ISSN 1009 - 3095 *Journal of Zhejiang University(SCIENCE)* V.1, No.3, P.249 - 253, July - Sep., 2000
http://www.chinainfo.gov.cn/periodical; http://www.zju.edu.cn;
http://lib.zju.edu.cn/English; http://www.zjupress.com

A ROBUST GAIN-SCHEDULING CONTROL BASED ON VSC AND FUZZY LOCAL CONTROLLER NETWORK*

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Received Mar.12, 1999; revision accepted Dec.12, 1999

Abstract: Based on variable structure control (VSC) and fuzzy local controller network (FLCN), a new design method of robust gain-scheduling control is proposed in this paper. The proper sliding-modes and the tendency-rates for general operation-points are introduced such that the system gets into the sliding-modes' motion as soon as possible and has the desired performance. Its good performance is due to the robustness of VSC. However, any local controller works well only in the local region of a specified operation-point. In this paper functions similar to the fuzzy-attributed function in fuzzy-systems are introduced to form FLCN. The simulation results showed that the presented method is feasible and acceptable.

Key words: gain-scheduling control, variable structure control (VSC), sliding-mode, tendency-rate, fuzzy local controller network (FLCN).
Document code: A **CLC number**: TP273

INTRODUCTION

In recent years, with the development of computer and control theory, gain-scheduling is widely used in many applications, such as chemical engineering, aircraft control etc. (Shamma et al., 1993; Wang et al. 1995). However, there are many problems, such as stability, which are still not solved well. Although many reseanchers attempted to deal with those difficult problems, the results were mainly qualitative, such as the modeling error for any operation point should be very small and the scheduling-variables should be varying-slowly with time (Lawrence et al, 1995; Shamma et al., 1997). In fact, the ordinary method required the scheduling-variable to capture the nonlinearity of the controlled system (Hunt et al, 1997), it is very difficult for reported gain-scheduling approaches to achieve this objective.

On the other hand, let us consider the new control scheme-fuzzy-control, which is widely used in many applications. The theory of fuzzy control is very similar to that of gain-scheduling control on the above difficult problem. So the above difficulty should not affect the research for gain-scheduling approaches, especially for the realization of a new control design method for the gain-scheduling. In fact, many reseanchers considered these problems and some good results had been obtained (Leith et al, 1997;1998).

This paper presents the design of a VSC based local controller, which chooses a suitable sliding-mode for any operation-point and introduces the tendency-rate for the chosen sliding-mode. We use the valid functions very similar to the fuzzy-attributed functions, to construct FLCN. The obtained controller is a robust gain-scheduling controller due to the fact that the valid function is the function of scheduling-variables, and the local controller is constructed by using the VSC method.

MODELING WITH LOCAL MODEL NETWORK

1. System description

Consider the following single-input single-output nonlinear system

$$y^{(n)} = f(y, \dot{y}, \ddot{y}, \cdots, y^{(n-1)}, u) \qquad (1)$$

* Project supported by the Natural Science Foundation of China (NSFC: 69934030) and the Zhejiang Provincial Natural Science Foundation of China (ZJPNSFC: ZD9905).

where $y \in Y \subset R$ is the output variable, $u \in U \in R$ is the control input variable, f is a smoothing nonlinear function.

For the system of Eq. (1), we define some fixed operation points,

$op_1, op_2 \cdots, op_m: op_i = [y_i, \dot{y}_i, \cdots, y_i^{(n-1)}, u_i^0](i = 1, \cdots, m)$

Remark 1: op_i need not be an equilibrium point of Eq. (1).

The system can be linearized around op_i as follows:

$$\dot{x}_{1i} = x_{2i}$$
$$\dot{x}_{2i} = x_{3i}$$
$$\cdots\cdots$$
$$\dot{x}_{ni} = c_i^T x + b_i u_i + e_i(i) \qquad (2)$$
$$x_{1i} = y - y_{op_i}$$
$$\boldsymbol{x}_i = [x_{1i}, \cdots, x_{ni}]^T$$
$$c_i^T = \left[\frac{\partial f}{\partial y}, \frac{\partial f}{\partial \dot{y}}, \cdots, \frac{\partial f}{\partial y^{(n-1)}}\right]\bigg|_{op_i}, b_i = \frac{\partial f}{\partial u}\bigg|_{op_i}$$

where $\boldsymbol{x}_i$ is the state vector, u_i is control increment of input signal u_i^0 at operation points, so the control input signal u can be written as $u = u_i^0 + u_i$. $e_i(t)$ is modeling error caused by linearization.

