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ROBUST STABILIZATION OF UNCERTAIN TIME-DELAY SYSTEMS CONTAINING NONLINEAR SATURATING ACTUATORS*

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Abstract: The robust stabilization problem for a class of uncertain linear time-delay systems containing sector saturating actuator is considered in this paper. The uncertain time-delay systems under consideration are described by state differential equations with time-varying unknown-but-bounded uncertain parameters and delayed state. The delay is assumed to be constant bounded but unknown. The new criterion of delay-dependent robust stabilizability for uncertain time-delay systems is presented and the corresponding robust memoryless state feedback controller is derived in terms of the solutions of several linear matrix inequalities (LMIs). Numerical example is presented to illustrate the obtained results.

Key words: linear time-delay systems, state feedback, uncertainty, saturating actuator, delay-dependent, linear matrix inequality (LMI)

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INTRODUCTION

Time-delays, due to transportation lags, finite calculation times, measurements times, etc., appear in numerous industrial and natural processes, often leading to oscillations and sometimes instability. The stability and stabilization problems of time-delay systems with or without uncertainties have been widely investigated during the past decades and numerous methods such as differential inequality techniques, finite spectrum assignment, matrix measure technique, Lyapunov theorem, Razumikhin theorem, quadratic cost optimal control, Riccati equation approach, etc. are presented to deal with the stability analysis or stabilization problem for time-delay systems with or without uncertainties (Yu et al., 1996; Su et al., 1998a, b, c; Niculescu et al., 1994). However, these results are given in terms of the solution of either a Lyapunov or Riccati equation and involve the tuning of scalars and/or positive definite symmetric matrices. To the best of our knowledge, no tuning procedure for such scalars and matrices is available, which makes the use of these methods somehow difficult and conservative. Recently, the linear matrix inequality (LMI) approach was proposed to treat the problem of robust stability analysis and robust stabilization synthesis for uncertain linear time-delay systems and less conservation results dependent on the size of delays had been obtained (Li et al., 1997). This approach has the advantage that no tuning of parameters and/or positive definite symmetric matrices is involved.

In many practical situations, among some non-linearities introduced by the actuator dynamics, a common one is the saturation. Generally, the physical limitations of the actuator are unavoidable in the operation of driving the actuator by the signals emitted from the designed controllers, thus causing actuator saturation, which not only deteriorates the control system performance, but can also lead to undesirable stability effects. If such non-linearity saturation is not taken into account in the control system design, an integral wind up or limit cycle may occur. Recently, special interest has been devoted to the robust stabilization problem for uncertain time-delay systems containing saturating actuator (Chou et al., 1989; Su et al., 1998d; Niculescu et al., 1996). For example, sufficient conditions for

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output feedback stabilization, independent of the size of delay, are given in Chou et al. (Chou et al., 1989) using the concept of matrix measure and the comparison theory for an uncertain linear time-delay system with single state delay and saturating actuator in time domain. Niculescu et al. (Niculescu et al., 1996) studied the problem of robust stabilization for a class of uncertain linear time-delay systems containing a saturating actuator by using the Razumikhin theorem. A delay-dependent criterion for robust stabilization via linear memoryless state feedback control law was obtained and the upper bound on time-delay is given such that the uncertain system is robustly stabilizable. But the obtained results are also given in terms of the solution of the algebraic Riccati equation and still need the tuning of the scalar and positive definite symmetric matrix.

In this paper, we extend the linear matrix inequality (LMI) method to time-varying uncertain linear dynamic systems with sector saturating actuator and unknown-but-bounded time-delay in state variable. It is assumed that perfect information of plant states is available for feedback. The robust stabilization problem addressed is to get the robust stabilizability criteria and design memoryless state feedback control laws such that the closed-loop system is asymptotically stable for all admissible uncertainties. Based on the linear matrix inequality (LMI) approach, new robust stabilizability criteria and corresponding robust stabilizing control laws are presented in terms of several LMIs. The obtained criteria and design approaches in this paper depend on the size of delay but does not involve any tuning of parameters, as in the case of the robust stability and stabilization method (Li et al., 1997), and can be computed effectively (Boyd et al., 1994).

