ISSN 1009 – 3095 *Journal of Zhejiang University(SCIENCE)* V.1, No.3, P.421 – 426, July – Sep., 2000
http://www.chinainfo.gov.cn/periodical; http://www.zju.edu.cn;
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WIND-INDUCED STOCHASTIC RESPONSE OF CONTROLLED TALL BUILDINGS BY COMPLEX-MODE ANALYSIS*

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Received Mar.26, 1999; revision accepted Dec.5, 1999

Abstract: This paper deals with the along-wind dynamic response of tall buildings passively controlled by Cross-Tensioned Spring Damper (CTSD) mechanisms. In order to estimate the dynamic response more accurately and efficiently, the building and its CTSD controllers were considered as a coupled non-classical damping system and a new stochastic response analysis method, complex-modal state-space method, was proposed based on the combination of complex modes of the structural system rather than on the real-modal decomposition as usual. As numerical examples, the displacement response of a 2-degree-of-freedom system to white-noise input is first considered and the validity and accuracy of the proposed method are demonstrated through comparison with the exact results. The wind-induced dynamic responses of a tall building controlled by two CTSD mechanisms are then analyzed. The calculation results showed that the proposed method is much more appropriate and accurate than the conventional real-modal decomposition method in the analysis of dynamic responses of passively controlled tall buildings.

Key words: tall buildings, wind-induced vibration, structural control, complex-mode analysis, state-space method

Document code: A **CLC number:** TU311.3, TU973

INTRODUCTION

Vibration control of tall buildings under wind loading is an important and active research topic in civil and structural engineering. Energy-dissipating mechanisms can be activated by the motion of the structure itself and are usually utilized as passive-control devices (Kareen, 1990). In contrast, active-control mechanisms are operated only through external energy supported by actuators or force generators (Leipholz, 1986; Soong, 1990). The commonly used passive and active control mechanisms in wind engineering include Tuned Mass Damper, Appendage and Cross-Tensioned Spring Damper (CTSD) recently proposed by the authors (Chen et al., 1992), etc. A CTSD mechanism, which consists of cross-tensioned springs (or cables) and dampers as shown in Fig. 1, is often installed between two different floors of tall buildings. Once the building suffers horizontal motion under wind loading, the CTSD causes an opposite action to restrain this motion. It is therefore a typical passive control mechanism.

The conventional real-modal decomposition method has been widely used in the analysis of stochastic dynamic responses of passively controlled structures under wind loading (Leipholz, 1986). The method assumes that the whole structural system including control mechanisms is an uncoupled dynamic system with classical or orthogonal damping, while the function of control mechanisms is just to provide action forces or to dissipate energy. Furthermore, the dynamic behavior of the controlled structure is also assumed to be equivalent to that of the uncontrolled structure. In many cases, however, these assumptions do not seem so reasonable, because: (1) when a control mechanism (e.g. a CTSD) is installed in the structure, the mechanism and the structure soon become an interactive and coupled system; the dynamic behavior of the system may be considerably changed compared to that of the original structure, and (2) in most cases, both the material properties and damping viscosity of the control mechanisms are greatly different from those of the original structure. That is to say, the effect of non-classical/orthogonal damping

* Project (59478034) supported by National Natural Science Foundation of China (NSFC).

and modal coupling between the structure and the control mechanisms may be significant and must be considered in the analysis. In this circumstance, the conventional real-modal decomposition method may be inappropriate so more appropriate and accurate approaches are required.

In this paper, a complex-mode analysis method is proposed for the study of the along-wind dynamic responses of tall buildings controlled by CTSD mechanisms. The tall building and its CTSD mechanisms are considered as an integral coupled system with non-classical/orthogonal damping in the analysis. Numerical examples are also presented to demonstrate the validity and accuracy of the proposed method.

