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EXACT SOLUTION OF AN EXTERNAL CIRCULAR CRACK IN A PIEZOELECTRIC SOLID SUBJECTED TO SHEAR LOADING*

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Abstract: A three-dimensional, exact analysis is presented in this paper for the problem of an external circular crack in a transversely isotropic piezoelectric medium subjected to arbitrary antisymmetric shear loading. A recently proposed general solution of three-dimensional piezoelectricity is employed. It is shown that four quasi harmonic functions involved in the general solution can be represented by just one complex potential. Previous results in potential theory are then used to obtain the exact solution of the problem. For point shear loading, Green's functions for the elastoelectric field are derived in terms of elementary functions.

Key words: piezoelectric material, potential theory, external circular crack, exact solution

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INTRODUCTION

Due to the inherent weakness or brittleness of piezoelectric ceramics, the fracture of piezoelectric materials has gained considerable interest. Most published works dealt with two-dimensional study of cracks in piezoelectric materials (Pak, 1990; Suo et al., 1992; Zhong and Meguid, 1997). For a penny-shaped crack in a transversely isotropic piezoelectric medium, Wang (1994) analyzed the mode I problem with the help of the Hankel transform. Kogan et al. (1996) derived the exact solutions for axisymmetric as well as antisymmetric far field uniform loadings from the solutions of a spheroidal piezoelectric inclusion through a limiting case.

New results have been developed for the application of potential theory in analyzing contact and crack problems in elasticity (Fabrikant, 1989). Recently, Chen and Shioya (1999) obtained the fundamental solution of a penny-shaped crack subjected to axisymmetric mechanical and electric loadings by extending Fabrikant's theory to piezoelectricity. It is noted that Fabrikant (1996) showed that exact solution for an external circular crack could be obtained for antisymmetric shear loading and particularly, complete solution was derived in terms of elementary functions for point loading.

This work aimed to investigate the problem of an external circular crack in a transversely isotropic piezoelectric medium subjected to arbitrary shear loading that is antisymmetric with respect to the crack plane. To this end, the general solution proposed by Ding et al. (1996) was employed. For the problem considered, it is shown that four quasi harmonic functions appearing in the general solution can be represented by only one complex potential. The satisfaction of the boundary conditions finally leads to an integro-differential equation with structure identical to that for elasticity (Fabrikant, 1996). The distinction lies in the definitions of the involved material constants that have no effect on the solution form. Thus existent results can be utilized to obtain the exact solution of the problem. Especially, for point shear loading, complete expressions for the elastoelectric field are exactly derived in terms of elementary functions. Such a solution had apparently not been presented before.

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GENERAL SOLUTION IN COMPLEX FORM

By introducing a tangential complex displacement $U = u + iv$ ($\mathrm{i} = \sqrt{-1}$), the coupled equations of a transversely isotropic piezoelectric medium can be rewritten in the following complex form in Cartesian coordinates (x, y, z), with the z-axis being normal to the plane of isotropy,

$$\begin{aligned}
&\frac{1}{2}(c_{11} + c_{66})\Delta U + c_{44}\frac{\partial^2 U}{\partial z^2} + \frac{1}{2}(c_{11} - c_{66})\Lambda^2 \overline{U} \\
&+ (c_{13} + c_{44})\Lambda\frac{\partial w}{\partial z} + (e_{15} + e_{31})\Lambda\frac{\partial \Phi}{\partial z} = 0, \\
&\frac{1}{2}(c_{13} + c_{44})\frac{\partial}{\partial z}(\overline{\Lambda} U + \Lambda \overline{U}) + c_{44}\Delta w + \\
&c_{33}\frac{\partial^2 w}{\partial z^2} + e_{15}\Delta\Phi + e_{33}\frac{\partial^2 \Phi}{\partial z^2} = 0, \qquad (1) \\
&\frac{1}{2}(e_{15} + e_{31})\frac{\partial}{\partial z}(\overline{\Lambda} U + \Lambda \overline{U}) + e_{15}\Delta w + \\
&e_{33}\frac{\partial^2 w}{\partial z^2} - \varepsilon_{11}\Delta\Phi - \varepsilon_{33}\frac{\partial^2 \Phi}{\partial z^2} = 0,
\end{aligned}$$

