

## ANALYSIS OF VERTICAL PRESSURE ON BURIED PIPELINE WITH CASE STUDY\*

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**Abstract:** This paper introduces a formula for the vertical pressure on a buried pipeline by using a modification of the basic assumptions of Marston's theory. The fill's cohesion is considered. The included angle between the slide surface above the pipeline and the horizontal surface is assumed to be equal to the fill's angle of friction. The friction is calculated by multiplying the active earth pressure on the outer column and the coefficient of the friction on the slide planes. It was found that the fill's cohesion had important influence on the vertical pressure, whose vertical pressure  $C_v$  decreases with increase of the fill depth, a relationship according with that observed in practice. At the end of the paper, the formula is employed to analyze a practical case.

**Key words:** pipeline, vertical pressure, failure, case study

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### INTRODUCTION

The buried pipeline can be classified into three different types. Due to the difference of the relative settlement between the pipe and the soil close to its two sides, the vertical pressure on the pipe varies differently. When the fill is high enough, the vertical pressure is the most important factor, which should be taken into consideration in pipe design and construction. The importance of determining vertical pressure in design and construction has resulted in a great demand for validated formula. Marston A. (1913) first developed a formula widely used in practice. Due to the difference of the compressibility between pipe and soil, there is a downward movement of soil close to the pipe, if no flexible material is filled above pipe. Marston (1913) assumed that the relative movement between pipe and soil induced two vertical slide planes (failure surfaces) in the soil above the pipe. Then the soil above pipe was divided into one inner and two outer columns (Fig.1

and Fig.2). A friction force acted on the interface on inner and outer columns. The friction caused by the active earth pressure of the inner column can be taken as the product of the active pressure and the coefficient of the friction on the surface. No cohesion of soil was considered by Marston (1913). Zeng (1960) assumed the friction was caused by the active pressure on the outer column. Considering soil cohesion, developed a theory for an analysis of vertical pressure. For analyzing dynamic response of a pipe, Zhang (1991) assumed the vertical slide plane was inclined at an angle (of friction of soil) to the horizontal surface. It was found in practice that the failure of underground pipe always induced large area ground settlement, which proved that the slide planes between the inner and outer columns should incline to the ground surface, (Fig.3).

Many other approximate theoretical formulas, some of which have been widely used in practice, have been proposed since 1913. Although a large number of formulas are available, it is also difficult for engineer to decide which formula should be used for a given case. There are many unsuccess-

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cessful cases (Gu, 1981). This simulates further studies in the field.

From the above analysis it is clear that there exist three problems in the assumptions of Marston's theory. The first one lies in the estimation of the slide plane's force, which is the product of the coefficient of friction and the active pressure on the inner column. The second one lies in the neglect of the cohesion of the soil. If the fill is cohesive, the assumption of Marston's theory is certainly unreasonable. The third one exists in the assumption of the vertical slide plane between inner and outer columns. It is clear in practice that, the slide plane is not vertical but oblique.

This paper presents the studies on the vertical

pressure on buried pipeline with and without taking unloading measures in the soil above the pipe. The assumptions of the slide planes and inner and outer columns are also made. It is assumed that: (1) the friction force on the slide plane is determined by the product of active pressure in outer column and the coefficient of friction on the surface; (2) the slide plane between inner and outer column is inclined to an angle  $\varphi$ , which is the soil's angle of friction and (3) the cohesion of soil is considered. The new formula helps engineers estimate the vertical pressure. In-situ measurement is used to check if the formula used is suitable for a given situation. At the end of the paper, it is employed to analyze a practical case.

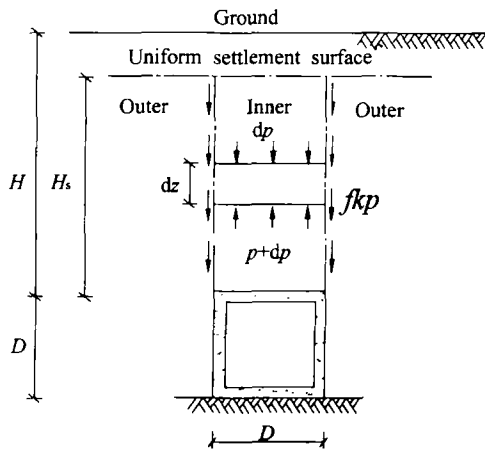


Fig. 1 Marston's model (1913)