2. Fuzzy local model network

The operation points play an important role in system modeling. The linear model of Eq. (2) is valid only very 'close' to the point op_i. In the following, we extend the basic linearization approach to the case of multi-points. By properly choosing valid functions $g_1(\xi), \cdots, g_m(\xi)$, we can get the following results

$$\dot{\tilde{x}}_1 = \tilde{x}_2$$
$$\dot{\tilde{x}}_2 = \tilde{x}_3$$
$$\cdots\cdots$$
$$\dot{\tilde{x}}_n = c^T \tilde{x} + bu + e(t) \qquad (3)$$
$$\tilde{x}_1 = y - \sum_{i=1}^{m} y_{op_i} g_i(\xi)$$
$$\tilde{\boldsymbol{x}} = [\tilde{x}, \cdots, \tilde{x}_n]^T$$
$$c^T = \sum_{i=1}^{m}\left[\frac{\partial f}{\partial y}, \frac{\partial f}{\partial \tilde{x}_2}, \cdots, \frac{\partial f}{\partial \tilde{x}_n}\right]\bigg|_{op_i}, g_i(\xi),$$
$$b = \sum_{i}^{m} \frac{\partial f}{\partial u}\bigg|_{op_i} g_i(\xi),$$
$$\sum_{i=1}^{m} g_i(\xi) = 1, g_i(\xi) \geq 0.$$

where ξ is scheduling-variable, and $g_1(\xi), \cdots, g_m(\xi)$ are fuzzy-attributed functions. Eq. (3) is called fuzzy local model network (FLMN) for Eq. (1).

Remark 2: From Eq. (3), the following points are very important:

(1) In general, the scheduling variable should vary slowly, but in our cases, this condition is not always needed because of the sliding-mode and tendency-rate.

(2) The modeling error should be considered in the controller design procedure .

(3) The valid function should be chosen carefully for each operation-point model. The basic requirement is that the valid functions should have the nonlinear properties of Eq. (1) as soon as possible.

CONTROLLER DESIGN

1. Local controller based on VSC

For Eq. (2) on op_i $(i = 1, \cdots, m)$, we choose the sliding-mode $S_i = x_{ni} + k_{n-1i}x_{n-1i} + \cdots + k_{1i}x_{1i} + k_{0i}\int_0^t x_{1i}dt \qquad (4)$

such that all eigenvalues of

$$p^n + k_{n-1i}p^{n-1} + \cdots + k_{1i}p + k_{0i} = 0$$

have a negative-real parts, where $k_{n-1i}, k_{n-2i}, k_{n-3i}, \cdots, k_{1i}, k_{0i}$ are constants and should be determined later.

The special case without modeling error is first considered. The tendency-rate for the chosen sliding-mode is introduced such that

$$\dot{S}_i = -\rho_i S_i, \quad \rho_i > 0 \qquad (5)$$

Further, the equality control law is obtained as

$$u_{eqi} = -\frac{1}{b_i}[(\boldsymbol{c}_i^T\boldsymbol{x} + \boldsymbol{k}_i^T\boldsymbol{x}) + \rho_i S_i] \quad (6)$$

where $\boldsymbol{k}_i = [k_{n-1i}, k_{n-2i}, k_{n-3i}, \cdots, k_{1i}, k_{0i}]^T$.

Finally, consider the existence of modeling error (Utkin, 1977), the VSC control law can be obtained as follows:

$$u_i = u_{\mathrm{eq}i} - \frac{M_i}{b_i}\mathrm{sgn}(S_i) \tag{7}$$

$$\text{where, } |e_i(t)| \le M_i$$

The control law of Eq. (7) is the local controller for system of Eq. (2).

Remark 3: For the control design at various operation-points, the parameters of sliding-mode and their tendency-rate should be chosen carefully. In fact, at each operation point the parameters may be different for special working conditions or working requirements. But the stability of the sliding-mode and the existence of sliding-mode motion are required for each of the operation points.

2. Fuzzy local controller network

From the local model network of Eq. (3) and local control law of Eq. (7), the fuzzy local controller network (FLCN) can be written as follows:

$$u(t) = -\sum_{i=1}^{m} g_i(\xi)\frac{M_i}{b}\mathrm{sgn}(S_i) + \sum_{i=1}^{m} u_{\mathrm{eq}i} g_i(\xi) \tag{8}$$

Then, we get the gain-scheduling control based on FLCN and VSC as follows:

$$u(t) = -\sum_{i=1}^{m} g_i(\xi)\frac{M_i}{b_i}\mathrm{sgn}(S_i) + \sum_{i=1}^{m}(u_{\mathrm{eq}i} + u_i^0) g_i(\xi) \tag{9}$$

$$u_{\mathrm{eq}i} = -\frac{1}{b_i}[(c_i^T x_i + k_i^T x_i) + \rho_i S_i]$$

Remark 4: $\sum_{i=1}^{m} y_{op_i} g_i(\xi)$ and its differentials will be replaced by the reference value of and its differentials for the realization of Eq.(9).

