

## Review:

# New developments in control design techniques of logical control networks\*

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**Abstract:** The control design problem plays a fundamental role in the study of logical control networks (LCNs). This paper presents a detailed survey on new developments in control design techniques of LCNs. First, some preliminary results on the semi-tensor product method and LCNs are reviewed. Then, we move on to some new developments for control design techniques of LCNs, including the reachable set approach, the pinning control technique, the control Lyapunov function approach, the event-triggered control technique, and the sampled-data control technique. Finally, an illustrative example is given to demonstrate the effectiveness of these techniques.

**Key words:** Logical control network; Control design; Semi-tensor product of matrices

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## 1 Introduction


Multi-valued logical systems were first proposed by Jan Łukasiewicz in 1920s for solving practical problems in computer and engineering. As a natural extension of Boolean networks (Cheng et al., 2011b),  $k$ -valued logical networks (KVLNs) have wide applications in the networked evolutionary game (Zhu B et al., 2016; Ding XY et al., 2017; Li HT et al., 2018b; Mao et al., 2018; Ding XY and Li, 2019), combinational circuit design (Liu ZB et al., 2014), feedback shift registers (Lu JQ et al., 2018a, 2018c), and fuzzy control (Moraga et al., 2003; Cheng et al., 2012; Fan et al., 2018; Wang SL and Li, 2019). In KVLNs, the values of states are taken from the set  $\mathcal{D}_k := \{0, 1, \dots, k-1\}$ , and thus the dynamics are

determined by  $k$ -valued logical mappings.

KVLNs with control inputs and outputs are called  $k$ -valued logical control networks (KVLNs) (Li HT et al., 2017c; Li YL et al., 2019), where the control variables can be directly manipulated. By virtue of specific control design techniques and suitable control policies, one can achieve various control objectives such as reachability, stabilization, synchronization, and optimal control (Liu Y et al., 2016b, 2017a; Lu JQ et al., 2016a; Yang QQ et al., 2017; Li HT et al., 2019a). Thus, the control design problem plays a fundamental role in the study of KVLNs. However, the control of KVLNs has been a challenging problem for a long time until the establishment of the algebraic state space representation (ASSR) approach (Cheng et al., 2011b). Using the ASSR approach, the dynamics of KVLNs can be converted into a bilinear form, which facilitates the application of classical control theory to KVLNs. In the last decade, many excellent results have been achieved for ASSR-based KVLNs, including stability (Li BW et al., 2019a; Liang et al., 2019),

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controllability (Liu Y et al., 2014, 2015; Zhong et al., 2019b; Zhu QX et al., 2019), observability (Zhu QX et al., 2018b; Yu et al., 2019c), stabilization (Liu RJ et al., 2019; Zhong et al., 2020), synchronization (Li FF, 2016a; Ding XY et al., 2018), disturbance decoupling (Zhang et al., 2014; Liu Y et al., 2016a; Yu et al., 2019b), optimal control (Zhu QX et al., 2018c), output tracking (Li HT et al., 2015, 2016; Li XH et al., 2018; Xu et al., 2018a; Li YL et al., 2019), and so on (Liu Y et al., 2018; Li BW et al., 2019c; Wang B and Feng, 2019). Details on the ASSR approach can be found in some recent surveys (Fornasini and Valcher, 2016; Lu JQ et al., 2017; Cheng et al., 2018; Li HT et al., 2018a; Li H et al., 2019).

It is noted that the above results have been extended to various kinds of KVLNs in some recent studies. Liu Y et al. (2016a, 2017a, 2018) introduced the ASSR approach to singular Boolean networks. The stability, controllability, and stabilization of probabilistic Boolean networks were considered in Tong et al. (2018a) and Liu JY et al. (2019). Boolean networks with time delays were investigated in Li XH et al. (2018), Liu YS et al. (2018), and Zhu SY et al. (2019). Boolean networks with impulsive effects were studied in Xu et al. (2018a), Li HT et al. (2019b), and Yang JJ et al. (2020). Switched Boolean networks were also well studied in Wang B and Feng (2019), Yu et al. (2019a), and Sun et al. (2020).

Recently, based on the ASSR approach, many efficient control design techniques have been introduced to solve control problems of KVLCNs. These include the reachable set approach, pinning control technique, control Lyapunov function approach, event-triggered control technique, and sampled-data control technique. The reachable set approach was first proposed by Fornasini and Valcher (2013) and Li R et al. (2013) for the study of stabilization of KVLCNs. Lu JQ et al. (2016b) first introduced the pinning control technique to KVLCNs and investigated the controllability of KVLCNs under pinning controllers. As an efficient method on stabilizer design of nonlinear control systems, the control Lyapunov function (CLF) approach was generalized to finite evolutionary games by Wang YH and Cheng (2017). Then, Li HT and Wang (2017b) established a novel framework concerning Lyapunov theory for KVLCNs using the ASSR approach. Li BW et al. (2018) first introduced the event-triggered control approach to

KVLCNs and considered the disturbance decoupling problem of KVLCNs. In addition, the sampled-data control approach was first generalized to KVLCNs by Liu Y et al. (2016c).

## 2 Preliminaries

### 2.1 Semi-tensor product of matrices

First, we present some notations in Table 1. Next, we present some useful preliminaries on semi-tensor product (STP) (Cheng et al., 2011b).

**Table 1** Notations used in this paper

Notation	Description
$\mathcal{D}_s$	$\mathcal{D}_s := \{0, 1, \dots, s-1\}$
$\mathcal{D}_s^r$	$\mathcal{D}_s^r := \underbrace{\mathcal{D}_s \times \mathcal{D}_s \times \dots \times \mathcal{D}_s}_r$
$\Delta_s$	$\Delta_s := \{\delta_s^j : j = 1, 2, \dots, s\}$
$\delta_s^j$	The $j^{\text{th}}$ column of the identity matrix $I_s$
$\mathbf{N}$	$\mathbf{N} = [\delta_s^{\alpha_1}, \delta_s^{\alpha_2}, \dots, \delta_s^{\alpha_r}] = \delta_s[\alpha_1, \alpha_2, \dots, \alpha_r]$ , where $\mathbf{N}$ is a logical matrix
$\mathcal{L}_{s \times r}$	Set of $s \times r$ logical matrices
$\text{Col}_j(\mathbf{N})$	The $j^{\text{th}}$ column of matrix $\mathbf{N}$
$\text{Row}_j(\mathbf{N})$	The $j^{\text{th}}$ row of matrix $\mathbf{N}$
$N_{s,r}$	The $(s, r)^{\text{th}}$ element of matrix $\mathbf{N}$

**Definition 1** The STP of matrices  $\mathbf{E} \in \mathbb{R}^{s \times t}$  and  $\mathbf{F} \in \mathbb{R}^{m \times n}$  is

$$\mathbf{E} \ltimes \mathbf{F} = (\mathbf{E} \otimes \mathbf{I}_{\frac{\mu}{t}})(\mathbf{F} \otimes \mathbf{I}_{\frac{\mu}{m}}),$$

where  $\mu = \text{lcm}(t, m)$  denotes the least common multiple of  $t$  and  $m$ . The symbol “ $\ltimes$ ” is often omitted since STP is a generalization of conventional matrix product.

Identify  $\mathcal{D}_k \sim \Delta_k$  with  $k - j \sim \delta_k^j$ ,  $j = 1, 2, \dots, k$ . Then, we have the following basic result for the matrix expression of logical functions:

**Lemma 1** Given a  $k$ -valued logical function  $g : \mathcal{D}_k^s \rightarrow \mathcal{D}_k$ , there exists a unique matrix  $\mathbf{G} \in \mathcal{L}_{k \times k^s}$  called the structural matrix of  $g$ , such that

$$g(x_1, x_2, \dots, x_s) = \mathbf{G} \ltimes x_1 \ltimes x_2 \ltimes \dots \ltimes x_s, \quad (1)$$

where  $x_j \in \Delta_k$ ,  $j = 1, 2, \dots, s$ .

**Lemma 2** Given a vector  $\mathbf{X} \in \mathcal{L}_{k \times 1}$ , then

$$\mathbf{X} \ltimes \mathbf{X} = \mathbf{R}_k^{\text{P}} \ltimes \mathbf{X},$$

where  $\mathbf{R}_k^{\text{P}} = \text{diag}(\delta_k^1, \delta_k^2, \dots, \delta_k^k)$  is called the power-reducing matrix.

## 2.2 Logical control networks

The dynamics of a  $k$ -valued logical network can be described as

$$x_i(t+1) = \tilde{f}_i(\mathbf{X}(t)), \quad (2)$$

where  $\mathbf{X}(t) = (x_1(t), x_2(t), \dots, x_n(t)) \in \mathcal{D}_k^n$  denotes the state and  $\tilde{f}_i : \mathcal{D}_k^n \rightarrow \mathcal{D}_k$  ( $i = 1, 2, \dots, n$ ) are the  $k$ -valued logical functions that determine the state evolution.

A KVLCN is a  $k$ -valued logical network with inputs and outputs, the dynamics of which can be represented as

$$\begin{cases} x_i(t+1) = f_i(\mathbf{X}(t), \mathbf{U}(t)), & i = 1, 2, \dots, n, \\ y_j(t) = g_j(\mathbf{X}(t)), & j = 1, 2, \dots, p, \end{cases} \quad (3)$$

where  $\mathbf{X}(t) = (x_1(t), x_2(t), \dots, x_n(t)) \in \mathcal{D}_k^n$  and  $\mathbf{U}(t) = (u_1(t), u_2(t), \dots, u_m(t)) \in \mathcal{D}_k^m$  denote the state and control input, respectively.  $y_j \in \mathcal{D}_k$  ( $j = 1, 2, \dots, p$ ) are the outputs.  $f_i : \mathcal{D}_k^n \rightarrow \mathcal{D}_k$  ( $i = 1, 2, \dots, n$ ) and  $g_j : \mathcal{D}_k^n \rightarrow \mathcal{D}_k$  ( $j = 1, 2, \dots, p$ ) are  $k$ -valued logical functions. Given an initial state  $\mathbf{X}(0) = (x_1(0), x_2(0), \dots, x_n(0)) \in \mathcal{D}_k^n$  and a control sequence  $\{\mathbf{U}(t) : t \in \mathbb{N}\} \subseteq \mathcal{D}_k^m$ ,  $\mathbf{X}(t; \mathbf{X}(0), \mathbf{U})$  denotes the state of system (3) at time  $t$ .

Using the vector form of logical variables, according to Lemma 1, one can obtain the structural matrix of  $\tilde{f}_i$ , denoted by  $\tilde{\mathbf{L}}_i \in \mathcal{L}_{k \times k^n}$ ,  $i = 1, 2, \dots, n$ . Then, system (2) can be expressed in the following algebraic form:

$$\mathbf{x}(t+1) = \tilde{\mathbf{L}}\mathbf{x}(t), \quad (4)$$

where  $\mathbf{x}(t) = \times_{i=1}^n x_i(t)$ ,  $\tilde{\mathbf{L}} = \tilde{\mathbf{L}}_1 * \tilde{\mathbf{L}}_2 * \dots * \tilde{\mathbf{L}}_n \in \mathcal{L}_{k^n \times k^n}$ , and  $*$  denotes the Khatri-Rao product. The Khatri-Rao product of two matrices  $\mathbf{E} \in \mathcal{L}_{m \times s}$  and  $\mathbf{F} \in \mathcal{L}_{n \times s}$  is defined as  $\mathbf{E} * \mathbf{F} = [\text{Col}_1(\mathbf{E}) \times \text{Col}_1(\mathbf{F}), \text{Col}_2(\mathbf{E}) \times \text{Col}_2(\mathbf{F}), \dots, \text{Col}_s(\mathbf{E}) \times \text{Col}_s(\mathbf{F})]$ . Similarly, the algebraic form of system (3) is

$$\begin{cases} \mathbf{x}(t+1) = \mathbf{L}\mathbf{u}(t)\mathbf{x}(t), \\ \mathbf{y}(t) = \mathbf{G}\mathbf{x}(t), \end{cases} \quad (5)$$

where  $\mathbf{x}(t) = \times_{i=1}^n x_i(t)$ ,  $\mathbf{u}(t) = \times_{i=1}^m u_i(t)$ ,  $\mathbf{y}(t) = \times_{i=1}^p y_i(t)$ ,  $\mathbf{L} \in \mathcal{L}_{k^n \times k^{m+n}}$ , and  $\mathbf{G} \in \mathcal{L}_{k^p \times k^n}$ .

