model with the brush hair element as the basic unit of movement and deformation. Thus, the model structure was simplified and the complexity reduced. When describing the dynamic control of the brush model, the complex physical equation is not used, but the dynamic change in the brush is obtained based on an empirical method, which can significantly reduce the computational load to improve the real-time interaction, and simulate the brush deformation and rendering effect very well.

3.2.7 Huang et al.'s brush model

Inspired by Chu et al.'s model, Huang et al. (2018) proposed a novel haptic decorating method which was applied to the surface of virtual clay (Fig. 11).

The brush surface is represented as triangular mesh surfaces, which are formed through a skin defined by the center skeleton and a series of outline controlling planes, from top to bottom. The rendering effect of their virtual brush is shown in Fig. 12.

The relationship between brush deformation and endured force was researched for the first time by applying the spring-mass model to construct the 3D brush. According to the actual characteristics of the brush, 3D brush models including geometric and mechanical models are constructed in their 3D painting system. The schematic of the deformation of the brush skeleton is shown in Fig. 13.

When using different pressures on the virtual brush, the brush deformations are shown in Fig. 14.

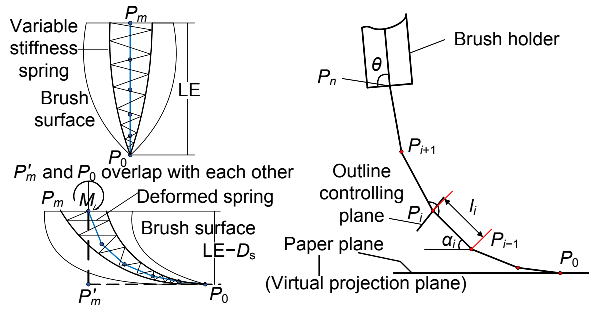

Fig. 11 Huang et al.'s brush model and deformation, including a geometric model and a skeleton model

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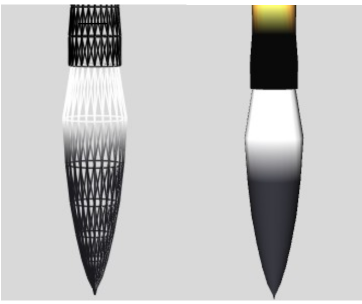

Fig. 12 The rendering effect of Huang et al.'s brush model

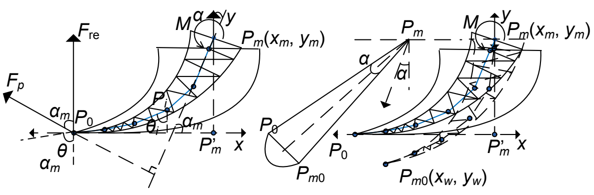

Fig. 13 Schematic of deformation of the brush skeleton

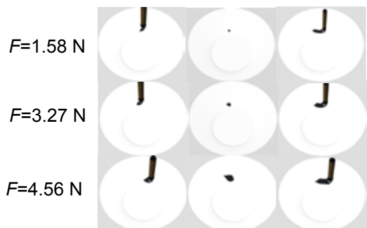

Fig. 14 Virtual brush deformations when exerting different pressures on the brush

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With this system, the 3D brush strokes with 3D half-dry and ink diffusion results can be painted with a Phantom Desktop haptic device, which effectively enhances reality for users. As a simple simulation example, the Chinese characters "七上八下" are painted on the exterior of a bowl (Fig. 15).

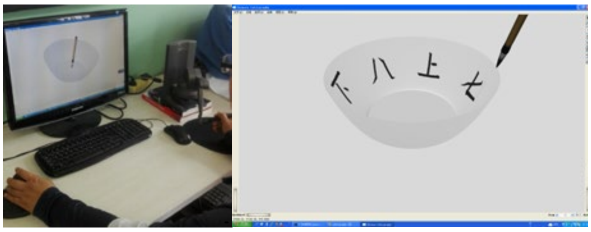

Fig. 15 A simple simulation example using Huang et al.'s haptic painting system

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4 Other related problems in virtual brush modeling

When putting forward virtual brush modeling methods, we found that researchers had not only focused on brush modeling, but also considered the influence of interactive factors for paper, ink, and input equipment on the painting effect, to simulate the brush effect naturally and vividly. This section gives a brief description of three aspects of the input equipment used in pen-paper, paper-ink, and human-computer interaction in virtual painting.