3. The valid function

For the realization of gain scheduling control expressed by Eq. (9), the choice of valid functions is very important. In fact, we may choose the typical functions, similar in form to those of fuzzy-attribution. The valid function given below is used in the next section for simulation study.

$$g_i(\xi) = \frac{h_i - \mathrm{sat}(\xi - \xi_{op_i})}{\sum_{i=1}^{m}(h_i - \mathrm{sat}(\xi - \xi_{op_i}))}$$

$$\mathrm{sat}(\xi - \xi_{op_i}) = \begin{cases} |\xi - \xi_{op_i}|, \text{for} |\xi - \xi_{op_i}| < h_i \\ 0, \text{for} |\xi - \xi_{op_i}| > h_i \end{cases} \tag{10}$$

where ξ is the scheduling-variables and ξ_{op_i} is are appropriate variables in χ_{op_i}.

As a valid function, the following conditions must be considered.

(1) For the case $\xi = \xi_{op_i}$, the valid function must be 1 or tend to 1.

(2) For any pairs of operation points (op_i, op_j) which are not the nearest points, we have

$$g_i(\xi) \cdot g_j(\xi) = 0 \tag{11}$$

(3) The value of each valid function must be changed smoothly.

(4) With the increasing of the distance between one fixed operation point and transient states, the valid function should tend to zero smoothly.

Obviously, the chosen function of Eq. (10) meets the above conditions.

NUMBERICAL EXAMPLE

The purpose of this section is to show the improvement of the presented controller design method based on VSC and FLCN over the other controller. Consider the following dynamic nonlinear system

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_1^2 + x_2^2 + u \\ y &= x_1 \end{aligned} \tag{12}$$

Here, we consider two LCNs, where the first LCN1 has the same operation-points as Hunt (1997)'s, but LCN2's is difference.

LCN1: LCN1 contains four local controllers designed on the basis of classical linearization around the equilibrium points:
$(x_1^e, x_2^e, u^e) \in \{(0.5, 0., -0.25), (1.5, 0, -2.25), (2.5, 0-6.25), (3.5, 0-12.25)\}$

The local controllers all have the same sliding-mode parameters and tendency-rate:

$$k_{1i} = 0.3, k_{0i} = 0.02, \rho_i = 50$$

The simulation results are given in Fig. 1. Obviously, our results are much better than those of Hunt (1997). For the experiments E_1, E_2, E_3 described by Hunt, our results are all stable, but Hunt's are not stable for E_2, E_3.

LCN2: LCN2 contains four local controllers based on off-equilibrium linearization about the points

$(x_1^e, x_2^e, u^e) \in \{(0.5, 0., 0), (1.5, 0, 0), (2.5, 0, 0), (3.5, 0, 0)\}$,

which covers some of the transient states only. We may find that there are less designing operation-points than those in the case LCN2 of Hunt (1997). The control parameters are the same as those for controller LCN1.

Fig.2 on the simulation results show they are acceptable. For the experiments E_1, E_2, E_3 Hunt (1997), the obtained results were all stable.

From the simulation results, we can get the following conclusions:

(1) The simulation results of a local controller network based on local equilibrium-points are better than those of a local controller network based on off-equilibrium-operation points.

(2) Comparison of our results with those of Hunt (1997), showed that our controller has similar characteristics but is much simpler.

(3) Comparison of the local-network with Hunt's shown that the local control network based on sliding-mode and its tendency-rate works well for the system of Eq. (12) due to the fast tending to sliding-mode and slow tending to its required value with high tendency-rate and suitable parameters for sliding-mode.