**Example 1** Consider the following apoptosis network (Li HT et al., 2014):

$$\begin{cases} \mathbf{x}_1(t+1) = \neg \mathbf{x}_2(t) \wedge \mathbf{u}(t), \\ \mathbf{x}_2(t+1) = \neg \mathbf{x}_1(t) \wedge \mathbf{x}_3(t), \\ \mathbf{x}_3(t+1) = \mathbf{x}_2(t) \vee \mathbf{u}(t), \end{cases} \quad (6)$$

where  $\mathbf{x}_1$ ,  $\mathbf{x}_2$ , and  $\mathbf{x}_3$  represent the concentration level of IAP, C3a, and C8a, respectively. In addition, the control input  $\mathbf{u}$  denotes the concentration level of TNF.

Setting  $\mathbf{x}(t) = \times_{i=1}^3 \mathbf{x}_i(t)$ , each equation in system (6) can be expressed in the following algebraic form:

$$\mathbf{x}_i(t+1) = \mathbf{M}_i \mathbf{u}(t) \mathbf{x}(t), \quad i = 1, 2, 3,$$

where

$$\begin{aligned} \mathbf{M}_1 &:= \mathbf{M}_c \mathbf{M}_n (\mathbf{1}_2^T \otimes \mathbf{I}_2 \otimes \mathbf{1}_2^T) \mathbf{W}_{[2,8]} \\ &= \delta_2[2, 2, 1, 1, 2, 2, 1, 1, 2, 2, 2, 2, 2, 2, 2], \\ \mathbf{M}_2 &:= \mathbf{M}_c \mathbf{M}_n (\mathbf{1}_2^T \otimes \mathbf{I}_2 \otimes \mathbf{1}_2^T) \\ &= \delta_2[2, 2, 2, 2, 1, 2, 1, 2, 2, 2, 2, 1, 2, 1, 2], \\ \mathbf{M}_3 &:= \mathbf{M}_d (\mathbf{1}_2^T \otimes \mathbf{I}_2 \otimes \mathbf{1}_2^T) \mathbf{W}_{[2,8]} \\ &= \delta_2[1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 2, 1, 1, 2, 2]. \end{aligned}$$

Then, the algebraic form of system (6) can be obtained as

$$\mathbf{x}(t+1) = \mathbf{L}\mathbf{u}(t)\mathbf{x}(t),$$

where  $\mathbf{L} := \mathbf{M}_1 * \mathbf{M}_2 * \mathbf{M}_3 = \delta_8[7, 7, 3, 3, 5, 7, 1, 3, 7, 7, 8, 8, 5, 7, 6, 8]$ .

## 3 Review of control design techniques for logical control networks

### 3.1 Reachable set approach

As one of the most fundamental issues among control problems of KVLCNs, the solution of the stabilization issue can not only guide medical scientists in designing suitable therapeutic interventions to steer a biological system to a desirable state, but also reveal how the structure of the system contributes to system stability. In the last decade, control design for stabilization of KVLCNs has attracted a lot of research interest, and many excellent results have been obtained (Cheng et al., 2011a; Li FF and Sun, 2012; Li HT et al., 2013b). It is worth pointing out that a general approach for the construction of stabilizers for KVLCNs is lacking until Fornasini and Valcher (2013) and Li R et al. (2013) proposed the reachable set approach to investigate the state feedback stabilization of Boolean control networks. In the following, we briefly introduce the idea of the reachable set approach in the state feedback stabilization control design of KVLCNs. For details, please refer to Fornasini and Valcher (2013) and Li R et al. (2013).

**Definition 2** Given an equilibrium  $\mathbf{x}_e \in \Delta_{k^n}$ , system (5) is said to be globally stabilizable to  $\mathbf{x}_e$ , if for any initial state  $\mathbf{x}_0$ , there exist a control sequence  $\{\mathbf{u}(t) : t \in \mathbb{N}\} \subseteq \mathcal{D}_{k^m}$  and a positive integer  $N$ , such that  $\forall t \geq N, \mathbf{x}(t; \mathbf{x}_0, \mathbf{u}) = \mathbf{x}_e$ .

Given an equilibrium  $\mathbf{x}_e \in \Delta_{k^n}$ , we aim to find a state feedback law in the form of

$$\mathbf{u}_i(t) = k_i(\mathbf{X}(t)), \quad i = 1, 2, \dots, m, \quad (7)$$

where  $k_i : \mathcal{D}_k^n \rightarrow \mathcal{D}_k, i = 1, 2, \dots, m$ , which makes system (5) globally stabilizable to  $\mathbf{x}_e$ .

Based on Lemma 1, one can obtain the following algebraic form of state feedback law (7):

$$\mathbf{u}(t) = \mathbf{K}\mathbf{x}(t), \quad (8)$$

where  $\mathbf{K} \in \mathcal{L}_{k^m \times k^n}$  is called the state feedback gain matrix.

Next, we investigate the state feedback stabilization of system (5) via the reachable set approach. To this end, we introduce the concept of the reachable set (Li R et al., 2013).

**Definition 3** Consider system (5) given a state  $\delta_{k^n}^\gamma \in \Delta_{k^n}$ . Let  $E_l^{(R)}(\delta_{k^n}^\gamma)$  denote the set of all the initial states  $\mathbf{x}_0 \in \Delta_{k^n}$  that can be driven to the state  $\delta_{k^n}^\gamma$  in  $l$  steps by a control sequence  $\mathbf{u}(0), \mathbf{u}(1), \dots, \mathbf{u}(l-1) \in \Delta_{k^m}$ .  $E_l^{(R)}(\delta_{k^n}^\gamma) = \{\mathbf{x}_0 \in \Delta_{k^n} : \text{there exists a control sequence } \mathbf{u}(0), \mathbf{u}(1), \dots, \mathbf{u}(l-1) \in \Delta_{k^m}, \text{ such that } \mathbf{x}(l; \mathbf{x}_0, \mathbf{u}(0), \mathbf{u}(1), \dots, \mathbf{u}(l-1)) = \delta_{k^n}^\gamma\}$ .  $E_l^{(R)}(\delta_{k^n}^\gamma)$  is called the  $l^{\text{th}}$  reachable set of system (5).

The following lemma presents some basic properties of reachable sets:

**Lemma 3** If  $\delta_{k^n}^\gamma \in E_1^{(R)}(\delta_{k^n}^\gamma)$ , then  $E_l^{(R)}(\delta_{k^n}^\gamma) \subseteq E_{l+1}^{(R)}(\delta_{k^n}^\gamma)$  holds  $\forall l \geq 1$ . If  $E_1^{(R)}(\delta_{k^n}^\gamma) = \{\delta_{k^n}^\gamma\}$ , then  $E_l^{(R)}(\delta_{k^n}^\gamma) = \{\delta_{k^n}^\gamma\}$  holds  $\forall l \geq 1$ . If  $\delta_{k^n}^\gamma \in E_1^{(R)}(\delta_{k^n}^\gamma)$  and  $E_{i+1}^{(R)}(\delta_{k^n}^\gamma) = E_i^{(R)}(\delta_{k^n}^\gamma)$  holds for some integer  $i \geq 1$ , then  $E_l^{(R)}(\delta_{k^n}^\gamma) = E_i^{(R)}(\delta_{k^n}^\gamma)$  holds  $\forall l \geq i$ .

Now, we are ready to present the result on the state feedback stabilization of system (5) via the reachable set approach.

**Theorem 1** Consider system (5) with  $\mathbf{L} = \delta_{k^n}[\alpha_1, \alpha_2, \dots, \alpha_{k^m+n}]$  and let an equilibrium  $\mathbf{x}_e = \delta_{k^n}^\xi \in \Delta_{k^n}$  be given. System (5) can be globally stabilized to  $\mathbf{x}_e$  by state feedback controller (8), if and only if (1)  $\delta_{k^n}^\xi \in E_1^{(R)}(\delta_{k^n}^\xi)$  and (2) there exists an integer  $N$  ( $1 \leq N \leq k^n - 1$ ), such that  $E_N^{(R)}(\delta_{k^n}^\xi) = \Delta_{k^n}$ .

Moreover, if conditions (1) and (2) hold, for each  $j$  ( $1 \leq j \leq k^n$ ) which corresponds to a unique integer  $l_j$  ( $1 \leq l_j \leq N$ ) such that  $\delta_{k^n}^j \in E_{l_j}^{(R)}(\delta_{k^n}^\xi) \setminus E_{l_j-1}^{(R)}(\delta_{k^n}^\xi)$ , where  $E_0^{(R)}(\delta_{k^n}^\xi) := \emptyset$ , let  $1 \leq \nu_j \leq k^m$  be such that

$$\begin{cases} \alpha_{(\nu_j-1)k^n+j} = \xi, & l_j = 1, \\ \delta_{k^n}^{\alpha_{(\nu_j-1)k^n+j}} \in E_{l_j-1}^{(R)}(\delta_{k^n}^\xi), & l_j \geq 2. \end{cases}$$

Then, the state feedback controller  $\mathbf{u}(t) = \mathbf{K}\mathbf{x}(t)$  with  $\mathbf{K} = \delta_{k^m}[\nu_1, \nu_2, \dots, \nu_{k^n}]$  globally stabilizes system (5) to  $\mathbf{x}_e = \delta_{k^n}^\xi$ .

Plugging state feedback controllers  $\mathbf{u}(t) = \mathbf{K}\mathbf{x}(t)$  obtained using the reachable set approach into system (5), we can obtain a closed-loop system (4). It is easy to see that in the closed-loop system constructed in this way, the  $l^{\text{th}}$  reachable set of system (5) is exactly the  $l^{\text{th}}$  basin of attraction of equilibrium  $\mathbf{x}_e$ .

Following Fornasini and Valcher (2013) and Li R et al. (2013), many valuable results on KVLNs have been obtained via the reachable set approach, including stabilization (Li HT and Wang, 2013, 2016a, 2016c, 2017a; Li R et al., 2014; Liu Y et al., 2016c; Yang QQ et al., 2016; Ding XY et al., 2017; Ding Y et al., 2017; Li FF, 2018; Liu RJ et al., 2018; Lu JQ et al., 2018b; Zhu SY et al., 2018a; Li XD et al., 2019a; Li YL et al., 2019; Wang LQ et al., 2019), set stabilization (Guo et al., 2015; Li FF and Tang, 2017; Li HT et al., 2017d; Liu RJ et al., 2017; Zheng et al., 2017; Li YL et al., 2018a; Li YY et al., 2018; Tong et al., 2018b; Xu et al., 2018b), synchronization (Zhong et al., 2014, 2017a; Li FF, 2016a,c; Liu Y et al., 2016b; Ding XY et al., 2018; Xu et al., 2018c; Yang JJ et al., 2019), output tracking (Li HT et al., 2015, 2016; Li HT and Wang, 2016b; Zhong et al., 2017b; Li XH et al., 2018; Liu YS et al., 2018; Xu et al., 2018a), and output regulation (Chen and Liang, 2017; Li HT et al., 2017a,b; Ding XY et al., 2019).

### 3.2 Pinning control

It was shown in Rosin et al. (2013) that synchronization patterns can be achieved by adjusting the refractory time of two out of 32 nodes, which means that one can achieve control goals of the global network via adjusting the properties of quite a small fraction of nodes. This is called the pinning control technique (Lukk et al., 2010; Müller and Schuppert,

2011; Müller et al., 2011). Lu JQ et al. (2016b) first introduced the pinning control technique to the investigation of KVL-CN. Generally speaking, there are two ideas on achieving pinning control of KVL-CN. One is to select pinned-nodes as control inputs (Lu JQ et al., 2016b), and the other is to change the columns of the state transition matrix (Li FF, 2016b).