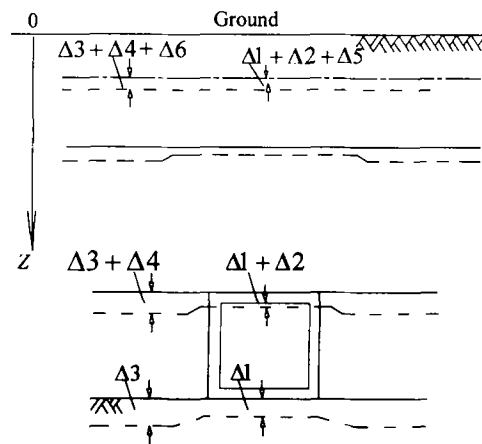
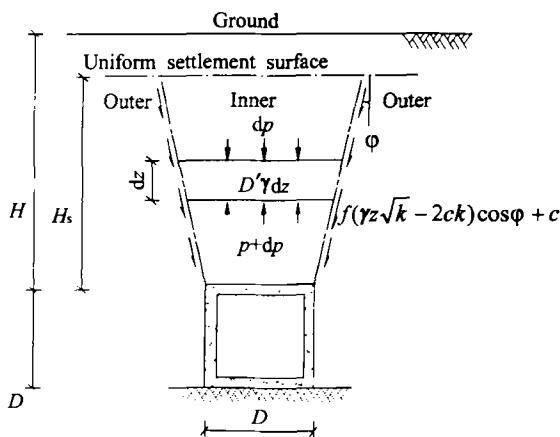
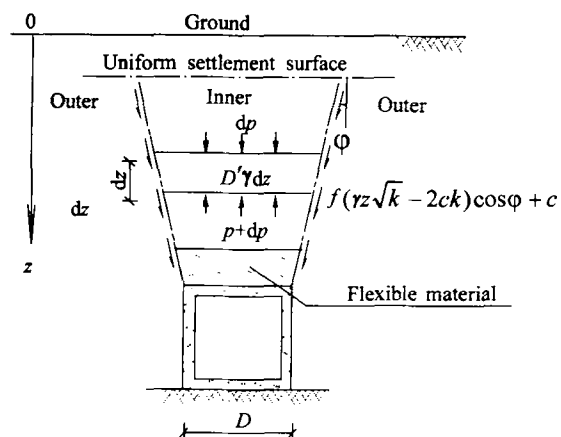


Fig. 2 Settlement of pipe



(a)



(b)

Fig. 3 Presented model (a) without unloading measures; (b) with unloading measures

MARSTON'S THEORY (1913)

In order to make the comparison clearer, the widely used in practice Marston's theory (1913) of vertical pressure is introduced. The basic assumptions of the theory are based on the different compressibility of pipe and soil along the pipe. Two slide planes above the pipe surface divide the soil into two outer columns and one inner column, as shown in Fig. 1. As pipe was always harder than soil, the compressibility of the inner column was less than that of the outer columns, which created friction force on the surface of the slide plane. The deformation of the inner column is greater in the middle part and less in the two slide planes, which decrease with the height of the inner column. The term "uniform settlement surface" is defined as the surface where the same settlement of inner and outer column occurs. The distance between the surface and the pipe top is denoted  $H_s$ . If the fill is not high enough, the theoretical uniform settlement surface locates above the fill surface. If  $H < H_s$ , vertical pressure  $P$  is given by

$$P = \frac{\gamma D}{2fk} e^{2fkH/D} \quad (1)$$

where  $f(f = \text{tg } \varphi)$  is the coefficient of friction on the slide plane,  $\varphi$  is the fill's angle of friction,  $k$  is the coefficient of active lateral pressure,  $k = \text{tg}^2(45^\circ - \varphi/2)$ ,  $D$  is the width of pipe and  $\gamma$  is unit weight of soil. And if  $H > H_s$ ,  $P$  is given by

$$P = C_c \gamma D \quad (2)$$

where  $C_c$  is the coefficient of vertical pressure and defined as

$$C_c = \frac{1}{2fk} (e^{2fkH_s/D} - 1) + \frac{H - H_s}{D} e^{2fkH_s/D} \quad (3)$$

PRESENTED THEORY

In this paper, the following assumptions are made: (1) the friction force on the slide plane is determined by the product of active pressure in the

outer column and the coefficient of the friction on the surface; (2) the slide plane between the inner and outer column is inclined at an angle  $\varphi$ , which is soil's angle of friction, and (3) soil's cohesion is considered.