SYSTEM DESCRIPTION AND DEFINITIONS

Consider the following uncertain time-delay systems described by

$$\begin{aligned} \dot{x}(t) &= A(t)x(t) + A_1(t)x(t-d) \\ &\quad + B(t)u'(t) \\ u'(t) &= \mathrm{sat}(u(t)), \\ \mathrm{sat}(u(t)) &= [\mathrm{sat}(u_1(t))\ \mathrm{sat}(u_2(t))\ \cdots \\ &\quad \mathrm{sat}(u_m(t))] \\ x(t) &= \phi(t), t \in [-\tau, 0] \end{aligned} \tag{1}$$

where $x(t) \in R^n$ is the state vector, $u(t) \in R^m$ is the control input vector to the actuator (generated from the designed controller), $u'(t) \in R^m$ is the control input vector to the plant; $A(t) = A + \Delta A(t)$, $A_1(t) = A_1 + \Delta A_1(t)$, $B(t) = B + \Delta B(t)$ and $A \in R^{n\times n}$, $A_1 \in R^{n\times n}$, $B \in R^{n\times m}$ are known constant matrices. The matrices $\Delta A(\cdot)$, $\Delta A_1(\cdot)$ and $\Delta B(\cdot)$ are real-valued continuous matrix functions representing time-varying parameter uncertainties in the system model with appropriate dimensions. The nonlinear saturation function is considered to be inside the sector $[\sigma \quad 1]$ and is shown in Fig. 1.

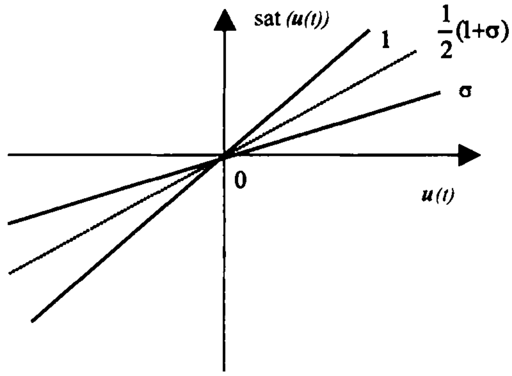

Fig. 1 Sector nonlinear saturation function

The d is unknown scalar denoting the delay in the state, and it is assumed that there exists a positive number τ such that

$$0 \le d \le \tau \tag{2}$$

holds for all t; $\phi(t)$ is a smooth vector-valued continuous initial function defined in the Banach space $C^n[-\tau, 0]$ of smooth functions

$$\Psi: [-\tau, 0] \to R^n \text{ with } \|\Psi\|_\infty := \sup_{-\tau \le \eta \le 0} \|\Psi(\eta)\|$$

In this paper, the admissible uncertainties are assumed to be of the form

$$\begin{aligned} \Delta A(t) &= H_1 F_1(t) E_1, \\ \Delta A_1(t) &= H_2 F_2(t) E_2, \\ \Delta B(t) &= H_3 F_3(t) E_3 \end{aligned} \tag{3}$$

where $F_i(t) \in R^{s_i \times q_i}$, $i = 1,2,3$ are unknown real timevarying matrices with Lebesgue measurable elements, satisfying

$$F_i^T(t)F_i(t) \leq I, i = 1,2,3 \quad (4)$$

and H_i, E_i, $i = 1,2,3$ are known real constant matrices with appropriate dimensions which characterize how the uncertain parameters in $F_i(t)$, $i = 1,2,3$ enter the nominal matrices A, A_1 and B.

Throughout this paper, we shall use the following concept of robust stable and robust stabilization for the uncertain time-delay system of the form (1) ~ (3).

Definition 1: The class of uncertain time-delay systems (1) ~ (3) is said to be robustly stable if the trivial solution $\boldsymbol{x}(t) \equiv 0$ of the functional differential equation associated to (1) with $\boldsymbol{u}(t) \equiv 0$ is globally uniformly asymptotically stable for all admissible uncertainties $\Delta A(t)$ and $\Delta A_1(t)$. The class of uncertain time-delay systems (1) ~ (3) is said to be robustly stabilizable if there exists a static linear state feedback control law $\boldsymbol{u}(t) = K\boldsymbol{x}(t)$ such that the resulting closed-loop system is robustly stable.