Assuming that the pinned-nodes are given as  $\{1, 2, \dots, r\}$ , the controllability of the following system is considered:

$$\begin{cases} x_i(t+1) = \hat{f}_i(u_i(t), x_1(t), x_2(t), \dots, x_n(t)), \\ \quad i = 1, 2, \dots, s, \\ x_j(t+1) = f_j(x_1(t), x_2(t), \dots, x_n(t)), \\ \quad j = s+1, 2, \dots, n, \end{cases} \quad (9)$$

where  $x_i \in \mathcal{D}_k$  ( $i = 1, 2, \dots, n$ ),  $u_i \in \mathcal{D}_k$  ( $i = 1, 2, \dots, s$ ) are  $k$ -valued logical variables, and  $\hat{f}_i : \mathcal{D}_k^{n+1} \rightarrow \mathcal{D}_k$  ( $i = 1, 2, \dots, s$ ) and  $f_j : \mathcal{D}_k^n \rightarrow \mathcal{D}_k$  ( $j = s+1, s+2, \dots, n$ ) are  $k$ -valued functions.

Using the vector form of logical variables and based on Lemma 1, system (9) can be expressed in the following algebraic form:

$$\begin{cases} \mathbf{x}^1(t+1) = \mathbf{F}_1 \mathbf{x}(t) \mathbf{u}(t), \\ \mathbf{x}^2(t+1) = \mathbf{F}_2 \mathbf{x}(t), \end{cases} \quad (10)$$

where  $\mathbf{x}^1(t) = \times_{i=1}^s \mathbf{x}_i(t)$ ,  $\mathbf{x}^2(t) = \times_{i=s+1}^n \mathbf{x}_i(t)$ ,  $\mathbf{u}(t) = \times_{i=1}^s \mathbf{u}_i(t)$ ,  $\mathbf{F}_1 \in \mathcal{L}_{k^s \times k^{n+s}}$ , and  $\mathbf{F}_2 \in \mathcal{L}_{k^{n-s} \times k^n}$ . Then, using the dummy matrix  $\mathbf{D}_r[k^n, k^s] = \mathbf{I}_{k^n} \otimes \mathbf{1}_{k^s}^T$ , system (10) can be converted into the following form:

$$\mathbf{x}(t+1) = \mathbf{F} \mathbf{x}(t) \mathbf{u}(t), \quad (11)$$

where  $\mathbf{F} = \mathbf{F}_1 * (\mathbf{F}_2 \mathbf{D}_r[k^n, k^s]) \in \mathcal{L}_{k^n \times k^{n+s}}$ .

Based on system (11), a series of results on pinning reachability and controllability were proposed by Lu JQ et al. (2016b).

By virtue of this idea, Chen et al. (2016) considered the controllability of autonomous Boolean control networks and proposed several criteria. The pinning output feedback stabilization of constrained Boolean control networks was studied in Yang QQ et al. (2016). The pinning output tracking of KVL-CN was considered in Li HT et al. (2017c) and Li XH et al. (2018).

In the following, we describe the idea of the pinning control technique proposed in Li FF (2016b). In

Li FF (2016b), the stabilization of Boolean control networks was considered by designing pinning controllers, where the global stability of the considered Boolean networks was first achieved via changing the columns of the state transition matrix, and then the algorithms for selecting pinning nodes and designing pinning controllers were successively established. In the remainder of this subsection, we consider only the case of  $k = 2$ .

Li FF (2016b) considered the global stability problem with respect to a state  $\mathbf{x}_e = \delta_{k^n}^\xi$  of system (4) and proposed Algorithm 1 to make system (4) globally stable at the state  $\mathbf{x}_e$ .

Then, by virtue of Algorithm 1, matrix  $\tilde{\mathbf{L}}$  is changed to a matrix  $\tilde{\mathbf{L}}'$ , and system (4) with state transition matrix  $\tilde{\mathbf{L}}'$  is globally stable to the equilibrium  $\mathbf{x}_e = \delta_{k^n}^\xi$ . Assume that the 1<sup>st</sup>, 2<sup>nd</sup>, ...,  $l^{\text{th}}$  columns of matrices  $\tilde{\mathbf{L}}_1, \tilde{\mathbf{L}}_2, \dots, \tilde{\mathbf{L}}_s$  alter, and assume that matrices  $\tilde{\mathbf{L}}_1, \tilde{\mathbf{L}}_2, \dots, \tilde{\mathbf{L}}_s$  are changed to  $\tilde{\mathbf{L}}'_1, \tilde{\mathbf{L}}'_2, \dots, \tilde{\mathbf{L}}'_s$ , respectively. Then, one can design the pinning control strategy to stabilize system (4) to the state  $\delta_{k^n}^\xi$  as

$$\begin{cases} \mathbf{x}_i(t+1) = u_i(\mathbf{X}(t)) \oplus_i \tilde{f}_i(\mathbf{X}(t)), \quad i = 1, 2, \dots, s, \\ \mathbf{x}_j(t+1) = \tilde{f}_j(\mathbf{X}(t)), \quad j = s+1, 2, \dots, n, \end{cases} \quad (12)$$

where  $u_i(t) = k_i(\mathbf{X}(t))$  ( $i = 1, 2, \dots, s$ ) are state feedback controls.

Denote the structure matrices of logical functions  $\oplus_1, \oplus_2, \dots, \oplus_s$  by  $\mathbf{M}_{\oplus_1}, \mathbf{M}_{\oplus_2}, \dots, \mathbf{M}_{\oplus_s}$ , respectively, and denote the structure matrices of feedback control functions  $k_1, k_2, \dots, k_s$  by  $\mathbf{K}_1, \mathbf{K}_2, \dots, \mathbf{K}_s$ , respectively. Then, the structure matrices  $\mathbf{M}_{\oplus_1}, \mathbf{M}_{\oplus_2}, \dots, \mathbf{M}_{\oplus_s}$  and  $\mathbf{K}_1, \mathbf{K}_2, \dots, \mathbf{K}_s$  can be obtained by

$$\mathbf{M}_{\oplus_i} \mathbf{K}_i (\mathbf{I}_{k^n} \otimes \tilde{\mathbf{L}}_i) \mathbf{R}_{k^n}^P = \tilde{\mathbf{L}}'_i, \quad i = 1, 2, \dots, s. \quad (13)$$

It was proved in Li FF (2016b) that Eq. (13) is solvable, which means that pinning control strategy (12) is feasible for stabilizing system (4) to the state  $\mathbf{x}_e = \delta_{k^n}^\xi$  (Li FF, 2016b; Lu JQ et al., 2016b).

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**Algorithm 1** Obtaining a desired system globally stable at state  $\mathbf{x}_e = \delta_{k^n}^\xi$

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- 1: Change the  $\xi^{\text{th}}$  column of matrix  $\tilde{\mathbf{L}}$  to  $\delta_{k^n}^\xi$
  - 2: Define  $E_\tau(\delta_{k^n}^\xi) := \{\mathbf{x}_0 \in \Delta_{k^n} : \mathbf{x}(\tau; \mathbf{x}_0) = \delta_{k^n}^\xi\}$  and  $E(\delta_{k^n}^\xi) = \bigcup_{\tau=1}^{k^n} E_\tau(\delta_{k^n}^\xi)$
  - 3: Find  $\delta_{k^n}^\gamma \notin E_\tau(\delta_{k^n}^\xi)$ ,  $\gamma = 1, 2, \dots, k^n$ , and change  $\text{Col}_\gamma(\tilde{\mathbf{L}})$  to one element of  $E(\delta_{k^n}^\xi)$
-

Using the idea of the pinning control technique proposed in Li FF (2016b), many excellent results on KVLCNs have been obtained, including controllability (Li FF et al., 2018), stabilization (Li FF, 2018), set stabilization (Liu RJ et al., 2017; Li YY et al., 2018; Li FF and Xie, 2019), synchronization (Li FF, 2016c), output tracking (Zhong et al., 2017b, 2019a), and disturbance decoupling (Liu Y et al., 2017b; Li BW et al., 2019b). In addition, the pinning control approach was generalized to the strategy consensus of networked evolutionary games in Li YL et al. (2018b), and an efficient algorithm was established to solve the minimal-agent consensus control problem.

### 3.3 Sampled-data control

Recently, the sampled-data control technique has attracted research interest for its application in networked control systems and radar tracking systems (Lee et al., 2013; Li HT et al., 2013a; Liu S et al., 2013; Wu ZG et al., 2014; Mu et al., 2015; Wang JY et al., 2016; Lee and Park, 2017). By virtue of sampled-data control, not only the system's anti-jamming capability and control accuracy can be improved, but also the controller's cost can be reduced. Recently, several novel results have been obtained on sampled-data control of KVLCNs. Liu Y et al. (2016c) first introduced the sampled-date control approach to KVLCNs and considered the sampled-data state feedback stabilization of KVLCNs. Li YL et al. (2019) further investigated sampled-data reachability and state feedback stabilization of constrained KVLCNs. Next, we take the results in Li YL et al. (2019) as an example to introduce the main idea of the sampled-data control technique.

Consider the following KVLCNs with input and state constraints:

$$\mathbf{x}_i(t+1) = f_i(\mathbf{X}(t), \mathbf{U}(t)), \quad i = 1, 2, \dots, n, \quad (14)$$

where  $\mathbf{X}(t) = (x_1(t), x_2(t), \dots, x_n(t)) \in C_x \subseteq \mathcal{D}_k^n$ ,  $\mathbf{U}(t) = (u_1(t), u_2(t), \dots, u_m(t)) \in C_u \subseteq \mathcal{D}_k^m$ ,  $C_x$  and  $C_u$  represent the state constraint set (SCS) and input constraint set (ICS), respectively, and  $f_i : C_x \times C_u \rightarrow \mathcal{D}_k$  ( $i = 1, 2, \dots, n$ ) are  $k$ -valued logical functions. Assuming  $|C_x| = p$  and  $|C_u| = q$ , SCS and ICS can be given as

$$C_x = \{\delta_{k^n}^{\alpha_1}, \delta_{k^n}^{\alpha_2}, \dots, \delta_{k^n}^{\alpha_p}\}, \quad (15)$$

$$C_u = \{\delta_{k^m}^{\beta_1}, \delta_{k^m}^{\beta_2}, \dots, \delta_{k^m}^{\beta_q}\}, \quad (16)$$

where  $\alpha_1 < \alpha_2 < \dots < \alpha_p$  and  $\beta_1 < \beta_2 < \dots < \beta_q$ .

**Definition 4** Let a set of sampling points be given as  $S = \{t_h : h \in \mathbb{N}\}$ ,  $t_0 = 0$ .  $\{\mathbf{U}(t) : t \in \mathbb{N}\} \subseteq C_u$  is said to be a sampled-data control with regard to  $S$ , if  $\mathbf{U}(t) = \mathbf{U}(t_h)$ ,  $t \in [t_h, t_h + 1)_{\mathbb{N}}$  holds for any  $h \in \mathbb{N}$ . A sampled-data control with regard to  $S$  is called a uniformly-sampled-data control, if there exists an integer  $\rho \in \mathbb{Z}_+$ , such that  $t_{h+1} - t_h = \rho$  holds for any  $h \in \mathbb{N}$ , where integer  $\rho$  is called the sampling period.

Fixing a set of sampling points  $S = \{t_h : h \in \mathbb{N}\}$ , Li YL et al. (2019) investigated the following sampled-data reachability and state feedback stabilization for system (14):

**Definition 5** Consider system (14) given the set of sampling points  $S = \{t_h : h \in \mathbb{N}\}$ . Then,  $\mathbf{X}_d \in C_x$  is constrained sampled-data reachable from  $\mathbf{X}_0 \in C_x$  at the sampling point  $t_h$ , if one can find a sampled-data control sequence  $\{\mathbf{U}(t) : t \in \mathbb{N}\} \subseteq C_u$  steering  $\mathbf{X}_0$  to  $\mathbf{X}_d$  at time  $t_h$  with  $\mathbf{X}(t; \mathbf{X}_0, \mathbf{U}) \in C_x, \forall t = 0, 1, 2, \dots, t_h$ .

