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Sampled data based containment control of second-order multi-agent systems under intermittent communications*

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Abstract: This paper studies the sampled data based containment control problem of second-order multi-agent systems with intermittent communications, where velocity measurements for each agent are unavailable. A novel controller for second-order containment is put forward via intermittent sampled position data measurement. Several necessary and sufficient conditions are derived to achieve intermittent sampled containment control by means of analyzing the relationship among control gains, eigenvalues of the Laplacian matrix, the sampling period, and the communication width. Finally, several simulation examples are used to testify the correctness and effectiveness of the theoretical results.

Key words: Containment control; Second-order multi-agent system; Sampled position data; Intermittent communication; Communication width

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1 Introduction

Recently, the distributed containment control problem of multi-agent systems has attracted considerable attention due to its wide applications in areas such as epidemic prevention and control, hazardous material handling, and smart grid dispatching. Typically, the study of containment control of multi-agent systems is devoted to drive each follower into a geometric area (convex hull) formed by multiple leaders. Up to now, the containment control problem of multi-agent systems has been persistently addressed, and many profound theoretical results have been established (Liu HY et al., 2012; Liu KE et al., 2014; Wen et al., 2016; Wang D et al., 2019; Zou and Xiang, 2019; Liu TF et al., 2020; Liu YF and Su, 2020a, 2020b; Shi et al., 2020; Wang FY et al., 2020a, 2020b; Sun et al., 2021; Xiao et al., 2021).

All of the abovementioned references address only the containment control problem under continuous communications. On one hand, since links of communication networks may change, interactions among autonomous agents are time-varying or discontinuous. On the other hand, due to sensor/actuator failures, network-injected packet losses, and the limitation of sensing ranges, the whole communication link among agents will be intermittent, and the control inputs will be zero during disconnected communication time intervals. In this scenario, Wang FY et al. (2018, 2019b) investigated the

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second-order containment control problem with intermittent communications. Note that the velocity measurements were typically required in the second-order containment control problem for the aforementioned works. However, the velocity information among agents may be difficult to achieve in practical applications. Moreover, to save space, cost, and reduce weight, agents might not be equipped with velocity sensors in many real systems. Considering this fact, Hong et al. (2008) and Li et al. (2012) designed distributed observers for leader-following consensus and containment control, respectively. Based on the distributed filter, Mei et al. (2013) solved the consensus control problem for second-order multi-agent systems with nonlinear dynamics using only relative position measurements. Furthermore, Wang FY et al. (2019a) designed a novel intermittent containment control protocol without velocity measurements by a filter-based method. However, higher-order system variables were involved in Hong et al. (2008), Li et al. (2012), Mei et al. (2013), and Wang FY et al. (2019a). To overcome this shortage, Yu WW et al. (2011) designed a second-order consensus control protocol based only on the current and some sampled position data. Zou et al. (2018) investigated the second-order leader-following consensus for nonlinear multi-agent systems via sampled data communication without velocity measurements. Unfortunately, position information monitoring needs to be continuous. Under intermittent communications, Yu ZY et al. (2018) designed a periodic intermittent sampled data control protocol for second-order consensus of multi-agent systems, using position and velocity information. In some real situations, the accurate velocity measurements may be difficult to achieve and the communication networks are usually intermittent. Based on this fact, consensus or containment control of multi-agent systems with intermittent communications and inaccurate velocity measurements is necessary to study.

Su et al. (2020) investigated the consensus problem for second-order multi-agent systems with intermittent sampled position data. However, for practical problems, multi-agent systems need multiple leaders to solve the more complex problems. Liu YF and Su (2021) obtained some necessary and sufficient conditions for containment of second-order multi-agent systems with intermittent sampled data. To use less information and save more energy, it is desirable to use only the intermittent sampled position data instead of velocity information in the design of the controller. This inspires us to investigate whether second-order containment control can still be achieved using only intermittent sampled position data measurements, even if the velocity measurements are unavailable.

The main contributions of this paper can be summarized as follows:

1. Under intermittent communications, an intermittent sampled data based control method for solving the second-order containment control problem without velocity measurements has been put forward for the first time.

2. To use less information and save more energy, a novel intermittent sampled containment controller is first put forward via intermittent sampled position data communications.

3. Necessary and sufficient conditions hinging on the sampling period, the communication width, system parameters, and the relationship between the network topology structures are obtained for solving the second-order intermittent sampled containment control problem with unavailable velocity measurements.

Notations used in this paper are as follows: $\mathbb{R}$, $\mathbb{R}^n$, and $\mathbb{R}^{n\times n}$ denote the sets of real numbers, n-dimensional real column vectors, and $n\times n$ real matrices, respectively. $\mathbb{N}$ represents the set of integer numbers. Let $\mathbf{0}_{m\times n}$ denote the $m\times n$ matrix with all zeros and $\boldsymbol{I}_n$ denote the identity matrix with dimension n. The Kronecker product is denoted by $\otimes$, the Euclidean norm is denoted by $\|\cdot\|$, and the module is denoted by $|\cdot|$.

2 Model formulation

Let $G=(V,\xi)$ be the interaction topology graph of n nodes, in which $V=\{v_1,v_2,\ldots,v_n\}$ denotes the set of nodes and $\xi\subseteq V\times V$ denotes the set of edges. Let $\boldsymbol{A}=[a_{ij}]\in\mathbb{R}^{n\times n}$ be the weighted adjacency matrix of graph G, where a_{ij} represents the adjacency elements. Furthermore, for all $i,\ j=1,2,\ldots,n$, $a_{ij}>0$ if $(w_j,w_i)\in\xi$ and $a_{ij}=0$ otherwise, and $a_{ii}=0$. A path from v_i to v_j is denoted by $\pi_{i,j}=\{(v_{i1},v_{i2}),(v_{i2},v_{i3}),\ldots,(v_{i(q-1)},v_{iq})\}$, where $v_{i1}=v_i$, $v_{iq}=v_j$, and $(v_{ip},v_{i(p+1)})\in\xi$, $p\in\{1,2,\ldots,q-1\}$. For an undirected graph, $(v_j,v_i)\in\xi\Leftrightarrow(v_i,v_j)\in\xi$ implies $a_{ij}=a_{ji}$. If there exists a path between any pair of different nodes, then the undirected graph is said to be connected. Moreover, the Laplacian matrix of graph G is defined as $\boldsymbol{L} = [l_{ij}] \in \mathbb{R}^{n\times n}$, where $l_{ii} = \sum\limits_{j=1,j\neq i}^{n} a_{ij}$, $i = 1, 2, \ldots, n$, and $l_{ij} = -a_{ij}$, $i \neq j$, $i,\ j = 1, 2, \ldots, n$.