MOTION EQUATIONS OF THE CONTROLLED TALL BUILDING

In order to control the possibly excessive dynamic responses of a tall building under wind loading, r sets of CTSD mechanisms are installed between some floors of the building. For example, the jth CTSD is installed between the kth and lth floors, as shown in Fig.1. If the

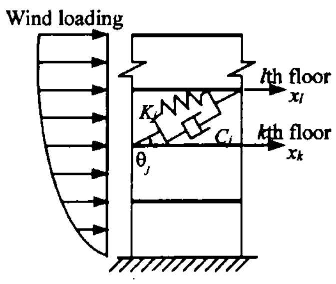

Fig.1 Tall building controlled by CTSD mechanisms

CTSD is assumed to be a linear damping system, the motion equation of the building coupled with r CTSD controllers may be written as

$$[M_0]\{\ddot{x}\} + [C_0]\{\dot{x}\} + [K_0]\{x\} = \{p\} + [H]\{u\} \tag{1}$$

where

$$\{u\} = -\text{diag}[K_j\cos^2\theta_j][H^T]\{x\} - \text{diag}[C_j\cos^2\theta_j][H]^T\{\dot{x}\} \tag{2}$$

and $[M_0]$, $[C_0]$ and $[K_0]$ are the mass, damping and stiffness matrices of the building; $\{x\}$ is the along-wind response vector of the building; $\{p\}$ is the load vector of along-wind fluctuation; $\{u\}$ is the control force vector; θ_j is the angle between the axis of the jth CTSD mechanism and the horizontal floor; K_j and C_j are the stiffness and damping coefficients of the jth CTSD, respectively; diag$[K_j]$ denotes a diagonal matrix with diagonal elements being K_1, K_2, etc. $[H]$ is the location matrix of CTSD mechanisms,

$$[H] = [\{H_1\}, \{H_2\}\cdots\{H_j\}\cdots\{H_r\}] \tag{3}$$

here $\{H_j\}$ is a vector with elements in the lth and kth lines being $+1$, -1 and other lines being zero.

As the building together with the installed CTSD mechanisms is a coupled structural system, the motion equation for the system can be easily obtained by substituting equation (2) into equation (1),

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{p\} \tag{4}$$

where $[M]$, $[C]$ and $[K]$ are the mass, damping and stiffness matrices of the integral structural system, respectively, and are

$$\left.\begin{aligned} [M] &= [M_0] \\ [C] &= [C_0] + [H]\text{diag}[C_j\cos^2\theta_j][H]^T \\ [K] &= [K_0] + [H]\text{diag}[K_j\cos^2\theta_j][H]^T \end{aligned}\right\} \tag{5}$$

Since the integral structural system is generally a non-classical/orthogonal damping system, the motion equation (4) cannot be de-coupled and further solved by the conventional real-modal decomposition method. Here we use a complex-mode analysis method instead. First introduce a state-space vector

$$\{Y\} = [\{x\}^T, \{\dot{x}\}^T]^T \tag{6}$$

Then equation (4) can be re-written as the following state-space expression

$$[A_1]\{\dot{Y}\} + [B_1]\{Y\} = [D_1]\{p\} \tag{7}$$

where

$$[A_1] = \begin{bmatrix} [C] & [M] \\ [M] & [0] \end{bmatrix},$$

$$[B_1] = \begin{bmatrix} [K] & [0] \\ [0] & -[M] \end{bmatrix},$$

$$[D_1]^T = [[I]^T \quad [0]^T] \tag{8}$$

It is easy to prove for a sub-critical damping system, that the characteristic equation of the state-space equation (7) contains complex-valued eigenvalues and eigenvectors (Singh, 1980; Singh and McCown, 1986). Based on the superposition principle of the complex modes, the system response vector $\{Y\}$ can be expressed as a linear combination of the complex modes (Singh and McCown, 1986), i.e.