1. Without unloading measures

If there is no flexible material filled above the pipe, the deformation of soil above the pipe is less than that along the pipe. The outer columns have a relative downward movement, so a downward friction force is acting on side surface of the inner column. In order to make the analysis easier, the soil element is assumed to be a cube. As shown in Fig. 4, the equilibrium equation of the soil element of inner column with depth of  $dz$ , is approximated as

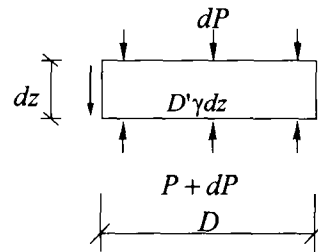


Fig.4 Vertical force acting on soil element without unloading measures

$$D'(P + dP) = D'P + \gamma D' dz + 2[f(\gamma z k - 2c\sqrt{k}) + c] dz \quad (4)$$

where  $D' = D + 2(H - z)\text{tg } \varphi$ . Simplification of Equation (4) yields

$$dP = \left[ \gamma + 2 \frac{f(\gamma z k - 2c\sqrt{k}) + c}{D + 2(H - z)\text{tg } \varphi} \right] dz \quad (5)$$

When  $H > H_s$ , integrating equation (5) gives

$$\int_{\gamma(H-H_s)}^P dP = \int_{(H-H_s)}^H \left[ \gamma + 2 \frac{f(\gamma z k - 2c\sqrt{k}) + c}{D + 2(H - z)\text{tg } \varphi} \right] dz \quad (6)$$

It is simplified and rewritten as

$$P = \gamma H \left[ 1 - k \frac{H_s}{H} - A_1 \ln \left( 1 + 2f \frac{H_s}{D} \right) \right] \quad (7)$$

where  $A_1 = \left[ \frac{2c}{\gamma H} (2f\sqrt{k} - 1) - k \left( \frac{D}{H} + 2f \right) \right] \frac{1}{2f}$ . Defining the coefficient of vertical pressure  $C_c$ .

as

$$C_c = 1 - k \frac{H_s}{H} - A_1 \ln \left( 1 + 2f \frac{H_s}{D} \right) \quad (8)$$

gives the vertical pressure without taking unloading measures  $P$  as

$$P = C_c \cdot \gamma H \quad (9)$$

When  $H < H_s$ , integrating Equation (5) gives

$$\int_0^P dp = \int_0^H \left[ \gamma + 2 \frac{f(\gamma 2k - 2c\sqrt{k}) + c}{D + 2(H-z)\text{tg}\varphi} \right] dz \quad (10)$$

It is simplified and rewritten as

$$P = \gamma H \left[ 1 - k - A_1 \ln \left[ 1 - \left( 1 + \frac{D}{2fH} \right)^{-1} \right] \right] \quad (11)$$

Defining the coefficient of vertical pressure  $C_c$  as

$$C_c = 1 - k - A_1 \ln \left[ 1 - \left( 1 + \frac{D}{2fH} \right)^{-1} \right] \quad (12)$$

also gives the vertical pressure  $P$  without taking unloading measures as

$$P = C_c \cdot \gamma H \quad (13)$$

The deformation of the inner and outer columns on the uniform settlement surface is assumed to be identical; then the deformation equilibrium equation at the surface is written as

$$\Delta 1 + \Delta 2 + \Delta 5 = \Delta 3 + \Delta 4 + \Delta 6 \quad (14)$$

where  $\Delta 1$  and  $\Delta 3$  are vertical deformation of pipe and soil at the same depth close to the pipe respectively;  $\Delta 2$  and  $\Delta 4$  are deformation of soil under pipe and soil on two sides respectively,  $\Delta 5$  and  $\Delta 6$  are the deformation of inner and outer columns respectively, as shown in Fig.2, can be estimated approximately, (Zeng, 1960) as

$$\Delta 4 \approx \frac{\gamma(H - H_s)}{E} D \quad (15)$$

where  $E$  is Young's modulus of soil. The expression obtained for  $\Delta 5$  and  $\Delta 6$  are

$$\Delta 5 \approx \int_0^H \frac{P}{E} dz$$

$$\Delta 6 \approx \frac{\gamma(H - H_s)}{E} H_s \quad (16)$$

The ratio of settlement  $\delta$  is defined as

$$\delta = \frac{(\Delta 3 + \Delta 4) - (\Delta 1 + \Delta 2)}{\Delta 4} \quad (17)$$