**Definition 6** Given an equilibrium  $\mathbf{X}_e \in C_x$  and the following constrained sampled-data control:

$$\mathbf{u}_i(t) = k_i(\mathbf{X}(t_h)), \quad t \in [t_h, t_{h+1})_{\mathbb{N}}, \quad (17)$$

where  $k_i : \mathcal{D}_k^n \rightarrow \mathcal{D}_k, i = 1, 2, \dots, m$  and  $(k_1, k_2, \dots, k_m) : C_x \rightarrow C_u$ . System (14) is stabilizable to  $\mathbf{X}_e$  under constrained sampled-data control (17), if there exists an integer  $T \in \mathbb{Z}_+$  such that  $\forall \mathbf{X}_0 \in C_x, \mathbf{X}(t; \mathbf{X}_0, \mathbf{U}) = \mathbf{X}_e, \forall t \geq T$ , and  $\mathbf{X}(t; \mathbf{X}_0, \mathbf{U}) \in C_x, \forall t \in \mathbb{N}$ .

Setting  $\mathbf{x}(t) = \times_{i=1}^n \mathbf{x}_i(t), \mathbf{u}(t) = \times_{j=1}^m \mathbf{u}_j(t)$ , the algebraic form of system (14) and constrained sampled-data control (17) can be obtained as

$$\mathbf{x}(t+1) = \mathbf{L}\mathbf{u}(t)\mathbf{x}(t), \quad (18)$$

and

$$\mathbf{u}(t) = \mathbf{K}\mathbf{x}(t_h), \quad t \in [t_h, t_{h+1})_{\mathbb{N}}, \quad (19)$$

respectively, where  $\mathbf{L} \in \mathcal{L}_{k^n \times k^{m+n}}$  is the state transition matrix and  $\mathbf{K} \in \mathcal{L}_{k^m \times k^n}$  is the state feedback gain matrix.

Next, we describe the results on uniform constrained sampled-data reachability proposed in Li YL et al. (2019).

Consider system (18) and split  $\mathbf{L}$  into  $k^m$  equal blocks as  $\mathbf{L} = [\text{Blk}_1(\mathbf{L}), \text{Blk}_2(\mathbf{L}), \dots, \text{Blk}_{k^m}(\mathbf{L})]$ . Define  $\widehat{\mathbf{L}} = [\widehat{\text{Blk}}_1(\mathbf{L}), \widehat{\text{Blk}}_2(\mathbf{L}), \dots, \widehat{\text{Blk}}_{k^m}(\mathbf{L})]$  and

$\mathbf{M}_i = \mathbf{N} \widehat{\text{Blk}}_i(\mathbf{L}) \mathbf{N}^T$ , where

$$\widehat{\text{Blk}}_i(\mathbf{L}) = \begin{cases} \text{Blk}_i(\mathbf{L}), & i \in \{\beta_1, \beta_2, \dots, \beta_q\}, \\ \mathbf{0}_{k^n \times k^n}, & \text{otherwise}, \end{cases}$$

and

$$\text{Row}_i(\mathbf{N}) = \begin{cases} (\delta_{k^n}^i)^T, & i \in \{\alpha_1, \alpha_2, \dots, \alpha_p\}, \\ \mathbf{0}_{k^n}^T, & \text{otherwise}. \end{cases}$$

For the set of sampling points  $\hat{S} = \{t_h : h \in \mathbb{N}\}$  with sampling period  $\rho$ , the constrained sampled-data controllability matrix is defined as

$$\mathbf{C} = \sum_{i=1}^{k^m} \mathbf{M}_i^\rho. \quad (20)$$

**Theorem 2** Consider system (14). Given the set of sampling points  $\hat{S} = \{t_h : h \in \mathbb{N}\}$  with sampling period  $\rho$  and an integer  $\lambda = \tau\rho \in \mathbb{Z}_+$ ,  $\mathbf{x}_d = \delta_{k^n}^{\alpha_\varphi} \in C_x$  is constrained sampled-data reachable from  $\mathbf{x}_0 = \delta_{k^n}^{\alpha_\theta} \in C_x$  at the  $\lambda^{\text{th}}$  step, if and only if  $(\mathbf{C}^\tau)_{\alpha_\varphi, \alpha_\theta} > 0$ .

If  $\lambda = \tau\rho + s$ ,  $0 \leq s \leq \rho$ , then Eq. (20) should be changed to  $(\widehat{\mathbf{C}}^s \mathbf{C}^\tau)_{\alpha_\varphi, \alpha_\theta} > 0$ , where  $\widehat{\mathbf{C}} = \sum_{i=1}^{k^m} \mathbf{M}_i$ .

Based on the results on uniform constrained sampled-data reachability, the constrained state feedback stabilization of system (14) under uniformly-sampled-data control (19) with sampling period  $\rho \in \mathbb{Z}_+$  has been investigated.

Given an equilibrium  $\mathbf{x}_e = \delta_{k^n}^{\alpha_\xi} \in C_x$  and defining a series of reachable sets as  $E_{\tau\rho}^{(S)}(\mathbf{x}_e) = \{\mathbf{x}_0 \in C_x : \text{there exists a sampled-data control sequence } \mathbf{u}(t_0), \mathbf{u}(t_1), \dots, \mathbf{u}(t_{\tau-1}) \in C_u, \text{ such that } \mathbf{x}(\tau\rho; \mathbf{x}_0, \mathbf{u}) = \mathbf{x}_e, \text{ and } \mathbf{x}(t; \mathbf{x}_0, \mathbf{u}) \in C_x, \forall t = 0, 1, \dots, \tau\rho\}$ , then based on reachable sets, the following theorem has been proposed:

**Theorem 3** System (14) with Eqs. (15) and (16) is stabilizable to  $\mathbf{x}_e = \delta_{k^n}^{\alpha_\xi} \in C_x$  by uniformly-sampled-data control (19), if and only if there exists an integer  $\gamma \in \{1, 2, \dots, q\}$  such that  $\mathbf{L} \delta_{k^m}^{\beta_\gamma} \mathbf{x}_e = \mathbf{x}_e$  and there exists a positive integer  $\nu \leq k^n$  such that  $E_{\nu\rho}^{(S)}(\mathbf{x}_e) = C_x$ . Then, feasible constrained sampled-data controllers to stabilize system (14) to  $\mathbf{x}_e$  are obtained.

Similar to the case of uniformly-sampled-data control, some results on nonuniform constrained sampled-data reachability and state feedback stabilization have been obtained. For details, please refer to Li YL et al. (2019).

In addition, there exist many excellent results on KVLCNs via the sampled-data control approach, including controllability (Yu et al., 2018; Zhu QX et al., 2018a), stabilization (Lu JQ et al., 2018b; Zhu SY et al., 2018b; Liu Y et al., 2019b), and so on (Tong et al., 2017; Lin et al., 2019; Liu Y et al., 2019a).

### 3.4 Event-triggered control

Event-triggered control was first introduced in Åström and Bernhardsson (1999). In the past few decades, event-triggered control has been applied in networked control systems and nonlinear systems (Postoyan et al., 2015; Li JN et al., 2017; Wu YQ et al., 2017). By virtue of event-triggered control, control execution time can be reduced and the computation cost can be greatly reduced. Li BW et al. (2018) first introduced the event-triggered control approach to KVLCNs and considered the disturbance decoupling problem of KVLCNs. Li YL et al. (2018a) developed an event-triggered control scheme for robust set stabilization of disturbed KVLCNs. In the following, we take the results in Li YL et al. (2018a) to illustrate the event-triggered control method.

Consider the following disturbed KVLCNs:

$$\begin{cases} \dot{x}_i(t) = f_i(\mathbf{X}(t), \mathbf{U}(t), \mathbf{\Gamma}(t)), & i = 1, 2, \dots, n, \\ \dot{y}_j(t) = g_j(\mathbf{X}(t)), & j = 1, 2, \dots, p, \end{cases} \quad (21)$$

where  $\mathbf{X}(t) = (x_1(t), x_2(t), \dots, x_n(t)) \in \mathcal{D}_k^n$ ,  $\mathbf{U}(t) = (u_1(t), u_2(t), \dots, u_m(t)) \in \mathcal{D}_k^m$ ,  $\mathbf{\Gamma}(t) = (\gamma_1(t), \gamma_2(t), \dots, \gamma_r(t)) \in \mathcal{D}_k^r$ , and  $\mathbf{Y}(t) = (y_j(t))_{j=1}^p \in \mathcal{D}_k^p$  denote the state, control input, disturbance input, and output at time  $t$ , respectively. Given an initial state  $\mathbf{X}(0) \in \mathcal{D}_k^n$ , a sequence of control inputs  $\{\mathbf{U}(t) : t \in \mathbb{N}\} \subseteq \mathcal{D}_k^m$ , and a disturbance input sequence  $\{\mathbf{\Gamma}(t) : t \in \mathbb{N}\} \subseteq \mathcal{D}_k^r$ , the state of system (21) at time  $t$  is denoted by  $\mathbf{X}(t; \mathbf{X}(0), \mathbf{U}, \mathbf{\Gamma})$ .

Li YL et al. (2018a) investigated the following robust set stabilization for system (21):

**Definition 7** Consider system (21). Let a nonempty set  $M \subseteq \mathcal{D}_k^n$  and an initial state  $\mathbf{X}(0) \in \mathcal{D}_k^n$  be given. System (21) is robustly stabilizable to  $M$ , if there exist a sequence of control inputs  $\{\mathbf{U}(t) : t \in \mathbb{N}\} \subseteq \mathcal{D}_k^m$  and an integer  $\tau \in \mathbb{Z}_+$  such that  $\mathbf{X}(t; \mathbf{X}(0), \mathbf{U}, \mathbf{\Gamma}) \in M$  holds for any  $t \geq \tau$  and any  $\{\mathbf{\Gamma}(t) : t \in \mathbb{N}\} \subseteq \mathcal{D}_k^r$ .

The following time-variant state feedback

control was considered:

$$u_i(t) = \varphi_i(t, \mathbf{X}(t)), i = 1, 2, \dots, m, \quad (22)$$

where  $\varphi_i : \mathbb{N} \times \mathcal{D}_k^n \rightarrow \mathcal{D}_k, i = 1, 2, \dots, m$  are time-variant logical functions.

By STP, systems (21) and (22) can be respectively converted into the equivalent algebraic forms as

$$\begin{cases} \mathbf{x}(t+1) = \mathbf{L}\mathbf{u}(t)\mathbf{x}(t)\gamma(t), \\ \mathbf{y}(t) = \mathbf{G}\mathbf{x}(t), \end{cases} \quad (23)$$

and

$$\mathbf{u}(t) = \Phi(t, \mathbf{x}(0))\mathbf{x}(t), \quad (24)$$

where  $\mathbf{x}(t) = \times_{i=1}^n \mathbf{x}_i(t), \mathbf{u}(t) = \times_{i=1}^m \mathbf{u}_i(t), \gamma(t) = \times_{i=1}^r \gamma_i(t), \mathbf{y}(t) = \times_{i=1}^p \mathbf{y}_i(t), \mathbf{L} \in \mathcal{L}_{k^n \times k^{m+n+r}}, \mathbf{G} \in \mathcal{L}_{k^p \times k^n},$  and  $\Phi(t, \mathbf{x}(0)) \in \mathcal{L}_{k^m \times k^n}.$

To reduce cost in the time-variant state feedback controller design, Li YL et al. (2018a) proposed an event-triggered control scheme for robust set stabilization of system (21).