In many practical systems, switching is a common phenomenon. Moreover, switching behaviors usually occur in the network topology due to the communication link creation or failure, abrupt parameter variations, and sensor failure. In this study, to describe variable communication topologies, we define a set of all switching signals $\varGamma$. Let $\sigma(t) \in \varGamma$ be a piecewise constant right continuous switching signal. Specifically, $\sigma(t) : [0,\infty) \to \varGamma = \{1, 2, \ldots, S\}$, $\sigma = \sigma(t) \in \varGamma$, where S represents the total number of all possible interconnection topology graphs. In this case, the communication topology graph at time t is denoted by $G_{\sigma(t)}$. The Laplacian matrix associated with the switching topology graph $G_{\sigma(t)}$ is denoted by $\boldsymbol{L}_{\sigma(t)}$. In view that the communication topology graph $G_{\sigma(t)}$ is time-varying, the adjacency elements $a_{ij}(t)$ $(i, j = 1, 2, \ldots, n)$ and Laplacian matrix $\boldsymbol{L}_{\sigma(t)}$ are also time-varying.

Definition 1 (Rockafellar, 1972) The convex hull composed of a finite set of points $z = \{z_1, z_2, \ldots, z_n\} \subseteq \mathbb{R}^n$ is defined as

$$\mathrm{CO}(z) = \left\{ \sum_{i=1}^{n} \epsilon_i z_i | z_i \in z, \epsilon_i \in \mathbb{R}, \epsilon_i \geq 0, \sum_{i=1}^{n} \epsilon_i = 1 \right\}.$$

Definition 2 (Liu HY et al., 2012) For a multi-agent system, an agent is called a leader if the agent has no neighbor, and a follower if the agent has at least one neighbor.

Lemma 1 (Horn and Johnson, 1985) For matrices $\boldsymbol{Q}_1$–$\boldsymbol{Q}_4$ with compatible dimensions, there hold that

(a) $(\boldsymbol{Q}_1 + \boldsymbol{Q}_2) \otimes \boldsymbol{Q}_3 = \boldsymbol{Q}_1 \otimes \boldsymbol{Q}_3 + \boldsymbol{Q}_2 \otimes \boldsymbol{Q}_3$,

(b) $(\boldsymbol{Q}_1 \otimes Q_2)(\boldsymbol{Q}_3 \otimes \boldsymbol{Q}_4) = (\boldsymbol{Q}_1\boldsymbol{Q}_3) \otimes (\boldsymbol{Q}_2\boldsymbol{Q}_4)$,

where $\otimes$ represents the Kronecker product.

Consider a multi-agent system composed of $N + M$ agents, where there are N followers and M leaders. The dynamics of leaders is given by

$$\begin{cases} \dot{\boldsymbol{q}}_l(t) = \boldsymbol{p}_l(t), \\ \dot{\boldsymbol{p}}_l(t) = \boldsymbol{0},\ l = 1, 2, \ldots, M, \end{cases} \tag{1}$$

and followers are governed by the following dynamics:

$$\begin{cases} \dot{\boldsymbol{q}}_i(t) = \boldsymbol{p}_i(t), \\ \dot{\boldsymbol{p}}_i(t) = \boldsymbol{u}_i(t),\ i = 1, 2, \ldots, N, \end{cases} \tag{2}$$

where $\boldsymbol{q}_l(t) \in \mathbb{R}^n$ and $\boldsymbol{p}_l(t) \in \mathbb{R}^n$ are the position and velocity of leader l, respectively, and $\boldsymbol{q}_i(t) \in \mathbb{R}^n$, $\boldsymbol{p}_i(t) \in \mathbb{R}^n$, and $\boldsymbol{u}_i(t) \in \mathbb{R}^n$ are the position, velocity, and control input of follower i, respectively.

For the considered multi-agent system, the interactions among $N + M$ agents can be described by a graph G, and the interactions among N followers can be described by an undirected graph $\hat{G}$, where $\hat{G}$ is a subgraph of graph G. The Laplacian matrix of graph G is defined as a block matrix $\boldsymbol{L} = \begin{bmatrix} \boldsymbol{L}_1 & \boldsymbol{L}_2 \\ \boldsymbol{0}_{M\times N} & \boldsymbol{0}_{M\times M} \end{bmatrix}$, where $\boldsymbol{L}_1 \in \mathbb{R}^{N\times N}$ and $\boldsymbol{L}_2 \in \mathbb{R}^{N\times M}$ represent the communication relationships among all the followers and between the leaders and followers, respectively.

Lemma 2 (Liu HY et al., 2012) Assume that graph $\hat{G}$ is undirected and connected. All the eigenvalues of $\boldsymbol{L}_1$ are positive, each entry of $-\boldsymbol{L}_1^{-1}\boldsymbol{L}_2$ is nonnegative, and the elements of each row of $-\boldsymbol{L}_1^{-1}\boldsymbol{L}_2$ have a sum equal to one.

Definition 3 For systems (1) and (2), intermittent containment control is achieved, if for any initial states, there exists an intermittent controller such that the followers' states converge into the dynamic convex hull consisting of the states of multiple leaders under intermittent communications, that is, $\lim_{t\to\infty} \boldsymbol{q}_i(t) \in \mathrm{CO}\{\boldsymbol{q}_l(t) | l = 1, 2, \ldots, M\}$, $i = 1, 2, \ldots, N$.

3 Main results

Inspired by the sampled controller design (Yu WW et al., 2011; Yu ZY et al., 2018; Zou et al., 2018), a new kind of intermittent containment control protocol is put forward via the current and sampled position data:

$$\boldsymbol{u}_i(t) = \begin{cases} \alpha \displaystyle\sum_{j=1}^{N+M} a_{ij}[\boldsymbol{q}_j(t) - \boldsymbol{q}_i(t)] \\ \quad - \beta \displaystyle\sum_{j=1}^{N+M} a_{ij}[\boldsymbol{q}_j(t_k) - \boldsymbol{q}_i(t_k)],\ t \in [t_k, t_k + \theta), \\ \boldsymbol{0},\ \ t \in [t_k + \theta, t_{k+1}), \end{cases} \tag{3}$$

where t_k $(k \in \mathbb{N})$ is the sampling sequence, θ is the communication width, and $\alpha > 0$ and $\beta > 0$ are the feedback gains to be determined. Moreover, $0 = t_0 < t_1 < t_2 < \ldots$ $(k > 1)$, $t_{k+1} - t_k = T$, and $0 < \theta \leq T$, where $T > 0$ represents the sampling period.