It has been proved in practice that if no unloading measures are taken, generally  $\delta$  varies from 0.5 to 0.8, (Zeng, 1960). Substituting Equation (15), (16), (17) into Equation (14) and then simplifying Equation (14) yields

$$\delta - \frac{H_s}{D} = \int_0^H \frac{C_c \gamma H}{(H - H_s) E} dz \quad (18)$$

If there are multiple soil layers, the settlement has to be calculated layer by layer. In the above equation only  $H_s$  is unknown that can be calculated with the method. After substitution of  $H_s$  into Equation (8) and combination with Equation (9),  $P$  can be calculated with the change of fill depth  $H$ .

## 2. With unloading measures

If the load of the fill above the pipe surface exceeds the bearing capacity of the pipe structure, flexible material is always filled above the pipe surface to a designed height, in order to increase the compressibility of the soil above pipe and decrease vertical pressure. Unit weight and height of the flexible material are denoted by  $\gamma_1$  and  $h_1$  respectively. The direction of the friction force acting on the inner column is reversed compared to the condition without taking unloading measures, (see Fig.5).

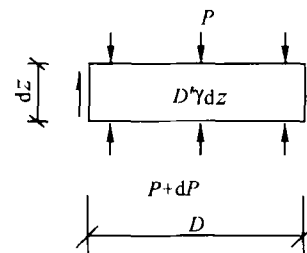


Fig.5 Vertical force acted on soil element with unloading measures

The equilibrium equation of soil element is as follows

$$D'(P' + dP) = D'P' + \gamma D' + \gamma D' dz - 2[f(\gamma zk - 2c\sqrt{k}) + c] dz \quad (19)$$

where  $P'$  is the vertical pressure of soil in the middle of the uniform settlement surface and the flexible material. Thus the vertical pressure acting on the pipe surface is expressed approximate-

ly as  $P = P' + \gamma_1 h_1$ . Equation (19) is simplified to give,

$$dP' = \left[ \gamma - 2 \frac{f(\gamma z k - 2c\sqrt{k} + c)}{D + 2(H - z)\text{tg}\varphi} \right] dz \quad (20)$$

When  $H > H_s$ , integrating equation (20) gives

$$\int_{\gamma(H-H_1)}^P dP' = \int_{(H-H_1-h_1)}^{(H-h_1)} \left[ \gamma - 2 \frac{f(\gamma z k - 2c\sqrt{k} + c)}{D + 2(H - z)\text{tg}\varphi} \right] dz \quad (21)$$

The coefficient of vertical pressure on the pipe surface is expressed as

$$C_c = 1 - \frac{h_1}{H} + K \frac{H_2}{H} + A_1 \ln\left(1 + 2f \frac{H_s}{D}\right) + \frac{\gamma_1 h_1}{\gamma H} \quad (22)$$

The vertical pressure on the pipe surface if unloading measures are taken is expressed as

$$P = C_c \cdot \gamma H$$

$H_s$  can be determined with the same method mentioned above. Then  $P$  and  $C_c$  can be obtained.

PARAMETERS STUDY

$P$  is interesting when  $H > H_s$ . In the following, the difference between Marston's, Zeng's, and the presented formula, and the influence of kinds of parameters on  $C_c$  are studied.

Figs 6 to 11 show the influence of the parameters on the coefficient of vertical pressure under normal conditions. Unit weight has little influence on  $C_c$  in Fig. 6. Fig. 7 shows that as ratio of deformation  $\sigma$  decreases, the coefficient of vertical pressure  $C_c$  increases slightly, which has been proved in practical measurements. Obviously the cohesion of soil has great influence on  $C_c$ , as shown in Fig. 8. Neglecting cohesion of soil, vertical pressure is overestimated.  $C_c$  decreases with the angle of friction of soil, in Fig. 9. The relationship of  $C_c$  and  $H$  presented in the paper is identical with that obtained in the in-situ measurement, showing that  $C_c$  decreases with  $H$  and that the gradient also decreases (Gu, 1981), see Fig. 12.