In this paper, we shall develop delay-dependent conditions for robust stabilizability of the uncertain time-delay system with sector saturating actuator (1) ~ (3) and the corresponding robust stabilizing control laws design approach. More specifically, we shall determine the bounds for the time-delay which ensure that the system (1) ~ (3) is robustly stabilizable and suitable memoryless state feedback control law will be developed. It will be shown later that the above problems can be solved by using linear matrix inequalities.

We introduce some useful lemmas (Su et al., 1998d), which will be essential for the proofs in the next section.

Lemma 1: Given any positive definite symmetric matrix R, vector $\boldsymbol{X}(t)$, $\boldsymbol{Y}(t)$ and matrix $F(t)$ with appropriate dimensions, and satisfying $F^T(t)F(t) \leq I$, then

$$2\boldsymbol{X}^T(t)F(t)\boldsymbol{Y}(t) \leq \boldsymbol{X}^T(t)R\boldsymbol{X}(t) + \boldsymbol{Y}^T(t)R^{-1}\boldsymbol{Y}(t) \quad (5)$$

Lemma 2: Let A, D and E be real constant matrices with appropriate dimensions, matrix $F(t)$ satisfies $F^T(t)F(t) \leq I$. Then we have:

(a) For any scalar $\varepsilon > 0$,

$$DF(t)E + E^TF^T(t)D^T \leq \varepsilon DD^T + \varepsilon^{-1}E^TE$$

(b) For any matrix $P > 0$ and scalar $\varepsilon > 0$ such that $\varepsilon I - EPE^T > 0$, then

$$(A + DF(t)E)P(A + DF(t)E)^T \leq APA^T + APE^T(\varepsilon I - EPE^T)^{-1}EPA^T + \varepsilon DD^T$$

(c) For any matrix $P > 0$ and scalar $\varepsilon > 0$ such that $P - \varepsilon DD^T > 0$, then

$$(A + DF(t)E)^TP^{-1}(A + DF(t)E) \leq A^T(P - \varepsilon DD^T)^{-1}A + \varepsilon^{-1}E^TE$$

DELAY-DEPENDENT ROBUST STABILIZING CONTROL LAW SYNTHESIS

In this section, we shall establish a delay-dependent robust stabilization sufficient condition and corresponding synthesis approach for uncertain linear time-delay systems with sector saturating actuator. Upper bound τ on the time delay d is given such that the uncertain linear time-delay system is robustly stabilizable for any time delay $0 \leq d \leq \tau$.

The main result is derived as follows:

Theorem 1: For the uncertain linear time-delay system (1) ~ (3), given scalar τ satisfying (2), this uncertain linear time-delay system is robustly stabilizable if there exist positive definite symmetric matrices $X > 0$, $T_{1j} > 0$, $j = 1,2,3$; $Q > 0$; matrix Y, and positive scalars $\alpha_i > 0$, $i = 1,2,3$; $\beta_j > 0$, $j = 1,2,3$; $\varepsilon > 0$ and $\varepsilon_1 > 0$ satisfying the following linear matrix inequalities (LMIs),

$$\begin{bmatrix} Q - BB^T - \varepsilon H_3H_3^T & BE_3^T \\ E_3B^T & \varepsilon I - E_3E_3^T \end{bmatrix} \geq 0$$

$$\begin{bmatrix} X & XA^T & XE_1^T \\ AX & T_{11} - \beta_1H_1H_1^T & 0 \\ E_1X & 0 & \beta_1 \end{bmatrix} \geq 0$$

$$\begin{bmatrix} X & XA_1^T & XE_2^T \\ A_1X & T_{12} - \beta_2H_2H_2^T & 0 \\ E_2X & 0 & \beta_2 \end{bmatrix} \geq 0$$

$$\begin{bmatrix} X & Y^TB^T & Y^TE_3^T \\ BY & T_{13} - \beta_3 H_3 H_3^T & 0 \\ E_3 Y & 0 & \beta_3 \end{bmatrix} \geq 0$$

$$\begin{bmatrix} S & M_1 & M_2 & M_3 & M_4 \\ M_1^T & -N_1 & 0 & 0 & 0 \\ M_2^T & 0 & -N_2 & 0 & 0 \\ M_3^T & 0 & 0 & -N_3 & 0 \\ M_4^T & 0 & 0 & 0 & -N_4 \end{bmatrix} < 0 \quad (6)$$

