

INTERFEROMETRIC MEASUREMENT OF AN ATOMIC WAVE FUNCTION BY USING A STANDING WAVE METHOD*

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Abstract: In this paper, the authors present the results of their study on the scheme of reconstructing the atomic wave function by using a standing wave laser beam. The scheme effectively avoids the initial random phase problem of the running light wave of lasers. The paper presents the relation between measured data and the atomic wave function, whose reconstruction procedure is also analyzed.

Key words: atomic wave function, reconstruct, standing wave laser beam.

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INTRODUCTION

The quantum state is an important concept in quantum mechanics and other related fields including quantum optics, atom optics, and the physics of trapped particles. In recent years, people devoted much time to produce various quantum states, and paid much attention to the problem of how to effectively measure a quantum state (Weigert, 1996; Jones, 1994; Raymer et al., 1994), because a quantum state is not as simple to observe as the state of a classical system, and to synthesize a quantum state requires suitable methods to probe it, one of which is reconstruction of the complete information encoded in a quantized system. The reconstruction of a quantum state not only has great significance in theory, but had also been achieved in experiments to reconstruct the vibratory states of a single ion (Bardroff et al., 1996) and a diatomic molecule (Dunn et al., 1993), and to reconstruct the quantum states of single atoms in the physics of a trapped particle (Leibfried et al., 1996).

There were several methods proposed to measure quantum states of light and quantum states of matter. For instance, the tomographic method (D'Ariano et al., 1995; Vogel et al., 1989) is a main method based on a set of probability distributions for a position observable, and is suit-

able for both kinds of wave functions of quantum states of light and matter. The endoscopy method (Bardroff et al., 1996; Bardroff et al., 1995) makes use of the excitation statistics of atoms that have interacted with the cavity field to reconstruct the quantum state, while the atomic deflection method (Herkommer et al., 1992) makes use of the momentum distributions of atoms.

In the reference (Freyberger et al., 1997), an interferometric method was proposed to probe the atomic wave function. This method has many advantages such as mathematical simplicity and experimental feasibility. The scheme consists of two counter propagating lasers 1 and 2, which locate at $z = 0$ and $z = l_1$ respectively. They interact resonantly with a beam of two-level atoms and their interaction regions are of the same length a . The relative phase between them is controlled by a phase shifter.

In order to obtain the full information of the interference term $\Psi^*(x, T)\Psi(x - \Delta x, T)$ in this scheme, it was proposed to measure a set of four position distributions of atoms on the screen, $w(x, \Delta x, \bar{\varphi})$, with $\bar{\varphi} = 0, \frac{\pi}{2}, \pi$, and $\frac{3\pi}{2}$ the variation of $\bar{\varphi}$ is achieved with the help of the phase shifter between laser beams 1 and 2.

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An important assumption in the reference is that the initial phase α_0 of the running light wave of the laser is 0. But if the initial phase α_0 in Eq. (2A) (Freyberger et al., 1997) is not omitted, then the Equation (3) should contain the factor α_0 , and the $\tilde{\varphi}$ in Eqs. (5) and (10) should contain the initial phase α_0 as well. Because the α_0 is completely random for each of the independent four times' measurements, the $\tilde{\varphi}$ cannot be determined to be $0, \frac{\pi}{2}, \pi$, and $\frac{3\pi}{2}$.

Since it is needed to extract the whole information on the atomic wave, especially on the phase information, and to exclude from the measured data the background which is not related with the interference term, it is unavoidable to carry out the independent measurements for several times, and then to analysis the data obtained from the measurements. Therefore, to solve the random problem of the initial phase seems very important to the present measurement scheme (Liu et al., 2000).

Note that in a standing wave of laser beam, the initial phase is actually determined by the intersection position of atomic flow and laser beam, such as at the wave valley or node, and this initial phase does not change with time. Through adjusting the intersection position at which the atomic flow traverses the laser beam, one may adjust the phase angle α of the standing wave field expressed as $\sin^2(kx + a)$. It is worthy to consider that the atomic flow interacts with a standing wave of laser beam.

In this paper, we will discuss this standing wave scheme as an improvement to the scheme in the reference (Freyberger et al., 1997).