where, $\Delta = \partial^2/\partial x^2 + \partial^2/\partial y^2$, $\Lambda = \partial/\partial x + \mathrm{i}\partial/\partial y$, and the overbar indicates the complex conjugate value. $(u, v, w)^{\mathrm{T}}$ and Φ are the displacement vector and electric potential, respectively. c_{ij}, ε_{ij}, and e_{ij} are the elastic, dielectric, and piezoelectric constants, respectively. The general solution to Eq. (1) proposed by Ding et al. (1996) can also be rewritten in complex form:

$$U = \Lambda\Big(\sum_{i=1}^{3} F_i + \mathrm{i}F_4\Big), w = \sum_{i=1}^{3}\alpha_{i1}\frac{\partial F_i}{\partial z_i},$$
$$\Phi = \sum_{i=1}^{3}\alpha_{i2}\frac{\partial F_i}{\partial z_i}, \qquad (2)$$

where α_{i1} and α_{i2} are material constants defined in Ding et al. (1997), and $z_i = s_i z$, $s_4^2 = c_{66}/c_{44}$, and $s_i^2 (i = 1,2,3)$ are roots of the following algebraic equation:

$$as^6 - bs^4 + cs^2 - d = 0, \qquad (3)$$

where expressions for a, b, c and d can be found in Ding et al. (1996). It is noted here that the general solution given in Eq. (2) is only valid for distinct s_i^2, while for other cases, different forms should be adopted (Ding et al., 1997). In addition, $F_i(z)$ should satisfy the following quasi harmonic equations, respectively

$$\Big(\Delta + \frac{\partial^2}{\partial z_i^2}\Big)F_i = 0, \ (i = 1,2,3,4). \qquad (4)$$

By virtue of the linear constitutive relations of a transversely isotropic piezoelectric body, the following expressions of stresses σ_i (τ_{ij}) and electric displacements D_i can be derived (Chen and Shioya, 1999):

$$\begin{aligned}
\sigma_1 &= 2\sum_{i=1}^{3}\frac{\partial^2}{\partial z_i^2} \\
&\quad [(c_{66} - c_{11}) + c_{13}s_i\alpha_{i1} + e_{31}s_i\alpha_{i2}]F_i, \\
\sigma_2 &= 2c_{66}\Lambda^2\Big(\sum_{i=1}^{3}F_i + \mathrm{i}F_4\Big), \\
\sigma_z &= \sum_{i=1}^{3}\frac{\partial^2}{\partial z_i^2}\gamma_{1i}F_i, \\
\tau_z &= \Lambda\Big(\sum_{i=1}^{3}\gamma_{1i}s_i\frac{\partial}{\partial z_i}F_i + \mathrm{i}s_4 c_{44}\frac{\partial}{\partial z_4}F_4\Big), \qquad (5) \\
D &= \Lambda\Big(\sum_{i=1}^{3}\gamma_{2i}s_i\frac{\partial}{\partial z_i}F_i + \mathrm{i}s_4 e_{15}\frac{\partial}{\partial z_4}F_4\Big), \\
D_z &= \sum_{i=1}^{3}\frac{\partial^2}{\partial z_i^2}\gamma_{2i}F_i,
\end{aligned}$$

where, $\sigma_1 = \sigma_x + \sigma_y$, $\sigma_2 = \sigma_x - \sigma_y + 2\mathrm{i}\tau_{xy}$, $\tau_z = \tau_{xz} + \mathrm{i}\tau_{yz}$, and $D = D_x + \mathrm{i}D_y$. Also,

$$\begin{aligned}
\gamma_{1i} &= -c_{13} + c_{33}s_i\alpha_{i1} + e_{33}s_i\alpha_{i2}, \\
\gamma_{2i} &= -e_{31} + e_{33}s_i\alpha_{i1} - \varepsilon_{33}s_i\alpha_{i2}.
\end{aligned} \qquad (6)$$