Substituting system (3) into system (2) and letting $\boldsymbol{x}_i(t) = [\boldsymbol{q}_i(t), \boldsymbol{p}_i(t)]^{\mathrm{T}}, i = 1, 2, \ldots, N+M$, we have

$$\dot{\boldsymbol{x}}_i(t) = \begin{cases} \boldsymbol{A}\boldsymbol{x}_i(t) - \alpha \sum\limits_{j=1}^{N+M} l_{ij}\boldsymbol{B}\boldsymbol{x}_j(t) \\ \quad + \beta \sum\limits_{j=1}^{N+M} l_{ij}\boldsymbol{B}\boldsymbol{x}_j(t_k),\ t \in [t_k, t_k+\theta), \\ \boldsymbol{A}\boldsymbol{x}_i(t),\ t \in [t_k+\theta, t_{k+1}), \end{cases} \tag{4}$$

where $\boldsymbol{A} = \begin{bmatrix} \boldsymbol{0}_n & \boldsymbol{I}_n \\ \boldsymbol{0}_n & \boldsymbol{0}_n \end{bmatrix}$ and $\boldsymbol{B} = \begin{bmatrix} \boldsymbol{0}_n & \boldsymbol{0}_n \\ \boldsymbol{I}_n & \boldsymbol{0}_n \end{bmatrix}$.

Let $\boldsymbol{x}(t) = [\boldsymbol{x}_{\mathrm{F}}^{\mathrm{T}}(t), \boldsymbol{x}_{\mathrm{R}}^{\mathrm{T}}(t)]^{\mathrm{T}}$, $\boldsymbol{x}_{\mathrm{F}}(t) = [\boldsymbol{x}_1^{\mathrm{T}}(t), \boldsymbol{x}_2^{\mathrm{T}}(t), \ldots, \boldsymbol{x}_N^{\mathrm{T}}(t)]^{\mathrm{T}}$, and $\boldsymbol{x}_{\mathrm{R}}(t) = [\boldsymbol{x}_{N+1}^{\mathrm{T}}(t), \boldsymbol{x}_{N+2}^{\mathrm{T}}(t), \ldots, \boldsymbol{x}_{N+M}^{\mathrm{T}}(t)]^{\mathrm{T}}$. Moreover, define $\tilde{\boldsymbol{x}}_{\mathrm{F}}(t) = (\boldsymbol{L}_2 \otimes \boldsymbol{I}_{2n})\boldsymbol{x}_{\mathrm{R}}(t) + (\boldsymbol{L}_1 \otimes \boldsymbol{I}_{2n})\boldsymbol{x}_{\mathrm{F}}(t)$. Then, from Lemma 1, system (4) can be transformed to a matrix form below:

$$\dot{\tilde{\boldsymbol{x}}}_{\mathrm{F}}(t) = \begin{cases} [(\boldsymbol{I}_N \otimes \boldsymbol{A}) - \alpha(\boldsymbol{L}_1 \otimes \boldsymbol{B})]\tilde{\boldsymbol{x}}_{\mathrm{F}}(t) \\ \quad + \beta(\boldsymbol{L}_1 \otimes \boldsymbol{B})\tilde{\boldsymbol{x}}_{\mathrm{F}}(t_k), t \in [t_k, t_k+\theta), \\ (\boldsymbol{I}_N \otimes \boldsymbol{A})\tilde{\boldsymbol{x}}_{\mathrm{F}}(t),\ t \in [t_k+\theta, t_{k+1}). \end{cases} \tag{5}$$

Lemma 3 Suppose that graph $\hat{G}$ is undirected and connected. The multi-agent systems (1) and (2) can reach second-order containment via control protocol (3) if and only if $\boldsymbol{q}_{\mathrm{F}}(t) \to -\boldsymbol{L}_1^{-1}\boldsymbol{L}_2\boldsymbol{q}_{\mathrm{R}}(t)$ as $t \to \infty$, where $\boldsymbol{q}_{\mathrm{F}}(t) = [\boldsymbol{q}_1(t), \boldsymbol{q}_2(t), \ldots, \boldsymbol{q}_N(t)]^{\mathrm{T}}$, $\boldsymbol{q}_{\mathrm{R}}(t) = [\boldsymbol{q}_{N+1}(t), \boldsymbol{q}_{N+2}(t), \ldots, \boldsymbol{q}_{N+M}(t)]^{\mathrm{T}}$.

Proof Sufficiency: Supposing that graph $\hat{G}$ is undirected and connected, it is easy to know that $-\boldsymbol{L}_1^{-1}\boldsymbol{L}_2\boldsymbol{q}_{\mathrm{R}}(t)$ is the convex hull of the set of $\{\boldsymbol{q}_{N+1}(t), \boldsymbol{q}_{N+2}(t), \ldots, \boldsymbol{q}_{N+M}(t)\}$ combining Definition 1 and Lemma 2. Afterwards, from Definition 3, we know that followers can move into the convex hull formed by leaders if $\boldsymbol{q}_{\mathrm{F}}(t) \to -\boldsymbol{L}_1^{-1}\boldsymbol{L}_2\boldsymbol{q}_{\mathrm{R}}(t)$ as $t \to \infty$. Therefore, the purpose of containment control can be realized.

Necessity: Assume that the communication topology graph G is unconnected. Then, there exists at least one follower that does not obtain any information from leaders; this means that this follower cannot communicate with any leader for all $t \geq 0$. In this case, the follower mentioned above may not converge to the convex hull formed by leaders. Thus, the considered containment control may not be achieved.

Theorem 1 Suppose that graph $\hat{G}$ is undirected and connected. For systems (1) and (2) with control protocol (3), the second-order containment control can be achieved if and only if

$$\begin{cases} 0 < \beta < \alpha, \\ 0 < \theta < \dfrac{\pi}{|\sqrt{\alpha\bar{\mu}}|}, \\ T < \varrho + \theta, \end{cases} \tag{6}$$

where $\bar{\mu} = \max_i\{\mu_i\}$, $\varrho = \min_i\left\{\frac{2(1+\cos(\sqrt{\alpha\mu_i}\theta))}{(1-\beta/\alpha)\sqrt{\alpha\mu_i}\sin(\sqrt{\alpha\mu_i}\theta)}\right\}$, and μ_i are the eigenvalues of $\boldsymbol{L}_1$, $i = 1, 2, \ldots, N$.