STANDING WAVE METHODS

We consider the two-level atom of mass m moving in the z direction at a velocity much higher than any velocity component in the x direction, that is, its total momentum $\hbar p_0$ is supposed to be much larger than any transverse momentum component in the x direction (Freyberger et al., 1997). The atomic beam is scattered elastically along a standing wave laser, and each atom has constant energy $E_0 \equiv \frac{\hbar^2 p_0^2}{2m}$. We solve

the scattering problem for the stationary incident atomic wave at $z = z_0$:

$$|\Psi(x, z_0, t)\rangle = \exp(-iE_0 t/\hbar) \left[\int \frac{dp}{2\pi} \phi_g \cdot (p, z_0) \exp(ipx) |g\rangle + \int \frac{dp}{2\pi} \phi_e(p, z_0) \exp(ipx) |e\rangle \right] \quad (1)$$

where $\phi_{g(e)}(p, z_0)$ is the wave function in momentum representation. The interaction with a standing wave laser of wave vector k in the region, $z_0 \leq z \leq z_0 + a$, is governed by the Hamiltonian:

$$\hat{H} = \frac{\hbar^2}{2m} (\hat{p}^2 + \hat{p}_z^2) - \Theta(z - z_0) \Theta(z_0 + a - z) \hbar \lambda \sin^2(kx + \alpha) (\hat{\sigma}_- + \hat{\sigma}_+), \quad (2)$$

where the momentum operator for the x degree of freedom is $\hat{p} = -i \frac{\partial}{\partial x}$ and the one for the z degree of freedom is $\hat{p}_z = -\frac{\partial}{\partial z}$; two Heaviside functions Θ switch the interaction on and off; λ is the coupling constant of atom and field. $\hat{\sigma}_-$ and $\hat{\sigma}_+$ are the Pauli spin operators describing transitions between the atomic states, and α can be changed by shifting the intersection position of the atomic beam and the standing wave laser.

In the interaction region the atomic wave can be generally expressed as:

$$|\Psi(x, z, t)\rangle = \exp(-iE_0 t/\hbar) \times \left[\int \frac{dp}{2\pi} \phi_g \cdot (p, z) \exp[ip_z(p)(z - z_0)] \cdot \exp(ipx) |g\rangle + \int \frac{dp}{2\pi} \phi_e(p, z) \cdot \exp[ip_z(p)(z - z_0)] \exp(ipx) |e\rangle \right], \quad (3)$$

Since the scattering process is elastic, then the total energy $E_0 \equiv \hbar^2 p_0^2/2m = \frac{\hbar^2}{2m} (p^2 + p_z^2)$ will remain constant, where the longitudinal momentum $p_z = p_z(p)$.

Substituting Eq. (3) into the Schrödinger equation:

$$i\hbar \frac{\partial}{\partial t} |\Psi\rangle = \hat{H} |\Psi\rangle, \quad (4)$$

we get the coupled equations:

$$i v_0 \frac{\partial}{\partial z} \phi_g(p, z) = -\frac{\lambda}{2} \{ \phi_e(p, z) - \frac{1}{2} [\phi_e(p - 2k, z) \exp(i2\alpha) + \phi_e(p + 2k, z) \exp(-i2\alpha)] \}, \quad (5)$$

$$i v_0 \frac{\partial}{\partial z} \phi_e(p, z) = -\frac{\lambda}{2} \{ \phi_g(p, z) - \frac{1}{2} [\phi_g(p - 2k, z) \exp(i2\alpha) + \phi_g(p + 2k, z) \exp(-i2\alpha)] \}, \quad (6)$$

where v_0 is the velocity of the atomic flow; if $2k$ is a small quantity in comparison with p , some approximation is adopted (Freyberger et al., 1997): $p_z(p + 2k) \approx p_z(p - 2k) \approx p_z(p) \approx p_0$; and since $\phi_g(p, z)$ and $\phi_e(p, z)$ are the slowly varying functions of z , then $\frac{\partial^2}{\partial z^2} \phi_g(p, z) \approx 0$ and $\frac{\partial^2}{\partial z^2} \phi_e(p, z) \approx 0$.