$$\{Y\} = [\Phi]\{q\} \tag{9}$$

where $[\Phi]$ is the complex-modal matrix and $\{q\}$ the complex-modal coordinate vector. Using the orthogonal behavior of the complex modes, a de-coupled modal equation set may thus be obtained

$$\{\dot{q}\} - \mathrm{diag}[\lambda_k]\{q\} = [D_2]\{p\} \tag{10}$$

in which $\mathrm{diag}[\lambda_k]$ denotes the diagonal matrix consisting of complex-valued eigenvalues λ_k ($k = 1,2,\cdots$) in the principal diagonal positions, and

$$[D_2] = ([\Phi]^T[A_1][\Phi])^{-1}[\Phi]^T[D_1] \tag{11}$$

The solution of the first-order differential equation (10) is the integral

$$q_k(t) = \int_0^t [D_2]_k\{p(\tau)\}e^{\lambda_k(t-\tau)}d\tau \quad (k = 1, 2, \cdots) \tag{12}$$

in which $[D_2]_k$ denotes the kth line of matrix $[D_2]$. Theoretically, $q_k(t)$ ($k=1, 2,\cdots$) in equation (12) may be found by using integral transformation and numerical integration and the statistical solutions of dynamic responses (e.g. displacement, velocity and acceleration) of the controlled structure can be further calculated. However this conventional approach is very complicated and requires a large amount of computational time. A new complex-modal state-space method is proposed in the next section.

COMPLEX-MODAL STATE-SPACE ANALYSIS

In the proposed method, each quantity of equation (10) is first separated into a real part and an imaginary part. A state response vector consisting of the real and imaginary parts of the modal coordinate vector $\{q\}$ is formed and the corresponding state-space equation is found. By introducing a filtering shape equation for the velocity spectrum of wind fluctuation, the de-coupled equation (10) then becomes a stable Lyapunov matrix equation with covariances of the structural response as unknowns, which can be solved more easily.

As the vector $\{q\}$ in equation (10) is in general a complex-valued vector, it is easy to separate $\{q\}$ into two parts: a real part $\{q\}_R$ and an imaginary part $\{q\}_I$. A new state-space vector $\{Q\}$ is introduced as follows

$$\{Q\}^T = [\{q\}_R^T \quad \{q\}_I^T] \tag{13}$$

where the subscripts R and I denote the real and imaginary parts, respectively. Let the real and imaginary parts in the left and right hand sides of equation (10) be identical and take the orthogonal behavior of the complex modes into consideration. A new state-space equation for the vector $\{Q\}$ can thus be found

$$\{Q\} = [A_2]\{Q\} + \{p^*\} \tag{14}$$

where

$$[A_2] = \begin{bmatrix} [\mathrm{diag}\lambda_k]_R & -[\mathrm{diag}\lambda_k]_I \\ [\mathrm{diag}\lambda_k]_I & [\mathrm{diag}\lambda_k]_R \end{bmatrix},$$

$$\{p^*\} = \begin{bmatrix} [D_2]_R\{p\} \\ [D_2]_I\{p\} \end{bmatrix} \tag{15}$$

Theoretical and experimental results indicate that wind fluctuation may be approximated by a Gaussian stationary process with zero mean value (Zhang, 1985). As a result, the generalized force vector of wind fluctuation $\{p^*\}$ in equation (14) can also be treated as a stationary process. Moreover, it has been confirmed that if the vector $\{p^*\}$ may be expressed as a rational function, $\{p^*\}$ can then be transformed to an input Gaussian white-noise vector $\{\eta\}$ by the following filtering equation (Cai, 1987)

$$\left.\begin{aligned} \{p^*\} &= [C_\eta]\{V\} \\ \{\dot{V}\} &= [A_\eta]\{V\} + [D_\eta]\{\eta\} \end{aligned}\right\} \tag{16}$$

where the coefficient matrices $[C_\eta]$, $[A_\eta]$ and $[D_\eta]$ can be found in the authors' previous work (Chen et al., 1992).

