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CONTINUOUS ROBUST TRACKING CONTROLLERS FOR A CLASS OF UNCERTAIN NONLINEAR DYNAMICAL SYSTEMS*

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Abstract: Robust tracking controller for a class of uncertain nonlinear dynamical systems, which are linearizable by input-output feedback with matching uncertainties, was investigated. In this study, uniform ultimate bound or uniformly asymptotic stability of tracking errors were obtained by different choice of the control gain. A simulation to determine the effectiveness of the proposed approach showed that the control performance was better than that of VSC (Variable Structure Control).

Key words: robust tracking control, uncertain nonlinear dynamical system, Lyapunov function
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INTRODUCTION

In recent years, robust control of dynamical systems with significant uncertainties was widely researched. Many new approaches have been proposed based on some desired performances. The Lyapunov function commomly used to synthesize the controller(Li, 1996; Yallapragada et al., 1996; Utkin, 1987) is generally based on the stabilizabilty of a normal system. Roughly speaking, a Lyapunov function candidate for the stable normal system is employed as a Lyapunov function candidate for the actual uncertain dynamical system, and an additive term is introduced for the controller such that the Lyapunov function decreased along every possible trajectory of the uncertain dynamical system.

Variable Structure Control (VSC) is an effective approach for the design of robust control. However, VSC is discontinuous control with significant chatting in control gain. So it is difficult to use in practical robust control problems (Itkis, 1976). Many new approaches have been proposed, such as approximate continuous, sliding-mode layer, tendency rate have been proposed to resolve this problem. But these approaches often result in the decreasing of control performance. Meanwhile, the continuous robust stabilizing controller may be obtained according to some stabilization of the Lyapunov function, if the bound of matching uncertainties is known (Andrey et al., 1996; Wu, 1996). But the authors of these references mainly investigated stabilization of the systems, and the obtained controller were without integrators.

This paper discusses the development of the robust tracking continuous controller with integrator for a class of input-output feedback linearizable system based on the combination of stabilization of Lyapunov function and VSC. Firstly, the sliding mode surface is chosen, which is a linear function of the tracking error, its derivative, and the integral of the tracking error. Secondly, for the derivation of a control law, a Lyapunov function is constructed, which is the quadratic of the sliding mode surface. Finally, the control law is obtained such that the derivative of the chosen Lyapunov function is less than zero.

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PROBLEM FORMULATION AND ASSUMPTION AND PRELIMINARIES

1. Problem formulation and assumption

Consider a class of uncertain nonlinear dynamical systems described by the following state equations:

$$\dot{x} = f(x) + g(x)u(t) + E(x,t)$$
$$y = h(x) \quad (1)$$

where, $t \in R$ is the time, $x(t) \in R^n$ is the state vector, $u(t) \in R^m$ is the control vector, and $E(x, t)$ the matching uncertainties with bound.

Let: $E(x,t) = g(x)\xi(x,t)$, $|\zeta_i(x,t)| < r_i, i = 1,\cdots,m$.

$$\xi(x,t) = [\zeta_1(x,t),\cdots,\zeta_m(x,t)]^T \quad (2)$$

$y(t) = h(x) = [h_1(x),\cdots,h_m(x)]^T \in R^m$ is output vector. Suppose f, g and h are smooth function (or matrices) with suitable dimension, and sufficiently high order derivatives.

Let:. $g(x) = [g_1(x),\cdots,g_m(x)]$. Eq (1) involves the following assumptions:

Assumption 1: System (1) has the relative degree vector $(r_1,\cdots,r_m)$ i.e,

$$L_gL_f^jh_i = 0,\ 0 \leqslant j \leqslant r_i - 2$$
$$L_gL_f^{r_i-1}h_i \neq 0 \quad (3)$$

Assumption 2: The following decoupling matrix:

$$D = \begin{bmatrix} L_{g_1}L_f^{r_1-1}h_1 & \cdots & L_{g_m}L_f^{r_1-1}h_1 \\ \cdots & \cdots & \cdots \\ L_{g_1}L_f^{r_m-1}h_m & \cdots & L_{g_m}L_f^{r_m-1}h_m \end{bmatrix} \quad (4)$$

is non-sigular for all $x \in R^n$.

According to the above two assumptions, we have

$$Y_r = \alpha + Du + D\xi\xi(t)x$$
$$Y_r = [y_1^{(r_1)}h_1,\cdots,y_m^{(r_m)}]^T$$
$$\alpha = [L_f^{r}h_1,\cdots,L_f^{(r_m)}h_m]^T$$
$$D = [(L_gL_f^{(r_1-1)}h_1)^T,\cdots,(L_gL_f^{(r_m-1)}h_m)^T]^T$$
$$\rho = D\xi(x,t) = [\rho_1,\cdots,\rho_m]^T \quad (5)$$

Assumption 3: $\rho_i, i = 1,\cdots,m$ in Eq(5) is bound such that there are $b_i, i = 1,\cdots,m$,

$$|\rho_i| < b_i, i = 1,\cdots,m \quad (6)$$

Now, the problem is to design the control u, such that $y_1,\cdots,y_m$ tracking their desired output $y_{c_1},\cdots,y_{c_m}$.