Then we have

$$\frac{\partial^2}{\partial z^2} \phi_g(p, z) = -\left(\frac{\lambda}{2v_0}\right)^2 \left\{ \frac{3}{2} \phi_g(p, z) - [\phi_g(p - 2k, z) \exp(i2\alpha) + \phi_g(p + 2k, z) \exp(-i2\alpha)] + \frac{1}{4} [\phi_g(p - 4k, z) \exp(i4\alpha) + \phi_g(p + 4k, z) \exp(-i4\alpha)] \right\}, \quad (7)$$

and

$$\frac{\partial^2}{\partial z^2} \phi_e(p, z) = -\left(\frac{\lambda}{2v_0}\right)^2 \left\{ \frac{3}{2} \phi_e(p, z) - [\phi_e(p - 2k, z) \exp(i2\alpha) + \phi_e(p + 2k, z) \exp(-i2\alpha)] + \frac{1}{4} [\phi_e(p - 4k, z) \exp(i4\alpha) + \phi_e(p + 4k, z) \exp(-i4\alpha)] \right\}. \quad (8)$$

Note that Eqs. (7) and (8) have similar forms, for simplicity, in the following derivation we omit the subscript $g(e)$:

Let $\alpha = 0$, have

$$\frac{\partial^2}{\partial z^2} \phi(p, z) = -\left(\frac{\Omega}{2}\right)^2 \phi^{(4)}(p + 4k, z) (2k)^4, \quad (9)$$

$\alpha = \frac{\pi}{2}$, have

$$\frac{\partial^2}{\partial z^2} \phi(p, z) = -\Omega^2 \{ 4\phi(p, z) + 2\phi''(p + 2k, z) (2k)^2 + \frac{1}{4} \phi^{(4)}(p + 4k, z) (2k)^4 \}, \quad (10)$$

$\alpha = \frac{\pi}{4}$, have

$$\frac{\partial^2}{\partial z^2} \phi(p, z) = -\Omega^2 \{ \phi(p, z) + i2\phi'(p, z) (2k) - \phi''(p, z) (2k)^2 - \frac{1}{4} \phi^{(4)}(p, z) (2k)^4 \}, \quad (11)$$

$\alpha = \frac{3\pi}{4}$, have

$$\frac{\partial^2}{\partial z^2} \phi(p, z) = -\Omega^2 \{ \phi(p, z) - i2\phi'(p, z) (2k) - \phi''(p, z) (2k)^2 - \frac{1}{4} \phi^{(4)}(p, z) (2k)^4 \}, \quad (12)$$

where $\Omega \equiv \frac{\lambda}{2v_0}$, and $\phi' \sim \phi^{(4)}$ are the first to fourth order difference derivative of $\phi(p, z)$ with respect to the parameter p in the difference step of $2k$, for instance, $\phi'(p, z) \equiv [\phi(p, z) - \phi(p - 2k, z)] / (2k)$. Because $2k$ is a small quantity, in the following solution to Eqs. (9) through (12), we omit those terms containing higher order small quantities such as $(2k)^2$ and $(2k)^4$. We obtain the expression of the wave function on a screen at $z = l$, when $\alpha = 0$,

$$\begin{aligned} |\Psi(x, l, t)\rangle &= \exp(-iE_0 t/\hbar) \left\{ \int \frac{dp}{2\pi} \phi_g(p, z_0) \exp[ip_z(p)l + ipx] + i\Omega a \int \frac{dp}{2\pi} \cdot \right. \\ &\quad \left. \phi_e(p, z_0) - \frac{1}{2} (\phi_e(p + 2k, z_0) + \phi_e(p - 2k, z_0)) \right\} \times \exp[ip_z(p)l + ipx] |g\rangle + \exp(-iE_0 t/\hbar) \\ &\quad \left\{ \int \frac{dp}{2\pi} \phi_e(p, z_0) \exp[ip_z(p)l + ipx] + i\Omega a \int \frac{dp}{2\pi} \phi_g(p, z_0) - \frac{1}{2} \right. \\ &\quad \left. (\phi_g(p + 2k, z_0) + \phi_g(p - 2k, z_0)) \right\} \exp[ip_z(p)l + ipx] |e\rangle. \quad (13) \end{aligned}$$

Assume that atoms are initially in the ground state, i. e., $\phi_g(p, z_0) = \phi(p)$, $\phi_e(p, z_0) = 0$.