where

$$\begin{aligned} S = {} & AX + XA^T + A_1 X + XA_1^T + \frac{1}{2}(1+\sigma)\cdot \\ & (BY + Y^TB^T) + \sum_{i=1}^{2} \alpha_i H_i H_i^T \\ & + \frac{1}{2}(1+\sigma)\alpha_3 H_3 H_3^T + Q + 2\tau X \\ & + \frac{1}{2}(1+\sigma)\tau X + \tau(A_1 T A_1^T + \epsilon_1 H_2 H_2^T) \end{aligned}$$

$$M_1 = [XE_1^T \quad XE_2^T], \quad N_1 = \mathrm{diag}(\alpha_1 I, \alpha_2 I)$$

$$M_2 = Y^T E_3^T, \quad N_2 = 2(1+\sigma)^{-1}\alpha_3 I$$

$$M_3 = \tau A_1 T E_2^T, \quad N_3 = \tau(\epsilon_1 I - E_2 T E_2^T)$$

$$M_4 = \frac{1}{2}(1-\sigma)(1+\tau)Y^T, \quad N_4 = (1+\tau)I$$

$$T = Q + \sum_{j=1}^{2} T_{1j} + \frac{1}{2}(1+\sigma)T_{13}$$

Moreover, a suitable delay-dependent robustly stabilizing control law is given by

$$u(t) = YX^{-1}x(t) \quad (7)$$

Proof: Introduce the control law $u(t) = Kx(t)$ for the uncertain linear time-delay system (1) ~ (3), where the control law gain matrix $K \in R^{m\times n}$ is to be found, the closed-loop system can be written as

$$\begin{aligned} \dot{x}(t) = {} & \Big[A(t) + \frac{1}{2}(1+\sigma)B(t)K\Big]x(t) \\ & + A_1(t)x(t-d) + B(t)\eta(t) \quad (8) \end{aligned}$$

$$x(t) = \phi(t), t \in [-\tau, 0]$$

where $\eta(t) = \mathrm{sat}(Kx(t)) - \frac{1}{2}(1+\sigma)Kx(t)$. Obviously, vector function $\eta(t)$ satisfies the following inequality

$$\eta^T(t)\eta(t) \leq \frac{1}{4}(1-\sigma)^2 x^T(t)K^TKx(t)$$

Since $x(t)$ is continuously differentiable for $t \geq 0$, using the Leibniz-Newton formula (Hale et al., 1971), we can write

$$\begin{aligned} x(t-d) = {} & x(t) - \int_{-d}^{0} \dot{x}(t+s)ds \\ = {} & x(t) - \int_{-d}^{0} [A(t+s)x(t+s) \\ & + A_1(t+s)x(t-d+s) \\ & + \frac{1}{2}(1+\sigma)B(t+s)Kx(t+s) \\ & + B(t+s)\eta(t+s)]ds \end{aligned}$$

for $t \geq d$. Then the system (8) can be written as

$$\begin{aligned} \dot{x}(t) = {} & [A(t) + \frac{1}{2}(1+\sigma)B(t)K \\ & + A_1(t)]x(t) + B(t)\eta(t) \\ & - \int_{-d}^{0} A_1(t)\{[A(t+s) + \frac{1}{2}\cdot \\ & (1+\sigma)B(t+s)K]x(t+s) \\ & + A_1(t+s)x(t-d+s) \\ & + B(t+s)\eta(t+s)\}ds \end{aligned}$$

$$x(t) = \varphi(t), \quad t \in [-2\tau, 0] \quad (9)$$

where $\varphi(t)$ is a smooth vector-valued continuous initial function. It is declared in (Hale et al., 1971) that the asymptotic stability of (9) can assure the asymptotic stability of the closed-loop system (8), since the system (8) is only a special case of the system dynamics described in (9). Therefore, for the sake of simplicity, we will use the system dynamics in (9) to obtain a bound τ for the time delay d such that the closed-loop uncertain time-delay system (8) still retains its asymptotic stability for any $0 \leq d \leq \tau$.