Proof System (5) can be rewritten as

$$\dot{\boldsymbol{y}}_i(t) = \begin{cases} (\boldsymbol{A} - \alpha\mu_i\boldsymbol{B})\boldsymbol{y}_i(t) + \beta\mu_i\boldsymbol{B}\boldsymbol{y}_i(t_k), \\ \qquad t \in [t_k, t_k+\theta), \\ \boldsymbol{A}\boldsymbol{y}_i(t),\ t \in [t_k+\theta, t_{k+1}), \end{cases} \tag{7}$$

where $[\boldsymbol{y}_1(t), \boldsymbol{y}_2(t), \ldots, \boldsymbol{y}_N(t)]^{\mathrm{T}} = \boldsymbol{y}(t) = \left(\boldsymbol{P}^{-1} \otimes \boldsymbol{I}_{2n}\right)\tilde{\boldsymbol{x}}_{\mathrm{F}}(t)$. $\boldsymbol{P}$ is a nonsingular matrix such that $\boldsymbol{L} = \boldsymbol{P}\boldsymbol{\Lambda}\boldsymbol{P}^{-1}$, where $\boldsymbol{\Lambda} = \mathrm{diag}(\mu_1, \mu_2, \ldots, \mu_N)$, $\mu_i > 0$.

When $t \to \infty$, $\boldsymbol{y}_i(t) \to \boldsymbol{0}$ and $\tilde{\boldsymbol{x}}_{\mathrm{F}}(t) \to \boldsymbol{0}$ are equivalent. Then, one knows that $\boldsymbol{x}_{\mathrm{F}}(t) \to -\left(\boldsymbol{L}_1^{-1}\boldsymbol{L}_2 \otimes \boldsymbol{I}_{2n}\right)\boldsymbol{x}_{\mathrm{R}}(t)$ if and only if $\boldsymbol{y}_i(t) \to \boldsymbol{0}$ as $t \to \infty$. Furthermore, from Lemma 3 and Definition 3, we can obtain that systems (1) and (2) with control protocol (3) can reach second-order containment if and only if $\boldsymbol{y}_i(t) \to \boldsymbol{0}$ as $t \to \infty$.

For $t \in [t_k, t_k+\theta)$, we have

$$\begin{aligned} \boldsymbol{y}_i(t) &= \exp[(\boldsymbol{A} - \alpha\mu_i\boldsymbol{B})(t-t_k)]\boldsymbol{y}_i(t_k) \\ &\quad + \int_{t_k}^{t} \exp[(\boldsymbol{A}-\alpha\mu_i\boldsymbol{B})(t-\sigma)]\beta\mu_i\boldsymbol{B}\boldsymbol{y}_i(t_k)\mathrm{d}\sigma \\ &= \begin{bmatrix} c_{1i}(t-t_k) & c_{2i}(t-t_k) \\ c_{3i}(t-t_k) & c_{4i}(t-t_k) \end{bmatrix}\boldsymbol{y}_i(t_k). \end{aligned}$$

For $t \in [t_k+\theta, t_{k+1})$, we have

$$\begin{aligned} \boldsymbol{y}_i(t) &= \exp[\boldsymbol{A}(t-(t_k+\theta))]\boldsymbol{y}_i(t_k+\theta) \\ &= \begin{bmatrix} d_{1i}(t-(t_k+\theta)) & d_{2i}(t-(t_k+\theta)) \\ d_{3i}(t-(t_k+\theta)) & d_{4i}(t-(t_k+\theta)) \end{bmatrix} \\ &\quad \cdot\boldsymbol{y}_i(t_k). \end{aligned}$$

Herein

$$c_{1i}(t-t_k) = \left(1 - \frac{\beta}{\alpha}\right)\cos(\vartheta_i(t-t_k)) + \frac{\beta}{\alpha},$$

$$c_{2i}(t-t_k) = \frac{1}{\vartheta_i}\sin(\vartheta_i(t-t_k)),$$

$$c_{3i}(t-t_k) = -\left(1 - \frac{\beta}{\alpha}\right)\vartheta_i\sin(\vartheta_i(t-t_k)),$$

$$c_{4i}(t-t_k)=\cos(\vartheta_i(t-t_k)),$$
$$d_{1i}(t-(t_k+\theta))=\Big(1-\frac{\beta}{\alpha}\Big)[\cos(\vartheta_i\theta)-\vartheta_i\sin(\vartheta_i\theta)\cdot(t-(t_k+\theta))]+\frac{\beta}{\alpha},$$
$$d_{2i}(t-(t_k+\theta))=\frac{1}{\vartheta_i}\sin(\vartheta_i\theta)+\cos(\vartheta_i\theta)(t-(t_k+\theta)),$$
$$d_{3i}(t-(t_k+\theta))=-\Big(1-\frac{\beta}{\alpha}\Big)\vartheta_i\sin(\vartheta_i\theta),$$
$$d_{4i}(t-(t_k+\theta))=\cos(\vartheta_i\theta).$$

Here, $\vartheta_i=\sqrt{\alpha\mu_i}$.

Thus, for $0=t_0<t_1<t_2<\ldots$ $(k>1)$ and $t_{k+1}-t_k=T$, we have

$$\boldsymbol{y}_i(t)=\begin{cases}\boldsymbol{C}_i(t-t_k)\boldsymbol{D}_i^k(T-\theta)\boldsymbol{y}_i(t_0),\ t\in[t_k,t_k+\theta),\\ \boldsymbol{D}_i(t-(t_k+\theta))\boldsymbol{D}_i^k(T-\theta)\boldsymbol{y}_i(t_0),\\ \quad t\in[t_k+\theta,t_{k+1}),\end{cases}\tag{8}$$

where

$$\begin{cases}\boldsymbol{C}_i(t)=\begin{bmatrix}c_{1i}(t) & c_{2i}(t)\\ c_{3i}(t) & c_{4i}(t)\end{bmatrix},\\ \boldsymbol{D}_i(t)=\begin{bmatrix}d_{1i}(t) & d_{2i}(t)\\ d_{3i}(t) & d_{4i}(t)\end{bmatrix}.\end{cases}$$