It is noted that the following identities have been utilized in Eq. (5):

$$\begin{aligned}
\gamma_{1i}s_i &= c_{44}(s_i + \alpha_{i1}) + e_{15}\alpha_{i2}, \\
\gamma_{2i}s_i &= e_{15}(s_i + \alpha_{i1}) - \varepsilon_{11}\alpha_{i2}.
\end{aligned} \qquad (7)$$

POTENTIAL THEORY METHOD

Consider an infinite transversely isotropic piezoelectric solid containing a flat crack S in the plane $z = 0$, which is subjected to an arbitrary shear loading that is antisymmetrically applied to the upper and lower crack faces. The problem can be described as a mixed boundary value problem of a half-space $z \geqslant 0$, with the following boundary conditions on the plane $z = 0$:

$$\begin{aligned}
&\tau_z = -\tau(x, y), \text{ for } (x, y) \in S; \\
&U = 0, \text{ for } (x, y) \notin S; \qquad (8) \\
&\sigma_z = D_z = 0, \text{ for } -\infty < (x, y) < \infty.
\end{aligned}$$

In a similar manner to pure elasticity (Fabrikant, 1996), conditions (8) can be satisfied by a representation of the general solution in terms of one complex harmonic function F, namely,

$$F_i(z) = c_i[\Lambda\bar{F}(z_i) + \bar{\Lambda}F(z_i)],(i = 1,2,3);$$
$$F_4(z) = c_4[\Lambda\bar{F}(z_4) - \bar{\Lambda}F(z_4)], \tag{9}$$

where $c_i(i = 1,2,3,4)$ are undetermined constants, and the function F is given by

$$F(\rho,\phi,z) = \iint_S \ln[R(M,N) + z]U(N)\mathrm{d}S_N, \tag{10}$$

where $R(M, N)$ is the distance between the points $M(\rho,\phi,z)$ and $N(r,\Psi,0), N \in S$, the integration is taken over the crack domain S. Hereafter, cylindrical coordinates (ρ,ϕ,z) are alternatively used for the sake of convenience. By assuming

$$\sum_{i=1}^{3} c_i\gamma_{1i} = 0, \quad \sum_{i=1}^{3} c_i\gamma_{2i} = 0, \tag{11}$$

the third condition in Eq. (8) is identically satisfied. It can be further verified that the second condition in Eq. (8) can be satisfied by setting

$$\sum_{i=1}^{3} c_i + \mathrm{i}c_4 = 0, \quad \sum_{i=1}^{3} c_i - \mathrm{i}c_4 = \frac{1}{2\pi}. \tag{12}$$

It is then obtained from Eqs. (11) and (12) that

$$\begin{Bmatrix} c_1 \\ c_2 \\ c_3 \end{Bmatrix} = \frac{1}{4\pi}\begin{bmatrix} \gamma_{11} & \gamma_{12} & \gamma_{13} \\ \gamma_{21} & \gamma_{22} & \gamma_{23} \\ 1 & 1 & 1 \end{bmatrix}^{-1}\begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix}, \quad c_4 = \frac{\mathrm{i}}{4\pi}. \tag{13}$$

Substituting Eq. (9) into Eq. (5) gives the expression of τ_z at $z = 0$ as

$$\tau_z\Big|_{z=0} = \left(\sum_{i=1}^{3} c_i\gamma_{1i}s_i - \frac{s_4 c_{44}}{4\pi}\right)\Lambda^2\frac{\partial}{\partial z}\bar{F}(z) + \left(\sum_{i=1}^{3} c_i\gamma_{1i}s_i + \frac{s_4 c_{44}}{4\pi}\right)\Delta\frac{\partial}{\partial z}F(z). \tag{14}$$