Let the following function be the Lyapunov functional candidate for the system (9)

$$\begin{aligned} V(x(t),t) = {} & x^T(t)Px(t) \\ & + \int_{-d}^{0}\Big[\int_{t+s}^{t} x^T(\theta)R_1 x(\theta)d\theta \\ & + \int_{t-d+s}^{t} x^T(\theta)R_2 x(\theta)d\theta\Big]ds \end{aligned} \quad (10)$$

where P, R_1, R_2 are positive definite symmetric matrices. The derivative of $V(x(t), t)$

along the trajectory of the system (9) with respect to time t is given by

$$\dot{V}(\boldsymbol{x}(t),t)=\boldsymbol{x}^T(t)\{[A(t)+\frac{1}{2}(1+\sigma)B(t)\cdot K+A_1(t)]^TP+P[A(t)+\frac{1}{2}(1+\sigma)B(t)K+A_1(t)]\}\boldsymbol{x}(t)+f(\boldsymbol{x}(t),t)+g(\boldsymbol{x}(t),t) \quad (11)$$

where

$$f(\boldsymbol{x}(t),t)=d\boldsymbol{x}^T(t)R_1\boldsymbol{x}(t)-\int_{-d}^{0}\boldsymbol{x}^T(t+s)\cdot R_1\boldsymbol{x}(t+s)\,ds+d\boldsymbol{x}^T(t)R_2\boldsymbol{x}(t)-\int_{-d}^{0}\boldsymbol{x}^T(t-d+s)R_2\boldsymbol{x}(t-d+s)\,ds$$

$$g(\boldsymbol{x}(t),t)=2\boldsymbol{x}^T(t)PB(t)\boldsymbol{\eta}(t)-2\boldsymbol{x}^T(t)P\{\int_{-d}^{0}A_1(t)[(A(t+s)+\frac{1}{2}(1+\sigma)B(t+s)K)\boldsymbol{x}(t+s)+A_1(t+s)\boldsymbol{x}(t-d+s)+B(t+s)\boldsymbol{\eta}(t+s)]ds\}$$

By using Lemma 1 to (11), we have

$$\dot{V}(\boldsymbol{x}(t),t)\le\boldsymbol{x}^T(t)W_1\boldsymbol{x}(t)+f(\boldsymbol{x}(t),t)+g(\boldsymbol{x}(t),t) \quad (12)$$

where

$$W_1=A^TP+PA+A_1^TP+PA_1+\frac{1}{2}(1+\sigma)K^TB^TP+\frac{1}{2}(1+\sigma)PBK+\sum_{i=1}^{2}\alpha_i^{-1}E_i^TE_i+P(\sum_{i=1}^{2}\alpha_iH_iH_i^T+\frac{1}{2}(1+\sigma)\alpha_3H_3H_3^T)P+\frac{1}{2}(1+\sigma)\cdot\alpha_3^{-1}K^TE_3^TE_3K$$

and

$$g(\boldsymbol{x}(t),t)\le\boldsymbol{x}^T(t)PB(t)B^T(t)P\boldsymbol{x}(t)+\frac{1}{4}(1-\sigma)^2\boldsymbol{x}^T(t)K^TK\boldsymbol{x}(t)+\sum_{j=1}^{2}\int_{-d}^{0}\boldsymbol{x}^T(t)PA_1(t)T_{1j}A_1^T(t)\cdot P\boldsymbol{x}(t)ds+\int_{-d}^{0}\boldsymbol{x}^T(t+s)A^T(t+s)T_{11}^{-1}A(t+s)\boldsymbol{x}(t+s)ds+\int_{-d}^{0}\boldsymbol{x}^T(t-d+s)A_1^T(t+s)\cdot T_{12}^{-1}A_1(t+s)\boldsymbol{x}(t-d+s)ds+\frac{1}{2}(1+\sigma)[\int_{-d}^{0}\boldsymbol{x}^T(t)PA_1(t)\cdot T_{13}A_1^T(t)P\boldsymbol{x}(t)ds+\int_{-d}^{0}\boldsymbol{x}^T(t+s)K^TB^T(t+s)T_{13}^{-1}B(t+s)\cdot K\boldsymbol{x}(t+s)ds]+\frac{1}{4}(1-\sigma)^2\cdot\int_{-d}^{0}\boldsymbol{x}^T(t+s)K^TK\boldsymbol{x}(t+s)ds+\int_{-d}^{0}\boldsymbol{x}^T(t)PA_1(t)B(t+s)\cdot B^T(t+s)A_1^T(t)P\boldsymbol{x}(t)ds \quad (13)$$