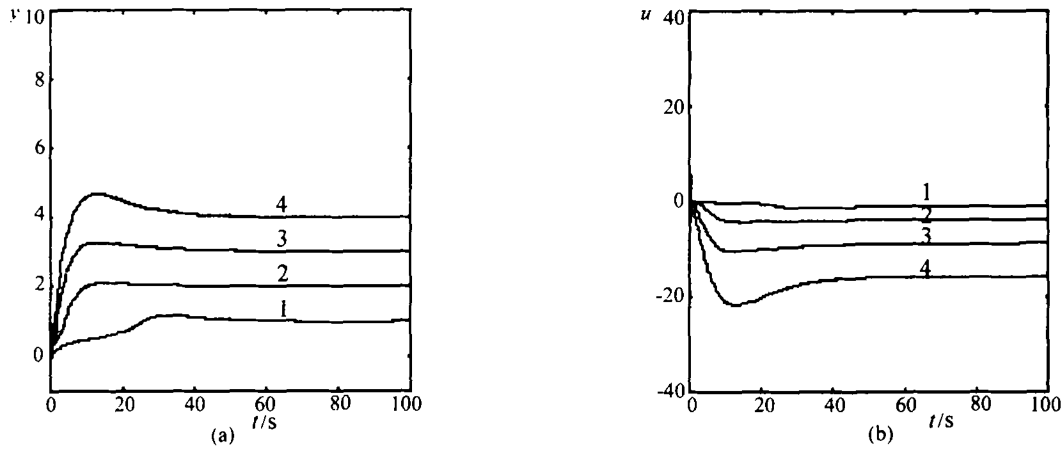

Fig.1 Simulation results with controller LCN1.
(a) the system output y for the desired values 1,2,3,4. (b) the control input u for different desired values 1,2,3,4.

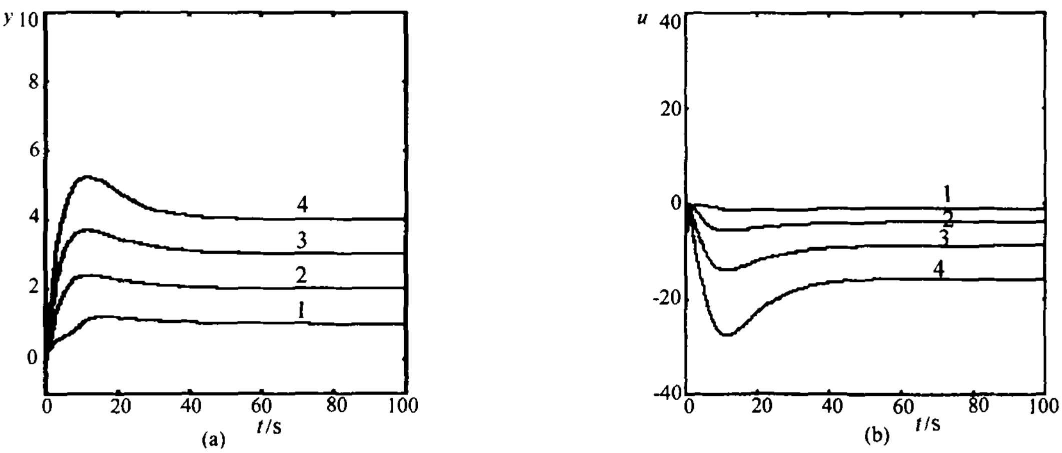

Fig.2 Simulation results with controller LCN2.
(a) the system output y for the desired values 1,2,3,4; (b) the control input u for different desired values 1,2,3,4.

CONCLUSION

This paper presents a new approach for gain-scheduling controller design based on local controller network using sliding-mode and its tendency-rate. The valid function similar in form to fuzzy-attribution, is introduced such that the designed local controllers are combined into FLCN. The simulation results showed that the designed controller works well for many cases, such as Hunt's (1997). One of the important results from simulation is that to achieve better performance, the tendency-rate and sliding-mode parameter should be different from each other.

References

Hunt, K. J. and Johansen, T. A., 1997. Design and analysis of gain-scheduling control using local controller networks. *INT. J. Control*, **66**(5): 619 - 651.

Lawrence, D. A. and Rugh W. J., 1995. Gain-scheduling dynamic linear controllers for a nonlinear plant. *Automatica*, **31**(3): 381 - 390.

Leith, D. J. and Leithead, W. E., 1998. Gain-scheduling controller design: an analytic framework directly incorporating non-equilibrium plant dynamics. *INT. J. Control*, **70**(2):249 - 269.

Leith, D. J. and Leithead W. E., 1998. Appropriate realization of MIMO gain-scheduling controllers, *INT. J. Control*, **70**(1):13 - 50.

Shamma, J. S. and Cloutier, J. R., 1993. Gain-scheduling missile autopilot design using linear parameter varying transformation. *Journal of guidance, control and dynamics*, **16**(2):256 - 263.

Shamma, J. S. and Tu, K. Y., 1997. Approximate set-valued observers for nonlinear systems. *IEEE Transaction on Automation Control*, **42**(5): 648 - 658.

Wang, J. L., 1995. Scheduling of local nonlinear control laws by exogenous signal-an application to flight control. *INT. J. Control*, **61**(4): 759 - 782.

Utkin, 1987. Discontinuous control state of art in theory and application. Proc. IFAC, München, p.75 - 93.