Substituting $p_z(p) \equiv \sqrt{p_0^2 - p^2} \approx p_0 - \frac{p^2}{2p_0}$, $p_z(p+2k) \approx p_0 - \frac{(p+2k)^2}{2p_0}$, and $p_z(p-2k) \approx p_0 - \frac{(p-2k)^2}{2p_0}$ into Eq. (13), we have

$$\begin{aligned} |\Psi(x, l, t)\rangle = & \exp(-iE_0 t/\hbar) \exp(ip_0 l) \times \\ & \left\{ \frac{1}{[1 + \Omega^2 a^2]^{\frac{1}{2}}} \Psi(x, T) |g\rangle \right. \\ & + \frac{i\Omega a}{[1 + \Omega^2 a^2]^{\frac{1}{2}}} \times [\Psi(x, T) \\ & - \frac{1}{2} [\exp[i(\varphi - 2kx)] \Psi(x \\ & + 2\Delta x, T) + \exp[i(\varphi \\ & + 2kx)] \Psi(x - 2\Delta x, T)]] \\ & \left. |e\rangle \right\}, \end{aligned} \quad (14)$$

where $\varphi \equiv -\frac{4k^2 l}{2p_0}$, $\Delta x \equiv \frac{kl}{p_0}$, $T \equiv \frac{ml}{\hbar p_0}$, and

$$\Psi(x, T) \equiv \int \frac{dp}{2\pi} \phi(p) \exp(ipx - i\frac{\hbar p^2}{2m} T).$$

The probability distribution for the position of the atoms reads:

$$\begin{aligned} w^{(0)}(x, \Delta x, T) = & |\Psi(x, T)|^2 + \frac{\Omega^2 a^2}{4(1 + \Omega^2 a^2)} \\ & [|\Psi(x + 2\Delta x, T)|^2 \\ & + |\Psi(x - 2\Delta x, T)|^2] \\ & + \frac{\Omega^2 a^2}{2(1 + \Omega^2 a^2)} \text{Re} \{ e^{i4kx} \Psi^* \cdot \\ & (x + 2\Delta x, T) \Psi(x - 2\Delta x, T) \\ & - \Psi^*(x, T) [e^{i(\varphi - 2kx)} \Psi \cdot \\ & (x + 2\Delta x, T) + e^{i(\varphi + 2kx)} \Psi \cdot \\ & (x - 2\Delta x, T)] \}. \end{aligned} \quad (15)$$

In the same way, when $\alpha = \frac{\pi}{2}$, we have

$$\begin{aligned} |\Psi(x, l, t)\rangle = & e^{-iE_0 t/\hbar} e^{ip_0 l} \{ \cos(2\Omega a) \Psi(x, T) | \\ & g\rangle - \frac{1}{2} i \sin(2\Omega a) \times \\ & [\Psi(x, T) + \frac{1}{2} [e^{i(\varphi - 2kx)} \Psi(x \\ & + 2\Delta x, T) + e^{i(\varphi + 2kx)} \Psi(x \end{aligned}$$

$$- 2\Delta x, T)]] |e\rangle, \quad (16)$$

and

$$\begin{aligned} w^{(\frac{\pi}{2})}(x, \Delta x, T) = & |\Psi(x, T)|^2 + \frac{\sin^2(2\Omega a)}{16} \\ & [|\Psi(x + 2\Delta x, T)|^2 + |\Psi \cdot \\ & (x - 2\Delta x, T)|^2 + \frac{1}{8} \sin^2 \cdot \\ & (2\Omega a) \text{Re} \{ \exp(i4kx) \Psi^* \cdot \\ & (x + 2\Delta x, T) \Psi(x - 2\Delta x, \\ & T) + \Psi^*(x, T) [\exp[i(\varphi \\ & - 2kx)] \Psi(x + 2\Delta x, T) \\ & + \exp[i(\varphi + 2kx)] \Psi(x \\ & - \Delta x, T)] \}. \end{aligned} \quad (17)$$

Note that $\Omega \equiv \frac{\lambda}{2v_0}$, so $\Omega a = \frac{\lambda a}{2v_0}$ is a small quantity, and the following formulas stand: $\sin^2 2\Omega a \approx 4\Omega^2 a^2$, and $\frac{\Omega^2 a^2}{1 + \Omega^2 a^2} \approx \Omega^2 a^2$.