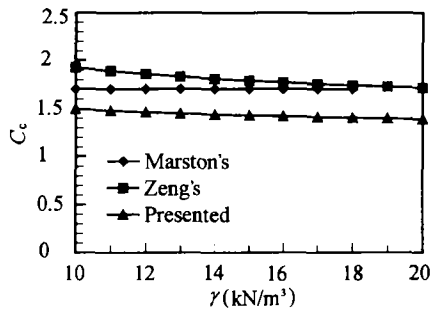


Fig. 6  $C_c$  versus soil unit weight

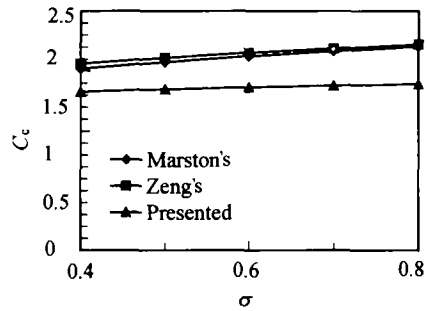


Fig. 7  $C_c$  versus ratio of deformation

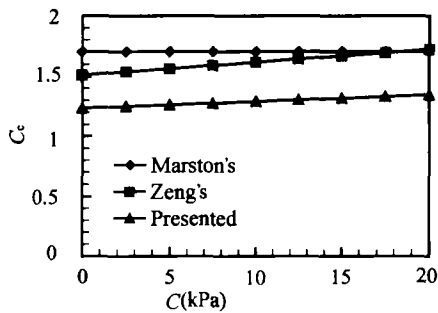


Fig. 8  $C_c$  versus cohesion of soil

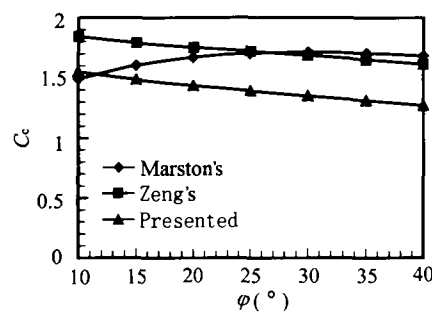


Fig. 9  $C_c$  versus angle of friction of soil

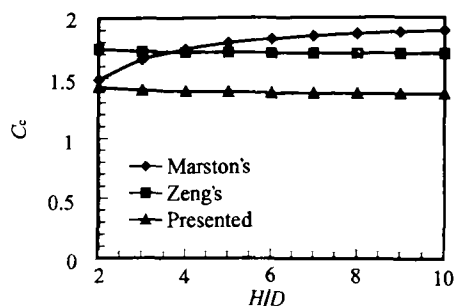
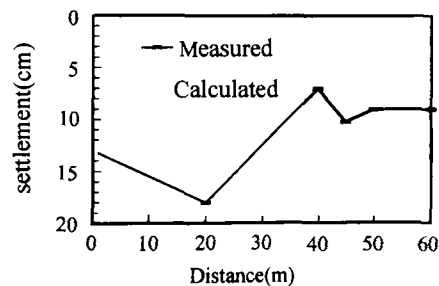
Fig. 10  $C_c$  versus fill height

Fig. 11 Settlement of pipe

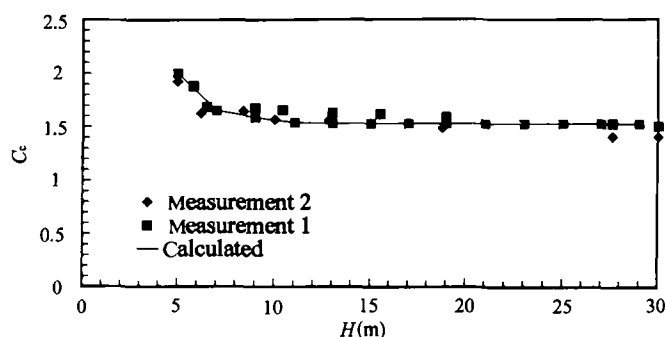


Fig. 12 Comparison of the calculated coefficient of vertical pressure with two in-situ measurements (presented by Gu, 1981)

## COMPARED WITH IN-SITU MEASUREMENT

Two in-situ measurements presented by Gu (1981) are quoted.

**Measurement 1:** The concrete pipe had 2 m outer diameter and 22 cm thickness. The unit weight of the fill was  $18 \text{ kN/m}^3$ , and soil's angle of friction was  $32.4^\circ$ . The elastic modulus of the pipe  $E_p$  was  $2.7 \times 10^4 \text{ MPa}$ . The measured coefficient of vertical pressure is shown in Fig. 12.