Given a nonempty set  $M \subseteq \Delta_{k^n},$  an initial state  $\mathbf{x}(0) = \delta_{k^n}^\theta,$  and a time-variant control  $\mathbf{u}(t) = \Phi(t, \mathbf{x}(0))\mathbf{x}(t),$  the event-triggered condition was formulated as

$$d_H(\Lambda(t+1), M) > 0, \quad (25)$$

where  $\Lambda(t+1) = \text{Col}(\text{Blk}_\theta(\times_{i=t}^0(\mathbf{L}\Phi(i, \mathbf{x}(0))\mathbf{R}_{k^n}^P)))$ , and  $d_H(\Lambda(t+1), M)$  represents the Hausdorff distance between  $\Lambda(t+1)$  and  $M.$  Given two nonempty sets  $A$  and  $B,$  the Hausdorff distance between  $A$  and  $B$  is defined as  $d_H(A, B) = \max_{a \in A} \{ \min_{b \in B} \{ d(a, b) \} \},$  where  $d(a, b)$  denotes the Euclidean distance between  $a$  and  $b.$  Denote the sequence of triggering time by  $t_1 < t_2 < \dots < t_\tau < \dots < t_\infty.$  Correspondingly, one can obtain a sequence of feedback control updates as  $\Phi(t_1, \mathbf{x}(0)), \Phi(t_2, \mathbf{x}(0)), \dots, \Phi(t_\tau, \mathbf{x}(0)), \dots, \Phi(t_\infty, \mathbf{x}(0)).$  Then, the event-triggered controllers can be designed as

$$\mathbf{u}(t) = \Phi(t_s, \mathbf{x}(0))\mathbf{x}(t), t \in [t_s, t_{s+1}) \cap \mathbb{N},$$

where  $s = 0, 1, 2, \dots, \tau, \dots, \infty$  with  $t_0 := 0.$

Based on event-triggered condition (25), Li YL et al. (2018a) proposed the following sufficient condition on the existence of the feedback event-triggered controller:

**Theorem 4** Consider system (21). Let a nonempty set  $M \subseteq \Delta_{k^n}$  and an initial state  $\mathbf{x}(0) = \delta_{k^n}^\theta$  be given. System (21) is robustly stabilizable to  $M$

under event-triggered condition (25), if  $M \subseteq \Psi_1(M)$  and  $\mathbf{x}(0) \in \Psi_1(M).$

Consider system (23). Split  $\mathbf{L}$  into  $k^m$  equal blocks, and further split each block into  $k^n$  equal blocks as

$$\mathbf{L} = [\mathbf{L}_1^1, \mathbf{L}_1^2, \dots, \mathbf{L}_1^{k^n}, \dots, \mathbf{L}_{k^m}^1, \mathbf{L}_{k^m}^2, \dots, \mathbf{L}_{k^m}^{k^n}].$$

For any  $s = 1, 2, \dots, \tau, \dots, \infty,$  define

$$\bar{\Lambda}(t_s) := \{ \mathbf{x}(t_s) \in \Lambda(t_s) \subseteq M : \text{Col}([\mathbf{L}\Phi(t_{s-1}, \mathbf{x}(0)) \times \mathbf{R}_{k^n}^P]\mathbf{x}(t_s)) \not\subseteq M \}. \quad (26)$$

Based on Theorem 4, Li YL et al. (2018a) proposed an efficient algorithm for the design of  $\Phi(t_s, \mathbf{x}(0)), s = 0, 1, 2, \dots, \tau, \dots, \infty.$

Recently, many excellent results on KVLCNs via the event-triggered control approach have been obtained, including synchronization (Li YL et al., 2018c; Tong et al., 2019; Yang JJ et al., 2019), stabilization (Zhu SY et al., 2018a; Zhu QX and Lin, 2019), and so on (Tong et al., 2018a), and the corresponding triggering mechanisms and methods for the design of feasible state feedback controllers were proposed.

### 3.5 Control Lyapunov function approach

Control Lyapunov functions (CLFs) were proposed for the design of state feedback stabilizers in nonlinear control systems (Goebel et al., 2009; Vaidya et al., 2010; Karafyllis and Jiang, 2013; Sanfelice, 2013; Lu YY and Zhang, 2017). Wang YH and Cheng (2017) generalized CLFs to finite evolutionary games. Li HT and Wang (2017b) first established a framework of Lyapunov stability theory for KVLCNs using STP. Later, Li HT and Ding (2019) presented a control Lyapunov approach to investigate feedback stabilization defined in Definition 2 of system (5), in which all possible state feedback stabilizers (8) and the corresponding CLFs were designed.

The following notations are used in what follows:

1. Denote  $U_{\theta, \varphi} = \{ \mathbf{u} \in \Delta_{k^m} : \delta_{k^n}^\varphi = \mathbf{L}\mathbf{u}\delta_{k^n}^\theta \}.$  For any  $\theta, \varphi \in \{1, 2, \dots, k^n\}$  with  $\theta \neq \varphi,$  if the inequality  $a_\varphi > a_\theta$  holds only when  $\mathbf{u} \in U_{\theta, \varphi},$  then we say  $\{a_\varphi > a_\theta\}|_{U_{\theta, \varphi}}.$

2.  $\{a_\gamma = a_\gamma\}|_{U_{\gamma, \gamma}}$  means the equality  $a_\gamma = a_\gamma$  holds only when  $\mathbf{u} \in U_{\gamma, \gamma}.$



3. For any  $\theta, \varphi \in \{1, 2, \dots, k^n\}$ , we define

$$Z_{\theta \rightarrow \varphi} := \begin{cases} \{a_\varphi > a_\theta\} |_{U_{\theta, \varphi}}, & \text{if } \theta \neq \varphi, U_{\theta, \varphi} \neq \emptyset, \\ \{a_\gamma = a_\theta\} |_{U_{\gamma, \theta}}, & \text{if } \theta = \varphi = \gamma, U_{\gamma, \theta} \neq \emptyset, \\ \emptyset, & \text{otherwise.} \end{cases}$$

**Definition 8** Consider system (5) and let an equilibrium  $\mathbf{x}_e = \delta_{k^n}^{\xi} \in \Delta_{k^n}$  be given.  $V(x) : \Delta_{k^n} \rightarrow \mathbb{R}$  is called a CLF of system (5), if (1)  $\exists \mathbf{u}^* \in \Delta_{k^m}$ , such that  $V(\mathbf{L}\mathbf{u}^*\mathbf{x}_e) - V(\mathbf{x}_e) = 0$ , and (2)  $\forall \mathbf{x} \in \Delta_{k^n}$  satisfying  $\mathbf{x} \neq \mathbf{x}_e$ ,  $\exists \mathbf{u}_x \in \Delta_{k^m}$  such that  $V(\mathbf{L}\mathbf{u}_x\mathbf{x}) - V(\mathbf{x}) > 0$ .

Li HT and Ding (2019) proposed the following theorem on the existence of feedback stabilizers for system (5):

**Theorem 5** System (5) is globally stabilizable to  $\mathbf{x}_e = \delta_{k^n}^{\xi}$  by state feedback control (8), if and only if system (5) has a control Lyapunov function  $V(x) = \mathbf{L}_v \mathbf{x}$ , where  $\mathbf{L}_v := [v_1, v_2, \dots, v_{k^n}] \in \mathbb{R}^{1 \times k^n}$  is the structural matrix of  $V(x)$ .

The following theorem has been proposed on the existence of CLFs:

**Theorem 6** System (5) has a CLF if and only if there exists a positive integer  $\tau \leq k^n$  such that

$$\text{Row}_r(\mathbf{S}^\tau) > 0,$$

where  $\mathbf{S} \in \mathbb{R}^{k^n \times k^n}$  is defined as

$$(\mathbf{S})_{\varphi, \theta} := \begin{cases} 1, & \text{if } Z_{\theta \rightarrow \varphi} \neq \emptyset, \\ 0, & \text{if } Z_{\theta \rightarrow \varphi} = \emptyset. \end{cases}$$

**Theorem 7** System (5) has a CLF if and only if there exists an integer  $\tau$  ( $0 < \tau \leq k^n$ ) such that

$$\begin{cases} \gamma \in \Theta_1(\gamma), \\ \Theta_\tau(\gamma) = \{1, 2, \dots, k^n\}, \end{cases} \quad (27)$$

where  $\Theta_1(\gamma) := \{\theta : Z_{\theta \rightarrow \gamma} \neq \emptyset, \theta \in \{1, 2, \dots, k^n\}\}$ , and  $\Theta_k(\gamma) := \{\theta : \exists \theta' \in \Theta_{k-1}(\gamma) \text{ such that } Z_{\theta \rightarrow \theta'} \neq \emptyset, \theta \in \{1, 2, \dots, k^n\}\}$ ,  $k = 2, 3, \dots, \infty$ .

In addition, denoting all of the admissible sets of control Lyapunov inequalities by  $\Phi_i$ ,  $i = 1, 2, \dots, l$ , the following result on the design of state feedback stabilizers has been presented:

**Theorem 8** Consider system (5) and assume that Eq. (27) in Theorem 7 holds. The set of all state feedback stabilizers is

$$\Psi = \bigcup_{j=1}^l \Psi_j,$$

where  $\Psi_j = \{\mathbf{K} = \delta_{k^m}[\nu_1, \nu_2, \dots, \nu_{k^n}] : \nu_\theta \in U_{\theta, \theta'}, \theta = 1, 2, \dots, k^n\}$ ,  $j = 1, 2, \dots, l$ .

## 4 Application

In this section, we present an example to illustrate the control design techniques considered in this survey.

**Example 2** Consider the reduced Boolean model of the lac operon in *Escherichia coli* (Veliz-Cuba and Stigler, 2011). The biological meaning of each node was given in Veliz-Cuba and Stigler (2011).

$$\begin{cases} x_1(t+1) = \neg u_1(t) \wedge (x_2(t) \vee x_3(t)), \\ x_2(t+1) = \neg u_1(t) \wedge u_2(t) \wedge x_1(t), \\ x_3(t+1) = \neg u_1(t) \wedge (u_2(t) \vee (u_3(t) \wedge x_1(t))). \end{cases} \quad (28)$$

First, we consider the global stabilization of system (28) via the reachable set approach. We take an equilibrium  $\mathbf{X}_e = (1, 0, 1) \sim \mathbf{x}_e = \delta_8^3$ , which corresponds to lac operon being ‘‘ON’’ (Veliz-Cuba and Stigler, 2011). Setting  $\mathbf{x}(t) = \times_{i=1}^3 \mathbf{x}_i(t)$  and  $\mathbf{u}(t) = \times_{j=1}^3 \mathbf{u}_j(t)$ , system (28) can be expressed in its algebraic form as

$$\mathbf{x}(t+1) = \mathbf{L}\mathbf{u}(t)\mathbf{x}(t),$$

where

$$\begin{aligned} \mathbf{L} = \delta_8[ & 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, \\ & 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8, \\ & 1, 1, 1, 5, 3, 3, 3, 7, 1, 1, 1, 5, 3, 3, 3, 7, \\ & 3, 3, 3, 7, 4, 4, 4, 8, 4, 4, 4, 8, 4, 4, 4, 8]. \end{aligned}$$

Note that  $E_1^{(R)}(\delta_8^3) = \{\delta_8^i : i = 1, 2, 3, 5, 6, 7\}$  and  $E_2^{(R)}(\delta_8^3) = \Delta_8$ . Therefore, conditions (1) and (2) in Theorem 1 are satisfied, which means that system (28) can be globally stabilized to  $\mathbf{x}_e$  by a state feedback controller.

For  $j = 1, 2, \dots, 8$ , let  $l_j$  and  $\nu_j$  be as those in Theorem 1. We have  $l_j = 1$ ,  $j = 1, 2, 3, 5, 6, 7$ ,  $l_j = 2$ ,  $j = 4, 8$ . Then, we can take  $p_j = 7$ ,  $j = 1, 2, 3$ , and  $p_j = 5$ ,  $j = 4, 5, 6, 7, 8$ . Thus, the feedback controller  $\mathbf{u}(t) = \mathbf{K}\mathbf{x}(t)$  with  $\mathbf{K} = \delta_8[7, 7, 7, 5, 5, 5, 5, 5]$  globally stabilizes system (28) to  $\mathbf{x}_e = \delta_8^3$ .

Next, we consider the sampled-data stabilization of system (28) with SCS  $C_x = \{\delta_8^1, \delta_8^2, \delta_8^3, \delta_8^4, \delta_8^5, \delta_8^7\}$  and ICS  $C_u = \{\delta_8^5, \delta_8^6, \delta_8^7, \delta_8^8\}$ . The equilibrium is also given as  $\mathbf{x}_e = \delta_8^3$ .