The first condition in Eq. (8) then gives rise to the following integro-differential equation:

$$\tau(N_0) = -\frac{1}{2\pi^2(G_1^2 - G_2^2)}\cdot \left[G_1\Delta\iint_S \frac{U(N)}{R(N,N_0)}\mathrm{d}S_N + G_2\Lambda^2\iint_S \frac{\bar{U}(N)}{R(N,N_0)}\mathrm{d}S_N\right], \tag{15}$$

where N_0, $N \in S$, and

$$G_1 = \beta + H, \quad G_2 = \beta - H, \quad \beta = \frac{1}{2\pi s_4 c_{44}},$$
$$H = \frac{1}{8\pi^2\sum_{i=1}^{3} c_i\gamma_{1i}s_i}. \tag{16}$$

Two material constants G_1 and G_2 are introduced so as to make the resulting integro-differential equation (15) have identical form with Eq. (13) in Fabrikant (1996), which had been solved for an external circular crack in a transversely isotropic elastic medium. Although G_1 and G_2 here are different from those for elasticity, they in fact have no effect on the form of the solution. Therefore, previous results can be used to obtain the complete solution for piezoelectric materials.

GREEN'S FUNCTIONS FOR AN EXTERNAL CIRCULAR CRACK

In case the crack occupies the exterior region of a circular domain with radius a, we can write down directly the solution to Eq. (15) as

$$U(\rho,\phi,0) = \frac{G_1}{\pi}\int_0^{2\pi}\int_a^{\infty}\left[\frac{1}{R}\tan^{-1}\left(\frac{\eta}{R}\right) - \frac{G_2^2}{G_1^2}\frac{t^2(1+t)\eta}{a^2(1-t)^2}\right]\cdot \tau(\rho_0,\phi_0)\rho_0\mathrm{d}\rho_0\mathrm{d}\phi_0 + \frac{G_2}{\pi}\int_0^{2\pi}\int_a^{\infty} \left[\frac{q}{R\bar{q}}\tan^{-1}\left(\frac{\eta}{R}\right) + \frac{\eta}{\bar{q}}\left(\frac{te^{\mathrm{i}\phi}}{\rho(1-t)} - \frac{\bar{t}e^{\mathrm{i}\phi_0}}{\rho_0(1-\bar{t})}\right)\right]\cdot \bar{\tau}(\rho_0,\phi_0)\rho_0 d\rho_0 d\phi_0, \tag{17}$$

where,

$$R = [\rho^2 + \rho_0^2 - 2\rho\rho_0\cos(\phi - \phi_0)]^{1/2},$$
$$\eta = (\rho^2 - a^2)^{1/2}(\rho_0^2 - a^2)^{1/2}/a,$$
$$q = \rho e^{\mathrm{i}\phi} - \rho_0 e^{\mathrm{i}\phi_0}, \quad t = (a^2/\rho\rho_0)e^{\mathrm{i}(\phi-\phi_0)}. \tag{18}$$

By substituting Eq. (17) into Eq. (10) and these, in turn, into Eqs. (2) and (5), exact expressions for the elastoelectric field can be derived.

Assume that the crack is subjected to a pair of concentrated shear forces $T = T_x + \mathrm{i}T_y$ that are applied to the crack faces antisymmetrically at the points $(\rho_0, \phi_0, 0^{\pm})$, $\rho_0 > a$. We can obtain the following Green's functions for the elastoelectric field by virtue of Fabrikant's results (Fabrikant, 1996):

$$U = \frac{H}{\pi}\sum_{i=1}^{3} C_i\Big\{\Big[g_2(z_i) + \frac{G_2}{G_1}\bar{g}_7(z_i)\Big]T - \Big[g_{16}(z_i) + \frac{G_2}{G_1}g_8(z_i)\Big]\bar{T}\Big\} + \frac{\beta}{\pi}\Big\{\Big[g_2(z_4) - \frac{G_2}{G_1}\bar{g}_7(z_4)\Big]T + \Big[g_{16}(z_4) - \frac{G_2}{G_1}g_8(z_4)\Big]\bar{T}\Big\},$$