Let: $e_1 = y_1 - y_{c_1},\cdots, e_m = y_m - y_{c_m}$, we have:

$$e_1^{(r_1)} = v_1 + \rho_1$$
$$\cdots \quad (7)$$
$$e_1^{(r_m)} = v_m + \rho_m$$

where:

$$\begin{bmatrix} v_1 \\ \cdots \\ v_m \end{bmatrix} = \begin{bmatrix} y_{c_1}^{(r_1)} \\ \cdots \\ y_{c_m}^{(r_m)} \end{bmatrix} + \alpha + Du \quad (8)$$

Assumption 4: When system (1) translate into Eq. (7), its internal dynamical system is input bounded output bound.

2. Preliminaries

Lemma 1: Let V ·,· : $R^n \times R \rightarrow R^+$ be a Lyapunov function candidate for a given continuous dynamical system

$$\dot{x} = f(x(t),t) \quad (9)$$

with the following properties

$$r_1(\|x\|) \leqslant V(x,t) \leqslant r_2(\|x\|)$$
$$\frac{\partial V(x,t)}{\partial t} + \nabla_x^T(x,t)f(x,t) \leqslant$$
$$-r_3(\|x\|) + 2\varepsilon \quad (10)$$

where $\varepsilon > 0$ is a constant, the function r_1, r_2 are strictly increasing continuous scalar functions. r_3 is a positive definite continuous function and

$$r_i(0) = 0, i = 1,2,3$$
$$\lim_{r\to\infty} r_j(r) = \infty, j = 1,2$$

Then if $\quad 2\varepsilon < \lim_{r\to\infty}\inf r_3(r) = l \quad (11)$

every solution $x(t, t_0, x^0)$ of dynamical system (9) is both uniformly bounded and uniformly ultimately bounded.

Lemma 2: Let $V(\cdot,\cdot): R^n \times R \rightarrow R^+$ be a Lyapunov function candidate for the given continuous dynamical system described by (9) with the following properties:

$$r_1(\|x\|) \leqslant V(x,t) \leqslant r_2(\|x\|)$$

$$\frac{\partial V(\boldsymbol{x},t)}{\partial t} + \nabla_x^{\mathrm{T}}(\boldsymbol{x},t)\boldsymbol{f}(\boldsymbol{x},t) \leqslant - r_3(\|\boldsymbol{x}\|) + 2\Psi(t)$$

where the functions r_1, r_2, r_3 are defined in Lemma 1, and $\Psi(\cdot)$ is a continuous function satisfying

$$\lim_{t\to\infty}\int_0^t \Psi(t)\mathrm{d}t \leqslant \overline{\Psi} < \infty \tag{12}$$

where $\overline{\Psi}$ is any constant. Then, system (9) is uniformly asymptotically stable. That is, for any solution $\boldsymbol{x}(t) = \boldsymbol{x}(t, t_0, \boldsymbol{x}^0)$ of (9)

$$\lim_{t\to\infty}\|\boldsymbol{x}(t)\| = 0$$

CONTINUOUS ROBUST TRACKING CONTROLLER FOR THE INPUT-OUTPUT FEEDBACK LINEARIZABLE NONLINEAR SYSTEM

Under the assumptions 1,2,3,4, the objective of robust control is to design v_i, $i = 1,\cdots,m$ for system(7), such that $e_1,\cdots,e_m$ are uniformly ultimate bounded or uniformly asymptotically stable.

Based on the idea of sliding-mode control, introduce the following complex error function:

$$\boldsymbol{S} = [s_1, s_2, \cdots, s_m]$$

$$s_i = e_i^{(r_i-1)} + K_{ir_i-1}e_i^{(r_i-2)} + \cdots + K_{i1} + K_{is}\int_0^t e_i\mathrm{d}t \tag{13}$$

Choose suitable parameters K_{ij}, K_{is}, $i = 1,\cdots, m$, $j = 1,\cdots, r_i - 1$, such that the following polynominals are Hurwitz.

$$P_i(\lambda) = \lambda^{r_i} + K_{ir_i-1}\lambda^{r_i-1} + \cdots + K_{i1}\lambda + K_{is},\quad i = 1,\cdots,m \tag{14}$$