Then

$$\begin{aligned} w^{(\frac{\pi}{2})} - w^{(0)} \equiv & (\Omega a)^2 \text{Re} \{ \Psi^*(xT) [\exp[i(\varphi \\ & - 2kx)] \Psi(x + 2\Delta x, T) \\ & + \exp[i(\varphi + 2kx)] \Psi \cdot \\ & (x - 2\Delta x, T)] \}. \end{aligned} \quad (18)$$

If without the approximation above, we let $A \equiv \frac{\sin^2(2\Omega a)(1 + \Omega^2 a^2)}{4\Omega^2 a^2}$, $\bar{w} \equiv |\Psi(x, T)|^2$, and

$\bar{w}^k \equiv w^k - \bar{w}$ ($k = \frac{\pi}{2}$, or 0), then we get:

$$\begin{aligned} w^{(\frac{\pi}{2})} - A\bar{w}^{(0)} = & \frac{1}{4} \sin^2(2\Omega a) \text{Re} \{ \Psi^*(x, T) \cdot \\ & [\exp[i(\varphi - 2kx)] \Psi(x \\ & + 2\Delta x, T) + \exp[i(\varphi + 2kx)] \cdot \\ & \Psi(x - 2\Delta x, T)] \}. \end{aligned} \quad (19)$$

If for the numerical processing of the measured data by computer, the parameter A is adjusted to get the clearest interference design after the deduction of the two sets of data above, then one may regard the choice of A is appropriate, and obtain the interference information of $\Psi^*(x, T) \cdot \Psi(x + 2\Delta x, T)$ and $\Psi^*(x, T) \Psi(x - 2\Delta x, T)$.

In the same way, when $\alpha = \frac{\pi}{4}$ and $\frac{3\pi}{4}$, one may obtain

$$\begin{aligned} |\Psi(x, l, t)\rangle &= \exp(-iE_0 t/\hbar) \exp(ip_0 l) \\ &\quad \left\{ \frac{S}{\Omega a} \Psi(x, T) |g\rangle - i \cdot \right. \\ &\quad \left. \left(\frac{1+C}{\Omega a} + \frac{1+C}{S} \right) \times \{ \Psi(x, \right. \\ &\quad T) \pm \frac{i}{2} [\exp(i(\varphi - 2kx))] \cdot \\ &\quad \Psi(x + 2\Delta x, T) - \exp[i \cdot \\ &\quad (\varphi + 2kx)] \Psi(x - 2\Delta x, \\ &\quad T) \} |e\rangle \}, \end{aligned} \quad (20)$$

where $C \equiv \cos \Omega a$, and $S \equiv \sin \Omega a$; ‘ \pm ’ correspond to the cases of $\alpha = \frac{\pi}{4}$ and $\frac{3\pi}{4}$, respectively.

And

$$\begin{aligned} w^{(\frac{\pi}{4})} - w^{(\frac{3\pi}{4})} &= \left(\frac{1+C}{\Omega a} + \frac{1+C}{S} \right) \text{Im} \{ \Psi^*(x, T) \\ &\quad \times [\exp[i(\varphi - 2kx)] \Psi(x \\ &\quad + 2\Delta x, T) - \exp[i(\varphi + 2kx)] \cdot \\ &\quad \Psi(x - 2\Delta x, T)] \}. \end{aligned} \quad (21)$$

Finally, we obtain

$$\begin{aligned} \exp(-i\varphi) \left[\frac{4}{\sin^2(2\Omega a)} (\bar{w}^{(\frac{\pi}{2})} - A\bar{w}^{(0)}) + i \cdot \right. \\ \left. \left(\frac{1+C}{\Omega a} + \frac{1+C}{S} \right)^{-2} (w^{(\frac{\pi}{4})} - w^{(\frac{3\pi}{4})}) \right] \\ = \Psi^*(x, T) [\exp(-i2kx) \Psi(x + 2\Delta x, T) \\ - \exp(i2kx) \Psi(x - \Delta x, T)]. \end{aligned} \quad (22)$$

In Eq. (22), the left hand side only relates with the experimental parameters and measured data, and the right hand side contains the interference terms such as $\Psi(x - 2\Delta x, T) \Psi^*(x, T)$. Thus, this equation builds up a connection between the atomic position distributions and the atomic wave function, so that it is a core equation which provides us a method for measuring the atomic wave function $\Psi(x, T)$ via the measured data of the recorded position distributions.