Combining equations (14) and (16) yields

$$\{\dot{Z}\} = [A]\{Z\} + [D]\{\eta\} \tag{17}$$

where

$\{Z\}^{\mathrm{T}} = [\{Q\}^{\mathrm{T}} \quad \{V\}^{\mathrm{T}}]$,

$[A] = \begin{bmatrix} [A_2] & [C_\eta] \\ [0] & [A_\eta] \end{bmatrix}$ and

$$[D]^{\mathrm{T}} = [[0] \quad [D_\eta]^{\mathrm{T}}] \tag{18}$$

Equation (18) is a state-space response equation with the input spectrum being a stationary white noise. The covariance matrix $[D_z]$ of the state vector $\{Z\}$ can be solved using the following Lyapunov algebraic equation (Hu, 1989)

$$[A][D_z] + [D_z][A]^{\mathrm{T}} + 2\pi[D][S_0^*][D]^{\mathrm{T}} = [0] \tag{19}$$

in which $[S_0^*]$ is the coefficient matrix of the cross-spectral density function of the generalized wind force vector $\{p^*\}$.

STATISTICAL COVARIANCES OF THE DYNAMIC RESPONSE

The dynamic response vector $\{Y\}$ has been expressed as a linear combination of the complex modes in equation (9). As $\{Y\}$ is a real-valued vector, it is easy to have

$$\{Y\} = [\Phi]_{\mathrm{R}}\{q\}_{\mathrm{R}} - [\Phi]_{\mathrm{I}}\{q\}_{\mathrm{I}} \tag{20}$$

According to the definition of covariance of a stochastic vector, the statistical covariance matrix of the response vector $\{Y\}$ can be derived as follows

$$\begin{aligned}[D_Y] &= [\Phi]_{\mathrm{R}}[D_{q\mathrm{R}}][\Phi]_{\mathrm{R}}^{\mathrm{T}} \\ &- [\Phi]_{\mathrm{R}}[D_{q\mathrm{R}q\mathrm{I}}][\Phi]_{\mathrm{I}}^{\mathrm{T}} - [\Phi]_{\mathrm{I}}[D_{q\mathrm{I}q\mathrm{R}}][\Phi]_{\mathrm{R}}^{\mathrm{T}} \\ &+ [\Phi]_{\mathrm{I}}[D_{q\mathrm{I}}][\Phi]_{\mathrm{I}}^{\mathrm{T}}\end{aligned} \tag{21}$$

where $[D_{q\mathrm{R}}]$, $[D_{q\mathrm{R}q\mathrm{I}}]$, $[D_{q\mathrm{I}q\mathrm{R}}]$ and $[D_{q\mathrm{I}}]$ are the upper-left, upper-right, lower-left and lower-right submatrices of $[D_Q]$ and $[D_Q]$ is the upper-left submatrix of $[D_Z]$, which are given in equation (22),

$$[D_Q] = \begin{bmatrix} [D_{q\mathrm{R}}] & [D_{q\mathrm{R}q\mathrm{I}}] \\ [D_{q\mathrm{I}q\mathrm{R}}] & [D_{q\mathrm{I}}] \end{bmatrix},$$

$$[D_Z] = \begin{bmatrix} [D_Q] & [D_{QV}] \\ [D_{VQ}] & [D_V] \end{bmatrix} \tag{22}$$

It may be noted that the covariance matrix of the displacement and velocity responses of the controlled tall building is given in equation (21). Through similar derivation the covariance matrix of the acceleration response vector $\{\ddot{x}\}$ can also be found

$$\begin{aligned}[D_{\ddot{x}}] &= [\Phi_2]_{\mathrm{R}}[D_{\dot{q}\mathrm{R}}][\Phi_2]_{\mathrm{R}}^{\mathrm{T}} \\ &- [\Phi_2]_R[D_{\dot{q}\mathrm{R}\dot{q}\mathrm{I}}][\Phi_2]_{\mathrm{I}}^{\mathrm{T}} - [\Phi_2]_{\mathrm{I}}[D_{\dot{q}\mathrm{I}\dot{q}\mathrm{R}}][\Phi_2]_{\mathrm{R}}^{\mathrm{T}} \\ &+ [\Phi_2]_{\mathrm{I}}[D_{\dot{q}\mathrm{I}}[\Phi_2]_{\mathrm{I}}^{\mathrm{T}}\end{aligned} \tag{23}$$