From Eqs.(8),(9),we have

$$\dot{s}_1 = K_{1r_1-1}e_1^{(r_1-1)} + \cdots + K_{11}e_1^{(1)} + K_{1s}e_1 + v_1 + \rho_1$$

$$\cdots$$

$$\dot{s}_i = K_{ir_i-1}e_i^{(r_i-1)} + \cdots + K_{i1}e_i^{(1)} + K_{is}e_i + v_i + \rho_i$$

$$\cdots$$

$$\dot{s}_m = K_{mr_m-1}e_m^{(r_m-1)} + \cdots + K_{m1}e_m^{(1)} + K_{ms}e_m + v_m + \rho_m \tag{15}$$

Corollary 1: For the system (15), if v_i ($i = 1,\cdots,m$) makes s_i ($i = 1,\cdots,m$) tend to zero, e_i ($i = 1,\cdots,m$) tends to zero too. Moreover, due to the integrator in Eq.(13), when s_i tends to a constant value, e_i tends to zero.

Consider the robust control for Eq. (15). Construct the Lyapunov function

$$V = \sum_{i=1}^{m}\frac{1}{2}s_i^2 \tag{16}$$

Choose v_i ($i = 1,\cdots,m$) such that

$$v_i = -(K_{ir_i-1}e_i^{r_i-1} + \cdots + K_{i1}e_i^{(1)} + K_{is}e_i) - K_is_i - K_{ip}b_i^2s_i \tag{17}$$

where $K_i > 0, K_{ip} > 0$.

Thus, the time derivative of the choosen Lyapunov function V along with Eq.(15).

$$\begin{aligned}\dot{V}_i = & -\sum_{i=1}^{m}K_is_i^2 - \sum_{i=1}^{m}K_{ip}s_i^2b_i^2 + \sum_{i=1}^{m}s_i\rho_i \leqslant \\ & -\sum_{i=1}^{m}K_is_i^2 - \sum_{i=1}^{m}K_{ip}(s_i^2b_i^2 - |s_i\rho_i|) \leqslant \\ & -\sum_{i=1}^{m}K_is_i^2 - \sum_{i=1}^{m}K_{ip}(s_i^2b_i^2 - |s_ib_i|) \leqslant \\ & -\sum_{i=1}^{m}K_is_i^2 + \frac{1}{4}\sum_{i=1}^{m}\frac{1}{K_{ip}}\end{aligned} \tag{18}$$

According to the preliminaries introduced in the above Section, the following results can be obtained for Eq.(15),

(1) If $\sum_{i=1}^{m}\frac{1}{4K_{ip}} \leqslant 2\varepsilon < l,\ K_{ip} > 0$ (19)

where l, ε are defined as in Lemma 1. For $\lim_{r\to\infty}K_ir^2 = \infty$ and if inequality (19) holds (eg. K_{ip} = constant), the closed-loop system comprised of Eq.(15) and Eq.(17) is uniformly ultimately bounded according to Lemma 1.

(2) If, $\lim_{t\to\infty}\sum_{i=1}^{m}\frac{1}{4K_{ip}} \leqslant \overline{K} < \infty$ (20)

The closed-loop system comprised of Eq. (15) and Eq. (17) is uniformly asymptically stable according to Lemma 2.

Corollary 2: For Closed-loop system comprised of Eq. (15), and Eq. (17) is unformly ultimately bounded or uniformly asymptotically stable by choosing the controller paramaters according to (19) or (20).

So, for system (1) with assumption (1), (2), (3), (4), we have the following theorem.

Theorem 1: For system (1) with assumptions (1), (2), (3), (4), the closed-loop nonlinear dynamical system comprised of Eq. (1), Eq. (9) and Eq. (17) has the following properties:

(a) If gain function $K_{ip}(t)$ is chosen such that $K_{ip} > 0, \frac{1}{K_{ip}(t)}$ is finite all the time, and satisfies (19), the closed-loop dynamical system is uniformly ultimately bounded for e_i, $i = 1, \cdots, m$.

(b) If the chosen gain function is such that $\lim_{t\to\infty}\sum_{i=1}^{m} \frac{1}{4K_{ip}} \leqslant \bar{K} < \infty$, the closed-loop dynamical system is uniformly asymptotically stable for e_i, $i = 1, \cdots, m$.

Proof:

(1) According to assumption (4), in the input bounded output bounded internal dynamical system, the following controller is achieved,

$$u = D^{-1}\left(\begin{bmatrix} v_1 + y_{c_1}^{(r_1)} \\ \cdots \\ v_m + y_{c_m}^{(r_m)} \end{bmatrix}\right) - \alpha \tag{21}$$

(2) According to Corollary 1, when $K_{ip} > 0$, $K_{ip}^{-1}(i = 1, \cdots, m)$, $|s_i|$ is bounded, and $|e_i|$ will be bounded too. When $K_{ip} > 0$ and $\lim_{t\to\infty}\sum_{i=1}^{m} \frac{1}{4_{ip}} \leqslant \bar{K} < \infty$, $|s_i|$ tends to zero, and $|e_i|$ tends to zero too. Therefore, theorem 1 is proved.