Given the set of sampling points  $\hat{S} = \{t_h : h \in \mathbb{N}\}$ , as for the uniformly sampled case, we take the sampling period  $\rho = 2$ . One can calculate that  $\mathbf{L}\delta_8^7\delta_8^3 = \delta_8^3$ ,  $E_2^{(S)}(\mathbf{x}_e) = \{\delta_8^1, \delta_8^2, \delta_8^3, \delta_8^4\}$ , and

$E_4^{(S)}(\mathbf{x}_e) = \{\delta_8^1, \delta_8^2, \delta_8^3, \delta_8^4, \delta_8^5, \delta_8^7\} = C_x$ . Then, according to Theorem 3, system (28) with SCS  $C_x$  and ICS  $C_u$  is stabilizable to  $\mathbf{x}_e = \delta_8^3$  by the uniformly-sampled-data control  $\mathbf{u}(t) = \mathbf{K}\mathbf{x}(t_h), t \in [t_h, t_{h+1})|_{\mathbb{N}}$  with

$$\mathbf{K} = \delta_8[\mu_1, \mu_2, \dots, \mu_8], \quad (29)$$

where  $\mu_j \in \{5\}, j = 4, 5, 7, \mu_j \in \{7\}, j = 1, 2, 3$ , and  $\mu_j \in \{1, 2, 3, 4, 5, 6, 7, 8\}, j = 6, 8$ .

As for the nonuniformly sampled case, assume that  $\rho_0 = 2, \rho_1 = 3, \rho_2 = 2, \rho_3 = 3, \dots$ . One can obtain that the nonuniformly-sampled-data control  $\mathbf{u}(t) = \mathbf{K}\mathbf{x}(t_h), t \in [t_h, t_{h+1})|_{\mathbb{N}}$  with Eq. (29) can also stabilize system (28) with SCS  $C_x$  and ICS  $C_u$  to  $\mathbf{x}_e = \delta_8^3$ .

## 5 Conclusions and future work

In this survey, we have reviewed a series of new developments for control design techniques of LCNs, including the reachable set approach, pinning control technique, control Lyapunov function approach, event-triggered control technique, and sampled-data control technique. Several other kinds of control methods can be introduced to the control design problems of LCNs in the future, including impulsive control (Li XD et al., 2015, 2019b; Vinodkumar et al., 2018; Li HT et al., 2019b) and observer-based control. Reducing the computational complexity of existing methods is an interesting topic. In the last decade, several methods have been introduced to reduce computational complexity, including the aggregation method (Zhao et al., 2016) and the matrix factorization approach (Li HT et al., 2015). It is hoped that more effective methods can be introduced to reduce the computational complexity of LCNs in the future. Extending the application fields of LCNs is also an interesting topic. In the past few decades, LCNs have found wide application in circuit design, finite automata, game theory, graph theory, fuzzy control, feedback register, and finite-field networks (Li HT et al., 2018a). In the future, one may apply the theory of LCNs to the modeling and control of engineering devices such as unmanned aerial vehicles and hybrid electric vehicles.

### Contributors

Xiang-shan KONG and Shu-ling WANG drafted the manuscript. Fuad E. ALSAADI helped organize the manuscript. Hai-tao LI revised and finalized the manuscript.

### Compliance with ethics guidelines

Xiang-shan KONG, Shu-ling WANG, Hai-tao LI, and Fuad E. ALSAADI declare that they have no conflict of interest.

### References

- Åström KJ, Bernhardsson B, 1999. Comparison of periodic and event based sampling for first order stochastic systems. Proc 14<sup>th</sup> IFAC World Congress, p.5006-5011.
- Chen HW, Liang JL, 2017. Output regulation of Boolean control networks with stochastic disturbances. *IET Contr Theory Appl*, 11(13):2097-2103. <http://doi.org/10.1049/iet-cta.2016.1675>
- Chen HW, Liang JL, Wang ZD, 2016. Pinning controllability of autonomous Boolean control networks. *Sci China Inform Sci*, 59(7):070107. <https://doi.org/10.1007/s11432-016-5579-8>
- Cheng DZ, Qi HS, Li ZQ, 2011a. Analysis and Control of Boolean Networks: a Semi-tensor Product Approach. Springer, London, UK.
- Cheng DZ, Qi HS, Li ZQ, et al., 2011b. Stability and stabilization of Boolean networks. *Int J Rob Nonl Contr*, 21(2):134-156. <https://doi.org/10.1002/rnc.1581>
- Cheng DZ, Feng JE, Lv HL, 2012. Solving fuzzy relational equations via semitensor product. *IEEE Trans Fuzzy Syst*, 20(2):390-396. <https://doi.org/10.1109/TFUZZ.2011.2174243>
- Cheng DZ, Qi HS, Liu ZQ, 2018. From STP to game-based control. *Sci China Inform Sci*, 61(1):010201. <https://doi.org/10.1007/s11432-017-9265-2>
- Ding XY, Li HT, 2019. Optimal control of random evolutionary Boolean games. *Int J Contr*, online. <https://doi.org/10.1080/00207179.2019.1585957>
- Ding XY, Li HT, Yang QQ, et al., 2017. Stochastic stability and stabilization of  $n$ -person random evolutionary Boolean games. *Appl Math Comput*, 306:1-12. <https://doi.org/10.1016/j.amc.2017.02.020>
- Ding XY, Li HT, Wang SL, 2018. Set stability and synchronization of logical networks with probabilistic time delays. *J Franklin Inst*, 355(15):7735-7748. <https://doi.org/10.1016/j.jfranklin.2018.08.009>
- Ding XY, Li HT, Alsaadi FE, 2019. Regulation of game result for  $n$ -person random evolutionary Boolean games. *Asian J Contr*, online. <https://doi.org/10.1002/asjc.2119>
- Ding Y, Guo YQ, Xie YF, et al., 2017. Time-optimal state feedback stabilization of switched Boolean control networks. *Neurocomputing*, 237:265-271. <https://doi.org/10.1016/j.neucom.2016.12.044>
- Fan HB, Feng JE, Meng M, et al., 2018. General decomposition of fuzzy relations: semi-tensor product approach. *Fuzzy Set Syst*, online. <https://doi.org/10.1016/j.fss.2018.12.012>
- Fornasini E, Valcher MM, 2013. On the periodic trajectories of Boolean control networks. *Automatica*, 49(5):1506-1509. <https://doi.org/10.1016/j.automatica.2013.02.027>
- Fornasini E, Valcher ME, 2016. Recent developments in Boolean networks control. *J Contr Dec*, 3(1):1-18. <https://doi.org/10.1080/23307706.2015.1098577>
- Goebel R, Priour C, Teel AR, 2009. Smooth patchy control Lyapunov functions. *Automatica*, 45(3):675-683. <https://doi.org/10.1016/j.automatica.2008.10.023>

- Guo YQ, Wang P, Gui WH, et al., 2015. Set stability and set stabilization of Boolean control networks based on invariant subsets. *Automatica*, 61:106-112. <https://doi.org/10.1016/j.automat.2015.08.006>
- Karafyllis I, Jiang ZP, 2013. Global stabilization of nonlinear systems based on vector control Lyapunov functions. *IEEE Trans Autom Contr*, 58(10):2550-2562. <https://doi.org/10.1109/TAC.2013.2264855>
- Lee TH, Park JH, 2017. Improved criteria for sampled-data synchronization of chaotic Lur'e systems using two new approaches. *Nonl Anal Hybr Syst*, 24:132-145. <https://doi.org/10.1016/j.nahs.2016.11.006>
- Lee TH, Park JH, Kwon OM, et al., 2013. Stochastic sampled-data control for state estimation of time-varying delayed neural networks. *Neur Netw*, 46:99-108. <https://doi.org/10.1016/j.neunet.2013.05.001>
- Li BW, Liu Y, Kou KI, et al., 2018. Event-triggered control for the disturbance decoupling problem of Boolean control networks. *IEEE Trans Cybern*, 48(9):2764-2769. <https://doi.org/10.1109/TCYB.2017.2746102>
- Li BW, Lu JQ, Zhong J, et al., 2019a. Fast-time stability of temporal Boolean networks. *IEEE Trans Neur Netw Learn Syst*, 30(8):2285-2294. <https://doi.org/10.1109/TNNLS.2018.2881459>
- Li BW, Lu JQ, Liu Y, et al., 2019b. The outputs robustness of Boolean control networks via pinning control. *IEEE Trans Contr Netw Syst*, online. <https://doi.org/10.1109/TCNS.2019.2913543>
- Li BW, Lou JG, Liu Y, et al., 2019c. Robust invariant set analysis of Boolean networks. *Complexity*, 2019:2731395. <https://doi.org/10.1155/2019/2731395>
- Li FF, 2016a. Feedback control design for the complete synchronisation of two coupled Boolean networks. *Int J Syst Sci*, 47(12):2996-3003. <https://doi.org/10.1080/00207721.2015.1053834>
- Li FF, 2016b. Pinning control design for the stabilization of Boolean networks. *IEEE Trans Neur Netw Learn Syst*, 27(7):1585-1590.
- Li FF, 2016c. Pinning control design for the synchronization of two coupled Boolean networks. *IEEE Trans Circ Syst*, 63(3):309-313. <https://doi.org/10.1109/TCSII.2015.2482658>
- Li FF, 2018. Stability of Boolean networks with delays using pinning control. *IEEE Trans Contr Netw Syst*, 5(1):179-185. <https://doi.org/10.1109/TCNS.2016.2585861>
- Li FF, Sun JT, 2012. Stability and stabilization of Boolean networks with impulsive effects. *Syst Contr Lett*, 61(1):1-5. <https://doi.org/10.1016/j.sysconle.2011.09.019>
- Li FF, Tang Y, 2017. Set stabilization for switched Boolean control networks. *Automatica*, 78:223-230. <https://doi.org/10.1016/j.automat.2016.12.007>
- Li FF, Xie LH, 2019. Set stabilization of probabilistic Boolean networks using pinning control. *IEEE Trans Neur Netw Learn Syst*, 30(8):2555-2561.
- Li FF, Yan HC, Karimi HR, 2018. Single-input pinning controller design for reachability of Boolean networks. *IEEE Trans Neur Netw Learn Syst*, 29(7):3264-3269. <https://doi.org/10.1109/TNNLS.2017.2705109>
- Li H, Li Y, Wang S, 2019. Recent development on analysis and control of finite-value dynamic systems. *J Shandong Norm Univ Nat Sci*, in press (in Chinese).
- Li HT, Ding XY, 2019. A control Lyapunov function approach to feedback stabilization of logical control networks. *SIAM J Contr Optim*, 57(2):810-831.
- Li HT, Wang YZ, 2013. Output feedback stabilization control design for Boolean control networks. *Automatica*, 49(12):3641-3645. <https://doi.org/10.1016/j.automat.2013.09.023>
- Li HT, Wang YZ, 2016a. Minimum-time state feedback stabilization of constrained Boolean control networks. *Asian J Contr*, 18(5):1688-1697. <https://doi.org/10.1002/asjc.1234>
- Li HT, Wang YZ, 2016b. Output tracking of switched Boolean networks under open-loop/closed-loop switching signals. *Nonl Anal Hybr Syst*, 22:137-146. <https://doi.org/10.1016/j.nahs.2016.04.001>
- Li HT, Wang YZ, 2016c. Robust stability and stabilisation of Boolean networks with disturbance inputs. *Int J Syst Sci*, 48(4):750-756. <https://doi.org/10.1080/00207721.2016.1212433>
- Li HT, Wang YZ, 2017a. Further results on feedback stabilization control design of Boolean control networks. *Automatica*, 83:303-308. <https://doi.org/10.1016/j.automat.2017.06.043>
- Li HT, Wang YZ, 2017b. Lyapunov-based stability and construction of Lyapunov functions for Boolean networks. *SIAM J Contr Optim*, 55(6):3437-3457. <https://doi.org/10.1137/16M1092581>
- Li HT, Xiao XF, Lei XY, et al., 2013a. Second-order consensus seeking in multi-agent systems with nonlinear dynamics over random switching directed networks. *IEEE Trans Circ Syst*, 60(6):1595-1607. <https://doi.org/10.1109/TCSI.2012.2226507>
- Li HT, Wang YZ, Liu ZB, 2013b. Simultaneous stabilization for a set of Boolean control networks. *Syst Contr Lett*, 62(12):1168-1174. <https://doi.org/10.1016/j.sysconle.2013.09.008>
- Li HT, Wang YZ, Liu ZB, 2014. Stability analysis for switched Boolean networks under arbitrary switching signals. *IEEE Trans Autom Contr*, 59(7):1978-1982. <https://doi.org/10.1109/TAC.2014.2298731>
- Li HT, Wang YZ, Xie LH, 2015. Output tracking control of Boolean control networks via state feedback: constant reference signal case. *Automatica*, 59:54-59. <https://doi.org/10.1016/j.automat.2015.06.004>
- Li HT, Wang YZ, Guo PL, 2016. State feedback based output tracking control of probabilistic Boolean networks. *Inform Sci*, 349-350:1-11. <https://doi.org/10.1016/j.ins.2016.02.035>
- Li HT, Wang YZ, Guo PL, 2017a. Output reachability analysis and output regulation control design of Boolean control networks. *Sci China Inform Sci*, 60(2):022202. <https://doi.org/10.1007/s11432-015-0611-4>
- Li HT, Xie LH, Wang YZ, 2017b. Output regulation of Boolean control networks. *IEEE Trans Autom Contr*, 62(6):2993-2998.
- Li HT, Song PP, Yang QQ, 2017c. Pinning control design for robust output tracking of  $k$ -valued logical networks. *J Franklin Inst*, 354(7):3039-3053. <https://doi.org/10.1016/j.jfranklin.2017.02.009>
- Li HT, Ding XY, Alsaedi A, et al., 2017d. Stochastic set stabilisation of  $n$ -person random evolutionary Boolean games and its applications. *IET Contr Theory Appl*, 11(13):2152-2160.
- Li HT, Zhao GD, Meng M, et al., 2018a. A survey on applications of semi-tensor product method in engineering. *Sci China Inform Sci*, 61(1):010202. <https://doi.org/10.1007/s11432-017-9238-1>