Assume that there exist scalar $\varepsilon>0$ and positive definite symmetric matrix $Q>0$ such that the following inequality is satisfied

$$BB^T+BE_3^T(\varepsilon I-E_3E_3^T)^{-1}E_3B^T+\varepsilon H_3H_3^T\le Q \quad (14)$$

where

$$\varepsilon I-E_3E_3^T>0 \quad (15)$$

Then using Lemma 2(b), we have

$$B(t)B^T(t)\le Q \quad (16)$$

$$B(t+s)B^T(t+s)\le Q \quad (17)$$

Let $T=Q+\sum_{j=1}^{2}T_{1j}+\frac{1}{2}(1+\sigma)T_{13}$ and assume that there also exists scalar $\varepsilon_1>0$ such that the following inequality is satisfied

$$\varepsilon_1 I-E_2TE_2^T>0 \quad (18)$$

Then using Lemma 2(b), we have

$$A_1(t)TA_1^T(t)\le A_1TA_1^T+A_1TE_2^T(\varepsilon_1 I-E_2TE_2^T)^{-1}E_2TA_1^T+\varepsilon_1H_2H_2^T \quad (19)$$

Assume that there exist scalars $\beta_j>0, j=1,2,3$ satisfying the following inequalities

$$A^T(T_{11} - \beta_1 H_1 H_1^T)^{-1} A + \beta_1^{-1} E_1^T E_1 \le P \tag{20}$$

$$A_1^T(T_{12} - \beta_2 H_2 H_2^T)^{-1} A_1 + \beta_2^{-1} E_2^T E_2 \le P \tag{21}$$

$$K^T B^T(T_{13} - \beta_3 H_3 H_3^T)^{-1} BK + \beta_3^{-1} K^T E_3^T E_3 K \le P \tag{22}$$

where

$$T_{1j} - \beta_j H_j H_j^T > 0, j = 1,2,3 \tag{23}$$

Then using Lemma 2(c), we have

$$A^T(t+s) T_{11}^{-1} A(t+s) \le P, \forall t \ge 0$$

$$A_1^T(t+s) T_{12}^{-1} A_1(t+s) \le P, \forall t \ge 0$$

$$K^T B^T(t+s) T_{13}^{-1} B(t+s) K \le P, \forall t \ge 0$$

Then applying (16), (17), (19) and (24) to (12) and letting

$$R_1 = P + \frac{1}{2}(1+\sigma)P + \frac{1}{4}(1-\sigma)^2 K^T K, \quad R_2 = P \tag{25}$$

we obtain

$$\dot{V}(\boldsymbol{x}(t),t) \le \boldsymbol{x}^T(t)[W_1 + PW_2P + W_3]\boldsymbol{x}(t) = \boldsymbol{x}^T(t) W_4 \boldsymbol{x}(t) \tag{26}$$

where

$$W_2 = Q + \tau[A_1 T A_1^T + A_1 T E_2^T(\varepsilon_1 I - E_2 T E_2^T)^{-1} E_2 T A_1^T + \varepsilon_1 H_2 H_2^T] \tag{27}$$

$$W_3 = 2\tau P + \frac{1}{2}(1+\sigma)\tau P + \frac{1}{4}(1-\sigma)^2(1+\tau)K^T K \tag{28}$$

$$\begin{aligned} W_4 = {} & A^T P + PA + A_1^T P + PA_1 + \frac{1}{2}(1+\sigma)K^T B^T P + \frac{1}{2}(1+\sigma)PBK \\ & + \sum_{i=1}^{2} \alpha_i^{-1} E_i^T E_i + P\left(\sum_{i=1}^{2} \alpha_i H_i H_i^T + \frac{1}{2}(1+\sigma)\alpha_3 H_3 H_3^T\right)P + \frac{1}{2}(1+\sigma) \cdot \\ & \alpha_3^{-1} K^T E_3^T E_3 K + P\{Q + \tau[A_1 T A_1^T + A_1 T E_2^T(\varepsilon_1 I - E_2 T E_2^T)^{-1} E_2 T A_1^T \\ & + \varepsilon_1 H_2 H_2^T]\}P + 2\tau P + \frac{1}{2}(1+\sigma)\tau P + \frac{1}{4}(1-\sigma)^2(1+\tau)K^T K \end{aligned} \tag{29}$$