4.1 Pen-paper interaction

Pen-paper interaction is a process where the brush states are displayed on the paper through ink and wash after touching the paper surface in the virtual painting process. Strassmann (1986) provided each brush hair with information on the location and ink volume. When the brush touches the paper surface, the information contained in the brush hairs was transmitted to the paper for display, and subsequent updates were made with the movement of the brush. Yu et al. (2002) used circles or rectangles representing the area of the footprint formed on paper by brushes. Wong and Ip (2000) adopted elliptic sets to represent the footprint of the brush remaining on the surface of the paper, but the tip of the brush penetrated the surface of the paper, affecting the performance of the vivid stroke effects in the painting process. Bai et al. (2008) simulated the writing effect using the impression produced by the interaction between the brush and paper. Xu et al. (2005) used an instantaneous interaction between paper and all basic elements in the brush model over time as the painting effects. Girshick (2004) used texture mapping to represent the trace of the brush left on paper, and prevented the brush hair from penetrating the paper surface, which would affect the painting effect. Mi et al. (2003) used a parametric raindrop model to simulate the contact area between the brush and the paper surface. When drawing brush strokes, Lee (1999) first specified the path, then used the Catmull-Rom fitting curve to generate boundary lines, and filled the inside of the strokes at the end. Chu and Tai (2004) used elliptical cross sections consisting of two semi-ellipses with the same short axis but different long axes to represent the brush strokes formed on paper. Sun et al. (2010) projected the key points and auxiliary points intersecting or penetrating the paper surface on the spine of the brush onto the paper surface to obtain the set of points. Then the outline of strokes can be obtained by curve fitting the point set, and the ink was used to fill the interior of the contour at the end.

4.2 Paper-ink interaction

Paper-ink interaction is a process of transmission, diffusion, and deposition of ink on the paper. Kallmes and Corte (1960) proposed a paper model with fiber structure where ink on the initial “paper element” can be transmitted to the adjacent “paper element” through the fiber structure. Guo et al. (2002) improved the paper model of Kallmes and Corte (1960) such that the diffusion of ink through fibers is simulated as a 1D filtration process. Yu et al. (2002) proposed a local equilibrium model to simulate the process of ink diffusion. Lee (2001) proposed a “wave” mechanism to simulate ink flow between fiber grids of paper. Chu and Tai (2004) proposed a novel fluid-flow model based on the Boltzmann lattice equation to simulate ink diffusion. Sun et al. (2005) proposed a rice paper based model and a simulation algorithm of ink transmission, which realized the simulation of ink diffusion and superposition of brush strokes naturally and vividly. Shi et al. (2003) proposed a particle system based approach to simulate the permeation and diffusion of ink. Zhang et al. (2014) used a fluid simulation model to simulate the diffusion effect of ink-water; on this basis, a fractal theory was applied to simulate the profile of ink-water diffusion. Guo et al. (2015) used a spring-mass model to build a model of the 3D Chinese brush and analyzed the resulting brush deformation.

4.3 Human-computer interactive devices

To verify the performance of the brush model, researchers employed different input devices for human-computer interaction. Strassmann (1986) and Wong and Ip (2000) used a combination of traditional mouse and keyboard to operate virtual brushes and implement parameter inputs. Although the needs of interaction can be met to some extent, the parameters that can be input are limited, and the operation is more complex. In addition to the combination of the mouse and keyboard, Chu and Tai (2004) used a pressure-sensitive pen to implement painting. Freedom information of the brush can be obtained by a pressure-sensitive pen combined with a digital board. Not only is it easy to operate, but it can make users experience the same feeling as when painting with a real brush. However, there are some limitations in detecting the tilt angle of the pen-holder. Zhu et al. (2009) employed a handwriting board, called the F-tablet, on which to write, which can not only acquire dynamic information such as 3D force and writing speed in the writing process, but also record static information such as pen shapes and strokes. However, the synthetic application of force components parallel to the paper surface needs to be improved. Chu and Tai (2005) installed ultrasonic devices and micro-gyroscopes on real brushes to obtain real-time six-degree-of-freedom (6-DOF) information of brushes during painting. However, the gyroscope is bulky, heavy, and inconvenient to operate. Huang et al. (2018) used a force feedback device with 6-DOF posture input for human-computer interaction, allowing users to perceive changes in pen-holding strength. However, this type of device is expensive and does not conform to the traditional pen-drawing method and usage habits.