When $t\in[t_k,t_k+\theta)$, $\boldsymbol{C}_i(t-t_k)$ is bounded; when $t\in[t_k+\theta,t_{k+1})$, $\boldsymbol{D}_i(t-(t_k+\theta))$ is bounded. Then, we can obtain $\lim\limits_{t\to\infty}\boldsymbol{y}_i(t)=\mathbf{0}$ if and only if $\|\lambda(\boldsymbol{D}_i(T-\theta))\|<1$. The eigenvalue of matrix $\boldsymbol{D}_i(T-\theta)$ is defined as $\check{\lambda}$. The characteristic equation of $\boldsymbol{D}_i(T-\theta)$ can be written as $\|\check{\lambda}\boldsymbol{I}_2-\boldsymbol{D}_i(T-\theta)\|=0$. Then, we have

$$\begin{aligned}&\det\big(\check{\lambda}\boldsymbol{I}_2-\boldsymbol{D}_i(T-\theta)\big)\\ &=\det\begin{pmatrix}\check{\lambda}-d_{1i}(T-\theta) & -d_{2i}(T-\theta)\\ -d_{3i}(T-\theta) & \check{\lambda}-d_{4i}(T-\theta)\end{pmatrix}\\ &=f_i(\check{\lambda}),\end{aligned}$$

where $f_i(\check{\lambda})=\check{\lambda}^2-b_i^1\check{\lambda}+b_i^0$. Here,

$$\begin{cases}b_i^1=\Big(2-\frac{\beta}{\alpha}\Big)\cos(\vartheta_i\theta)\\ \qquad -\Big(1-\frac{\beta}{\alpha}\Big)(T-\theta)\vartheta_i\sin(\vartheta_i\theta),\\ b_i^0=\frac{\beta}{\alpha}\cos(\vartheta_i\theta)+\Big(1-\frac{\beta}{\alpha}\Big).\end{cases}$$

Moreover, it is easy to check that $1-b_i^1+b_i^0\neq 0$.

Let $\check{\lambda}=\frac{s+1}{s-1}$. We have

$$g_i(s)=s^2+r_i^1s+r_i^0,\tag{9}$$

where

$$\begin{cases}r_i^1=\dfrac{2\frac{\beta}{\alpha}[1-\cos(\vartheta_i\theta)]}{(1-\frac{\beta}{\alpha})[2(1-\cos(\vartheta_i\theta))+(T-\theta)\vartheta_i\sin(\vartheta_i\theta)]},\\ r_i^0=\dfrac{2[1+\cos(\vartheta_i\theta)]-(1-\frac{\beta}{\alpha})(T-\theta)\vartheta_i\sin(\vartheta_i\theta)}{(1-\frac{\beta}{\alpha})[2(1-\cos(\vartheta_i\theta))+(T-\theta)\vartheta_i\sin(\vartheta_i\theta)]}.\end{cases}$$

Then, it is not difficult to see that $f_i(\check{\lambda})$ is Schur stable if and only if $g_i(s)$ is Hurwitz stable. From the Routh criterion, it is easy to obtain that $g_i(s)$ is Hurwitz stable if $r_i^1>0$ and $r_i^0>0$. Therefore, $\lim\limits_{t\to\infty}\boldsymbol{y}_i(t)=\mathbf{0}$ if and only if condition (6) holds. The proof has been completed.

Remark 1 Yu ZY et al. (2018) designed a periodic intermittent sampled data control protocol for second-order consensus of multi-agent systems. Unlike both the position information and velocity information were used in Yu ZY et al. (2018), in this study, we design a novel controller with less information, in which only intermittent sampled data and current position data are used in the designed control protocol, reducing the cost of the controller and the load of the agents.

Corollary 1 Under continuous communications, i.e., $\theta=T$, for systems (1) and (2) with control protocol (3), the second-order containment can be achieved if and only if graph $\hat{G}$ is undirected and connected and $0<\beta<\alpha$ and $T<\frac{\pi}{|\sqrt{\alpha\bar{\mu}}|}$ are satisfied.

Remark 2 In the process of the proof of Theorem 1, from $r_i^1>0$, we have $\theta\neq\frac{k\pi}{|\sqrt{\alpha\mu_i}|}$, $k=1,2,\ldots,$ $i=1,2,\ldots,N$. In general, the convergence rate around the critical points $\theta=\frac{k\pi}{|\sqrt{\alpha\mu_i}|}$ is very low. Therefore, for a large communication width θ, it is hard to achieve better performance in a large-scale communication network. In this study, condition (6) gives the appropriate critical values of communication width and sampling period.

In the above discussion, we consider a fixed network communication topology in multi-agent systems. However, in the real word, switching behaviors usually occur in network topology. Moreover, due to the unreliability of communication channels, limitations of sensing ranges, and the failure of physical devices, the interaction network topology of multi-agent systems may be time-varying or intermittent.

Next, we will give the results for the intermittent sampled containment control of second-order multi-agent systems under switching topology.

Under switching communication topology, the following time-varying control protocol is proposed:

$$\boldsymbol{u}_i(t)=\begin{cases}\alpha\displaystyle\sum_{j=1}^{N+M}a_{ij}(t)[\boldsymbol{q}_j(t)-\boldsymbol{q}_i(t)]\\ \quad-\beta\displaystyle\sum_{j=1}^{N+M}a_{ij}(t)[\boldsymbol{q}_j(t_k)-\boldsymbol{q}_i(t_k)],\\ \quad t\in[t_k,t_k+\theta),\\ \boldsymbol{0},\quad t\in[t_k+\theta,t_{k+1}).\end{cases}\tag{10}$$

Then, applying control protocol (10) for systems (1) and (2), we have

$$\dot{\tilde{\boldsymbol{x}}}_{\mathrm{F}}(t)=\begin{cases}[(\boldsymbol{I}_N\otimes\boldsymbol{A})-\alpha\,(\boldsymbol{L}_{\sigma1}\otimes\boldsymbol{B})]\,\tilde{\boldsymbol{x}}_{\mathrm{F}}(t)\\ \quad+\beta\,(\boldsymbol{L}_{\sigma1}\otimes\boldsymbol{B})\,\tilde{\boldsymbol{x}}_{\mathrm{F}}(t_k),\ t\in[t_k,t_k+\theta),\\ (\boldsymbol{I}_N\otimes\boldsymbol{A})\,\tilde{\boldsymbol{x}}_{\mathrm{F}}(t),\ t\in[t_k+\theta,t_{k+1}),\end{cases}\tag{11}$$

where $\boldsymbol{L}_{\sigma1}$ is the Laplacian matrix associated with the switching topology subgraph $\hat{G}_\sigma,\ \sigma\in\Gamma$.