Remark 1: From the above discussion, the proposed approach is better than VSC because of the continuous controller. It is useful for engineering. But it is worth pointing out that K_{ip} satisfying (20) is infinity as time tends to infinity, which is impossible for practical control problems. In fact, we may choose the gains K_{ip} as finite constants, such that s_i is uniformly ultimately bounded and e_i will be uniformly ultimately bounded too. However we may choose the gains K_{ip} satisfying (20). Either stopping the control at some finite time after the required performance of the tracking is achieved, or maintaining K_{ip} after the required performance of the tracking is achieved.

SIMULATION

Consider the following nonlinear system with matching uncertainty:

$$\dot{x}_1 = \frac{-10x_1 - x_1x_2 + x_2}{10 + x_2} + u_1$$

$$\dot{x}_2 = 0.1x_2(10 + x_2) + u_2 + 0.5\vartheta x_1 x_2^2$$

Denote $y = [x_1, x_2]$ and design control u_1 and u_2, such that x_1, x_2 track x_{c_1}, x_{c_2}. Letting $e_1 = x_1 - x_{c_1}$, $e_2 = x_2 - x_{c_2}$, we have the output tracking control as follows,

$$s_1 = e_1 + k_1\int_0^t e_1 \mathrm{d}t, \; k_1 > 0$$

$$s_2 = e_2 + k_2\int_0^t e_2 \mathrm{d}t, \; k_2 > 0$$

$$u_1 = v_1 + \dot{x}_{c_1} - \frac{10x_1 - x_1x_2 + x_2}{10 + x_2}$$

$$u_2 = v_2 + \dot{x}_{c_2} + 0.1x_2(10 + x_2)$$

$$v_1 = -k_1e_1 - c_1s_1$$

$$v_2 = -k_2e_2 - c_2s_2 - 0.25c_{2\rho}s_2 \mid x_1x_2^2 \mid^2$$

In simulation, we choose,

$$k_1 = k_2 = 5.0, x_{c_1} = \cos(\pi t),$$

$$x_{c_2} = \sin(\pi t), c_1 = c_2 = 2.0$$

$$c_{2\rho} = 0.02\exp(0.5t)$$

The simulation results are given in Fig. 1 (see next page). From the results, we can say that the result is acceptable.

CONCLUSIONS

The robust tracking control problem of a class of uncertain nonlinear dynamical systems has been discussed. The obtained controller is very easy to be implemented due to its simple structure. Compared to the proposed controller with VSC, the closed-loop system has similar robustness and avoids the chatting of control value. The simulation results showed that the proposed controller may guarante the uniform ultimate bound and uniform asymptotic stability of an uncertain nonlinear dynamical system by choosing different gain functions of the controller.

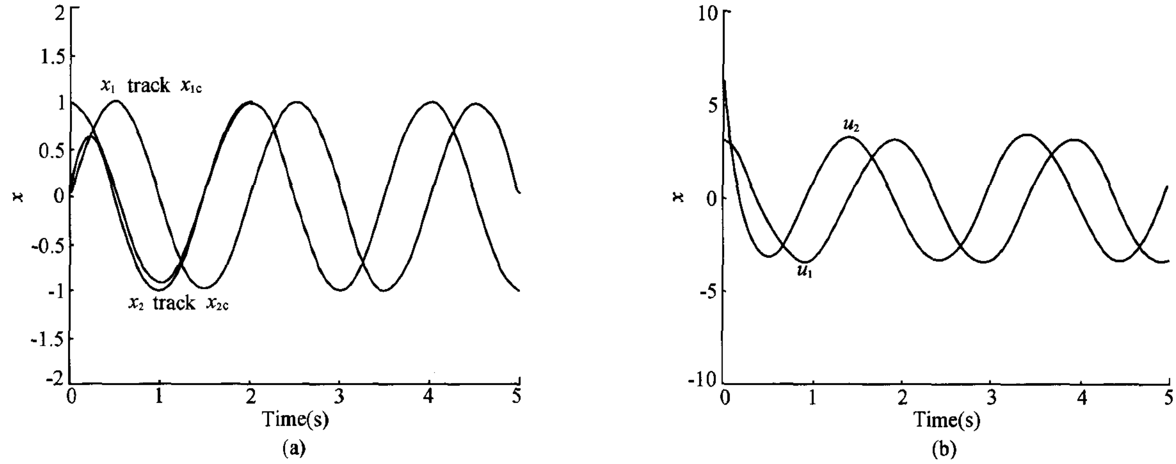

Fig. 1 The result of simulation
(a) the results of the state tracking;
(b) the results of control inputs

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