5 Prospects

Research on virtual brush modeling has undergone nearly 30 years and remarkable progress has been achieved. However, there is still a certain discrepancy with respect to the actual style of expression and the painting effect in Chinese painting and calligraphy. According to the research on virtual brush modeling and related problems, we infer that future research can be carried out in the following respects:

1. More in-depth research on the brush model. The existing brush models have many shortcomings, and some brush characteristics and special effects cannot be simulated well, for instance, different degrees of softness and hardness of the brush hairs for different types of brushes, random bifurcation of brush hairs, and twisting and distortion of brushes. Different types of virtual brushes can be constructed according to various actual brushes. In the process of brush modeling, a brush modeling method based on a combination of physical simulation and the brush modeling method based on empirical simulation can be considered. Through complementary advantages, different brush painting effects can be simulated naturally while ensuring computational efficiency and real-time performance.

2. Further research on paper-ink interaction. The existing paper model and the ink diffusion model can simulate the general effect of paper-ink interaction. However, there is still difficulty in the simulation of more complex painting effects, such as the superposition of strokes and the gradual change of ink color in the strokes. The fiber structure of rice paper and the transmission mechanism of ink on rice paper should be studied in depth in the future, and the paper-ink interaction can be realized by constructing a more effective rice paper model and ink diffusion model.

3. Construction of the color model. Because of the rich color changes in Chinese painting, how to use color reasonably, simulate the bending of various pigments, and synthesize multi-layer pigments in the computer are also challenges. We can construct a new color mixing model and design a new color simulation algorithm based on Kubelka-Munk theory. In addition, multi-color simulation in painting can be realized through the construction of a palette module.

4. Stylization and automatic generation. At present, research related to the writing of different styles of fonts, realization of special effects in calligraphy and painting, and automatic stylization of paintings has not been conducted. In the future, by studying the rules governing the creation of painting and calligraphy, creative styles, and the formation mechanism of special effects, automatic stylization of calligraphy and painting creation can be achieved using artificial intelligence technology. Furthermore, calligraphy and painting works of different styles can be automatically generated using a computer through the establishment of a database containing different styles and characteristics of painting.

5. Real-time performance. In the process of virtual painting, the dynamic deformation of the brush, diffusion of ink, and change of color need to be achieved by calculating a great deal of data, which leads to inefficiency and degradation of real-time performance. We can begin with two aspects to improve the computational efficiency and real-time performance: introducing multithread parallel processing technology to improve the system sampling frequency and using GPU-based hardware acceleration technology to speed up graphics processing and enhance the reality of the painting.

6. A new type of input device. The forces acting on the brush and the posture of the brush are important factors that affect calligraphy and painting effects in the calligraphy and painting processes. With the rapid development and wide application of microelectromechanical systems (MEMS) technology, a MEMS force sensor and a MEMS inertial measurement unit can be integrated to construct a simulation brush based on MEMS that is used for real-time detection of pressure and gesture of the brush in the painting and calligraphy processes. The influence of pressure and posture of the simulated brush on virtual brush deformation should be further studied to control the virtual painting process and enhance the sense of reality in the painting process.

6 Conclusions

The computer simulation of Chinese painting and calligraphy has achieved progress. However, there are still some problems in brush modeling, construction of the ink diffusion model, and the formation mechanism of Chinese painting and calligraphy. In the future, interdisciplinary research in computer graphics rendering technology, artificial intelligence technology, and MEMS technology should be considered comprehensively to promote the computer simulation of Chinese calligraphy and painting.

Compliance with ethics guidelines

Lei HUANG, Zeng-xuan HOU, You-hang ZHAO, and Di-jing ZHANG declare that they have no conflict of interest.

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