- Li HT, Ding XY, Yang QQ, et al., 2018b. Algebraic formulation and Nash equilibrium of competitive diffusion games. *Dynam Game Appl*, 8(2):423-433. <https://doi.org/10.1007/s13235-017-0228-4>
- Li HT, Zheng YT, Alsaadi FE, 2019a. Algebraic formulation and topological structure of Boolean networks with state-dependent delay. *J Comput Appl Math*, 350:87-97. <https://doi.org/10.1016/j.cam.2018.10.003>
- Li HT, Xu XJ, Ding XY, 2019b. Finite-time stability analysis of stochastic switched Boolean networks with impulsive effect. *Appl Math Comput*, 347:557-565. <https://doi.org/10.1016/j.amc.2018.11.018>
- Li JN, Modares H, Chai TY, et al., 2017. Off-policy reinforcement learning for synchronization in multiagent graphical games. *IEEE Trans Neur Netw Learn Syst*, 28(10):2434-2445. <https://doi.org/10.1109/TNNLS.2016.2609500>
- Li R, Yang M, Chu TG, 2013. State feedback stabilization for Boolean control networks. *IEEE Trans Autom Contr*, 58(7):1853-1857. <https://doi.org/10.1109/TAC.2013.2238092>
- Li R, Yang M, Chu TG, 2014. State feedback stabilization for probabilistic Boolean networks. *Automatica*, 50(4):1272-1278. <https://doi.org/10.1016/j.automatica.2014.02.034>
- Li XD, Shen JH, Akca H, et al., 2015. LMI-based stability for singularly perturbed nonlinear impulsive differential systems with delays of small parameter. *Appl Math Comput*, 250:798-804. <https://doi.org/10.1016/j.amc.2014.10.113>
- Li XD, Li HT, Zhao GD, 2019a. Function perturbation impact on feedback stabilization of Boolean control networks. *IEEE Trans Neur Netw Learn Syst*, 30(8):2548-2554. <https://doi.org/10.1109/TNNLS.2018.2881168>
- Li XD, Yang XY, Huang TW, 2019b. Persistence of delayed cooperative models: impulsive control method. *Appl Math Comput*, 342:130-146. <https://doi.org/10.1016/j.amc.2018.09.003>
- Li XH, Lu JQ, Chen XY, et al., 2018. Robust output tracking of delayed Boolean networks under pinning control. *IEEE Trans Circ Syst*, 65(9):1249-1253. <https://doi.org/10.1109/TCSII.2018.2827931>
- Li YL, Li HT, Sun WW, 2018a. Event-triggered control for robust set stabilization of logical control networks. *Automatica*, 95:556-560. <https://doi.org/10.1016/j.automatica.2018.06.030>
- Li YL, Li HT, Xu XJ, et al., 2018b. Semi-tensor product approach to minimal-agent consensus control of networked evolutionary games. *IET Contr Theory Appl*, 12(16):2269-2275. <https://doi.org/10.1049/iet-cta.2018.5230>
- Li YL, Li HT, Duan PY, 2018c. Synchronization of switched logical control networks via event-triggered control. *J Franklin Inst*, 355(12):5203-5216. <https://doi.org/10.1016/j.jfranklin.2018.04.028>
- Li YL, Li HT, Wang SL, 2019. Constrained sampled-data reachability and stabilization of logical control networks. *IEEE Trans Circ Syst*, 66(12):2002-2006. <https://doi.org/10.1109/TCSII.2019.2892357>
- Li YY, Li BW, Liu Y, et al., 2018. Set stability and stabilization of switched Boolean networks with state-based switching. *IEEE Access*, 6:35624-35630. <https://doi.org/10.1109/ACCESS.2018.2851391>
- Liang S, Zhao GD, Li HT, et al., 2019. Structural stability analysis of gene regulatory networks modeled by Boolean networks. *Math Meth Appl Sci*, 42(7):2221-2230. <https://doi.org/10.1002/mma.5488>
- Lin L, Zhu SY, Liu Y, et al., 2019. Output regulation of Boolean control networks with nonuniform sampled-data control. *IEEE Access*, 7:50691-50696. <https://doi.org/10.1109/ACCESS.2019.2910593>
- Liu JY, Liu Y, Guo YQ, et al., 2019. Sampled-data state-feedback stabilization of probabilistic Boolean control networks: a control Lyapunov function approach. *IEEE Trans Cybern*, online. <https://doi.org/10.1109/TCYB.2019.2932914>
- Liu RJ, Lu JQ, Lou JG, et al., 2017. Set stabilization of Boolean networks under pinning control strategy. *Neurocomputing*, 260:142-148. <https://doi.org/10.1016/j.neucom.2017.04.005>
- Liu RJ, Lu JQ, Liu Y, et al., 2018. Delayed feedback control for stabilization of Boolean control networks with state delay. *IEEE Trans Neur Netw Learn Syst*, 29(7):3283-3288. <https://doi.org/10.1109/TNNLS.2017.2659386>
- Liu RJ, Lu JQ, Zheng WX, et al., 2019. Output feedback control for set stabilization of Boolean control networks. *IEEE Trans Neur Netw Learn Syst*, online. <https://doi.org/10.1109/TNNLS.2019.2928028>
- Liu S, Li T, Xie LH, et al., 2013. Continuous-time and sampled-data-based average consensus with logarithmic quantizers. *Automatica*, 49(11):3329-3336. <https://doi.org/10.1016/j.automatica.2013.07.016>
- Liu Y, Lu JQ, Wu B, 2014. Some necessary and sufficient conditions for the output controllability of temporal Boolean control networks. *ESAIM Contr Optim Calc Var*, 20(1):158-173. <https://doi.org/10.1051/cocv/2013059>
- Liu Y, Chen HW, Lu JQ, et al., 2015. Controllability of probabilistic Boolean control networks based on transition probability matrices. *Automatica*, 52:340-345. <https://doi.org/10.1016/j.automatica.2014.12.018>
- Liu Y, Li BW, Lou JG, 2016a. Disturbance decoupling of singular Boolean control networks. *IEEE/ACM Trans Comput Biol Bioinform*, 13(6):1194-1200. <https://doi.org/10.1109/TCBB.2015.2509969>
- Liu Y, Sun LJ, Lu JQ, et al., 2016b. Feedback controller design for the synchronization of Boolean control networks. *IEEE Trans Neur Netw Learn Syst*, 27(9):1991-1996. <https://doi.org/10.1109/TNNLS.2015.2461012>
- Liu Y, Cao JD, Sun LJ, et al., 2016c. Sampled-data state feedback stabilization of Boolean control networks. *Neuro Comput*, 28(4):778-799. [https://doi.org/10.1162/NECO\\_a\\_00819](https://doi.org/10.1162/NECO_a_00819)
- Liu Y, Li BW, Chen HW, et al., 2017a. Function perturbations on singular Boolean networks. *Automatica*, 84:36-42. <https://doi.org/10.1016/j.automatica.2017.06.035>
- Liu Y, Li BW, Lu JQ, et al., 2017b. Pinning control for the disturbance decoupling problem of Boolean networks. *IEEE Trans Autom Contr*, 62(12):6595-6601. <https://doi.org/10.1109/TAC.2017.2715181>
- Liu Y, Cao JD, Li BW, et al., 2018. Normalization and solvability of dynamic-algebraic Boolean networks. *IEEE Trans Neur Netw Learn Syst*, 29(7):3301-3306.
- Liu Y, Tong LY, Lou JG, et al., 2019a. Sampled-data control for the synchronization of Boolean control networks. *IEEE Trans Cybern*, 49(2):726-732.