$$w = -\frac{H}{\pi}\sum_{i=1}^{3} C_i\alpha_{i1}\Big\{\Big[\bar{g}_1(z_i) + \frac{G_2}{G_1}\bar{g}_9(z_i)\Big]T + \Big[g_1(z_i) + \frac{G_2}{G_1}g_9(z_i)\Big]\bar{T}\Big\},$$

$$\Phi = -\frac{H}{\pi}\sum_{i=1}^{3} C_i\alpha_{i2}\Big\{\Big[\bar{g}_1(z_i) + \frac{G_2}{G_1}\bar{g}_9(z_i)\Big]T + \Big[g_1(z_i) + \frac{G_2}{G_1}g_9(z_i)\Big]\bar{T}\Big\},$$

$$\sigma_1 = -\frac{2H}{\pi}\sum_{i=1}^{3} C_i \cdot [(c_{66} - c_{11}) + c_{13}s_i\alpha_{i1} + e_{31}s_i\alpha_{i2}] \cdot \Big\{\Big[\bar{g}_5(z_i) + \frac{G_2}{G_1}\bar{g}_{10}(z_i)\Big]T + \Big[g_5(z_i) + \frac{G_2}{G_1}g_{10}(z_i)\Big]\bar{T}\Big\},$$

$$\sigma_2 = \frac{2c_{66}H}{\pi}\sum_{i=1}^{3} C_i\Big\{\Big[g_5(z_i) + \frac{G_2}{G_1}\bar{g}_{13}(z_i)\Big]T + \Big[g_{11}(z_i) + \frac{G_2}{G_1}g_{12}(z_i)\Big]\bar{T}\Big\} - \frac{s_4}{\pi^2}\Big\{\Big[-g_5(z_4) + \frac{G_2}{G_1}\bar{g}_{13}(z_4)\Big]T + \Big[g_{11}(z_4) - \frac{G_2}{G_1}g_{12}(z_4)\Big]\bar{T}\Big\},$$

$$\sigma_z = -\frac{H}{\pi}\sum_{i=1}^{3} C_i\gamma_{1i}\Big\{\Big[\bar{g}_5(z_i) + \frac{G_2}{G_1}\bar{g}_{10}(z_i)\Big]T + \Big[g_5(z_i) + \frac{G_2}{G_1}g_{10}(z_i)\Big]\bar{T}\Big\}, \quad (19)$$

$$\tau_z = \frac{H}{\pi}\sum_{i=1}^{3} C_i\gamma_{1i}s_i\Big\{\Big[g_3(z_i) + \frac{G_2}{G_1}\bar{g}_{14}(z_i)\Big]T + \Big[-g_4(z_i) + \frac{G_2}{G_1}g_{15}(z_i)\Big]\bar{T}\Big\} + \frac{1}{2\pi^2}\Big\{\Big[g_3(z_4) - \frac{G_2}{G_1}\bar{g}_{14}(z_4)\Big]T + \Big[g_4(z_4) + \frac{G_2}{G_1}g_{15}(z_4)\Big]\bar{T}\Big\},$$

$$D = \frac{H}{\pi}\sum_{i=1}^{3} C_i\gamma_{2i}s_i\Big\{\Big[g_3(z_i) + \frac{G_2}{G_1}\bar{g}_{14}(z_i)\Big]T + \Big[-g_4(z_i) + \frac{G_2}{G_1}g_{15}(z_i)\Big]\bar{T}\Big\} + \frac{e_{15}}{2\pi^2 c_{44}}\Big\{\Big[g_3(z_4) - \frac{G_2}{G_1}\bar{g}_{14}(z_4)\Big]T + \Big[g_4(z_4) + \frac{G_2}{G_1}g_{15}(z_4)\Big]\bar{T}\Big\},$$

$$D_z = -\frac{H}{\pi}\sum_{i=1}^{3} C_i\gamma_{2i}\Big\{\Big[\bar{g}_5(z_i) + \frac{G_2}{G_1}\bar{g}_{10}(z_i)\Big]T + \Big[g_5(z_i) + \frac{G_2}{G_1}g_{10}(z_i)\Big]\bar{T}\Big\},$$

where, $C_i = 4\pi c_i$, $g_1 - g_5$ and $g_7 - g_{16}$ are given in Eqs. (43) – (57) in Fabrikant (1996). They are not repeated here to save space. It is noted that these expressions are all in terms of elementary functions, that is to say, the Green's functions of an external circular crack in a transversely isotropic piezoelectric medium due to point shear loading can be exactly obtained in elementary functions.