Then if for some scalar $\tau > 0$, there exist positive definite symmetric matrices P, $T_{1j}, j = 1,2,3, Q$; matrix K and positive scalars $\alpha_i > 0$, $i = 1,2,3$; $\beta_j > 0, j = 1,2,3$; $\varepsilon > 0$ and $\varepsilon_1 > 0$ satisfying the inequalities (14), (15), (18), (20) ~ (23) and $W_4 < 0$, then for any $0 \le d \le \tau$, we have

$$\dot{V}(\boldsymbol{x}(t),t) \le -\alpha \|\boldsymbol{x}(t)\|^2$$

where $\alpha = -\lambda_{\max}(W_4) > 0$ and $\lambda_{\max}(W_4)$ denotes the maximum eigenvalue of matrix W_4. Therefore, it follows from Lyapunov theorem and Definition 1 that the system (9) is asymptotically stable for any $0 \le d \le \tau$ and for any admissible uncertainties $F_i(t)$ satisfying (4). This implies that the closed-loop system (8) is robustly stable and the original uncertain time-delay system (1) ~ (3) is robustly stabilizable for any $0 \le d \le \tau$ and for any admissible uncertainties $F_i(t)$ satisfying (4).

Introduce the new variable X, let

$$X = P^{-1} \tag{30}$$

and let $W_5 = XW_4X$, then equation (29) can be rewritten as

$$\begin{aligned} W_5 = {} & XA^T + AX + XA_1^T + A_1 X + \frac{1}{2} \cdot (1+\sigma)XK^T B^T + \frac{1}{2}(1+\sigma)BKX \\ & + \sum_{i=1}^{2} \alpha_i^{-1} X E_i^T E_i X + \sum_{i=1}^{2} \alpha_i H_i H_i^T + \frac{1}{2}(1+\sigma)\alpha_3 H_3 H_3^T + \frac{1}{2}(1+\sigma) \cdot \\ & \alpha_3^{-1} X K^T E_3^T E_3 K X + Q + \tau[A_1 T A_1^T + A_1 T E_2^T(\varepsilon_1 I - E_2 T E_2^T)^{-1} E_2 T A_1^T \\ & + \varepsilon_1 H_2 H_2^T] + 2\tau X + \frac{1}{2}(1+\sigma)\tau X + \frac{1}{4}(1-\sigma)^2(1+\tau) X K^T K X \end{aligned} \tag{31}$$

Let $Y = KX$ and using schur complements, we obtain that inequalities (14), (15), (18),

(20) ~ (23) and $W_5 < 0$ are equivalent to those LMIs in (6).

Remark 1: Theorem 1 provides a delay-dependent condition for the robust stabilizability of the uncertain linear time-delay system (1) ~ (3) with sector nonlinear saturating actuator and the corresponding delay-dependent linear memoryless state feedback control law synthesis approach based on LMI technique. Since the robustly stabilizing control law design approach is dependent on the size of the time-delays, in general, it is expected to be less conservative than the delay-independent robust stabilizing control law design methods. In contrast with the results of Niculescu et al. (1996), who developed delay-dependent robust stabilization method for uncertain time-delay systems with constrained input in terms of the solution of Riccati equations, the obtained results in Theorem 1 is given in terms of the solutions of linear matrix inequalities, does not need any tuning of parameters, and can be calculated very effectively by using interior point algorithms. On the other hand, since the saturating actuator considered in Niculescu et al. (1996) is inside the bound of ± 1 while in this paper it is inside the sector $[\sigma, 1]$, the obtained results in Theorem 1 must be less conservative than the results in Niculescu et al. (1996).

Remark 2: Based on the Theorem 1, the upper bound τ for time delay d which ensures that the uncertain linear time-delay system (1) ~ (3) is robustly stabilizable for any $0 \le d \le \tau$ can be determined by solving the following quasi-convex optimization problems:

maximize τ Subject to LMIs (6) and $X > 0$, $T_{1j} > 0$, $j = 1,2,3$; $Q > 0$; matrix Y, and positive scalars, $\alpha_i > 0$, $i = 1,2,3$; $\beta_j > 0$, $j = 1,2,3$; $\varepsilon > 0$ and $\varepsilon_1 > 0$.