Theorem 2 Suppose that each communication graph $\hat{G}_\sigma$ $(\sigma\in\Gamma)$ is undirected and connected. Control protocol (10) can solve the intermittent sampled containment control problem for second-order multi-agent systems (1) and (2) under switching topology, if and only if

$$\begin{cases}0<\beta<\alpha,\\ 0<\theta<\dfrac{\pi}{|\sqrt{\alpha\bar{\mu}_\sigma}|},\\ T<\tilde{\varrho}+\theta,\end{cases}\tag{12}$$

where

$$\begin{cases}\bar{\mu}_\sigma=\max\limits_i\{\mu_{\sigma i}\},\\ \tilde{\varrho}=\min\limits_i\left\{\dfrac{2[1+\cos(\sqrt{\alpha\mu_{\sigma i}}\theta)]}{(1-\frac{\beta}{\alpha})\sqrt{\alpha\mu_{\sigma i}}\sin(\sqrt{\alpha\mu_{\sigma i}}\theta)}\right\},\end{cases}$$

and $\mu_{\sigma i}$ are the eigenvalues of $L_{\sigma1}$, $i=1,2,\ldots,N$.

Based on the process of the proof of Theorem 1, it is easy to obtain Theorem 2. Thus, the proof of Theorem 2 is omitted here.

Remark 3 In switching communication networks, the Laplacian matrix $\boldsymbol{L}_{\sigma(t)}$ associated with the switching topology graph $G_{\sigma(t)}$ is time-varying, and the Laplacian matrix $\boldsymbol{L}_{\sigma1(t)}$ associated with the subgraph $\hat{G}_{\sigma(t)}$ is also time-varying. Then, based on Theorem 2, the intermittent sampled containment control problem under switching topology can still be solved through choosing the appropriate eigenvalues of $\boldsymbol{L}_{\sigma1(t)}$, $\sigma\in\Gamma$. Therefore, the result in Theorem 1 can be extended to the case with switching topology.

Remark 4 Under intermittent communications, Wang FY et al. (2019a) introduced a filter-based method for second-order containment control with unavailable velocity measurements at the price of having some additional variables, leading to the study of higher-order dynamical systems. Different from Wang FY et al. (2019a), in this work, a novel controller is first designed for second-order containment via intermittent sampled position data measurements, which circumvents the complexity of studying higher-order systems caused by auxiliary filters. The controller design uses less information and saves more energy since only intermittent sampled data and current position data are used in our proposed control protocol. Moreover, necessary and sufficient conditions are obtained for sampled data based containment control under intermittent communications.

4 Simulation examples

Consider a multi-agent system with eight agents, where there are five followers and three leaders (colored as yellow ones). The initial states (positions) of followers can be chosen as $\boldsymbol{x}_1(0)=[1,1]^{\mathrm{T}}$, $\boldsymbol{x}_2(0)=[1,5]^{\mathrm{T}}$, $\boldsymbol{x}_3(0)=[1,10]^{\mathrm{T}}$, $\boldsymbol{x}_4(0)=[5,1]^{\mathrm{T}}$, and $\boldsymbol{x}_5(0)=[10,1]^{\mathrm{T}}$, and the initial states (positions) of leaders can be selected as $\boldsymbol{x}_6(0)=[15,15]^{\mathrm{T}}$, $\boldsymbol{x}_7(0)=[15,20]^{\mathrm{T}}$, and $\boldsymbol{x}_8(0)=[20,20]^{\mathrm{T}}$. The multi-agent system's interconnection topology G is shown in Fig. 1, where G is a fixed topology graph. Fig. 2 shows the switching communication topology, where the communication network starts with topology G_1 and switches to another topology randomly chosen within $\{G_1,G_2,G_3\}$ at time $\hat{k}\hat{T}$, $\hat{k}=1,2,\ldots$, $\hat{T}=0.5$. Moreover, in this study, the connection weights of each edge in the communication topology graph are all set to 1.

From the topology of graph G in Fig. 1, one has

$$\boldsymbol{L}_1=\begin{bmatrix}2&-1&0&0&0\\-1&2&-1&0&0\\0&-1&3&-1&0\\0&0&-1&2&-1\\0&0&0&-1&2\end{bmatrix}.$$

By simple calculation, we have $\mu_1 = 0.5188$, $\mu_2 = 1.0000$, $\mu_3 = 2.3111$, $\mu_4 = 3.0000$, $\mu_5 = 4.1701$, and $\bar{\mu} = 4.1701$, where $\bar{\mu} = \max_i\{\mu_i\}$. Moreover, we choose $\alpha = 1.0$ and $\beta = 0.5$, where α and β are the feedback gains for the designed control protocol. According to condition (6) in Theorem 1, one knows that the communication width $\theta < 1.5384$, and we can take $\theta = 1.5$. Then, one obtains $T < 2.1432$. Based on Theorem 1, we can obtain that the right interval of the sampling period is $\theta \le T < 2.1432$. The position states of all agents under the fixed communication topology are presented in Figs. 3 and 4, where qx and qy represent the horizontal and vertical axes, respectively. It is obvious to see that when $\theta = 1.5$ and $T = 2$, all followers can converge to the convex hull formed by multiple leaders under the fixed topology. When $T = 2.3$, however, the convergence is not guaranteed.