where $[\Phi_2]_{\mathrm{R}}$ and $[\Phi_2]_{\mathrm{I}}$ are the lower-half submatrices of $[\Phi]_{\mathrm{R}}$ and $[\Phi]_{\mathrm{I}}$; $[D_{\dot{q}\mathrm{R}}]$, $[D_{\dot{q}\mathrm{R}\dot{q}\mathrm{I}}]$, $[D_{\dot{q}\mathrm{R}\dot{q}\mathrm{I}}]$ and $[D_{\dot{q}I}]$ are the upper-left, upper-right, lower-left and lower-right sub-matrices of $[D_{\dot{Q}}]$, respectively. Using equation (17), the matrix $[D_{\dot{Q}}]$ can be derived,

$$\begin{aligned}[D_{\dot{Q}}] &= [A_2][D_Q][A_2]^{\mathrm{T}} + [A_2][D_{QV}][C_\eta]^{\mathrm{T}} \\ &+ [C_\eta][D_{VQ}][A_2]^{\mathrm{T}} + [C_\eta][D_V][C_\eta]^{\mathrm{T}}\end{aligned} \tag{24}$$

NUMERICAL EXAMPLES

1. Two-degree-of-freedom system

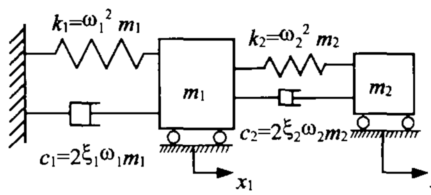

Fig. 2 Two-degree-of-freedom example system

In order to demonstrate the validity and accuracy of the proposed method, a simple 2-degree-of-freedom system as shown in Fig. 2 is first considered. If the ratio of the spring stiffness k_1/k_2 is not equal to that of the damping coefficients c_1/c_2, then strictly speaking, the system is non-classically damped. This system may be observed as a soil-structure interaction where the damping ratios of the soil (m_1) and the structure (m_2) are significantly different. The system is assumed to be subject to white-noise input. Igusa et al. (1984) studied the same problem using a complex-modal decomposition procedure. They presented the exact solutions of the system including the mean square of the relative displacement between two masses. In their study the following parameters were used: average damping ratio $(\xi_1 + \xi_2)/2 = 0.2$, mass ratio $m_2/m_1 = 0.3$ and frequency ratio $\omega_2/\omega_1 = 1.0$. The same parameters were adopted in the present study.

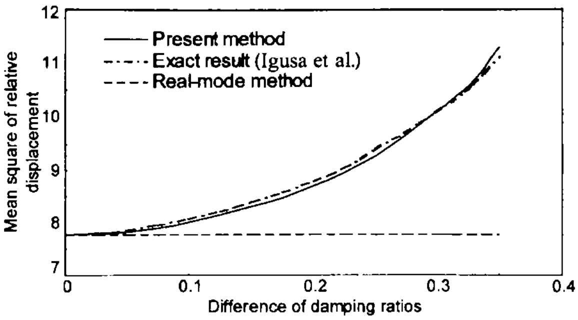

Fig.3 Mean square response of a 2-degree-of-freedom system

When the difference of damping ratios $(\xi_1 - \xi_2)$ changes from 0 to 0.35, the mean square of the relative displacement d_s is given in Fig. 3, showing that the result using the present complex-modal state-space method matches the exact solution closely whether the difference of damping ratios is small or large. This demonstrates the feasibility and accuracy of the present method in the analysis of non-classical damping systems. On the other hand, the result of the classical damping approximation using the real-modal decomposition method becomes more inaccurate with larger difference of damping ratios.