STRESS INTENSITY FACTOR AND NUMERICAL EXAMPLE

Define the complex stress intensity factor, as follows

$$k_{\rm II} + \mathrm{i}k_{\rm III} = \lim_{\rho\to a}\{(a-\rho)^{1/2}e^{-\mathrm{i}\phi}\tau_z|_{z=0}, \quad (20)$$

where $k_{\rm II}$ and $k_{\rm III}$ actually correspond to the mode II and mode III intensity factors of an external circular crack, respectively.

One can obtain the expression of τ_z at $z = 0$ for $\rho < a$ from Eq. (19) as follows:

$$\tau_z|_{z=0} = \frac{1}{\pi^2 G_1}[G_1 g_3(0)T + G_2 g_{15}(0)\bar{T}], \quad (21)$$

Noticing the following property:

$l_{1i} \to \min(a, \rho)$, and $l_{2i} \to \max(a, \rho)$, when $z = 0$, (22)

$g_3(0)$ and $g_{15}(0)$ in Eq. (21) can be calculated from Fabrikant (1996)

$$g_3(0) = \frac{(\rho_0^2 - a^2)^{1/2}}{R^2(a^2 - \rho^2)^{1/2}},$$

$$g_{15}(0) = \frac{(\rho_0^2 - a^2)^{1/2}}{(a^2 - \rho^2)^{1/2}} \frac{(\rho_0 e^{-i\phi_0} + \rho e^{-i\phi}) e^{i\phi_0}}{\rho_0 (\rho_0 e^{-i\phi_0} - \rho e^{-i\phi})^2} . \tag{23}$$

The complex stress intensity factor thus can be obtained for point loading as follows:

$$k_{II} + ik_{III} = \frac{(\rho_0^2 - a^2)^{1/2}}{\pi^2 \sqrt{2a}} \left[\frac{Te^{-i\phi}}{\rho_0^2 + a^2 - 2a\rho_0 \cos(\phi - \phi_0)} + \frac{G_2}{G_1} \frac{e^{-i(\phi - \phi_0)}(\rho_0 e^{-i\phi_0} + ae^{-i\phi})\bar{T}}{\rho_0(\rho_0 e^{-i\phi_0} - ae^{-i\phi})^2} \right] . \tag{24}$$

It can be seen that only two material constants G_1 and G_2 are involved in the expression of the complex stress intensity factor. Their values are given in Table 1 for several materials whose elastic, dielectric and piezoelectric constants can be found in Dunn and Taya (1994).

Table 1 G_1 and G_2 for piezoelectric materials

Materials	$G_1(10^{-12}\mathrm{m}^2/\mathrm{N})$	$G_2(10^{-12}\mathrm{m}^2/\mathrm{N})$	G_2/G_1
PZT-4	8.6144	2.7585	0.3202
PZT-5	10.798	3.7142	0.3440
PZT-7A	8.0470	2.4941	0.3099
$BaTiO_3$	6.1322	1.2724	0.2075

It is also noted that the complex intensity factor obtained here for piezoelectricity is similar to that for elasticity. The two differ from each other by the definitions of G_1 and G_2 only.