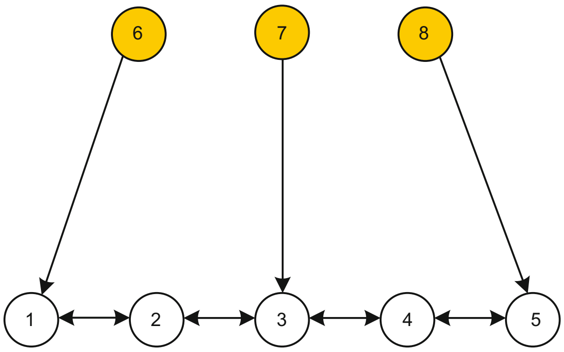

Fig. 1 Fixed communication topology graph G

References to color refer to the online version of this figure

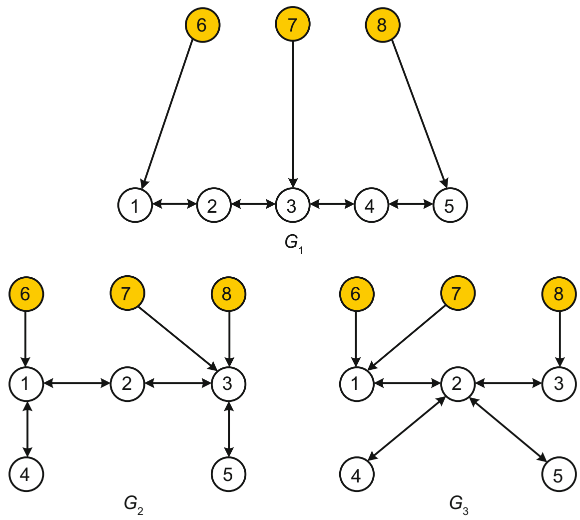

Fig. 2 Switching communication topology graph

References to color refer to the online version of this figure

From the topology of graph G_σ, $\sigma \in \Gamma = \{1, 2, 3\}$, one obtains

$$\boldsymbol{L}_{1\sigma_1} = \begin{bmatrix} 2 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 3 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 2 \end{bmatrix},$$

$$\boldsymbol{L}_{1\sigma_2} = \begin{bmatrix} 3 & -1 & 0 & -1 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 4 & 0 & -1 \\ -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 1 \end{bmatrix},$$

$$\boldsymbol{L}_{1\sigma_3} = \begin{bmatrix} 3 & -1 & 0 & 0 & 0 \\ -1 & 4 & -1 & -1 & -1 \\ 0 & -1 & 2 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 1 \end{bmatrix}.$$

Similarly, we have $\bar{\mu}_\sigma = 5.2306$, $\sigma \in \Gamma = \{1, 2, 3\}$. According to condition (12) in Theorem 2, one knows

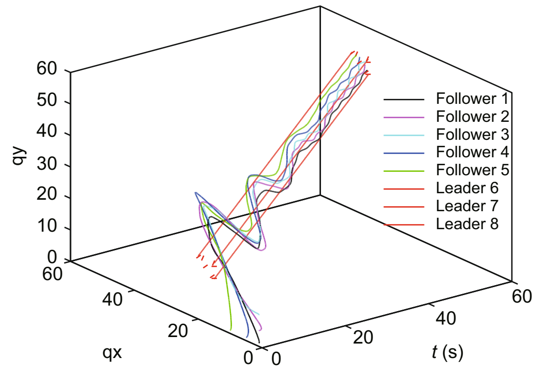

Fig. 3 State trajectories of agents under the fixed communication topology when $\theta = 1.5$ and $T = 2$

References to color refer to the online version of this figure

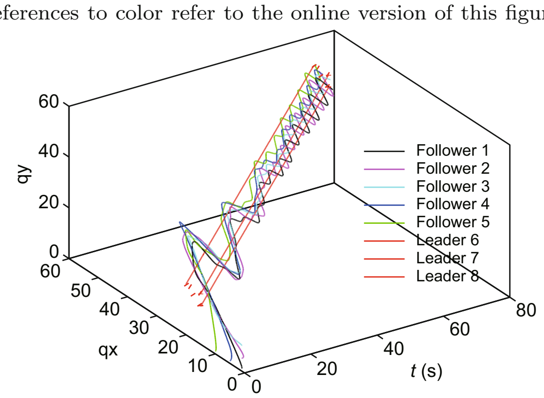

Fig. 4 State trajectories of agents under the fixed communication topology when $\theta = 1.5$ and $T = 2.3$

References to color refer to the online version of this figure that the communication width $\theta < 1.3736$, and then we take $\theta = 1.3$. Then, it follows from Theorem 1 that $T < 1.5212$. Under switching communication topology, the simulation results are presented in Figs. 5 and 6. It can be seen that when $\theta = 1.3$ and $T = 1.5$, all followers can converge to the convex hull formed by multiple leaders under switching topology. When $T = 1.7$, however, the convergence is not guaranteed.

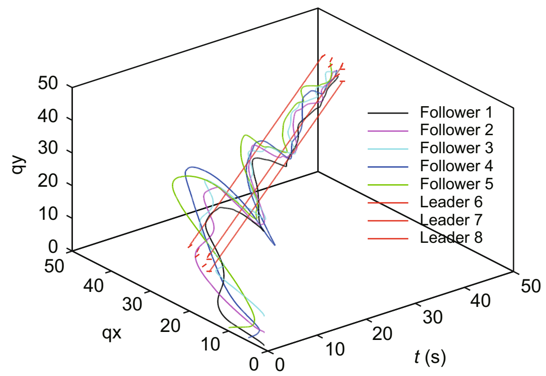

Fig. 5 State trajectories of agents under the switching communication topology when $\theta = 1.3$ and $T = 1.5$

References to color refer to the online version of this figure

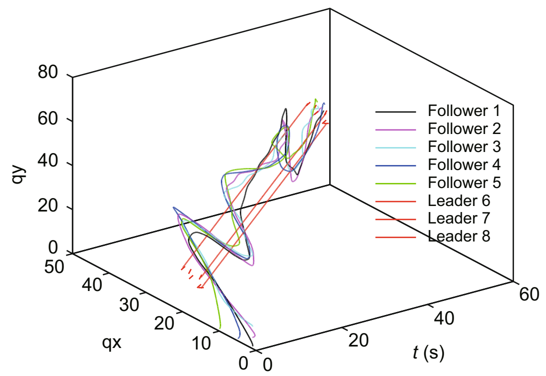

Fig. 6 State trajectories of agents under the switching communication topology when $\theta = 1.3$ and $T = 1.7$

References to color refer to the online version of this figure

5 Conclusions

This study investigates the containment control problem in the context of second-order multi-agent systems under intermittent sampled communications. For second-order containment of multi-agent systems without velocity measurements, a novel distributed control protocol is put forward via only intermittent sampled position data communications. Necessary and sufficient conditions hinging on the feedback gains, eigenvalues of the Laplacian matrix, the sampling period, and the communication width are derived under undirected and connected communication networks. In the future, general directed communication networks will be considered.