- Liu Y, Wang LQ, Lu JQ, et al., 2019b. Sampled-data stabilization of probabilistic Boolean control networks. *Syst Contr Lett*, 124:106-111. <https://doi.org/10.1016/j.sysconle.2018.12.012>
- Liu YS, Zheng YT, Li HT, et al., 2018. Control design for output tracking of delayed Boolean control networks. *J Comput Appl Math*, 327:188-195. <https://doi.org/10.1016/j.cam.2017.06.016>
- Liu ZB, Wang Y, Li H, 2014. New approach to derivative calculation of multi-valued logical functions with application to fault detection. *IET Contr Theory Appl*, 8(8): 554-560. <https://doi.org/10.1049/iet-cta.2013.0104>
- Lu JQ, Zhong J, Ho DWC, et al., 2016a. On controllability of delayed Boolean control networks. *SIAM J Contr Optim*, 54(2):475-494. <https://doi.org/10.1137/140991820>
- Lu JQ, Zhong J, Huang C, et al., 2016b. On pinning controllability of Boolean control networks. *IEEE Trans Autom Contr*, 61(6):1658-1663. <https://doi.org/10.1109/TAC.2015.2478123>
- Lu JQ, Li HT, Liu Y, et al., 2017. Survey on semi-tensor product method with its applications in logical networks and other finite-valued systems. *IET Contr Theory Appl*, 11(13):2040-2047. <http://doi.org/10.1049/iet-cta.2016.1659>
- Lu JQ, Li ML, Liu Y, et al., 2018a. Nonsingularity of Grain-like cascade FSRs via semi-tensor product. *Sci China Inform Sci*, 61(1):010204. <https://doi.org/10.1007/s11432-017-9269-6>
- Lu JQ, Sun LJ, Liu Y, et al., 2018b. Stabilization of Boolean control networks under aperiodic sampled-data control. *SIAM J Contr Optim*, 56(6):4385-4404. <https://doi.org/10.1137/18M1169308>
- Lu JQ, Li ML, Huang TW, et al., 2018c. The transformation between the Galois NLF SRs and the Fibonacci NLF SRs via semi-tensor product of matrices. *Automatica*, 96:393-397. <https://doi.org/10.1016/j.automatica.2018.07.011>
- Lu YY, Zhang W, 2017. A piecewise smooth control-Lyapunov function framework for switching stabilization. *Automatica*, 76:258-265. <https://doi.org/10.1016/j.automatica.2016.09.029>
- Lukk M, Kapushesky M, Nikkilä J, et al., 2010. A global map of human gene expression. *Nat Biotechnol*, 28(4):322-324. <https://doi.org/10.1038/nbt0410-322>
- Mao Y, Wang LQ, Liu Y, et al., 2018. Stabilization of evolutionary networked games with length- $r$  information. *Appl Math Comput*, 337:442-451. <https://doi.org/10.1016/j.amc.2018.05.027>
- Moraga C, Trillas E, Guadarrama S, 2003. Multiple-valued logic and artificial intelligence fundamentals of fuzzy control revisited. *Artif Intell Rev*, 20(3-4):169-197.
- Mu NK, Liao XF, Huang TW, 2015. Leader-following consensus in second-order multiagent systems via event-triggered control with nonperiodic sampled data. *IEEE Trans Circ Syst*, 62(10):1007-1011. <https://doi.org/10.1023/10.1109/TCSII.2015.2458036>
- Müller FJ, Schuppert A, 2011. Few inputs can reprogram biological networks. *Nature*, 478(7369):E4. <https://doi.org/10.1038/nature10543>
- Müller FJ, Schuldt BM, Williams R, et al., 2011. A bioinformatic assay for pluripotency in human cells. *Nat Meth*, 8(4):315-317. <https://doi.org/10.1038/nmeth.1580>
- Postoyan R, Tabuada P, Nešić D, et al., 2015. A framework for the event-triggered stabilization of nonlinear systems. *IEEE Trans Autom Contr*, 60(4):982-996. <https://doi.org/10.1109/TAC.2014.2363603>
- Rosin DP, Rontani D, Gauthier DJ, et al., 2013. Control of synchronization patterns in neural-like Boolean networks. *Phys Rev Lett*, 110(5):104102. <https://doi.org/10.1103/PhysRevLett.110.104102>
- Sanfelice RG, 2013. On the existence of control Lyapunov functions and state-feedback laws for hybrid systems. *IEEE Trans Autom Contr*, 58(12):3242-3248. <https://doi.org/10.1109/TAC.2013.2264851>
- Sun LJ, Lu JQ, Ching WK, 2020. Switching-based stabilization of aperiodic sampled-data Boolean control networks with all subsystems unstable. *Front Inform Technol Electron Eng*, 21(2):260-267. <https://doi.org/10.1631/FITEE.1900312>
- Tong LY, Liu Y, Alsaadi FE, et al., 2017. Robust sampled-data control invariance for Boolean control networks. *J Franklin Inst*, 354(15):7077-7087. <https://doi.org/10.1016/j.jfranklin.2017.08.017>
- Tong LY, Liu Y, Li YY, et al., 2018a. Robust control invariance of probabilistic Boolean control networks via event-triggered control. *IEEE Access*, 6:37767-37774. <https://doi.org/10.1109/ACCESS.2018.2828128>
- Tong LY, Liu Y, Lou JG, et al., 2018b. Static output feedback set stabilization for context-sensitive probabilistic Boolean control networks. *Appl Math Comput*, 332:263-275. <https://doi.org/10.1016/j.amc.2018.03.043>
- Tong LY, Liang JL, Chen HW, 2019. State feedback controller design for anti-synchronization of Boolean control networks: an event-based idea. *Asian J Contr*, online. <https://doi.org/10.1002/asjc.2105>
- Vaidya U, Mehta PG, Shanbhag UV, 2010. Nonlinear stabilization via control Lyapunov measure. *IEEE Trans Autom Contr*, 55(6):1314-1328. <https://doi.org/10.1109/TAC.2010.2042226>
- Veliz-Cuba A, Stigler B, 2011. Boolean models can explain bistability in the lac operon. *J Comput Biol*, 18(6):783-794. <https://doi.org/10.1089/cmb.2011.0031>
- Vinodkumar A, Senthilkumar T, Li XD, 2018. Robust exponential stability results for uncertain infinite delay differential systems with random impulsive moments. *Adv Differ Equat*, 2018:39. <https://doi.org/10.1186/s13662-018-1488-z>
- Wang B, Feng JE, 2019. On detectability of probabilistic Boolean networks. *Inform Sci*, 483:383-395. <https://doi.org/10.1016/j.ins.2019.01.055>
- Wang JY, Feng JW, Xu C, et al., 2016. The synchronization of instantaneously coupled harmonic oscillators using sampled data with measurement noise. *Automatica*, 66:155-162. <https://doi.org/10.1016/j.automatica.2016.01.012>
- Wang LQ, Liu Y, Wu ZG, et al., 2019. Stabilization and finite-time stabilization of probabilistic Boolean control networks. *IEEE Trans Syst Man Cybern Syst*, online. <https://doi.org/10.1109/TSMC.2019.2898880>
- Wang SL, Li HT, 2019. Column stacking approach to resolution of systems of fuzzy relational inequalities. *J Franklin Inst*, 356(6):3314-3332. <https://doi.org/10.1016/j.jfranklin.2019.02.007>

- Wang YH, Cheng DZ, 2017. Stability and stabilization of a class of finite evolutionary games. *J Franklin Inst*, 354(3):1603-1617. <https://doi.org/10.1016/j.jfranklin.2016.12.007>
- Wu YQ, Meng XY, Xie LH, et al., 2017. An input-based triggering approach to leader-following problems. *Automatica*, 75:221-228. <https://doi.org/10.1016/j.automatica.2016.09.040>
- Wu ZG, Shi P, Su HY, et al., 2014. Local synchronization of chaotic neural networks with sampled-data and saturating actuators. *IEEE Trans Cybern*, 44(12):2635-2645. <https://doi.org/10.1109/TCYB.2014.2312004>
- Xu XJ, Li HT, Li YL, et al., 2018a. Output tracking control of Boolean control networks with impulsive effects. *Math Meth Appl Sci*, 41(4):1554-1564. <https://doi.org/10.1002/mma.4685>
- Xu XJ, Liu YS, Li HT, et al., 2018b. Robust set stabilization of Boolean control networks with impulsive effects. *Nonl Anal Model Contr*, 23(4):553-567. <https://doi.org/10.15388/NA.2018.4.6>
- Xu XJ, Liu YS, Li HT, et al., 2018c. Synchronization of switched Boolean networks with impulsive effects. *Int J Biomath*, 11(6):1850080.
- Yang JJ, Lu JQ, Li LL, et al., 2019. Event-triggered control for the synchronization of Boolean control networks. *Nonl Dynam*, 96(2):1335-1344. <https://doi.org/10.1007/s11071-019-04857-2>
- Yang JJ, Lu JQ, Lou JG, et al., 2020. Synchronization of drive-response Boolean control networks with impulsive disturbances. *Appl Math Comput*, 364:124679. <https://doi.org/10.1016/j.amc.2019.124679>
- Yang QQ, Li HT, Liu YS, 2016. Pinning control design for feedback stabilization of constrained Boolean control networks. *Adv Differ Equat*, 2016:182. <https://doi.org/10.1186/s13662-016-0909-0>
- Yang QQ, Li HT, Song PP, et al., 2017. Global convergence of serial Boolean networks based on algebraic representation. *J Differ Equat Appl*, 23(3):633-647. <https://doi.org/10.1080/10236198.2016.1270275>
- Yu YY, Feng JE, Wang B, et al., 2018. Sampled-data controllability and stabilizability of Boolean control networks: nonuniform sampling. *J Franklin Inst*, 335(12):5324-5335. <https://doi.org/10.1016/j.jfranklin.2018.05.036>
- Yu YY, Meng M, Feng JE, et al., 2019a. An adjoint network approach to design stabilizable switching signals of switched Boolean networks. *Appl Math Comput*, 357:12-22. <https://doi.org/10.1016/j.amc.2019.02.067>
- Yu YY, Feng JE, Pan JF, et al., 2019b. Block decoupling of Boolean control networks. *IEEE Trans Autom Contr*, 64(8):3129-3140. <https://doi.org/10.1109/TAC.2018.2880411>
- Yu YY, Wang B, Feng JE, 2019c. Input observability of Boolean control networks. *Neurocomputing*, 333:22-28. <https://doi.org/10.1016/j.neucom.2018.12.014>
- Zhang LQ, Feng JE, Feng XH, et al., 2014. Further results on disturbance decoupling of mix-valued logical networks. *IEEE Trans Autom Contr*, 59(6):1630-1634. <https://doi.org/10.1109/TAC.2013.2292733>
- Zhao Y, Ghosh BK, Cheng DZ, 2016. Control of large-scale Boolean networks via network aggregation. *IEEE Trans Neur Netw Learn Syst*, 27(7):1527-1536. <https://doi.org/10.1109/TNNLS.2015.2442593>
- Zheng YT, Li HT, Ding XY, et al., 2017. Stabilization and set stabilization of delayed Boolean control networks based on trajectory stabilization. *J Franklin Inst*, 354(17):7812-7827.
- Zhong J, Lu JQ, Liu Y, et al., 2014. Synchronization in an array of output-coupled Boolean networks with time delay. *IEEE Tran Neur Netw Learn Syst*, 25(12):2288-2294. <https://doi.org/10.1109/TNNLS.2014.2305722>
- Zhong J, Lu JQ, Huang TW, et al., 2017a. Controllability and synchronization analysis of identical-hierarchy mixed-valued logical control networks. *IEEE Trans Cybern*, 47(11):3482-3493. <https://doi.org/10.1109/TCYB.2016.2560240>
- Zhong J, Ho DWC, Lu JQ, et al., 2017b. Switching-signal-triggered pinning control for output tracking of switched Boolean networks. *IET Contr Theory Appl*, 11(13):2089-2096. <https://doi.org/10.1049/iet-cta.2017.0260>
- Zhong J, Ho DWCH, Lu JQ, et al., 2019a. Pinning controllers for activation output tracking of Boolean network under one-bit perturbation. *IEEE Trans Cybern*, 49(9):3398-3408. <https://doi.org/10.1109/TCYB.2018.2842819>
- Zhong J, Liu Y, Kou KI, et al., 2019b. On the ensemble controllability of Boolean control networks using STP method. *Appl Math Comput*, 358:51-62. <https://doi.org/10.1016/j.amc.2019.03.059>
- Zhong J, Li BW, Liu Y, et al., 2020. Output feedback stabilizer design of Boolean networks based on network structure. *Front Inform Technol Electron Eng*, 21(2):247-259. <https://doi.org/10.1631/FITEE.1900229>
- Zhu B, Xia XH, Wu Z, 2016. Evolutionary game theoretic demand-side management and control for a class of networked smart grid. *Automatica*, 70:94-100. <https://doi.org/10.1016/j.automatica.2016.03.027>
- Zhu QX, Lin W, 2019. Stabilizing Boolean networks by optimal event-triggered feedback control. *Syst Contr Lett*, 126:40-47. <https://doi.org/10.1016/j.sysconle.2019.03.002>
- Zhu QX, Liu Y, Lu JQ, et al., 2018a. Controllability and observability of Boolean control networks via sampled-data control. *IEEE Trans Contr Netw Syst*, online. <https://doi.org/10.1109/TCNS.2018.2885273>
- Zhu QX, Liu Y, Lu JQ, et al., 2018b. Observability of Boolean control networks. *Sci China Inform Sci*, 61(9):092201. <https://doi.org/10.1007/s11432-017-9135-4>
- Zhu QX, Liu Y, Lu JQ, et al., 2018c. On the optimal control of Boolean control networks. *SIAM J Contr Optim*, 56(2):1321-1341. <https://doi.org/10.1137/16M1070281>
- Zhu QX, Liu Y, Lu JQ, et al., 2019. Further results on the controllability of Boolean control networks. *IEEE Trans Autom Contr*, 64(1):440-442. <https://doi.org/10.1109/TAC.2018.2830642>
- Zhu SY, Lou JG, Liu Y, et al., 2018a. Event-triggered control for the stabilization of probabilistic Boolean control networks. *Complexity*, 2018:9259348. <https://doi.org/10.1155/2018/9259348>
- Zhu SY, Liu Y, Lou JG, et al., 2018b. Sampled-data state feedback control for the set stabilization of Boolean control networks. *IEEE Trans Syst Man Cybern Syst*, online. <https://doi.org/10.1109/TSMC.2018.2852703>
- Zhu SY, Lu JQ, Ho DWC, 2019. Topological sorting for finite-time stability of probabilistic logical networks. *IEEE Trans Circ Syst*, online. <https://doi.org/10.1109/TCSII.2019.2919018>