In the case when the time-delay system (1) ~ (3) does not involve uncertainties, i.e. $H_i = 0$ and $E_i = 0$, $i = 1,2,3$, we have the following result.

Corollary 1: For the uncertain time-delay system (1) ~ (3) with $H_i = 0$ and $E_i = 0$, $i = 1,2,3$, given scalar τ satisfying (2), this linear time-delay system is robustly stabilizable if there exist positive definite symmetric matrices $X > 0$, $Q > 0$, $T_{1j} > 0$, $j = 1,2,3$ and matrix Y satisfying the following linear matrix inequalities (LMIs),

$$Q - BB^T \ge 0$$

$$\begin{bmatrix} X & XA^T \\ AX & T_{11} \end{bmatrix} \ge 0, \quad \begin{bmatrix} X & XA_1^T \\ A_1 X & T_{12} \end{bmatrix} \ge 0,$$

$$\begin{bmatrix} X & Y^T B^T \\ BY & T_{13} \end{bmatrix} \ge 0$$

$$\begin{bmatrix} (A + A_1)X + X(A + A_1)^T + \frac{1}{2}(1+\sigma)(BY + Y^T B^T) + Q + 2\tau X + \frac{1}{2}(1+\sigma)\tau X + \tau A_1 T A_1^T & \frac{1}{2}(1-\sigma)(1+\tau)Y^T \\ \frac{1}{2}(1-\sigma)(1+\tau)Y & -(1+\tau)I \end{bmatrix} < 0$$

where $T = Q + \sum_{j=1}^{2} T_{1j} + \frac{1}{2}(1+\sigma)T_{13}$. Moreover, a suitable delay-dependent robustly stabilizing control law is given by $u(t) = YX^{-1}x(t)$.

NUMERICAL EXAMPLE

Consider an uncertain time-delay system (1) ~ (3), whose saturating actuator is inside the sector [1/3, 1] and whose dynamics are described as follows,

$$A = \begin{bmatrix} -2 & 0 \\ 1 & -3 \end{bmatrix}, \quad A_1 = \begin{bmatrix} -1 & 0 \\ -0.8 & -1 \end{bmatrix},$$

$$B = \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$$

$$H_i = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, E_i = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, i = 1,2,3$$

$$F_i(t) = \begin{bmatrix} \sin(t) & 0 \\ 0 & \cos(t) \end{bmatrix}, i = 1,2,3$$

and $F_i^T(t) F_i(t) \le I$, $i = 1,2,3$.

Applying Theorem 1 to this uncertain time-delay system, it is found, using the software package LMI Lab, that this system is robustly stabilizable for any time-delay $d \le 0.2961$. If there are no uncertainties in this system, i.e. $H_i = 0$ and $E_i = 0$, $i = 1,2,3$, the system is robustly stabilizable for any time-delay $d \le 0.6317$ by using the software package LMI Lab based on Corollary 1.

In the sequel, the results in Niculescu et al. (1996) will be applied to this uncertain time-delay system. We note that these results are based on the solution of a Riccati equation and involve the tuning of positive real scalars and 2×2 positive definite symmetric matrices. In view of the difficulty in tuning such multiple parameters (scalars and matrices) in order to maximize the bound for the time-delay, scalar 1 and an identity matrix are used. However, we cannot find the upper bound of time-delay such that the uncertain time-delay system with saturating actuator is robustly stabilizable via memoryless state feedback by using the Theorem 1 in Niculescu et al. (1996). When the system has no uncertainties, it is robustly stabilizable for any time-delay $d \leq 0.3819$ by using Remark 2 in Niculescu et al. (1996). Observe that this bound for the time-delay is not necessarily the optimal one; however, to the best of our knowledge, no optimization procedure is available. From comparison with the previous results, the robust stabilization approach presented in this paper is less conservative than the method in Niculescu et al. (1996).

CONCLUSION

This paper deals with the problem of robust stabilization synthesis for a class of uncertain linear time-delay systems with sector nonlinear saturating actuator. LMI based methods for analyzing the robust stabilizability and designing linear memoryless state feedback control laws have been developed. New criterion of delay-dependent robust stabilizability for uncertain time-delay systems is given in terms of linear matrix inequality (LMI). Numerical example shows that the presented method is feasible and less conservative than the reported results.

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