2. Tall building passively controlled by CTSD mechanisms

The wind dynamic responses of a tall building, with 40 floors, a rectangular cross-section of 45m × 50m and a height of 124m are analyzed here. The building is subjected to a mean wind pressure of 0.75 kN/m^2 and the terrain coefficient α is equal to 0.16. The damping ratio of the structure is 0.05. In the analysis, the following Devenport wind-velocity spectral function was used as the power spectral density function of the along-wind fluctuation (Zhang, 1985),

$$\left.\begin{aligned} S_v(n) &= 4Kv_{10}^2 \frac{x_0^2}{n(1+x_0^2)^{4/3}} \\ x_0 &= \frac{1200n}{v_{10}} \end{aligned}\right\} \qquad (25)$$

where n is the frequency of the wind fluctuation, K is a parameter which depends on the terrain coefficient α and v_{10} is the mean wind speed at the height of 10 m.

Two CTSD mechanisms are installed to reduce the wind-induced dynamic responses of the building. The first CTSD with stiffness and damping coefficients $k_1 = 20.29 \times 10^6$ N/m and $c_1 = 193.52 \times 10^6$ kg/s locates between the foundation and the 6th floor. The second CTSD with $k_2 = 21.72 \times 10^6$ N/m and $c_2 = 225.00 \times 10^6$ kg/s locates between the 6th and 12th floors. The values of the CTSD parameters were optimized using the equivalent sub-optimal algorithm proposed in the authors' previous work (Chen et al., 1992).

The root-mean-square responses (including displacement, velocity and acceleration) of some floors of the building computed by the present complex-modal state-space method are listed in Table 1. For convenience of comparison, the results of the classical damping approximation using the real-modal decomposition method and the corresponding uncontrolled responses are also shown in the table.

Table 1 Root-mean-square responses of the building

Floor No.		1	6	12	18	24	30	40
Present method	Displacement	0.10	0.57	1.10	1.59	1.98	2.27	2.43
	Velocity	0.21	1.12	2.16	3.06	3.81	4.40	4.82
	Acceleration	0.88	4.20	6.92	9.11	10.95	12.40	14.24
Real-modal method	Displacement	0.09	0.56	1.08	1.54	1.92	2.19	2.38
	Velocity	0.19	1.09	2.11	3.01	3.76	4.33	4.75
	Acceleration	0.71	3.97	6.81	9.02	10.85	12.27	14.11
Uncontrolled results	Displacement	0.15	0.92	1.77	2.52	3.13	3.58	3.89
	Velocity	0.39	2.31	4.39	6.16	7.60	8.70	9.53
	Acceleration	1.59	8.89	14.80	18.32	21.12	23.84	27.73

Displacement: cm; Velocity: cm/s; Acceleration: cm/s^2

It is apparent that the mean responses of the building by the present method are generally larger than those by the real-mode method, particularly for the acceleration responses at lower floors. This indicates that if the effect of non-classical damping of the structural system is ignored, the actual responses of the building may be underestimated. The situation is similar to that in the first example of a 2-degree-of-freedom system, in which the classical damping approximation greatly under-predicted the relative displacement response of the system when the difference of damping ratios is relatively large.

Further investigation showed that when the damping ratio of the building is assumed to be smaller, the difference between the results of the present complex-modal state-space method and those of the classical real-modal decomposition method becomes more significant. This further confirms that when the damping ratios of the structure and its controllers are considerably different, the classical damping approximation will become very inaccurate and inappropriate. In contrast, the present method remains reasonable and accurate.

CONCLUSIONS

(1) The validity and accuracy of the proposed complex-modal state-space method have been demonstrated through the illustration and comparison with the exact results of a simple 2-degree-of-freedom system.

(2) When the difference of damping ratios of the original structure and its controllers turns larger, the classical damping approximation using the real-modal decomposition method becomes more inaccurate and inappropriate.

(3) For a controlled tall building with non-classical damping, the conventional real-modal decomposition method usually underestimates the actual responses of the building under wind loading. The proposed complex-modal state-space method is feasible and accurate in handling this situation.

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