Contributors

Fuyong WANG designed the research. Fuyong WANG, Zhongxin LIU, and Zengqiang CHEN contributed to the idea, technical discussion, and measurements. Fuyong WANG drafted the manuscript. Zhongxin LIU and Zengqiang CHEN helped organize the manuscript. Fuyong WANG, Zhongxin LIU, and Zengqiang CHEN revised and finalized the paper.

Compliance with ethics guidelines

Fuyong WANG, Zhongxin LIU, and Zengqiang CHEN declare that they have no conflict of interest.

References

Hong YG, Chen GR, Bushnell L, 2008. Distributed observers design for leader-following control of multi-agent networks. *Automatica*, 44(3):846-850. https://doi.org/10.1016/j.automatica.2007.07.004

Horn RA, Johnson CR, 1985. Matrix Analysis. Cambridge University Press, Cambridge, UK.

Li JZ, Ren W, Xu SY, 2012. Distributed containment control with multiple dynamic leaders for double-integrator dynamics using only position measurements. *IEEE Trans Autom Contr*, 57(6):1553-1559. https://doi.org/10.1109/TAC.2011.2174680

Liu HY, Xie GM, Wang L, 2012. Necessary and sufficient conditions for containment control of networked multi-agent systems. *Automatica*, 48(7):1415-1422. https://doi.org/10.1016/j.automatica.2012.05.010

Liu KE, Xie GM, Wang L, 2014. Containment control for second-order multi-agent systems with time-varying delays. *Syst Contr Lett*, 67:24-31. https://doi.org/10.1016/j.sysconle.2013.12.013

Liu TF, Qi J, Jiang Z, 2020. Distributed containment control of multi-agent systems with velocity and acceleration saturations. *Automatica*, 117:108992. https://doi.org/10.1016/j.automatica.2020.108992

Liu YF, Su HS, 2020a. Necessary and sufficient conditions for containment in fractional-order multiagent systems via sampled data. *IEEE Trans Syst Man Cybern Syst*, online. https://doi.org/10.1109/TSMC.2020.2997294

Liu YF, Su HS, 2020b. Some necessary and sufficient conditions for containment of second-order multi-agent systems with sampled position data. *Neurocomputing*, 378:228-237. https://doi.org/10.1016/j.neucom.2019.10.031

Liu YF, Su HS, 2021. Some necessary and sufficient conditions for containment of second-order multi-agent systems with intermittent sampled data. *ISA Trans*, 108:154-163. https://doi.org/10.1016/j.isatra.2020.08.014

Mei J, Ren W, Ma GF, 2013. Distributed coordination for second-order multi-agent systems with nonlinear dynamics using only relative position measurements. *Automatica*, 49(5):1419-1427. https://doi.org/10.1016/j.automatica.2013.01.058

Rockafellar RT, 1972. Convex Analysis. Princeton University Press, Princeton, USA.

Shi L, Xiao Y, Shao JL, et al., 2020. Containment control of asynchronous discrete-time general linear multiagent systems with arbitrary network topology. *IEEE Trans Cybern*, 50(6):2546-2556. https://doi.org/10.1109/TCYB.2019.2915941

Su HS, Liu YF, Zeng ZG, 2020. Second-order consensus for multi-agent systems via intermittent sampled position data control. *IEEE Trans Cybern*, 50(5):2063-2072. https://doi.org/10.1109/TCYB.2018.2879327

Sun YN, Zou WC, Guo J, 2021. Containment control for heterogeneous nonlinear multi-agent systems under distributed event-triggered schemes. *Front Inform Technol Electron Eng*, 22(1):107-119. https://doi.org/10.1631/FITEE.2000034

Wang D, Wang D, Wang W, 2019. Necessary and sufficient conditions for containment control of multi-agent systems with time delay. *Automatica*, 103(5):418-423. https://doi.org/10.1016/j.automatica.2018.12.029

Wang FY, Liu ZX, Chen ZQ, 2018. Distributed containment control for second-order multiagent systems with time delay and intermittent communication. *Int J Rob Nonl Contr*, 28(18):5730-5746. https://doi.org/10.1002/rnc.4341

Wang FY, Liu ZX, Chen ZQ, 2019a. Containment control for second-order nonlinear multi-agent systems with aperiodically intermittent position measurements. *J Franklin Inst*, 356(15):8706-8725. https://doi.org/10.1016/j.jfranklin.2018.11.057

Wang FY, Liu ZX, Chen ZQ, et al., 2019b. Containment control for second-order nonlinear multi-agent systems with intermittent communications. *Int J Syst Sci*, 50(5):919-934. https://doi.org/10.1080/00207721.2019.1585997

Wang FY, Ni YH, Liu ZX, et al., 2020a. Containment control for general second-order multiagent systems with switched dynamics. *IEEE Trans Cybern*, 50(2):550-560. https://doi.org/10.1109/TCYB.2018.2869706

Wang FY, Ni YH, Liu ZX, et al., 2020b. Fully distributed containment control for second-order multi-agent systems with communication delay. *ISA Trans*, 99:123-129. https://doi.org/10.1016/j.isatra.2019.09.009

Wen GH, Zhao Y, Duan ZS, et al., 2016. Containment of higher-order multi-leader multi-agent systems: a dynamic output approach. *IEEE Trans Autom Contr*, 61(4):1135-1140. https://doi.org/10.1109/TAC.2015.2465071

Xiao WB, Ren HR, Zhou Qi, et al., 2021. Distributed finite-time containment control for nonlinear multiagent systems with mismatched disturbances. *IEEE Trans Cybern*, online. https://doi.org/10.1109/TCYB.2020.3042168

Yu WW, Zheng WX, Chen GR, et al., 2011. Second-order consensus in multi-agent dynamical systems with sampled position data. *Automatica*, 47(7):1496-1503. https://doi.org/10.1016/j.automatica.2011.02.027

Yu ZY, Jiang HJ, Hu C, 2018. Second-order consensus for multiagent systems via intermittent sampled data control. *IEEE Trans Syst Man Cybern Syst*, 48(11):1986-2002. https://doi.org/10.1109/TSMC.2017.2687944

Zou WC, Xiang ZR, 2019. Event-triggered containment control of second-order nonlinear multi-agent systems. *J Franklin Inst*, 356(17):10421-10438. https://doi.org/10.1016/j.jfranklin.2018.05.060

Zou WC, Guo J, Xiang ZR, 2018. Sampled-data leader-following consensus of second-order nonlinear multiagent systems without velocity measurements. *Int J Rob Nonl Contr*, 28(17):5634-5651. https://doi.org/10.1002/rnc.4340