**Measurement 2:** The in-situ measurement was taken on a flood drainpipe under a dam of gangue. The 3.2 m outer diameter drainpipe was located on hard rock. The unit weight of the gangue was  $20 \text{ kN/m}^3$ . The gangue was cohesionless, and soil's angle of friction was  $32^\circ$ . The elastic modulus of the pipe  $E_p$  was  $2.7 \times 10^4 \text{ MPa}$ . The measured coefficient of vertical pressure is also shown in Fig. 12.

With the parameters of soil and pipe obtained from two measurements, the calculated coefficient of vertical pressure is shown in Fig.

12. It can be seen that the calculated coefficient of vertical pressure agrees well with the first measurement. When the height of the fill was lower than 15 m, the calculated results also coincided with the second measurement. Deviation of calculated result from measured result occurred with increase of fill height. The calculated coefficient was almost greater than the first measurement by 0.2 when the height of the fill was greater than 15 m.

## CASE STUDY

A sewer system was constructed to prevent the Grand Canal in Hangzhou from being severely polluted. The concrete sewer pipe had rectangle cross-section of  $2.35 \text{ m} \times 2.2 \text{ m}$ , thickness of 0.30 m; and the grade number of the concrete was C30. The pipe bottom was embedded in medium silty sand, with soft silty intercalation, under the ground at 5 m depth. Due to the high level of ground water, two rows of well-point system still could not dewater effectively the worksite during the excavation. The piping had

made the soil severely disturbed. After completion of the pipe, it was surprising to find a 60 m long pipe had a settlement of 6 ~ 18 cm. One part of the pipe had cracked with the thickness of crack of 1.0 mm. Because the pipe was designed to withstand inner pressure of 13 kPa, the cracked concrete had to be punched and reconstructed. During the second time of excavation, although the depth of the pit was only 5.0 m, use of two rows of well point system still could not effectively drain the pit site. When it was dug to a depth of 4 m, the piping problem made it impossible to go on with further construction and soil improvement measures had to be taken.

The vertical pressure on the pipe surface was estimated with the equation  $P = 75 \text{ kN/m}^2$ . The e-p curve obtained in laboratory test demonstrated well the compressibility of the soil because of the condition that specimen and the insitu soil were all disturbed. The due to vertical pressure settlement of the pipe calculated by the layer wise summation method was approximately 12.5 cm, which was identical with the insitu measurement (Fig. 11). After the pipe is put into use, the additional stress will increase to 20  $\text{kN/m}^2$  and the settlement will increase by 3.5 cm.

Winkler's model was used to analyze the inner force of the cracked pipe. With the vertical pressure  $75 \text{ kN/m}^2$  acting on the pipe surface, the foundation pressure is calculated with the measured settlement in the longitudinal axis of the pipe. Then coefficient of subgrade reaction is given by dividing the foundation pressure by the settlement at the point. During the calculation, inner force is also obtained.

It is shown that (1) coefficient of subgrade reaction  $K$  varied greatly and (2) differential settlement generated additional moment in the pipe. It is known that  $K$  reflects soil hardness. The settlement was greater where  $K$  was less. The greater the differential settlement, the greater was the additional moment and the more severely cracked was the pipe. The maximum additional moment was  $602 \text{ kN}\cdot\text{m}$ , while the maximum moment without cracking was only  $160 \text{ kN}\cdot\text{m}$ . So it

is certain the pipe would crack under the vertical pressure. The maximum thickness of the crack under the maximum additional moment was 0.12 m, which accorded with the measured value. In addition to other soil improvement methods, chemical grouting was employed to improve the soil. The soil improvement aimed to mainly (1) improve the bearing capacity of soft soil and decrease the settlement after the pipe is put into use and (2) prevent piping and make it easy to finish the remedial work. These two aims were achieved after the chemical grouting method was put into use.

## CONCLUSIONS

In this paper a new formula for estimating the vertical pressure on a pipe is presented. And the influence of the parameters on the coefficient of vertical pressure is dealt with. At the end of the paper the formula is employed to analyze a practical case. It can be concluded that:

1. The soil cohesion has apparent influence on the vertical pressure. Without consideration of the cohesion, the vertical pressure will be overestimated.
2. The coefficient of vertical pressure decreases with increase of fill height, the gradient also gradually decreases.

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