In the following, we will discuss how to reconstruct the whole wave function $\Psi(x, T) = |\Psi(x, T)\rangle \exp[i\phi(x, T)]$. One should first determine the modulus $|\Psi(x, T)|$, through

which we are going to learn in which position interval $[x_m, x_n]$ the wave function is mainly distributed. Then we may neglect the area not within this interval, where the modulus is close to zero. We apply Eq. (22) into this measurement procedure; at position $x = x_m$, we may evaluate the phase difference $\phi(x_m, T) - \phi(x_m + 2\Delta x, T)$ of $\Psi(x_m, T)$ and $\Psi(x_m + 2\Delta x, T)$; another contribution which contains $\Psi(x_m - 2\Delta x, T)$ is negligible, for its position is not within the interval. Then, we go $2\Delta x$ step further to the position $x_m + 2\Delta x$ and apply Eq. (22) again. Since this time we already know the second term on the right hand side we can extract the phase difference $\phi(x_m + 2\Delta x, T) - \phi(x_m + 4\Delta x, T)$ of $\Psi(x_m + 2\Delta x, T)$ and $\Psi(x_m + 4\Delta x, T)$. Thus, we can go on step by step according to this method till we cover the full interval $[x_m, x_n]$. Eventually, we will achieve a sequence of the phase differences in that interval as $\{\phi(x_m, T) - \phi(x_m + 2\Delta x, T), \phi(x_m + 2\Delta x, T) - \phi(x_m + 4\Delta x, T), \dots\}$.

Clearly, the above sequence carries the information about the phase encoded in $\Psi(x, T)$, with spacing $2\Delta x$. Moreover, if we want to determine the individual phases via this sequence, we should first set the unknown phase at the position x_m as zero, i. e., $\phi^{(\text{rec})}(x_m, T) \equiv 0$, so that the phases $\phi^{(\text{rec})}(x_m + 2\Delta x, T)$, $\phi^{(\text{rec})}(x_m + 4\Delta x, T)$, \dots can be recursively calculated. The reconstructed wave function satisfies:

$$\Psi^{(\text{rec})}(x, T) = \Psi(x, T) \exp[-i\phi(x_m, T)], \quad (23)$$

whose phase is determined on a grid with the spacing $2\Delta x$ in the interval $[x_m, x_n]$. The phase factor $\exp[-i\phi(x_m, T)]$ is multiplied as a constant phase factor.

If the grid spacing $2\Delta x$ is fine enough, then we can propagate $\Psi^{(\text{rec})}(x, T)$ time-inversely according to the relation:

$$\begin{aligned} \Psi(x, t) &\equiv \exp\left(-\frac{i\hat{p}^2}{\hbar} T\right) \Psi(x) \\ &= \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} dp \exp\left[\frac{i}{\hbar} (px - \frac{\hat{p}^2}{2m} T)\right] \cdot \\ &\quad \bar{\Psi}(p). \end{aligned} \quad (24)$$

Thus we get $\Psi^{(\text{rec})}(x) = \Psi(x) \exp[-i\phi(x_m,$

$T)$], which corresponds to the reconstructed wave-function at $z = z_0$. In this way, we may back project all the discrete wave functions and obtain the sequence $\{\Psi^{(\text{rec})}(x_m), \Psi^{(\text{rec})}(x_m + 2\Delta x), \dots\}$.

It should be noted that this method is subject to the limitation of the grid parameter $2\Delta x$, so the reconstruction cannot be arbitrarily fine. If the structure of the atomic wave function is finer than the spacing $2\Delta x$, then this reconstruction will become less effective.

CONCLUSION

In conclusion, we applied the standing wave laser beam for interferometric measurement of atomic wave function, found that the method can effectively solve the initial random phase problem of the running light wave of the laser (Freyberger, et al., 1997). We presented the relation between the measurement data (atomic position distributions) and the atomic wave function. This method is quite applicable when the spacing $2\Delta x$ is finer than the atomic wave function structure.

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