The corresponding stress intensity factor for an arbitrarily distributed loading $\tau(\rho, \phi)$ can obviously be obtained by integrating Eq. (24):

$$k_{II} + ik_{III} = \frac{e^{-i\phi}}{\pi^2 \sqrt{2a}} \int_0^{2\pi} \int_a^{\infty} (\rho_0^2 - a^2)^{1/2} \left\{ \frac{\tau(\rho_0, \phi_0)}{\rho_0^2 + a^2 - 2a\rho_0 \cos(\phi - \phi_0)} + \frac{G_2}{G_1} \frac{(\rho_0 + ae^{-i(\phi - \phi_0)})\bar{\tau}(\rho_0, \phi_0)}{\rho_0(\rho_0 e^{-i\phi_0} - ae^{-i\phi})^2} \right\} \rho_0 \mathrm{d}\rho_0 \mathrm{d}\phi_0 \tag{25}$$

As numerical examples, Figs. 1 and 2 display curves of the nondimensional mode II and mode III stress intensity factors ($K_i = k_i \pi^2 a \sqrt{2a}/T, i = \mathrm{II}, \mathrm{III}$) versus the circular angle ϕ along the crack tip, respectively. The crack is assumed to be subjected to a pair of antisymmetric point shear forces $T(=T_x)$ applied at the points $(1.5a, 0, 0^{\pm})$ along the x-axis. The piezoelectric material is taken to be PZT-5, for which the ratio $G_2/G_1 = 0.3440$. For comparison, results for the corresponding elastic material ($G_2/G_1 = 0.2267$) in which the piezoelectric effect in PZT-5 is ignored [indicated by (E) in both figures] as well as for an isotropic elastic material (Poisson ratio equals 0.3, $G_2/G_1 = 0.1765$) are simultaneously given in the two figures. From the results, it can be found that, for the case considered, the piezoelectric effect has a certain influence on the crack stress intensity factors, especially on the mode III one. This fact is however different from that for the axisymmetric (mode I) problem of a penny-shaped crack in a piezoelectric solid that the piezoelectric effect contributes nothing to the singular field (Chen and Shioya, 1999).

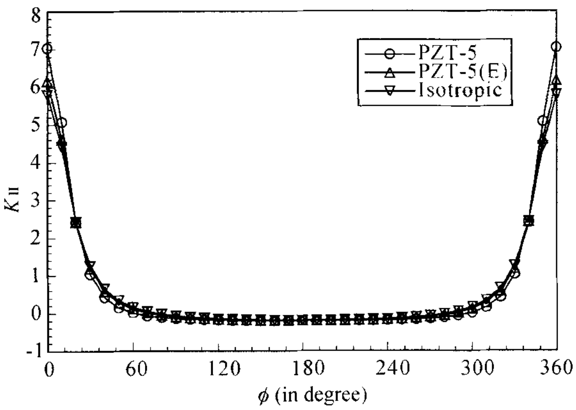

Fig.1 K_{II} along the crack tip

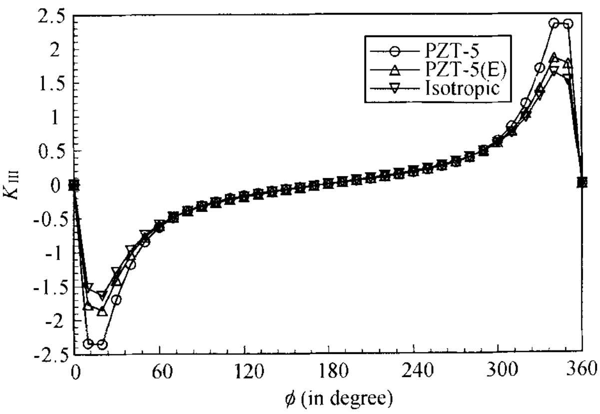

Fig.2 K_{III} along the crack tip

CONCLUSIONS

This paper gives an exact solution to the problem of an external circular crack in a transversely isotropic piezoelectric medium subjected to antisymmetric shear loading. The analysis is based on the general solution proposed by Ding et al. (1996, 1997). The potential theory method is then employed to derive an integro-differential equation governing the problem. Previous results in potential theory are used to give the Green's functions for the elastoelectric field when the crack is subjected to point shear loading.

The complex stress intensity factor (relevant to mode II and mode III) is also presented in an exact manner. Numerical results show that the piezoelectric effect has certain influence on the mode II and III stress intensity factors. This observation is obviously different from the mode I problem for which the electric field has no effect at all on the singularity behavior near the crack tip (Chen and Shioya, 1999).

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